STEEL DESIGNERS' MANUAL

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FOREWORD

This Manual on Steel Design has been written and compiled by the uthors at the instigation of the British Steel Producers' Conference and the ritish Iron and Steel Federation.

The Manual is intended to bridge the gap between the normal textbook on e theory of design and its practical application in constructional engineerg. Advanced methods of design as applied to modern structures are plained, and practical examples form an important part of the Manual, the levant data being given largely in tabular form.

This is a Manual for designers, to be used for reference to the latest design chniques and for general information not otherwise published in any one lume.

We are sure that the Manual will render a useful service to all who use eel and who must have recourse to authoritative data in the interests of onomic design.

The Steelmakers of Great Britain are pleased to be concerned with a ublication of this character and intend to arrange for its revision from time o time as the science and technique of construction advance.

RESIDENT,
BRITISH IRON AND STEEL
EDERATION.

CHAIRMAN,
BRITISH STEEL PRODUCERS'
CONFERENCE.

October, 1954

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INTRODUCTION

This Manual has been prepared to make available in concise form the best of present-day knowledge concerning the design of steel framed buildings and to present complete data, tables and diagrams for use by the practising designer. It is not a text-book of structural theory, but, where necessary, basic principles are enunciated to amplify the worked examples which form a special feature of the book. British Standard 449, The Use of Structural Steel in Building, has been used as the basis for the practical examples.

The use of higher mathematics has been reduced to a minimum and the major portion of the work involves arithmetic only, while graphical methods are used where they can be applied with ease. Many of the worked examples have been solved by more than one method in order that the relative merits of the methods may be assessed.

The analysis of single bay single storey rigid frames has been treated at great length in view of the increasing interest shown in this form of construction. It should be noted that the charts on pages 346 to 398 inclusive have been drawn for roof pitches of zero, 1 in 5 and 1 in 2½. Interpolation can be readily made for intermediate pitches and this, with the wide range of height to span ratios and the numerous types of loading included, enable moments, shears and thrusts for any practical case to be found by simple arithmetic. The time involved in the preparation of designs and estimates for rigid frames is an important factor and the data here presented will effect a marked saving in this respect.

Tables of safe loads on beams and stanchions are given in the handbooks issued by steelmakers and constructional engineers and, in consequence, are not included here. Similarly, to conserve space, B.S. 449 has not been reproduced in full.

The authors wish to pay tribute to the work of the Committee representing the British Steel Producers' Conference and the British Constructional Steelwork Association who drew up an outline of the contents of this Manual which has been of considerable assistance to them.

By kind permission of the British Standards Institution references are made to British Standards throughout the Manual. Copies of the Standards may be obtained from the Institution at 2, Park Street, London, W.1.

C. S. G.
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London,
1954.

1. ORDINARY BEAM THEORY

The theory involved in the derivation of the formula for the moment of resistance (M.R.) of a beam section is based on the following assumptions:

(i) The beam section must have one axis of symmetry, say, a vertical axis, in the plane of which a bending moment is applied.
(ii) Simple or circular bending is assumed. This is the type produced by equal and opposite pure couples and in which shear is absent, as depicted by the portion AB of the beam shown in Fig. 1.
(iii) Sections of the beam which are plane before bending remain plane after bending.
(iv) The stress in any fibre is proportional to its strain.
(v) Young's Modulus of Elasticity (E) is constant.

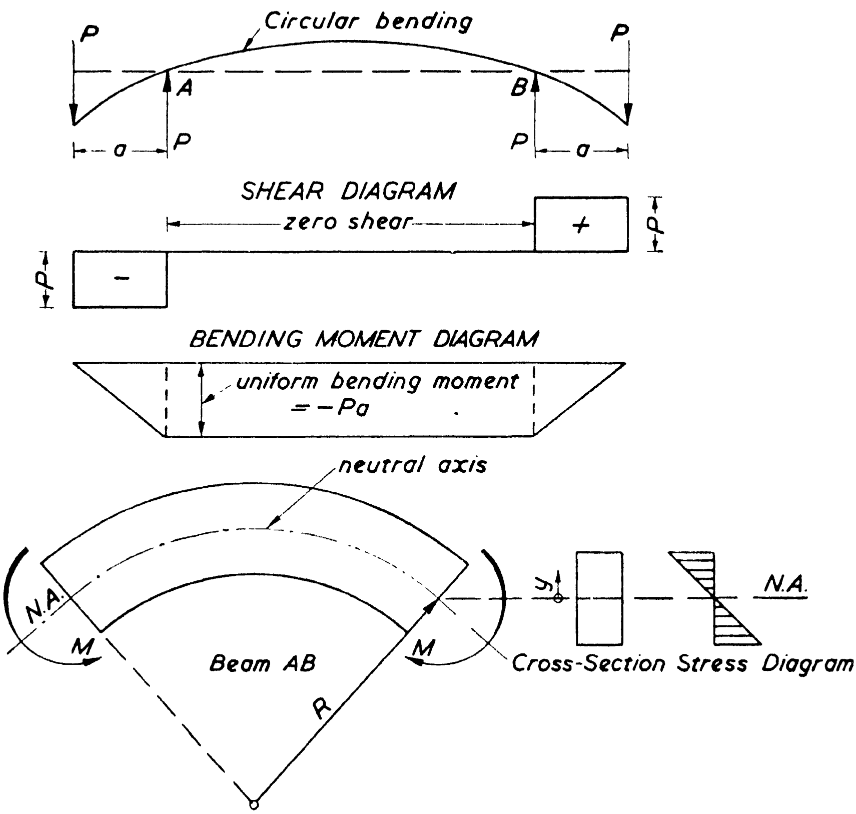


FIG. 1

The results of the beam theory can be expressed as follows.

$$\frac{M}{I}=\frac{f}{y}=\frac{E}{R},$$

where M = the bending moment,
I = the moment of inertia,
f = the stress at any point,
y = the vertical distance from the neutral axis to the point under consideration
and R = the radius of curvature of the beam.

In using this expression f will normally represent a maximum stress, so that y, in that case, will be the distance from the neutral axis to an extreme fibre, top or bottom of the section as the case may be.

The expression $$\frac{M}{I}=\frac{f}{y}$$

can be rewritten $$M=f\frac{I}{y}.$$

But $\frac{I}{y}=Z$, the modulus of section Hence:

$$M=fZ.$$

The foregoing remarks apply, in theory, only to beams subjected to circular bending When shear is present there are shear strains, but as such strains are very small compared with those due to bending, they are normally neglected in practice Consequently, the bending stresses in any beam with one axis of symmetry and subjected to bending only may be found from the standard formula, $M=fZ$

More detailed information is given in Section 17—Bending and Axial Stresses.

2. SIMPLY SUPPORTED BEAMS

ALTHOUGH it is assumed that every reader of this book has an elementary knowledge of the theory of structures, it is advisable to consider the calculations required in obtaining the shear forces (S.F.s) and bending moments (B.M.s) in simply supported beams, not so much for their own sake but because these quantities, obtainable by simple statics, form the basis of many other calculations required for the analysis of built-in beams, continuous beams and other indeterminate structures.

It should be noted that appropriate formulæ for simple beams and cantilevers under various types of loads are presented in tabular form on pp. 5–24.

In the case of simple beams, it is necessary to calculate the support reactions before the bending moments can be evaluated. This procedure is reversed for built-in or continuous beams. The following rules relate to the S.F. and B.M. diagrams for beams, viz.:

1. The shear force at any section is the algebraic sum of normal forces acting to one side of the section.
2. Shear is considered positive when the shear force calculated as above is upwards to the left of the section.
3. The B.M. at any section is the algebraic summation of the moments about that section of all forces to one side of the section.
4. Moments are considered positive when the middle of a beam sags with respect to its ends or when tension occurs in the lower fibres of the beam.
5. Under point loads only, the S.F. diagram will consist of a series of horizontal and vertical lines, whilst the B.M. diagram will consist of sloping straight lines, changes of slope occurring only at the loads.
6. For uniformly distributed loads (U.D.L.s) the S.F. diagram will consist of sloping straight lines, whilst the B.M. diagram will consist of second-degree parabolas.
7. The maximum B.M. occurs at the point of zero shear, where such exists, or at the point where the shear-force curve crosses the base line.

Loading, Shear Force and Bending Moment

The relationship between loading, S.F. and B.M. may be derived graphically by considering Fig. 2.

If ABC represents the intensity of loading upon a beam simply supported at A and B, then the total area enclosed by the curve and the base AB will represent the total load on the span.

If a second curve DEF is constructed on a base line GD such that the ordinate X at any point represents the area of the load intensity curve to, say, the right of the point, then it follows that the curve DEF represents the total load to any point and the ordinate FG will equal the total load on the

beam. *If this* curve *is* then amended by the addition of the reactions as shown, then by the definition given earlier the curve *DEFHJ* will be the shear force diagram for the given loading, since the ordinate at any point will be the algebraic sum to the right of the section of the downward load and the upward reaction.

If then the total load curve *DEFG* is treated in a similar manner and a third curve *KLM* is constructed on a base line *KN* such that the ordinate *Y*

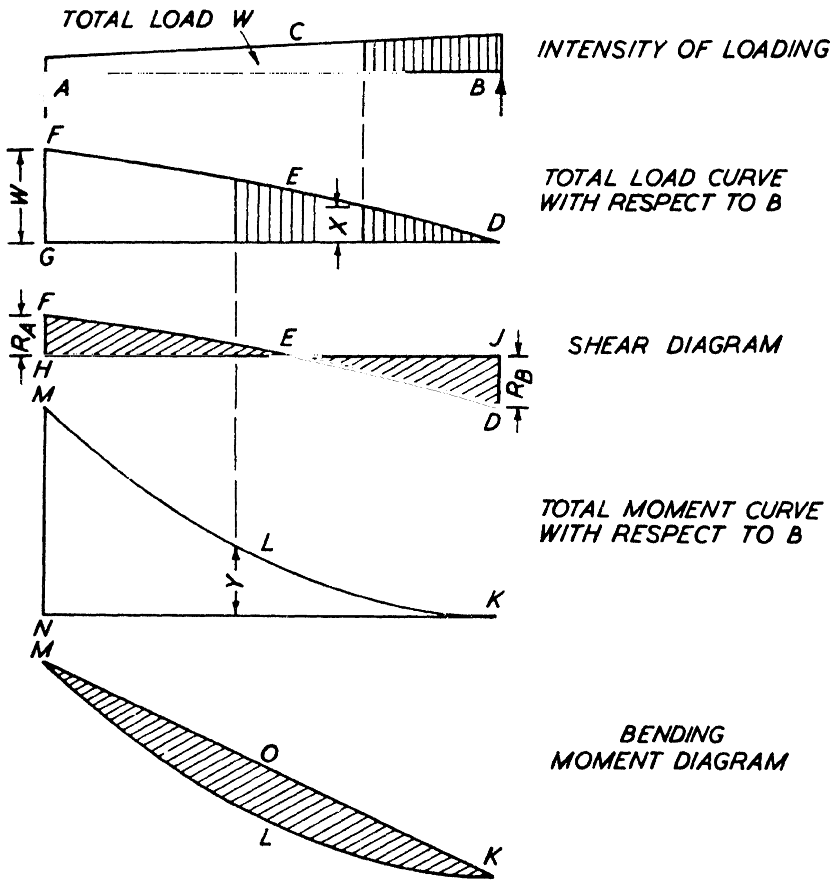


FIG. 2

at any point represents the area of the total load curve to the right of the point, then the curve *KLM* represents the total moment about *B* of the load to the right of the point and the ordinate *MN* represents the moment of *W* about *B*. If this curve is then corrected by the line *KM* representing the effect of the moment of R_A about *B*, then the curve *KLMO* will be the bending moment diagram for the beam.

If the load intensity curve be called a primitive curve, then the shear curve is known as the sum curve of the load intensity curve and the bending

moment curve will be the sum curve of the shear curve. The proof of this is as follows.

Consider any section XX and a portion of load AB distant x from the section as shown in Fig. 3.

The shear at XX due to this portion of the load will be the hatched area $ABCD$, and hence the total load up to XX will be represented by the total area of the load intensity curve to that point. Hence the sum curve of the load intensity curve will be, by definition, the total shear curve.

Consider the bending moment at XX. The B.M. at this point due to the portion of load AB = the given portion of load $\times x$. Then if E and F are

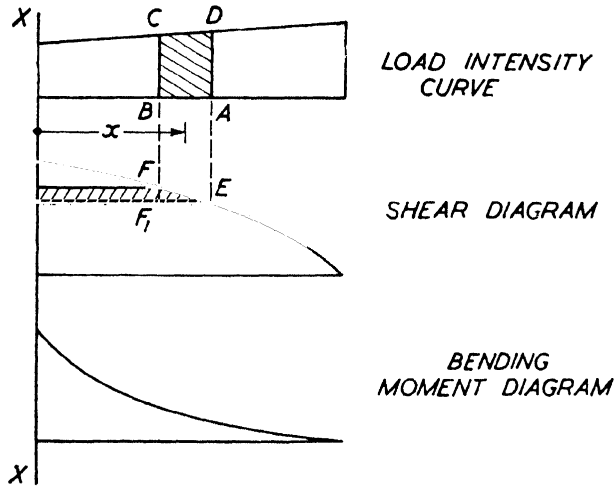


FIG. 3

the corresponding points on the shear curve, the difference of the ordinates at E and F represents the load on the portion AB.

Therefore load on $AB = FF_1$,

and B.M. at XX due to this load $= FF_1 \times x$.

That is to say, the shaded portion of the shear curve represents the B.M. at XX due to the portion AB of the load. Hence the total B.M. at XX due to the load to the right equals the total area of the shear diagram to XX.

The sum curve of a horizontal straight line is a sloping straight line and the sum curve of a sloping straight line is a parabola of the second degree, whilst the sum curve of this will be a parabola of the third degree, etc.

The sum curve is obtained graphically as shown in Fig. 4.

Divide the primitive curve into a number of small parts 1, 2, 3, ... Find the mid-points of these divisions and project horizontally to a vertical line AB passing through the origin. From any point P on the base line CA produced draw a corresponding ray diagram 1, 2, 3, ... For the shear diagram draw parallels to the respective rays across the relevant spaces, to give a connected line as shown, which is the approximate first sum curve of

the primitive curve. The true sum curve is a line joining the mid-points of each of the divisions of the sum curve illustrated.

The procedure is then repeated to produce the moment diagram which is the second sum curve.

The scales of the respective load, shear and moment diagrams are related as follows. Let 1 in. $= W$ tons per foot be the scale of the load intensity diagram, and let p_1, the polar distance used for the construction of the shear curve, be measured on the space scale. Then the shear force scale is 1 in. $= p_1 W$ tons. Similarly, if p_2 is the polar distance used for the construction of

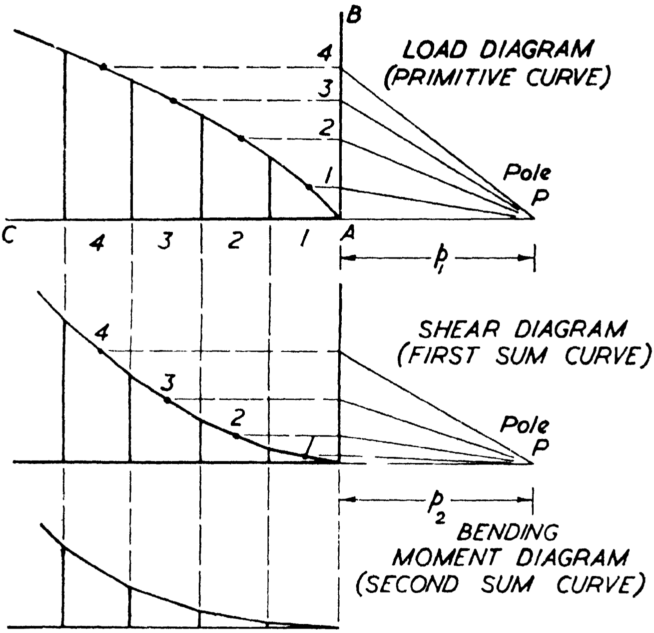


FIG. 4

the moment curve, again measured on the space scale, the B.M. scale is 1 in. $= p_2 p_1 W$ tons ft.

Point of Maximum B.M.

At the point where the B.M. is a mathematical maximum (or minimum) the tangent to the B.M. curve must be horizontal and therefore the ray corresponding to this must also be horizontal. This ray, however, is related to the ordinate of the shear diagram, and since the pole is taken on the base line produced it follows that the shear force ordinate at this point must be zero for the line to the pole to be horizontal. Thus we have the rule that the maximum B.M. occurs where the shear is zero.

Mathematical Approach

The foregoing relationship between loading, shear force and bending moment, derived graphically, may be expressed mathematically as follows:

Let the intensity of load at a distance x from the origin be $F(x)$. Then:

$$\text{the shear at the point} = \int F(x)\,dx + C_1$$

and
$$\text{the B.M.} = \int F(x)x\,.\,dx + C_1x + C_2.$$

The integration constants C_1 and C_2 represent the effects of the base lines *JH* and *KOM* in Fig. 2.

Areas and Centres of Gravity

It has been noted earlier that the area of the load intensity diagram represents the total load on the beam. The area and the position of its centre of gravity may be found by link and vector polygons or by Simpson's Rule. Using this information, the reactions may be calculated.

To apply Simpson's Rule it is necessary to divide the loaded length into an even number of equal parts and the required area is approximately (to within about 0·1 per cent of the true value as given by integration) equal to the sum of the extreme ordinates plus four times the sum of the odd ordinates plus twice the sum of the remaining even ordinates all multiplied by one-third of the common distance between them.

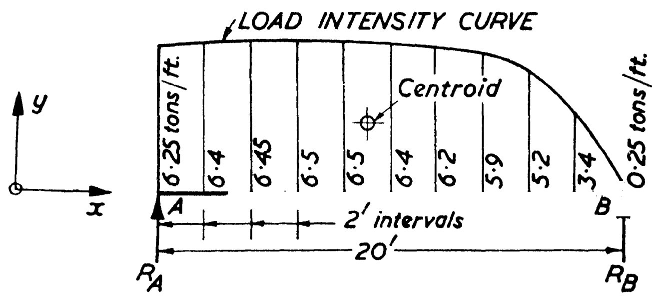


FIG. 5

Thus if the base is divided into n equal parts of length D, and the extreme ordinates are y_0 and y_n we have:

$$\text{Area} = \tfrac{1}{3}D\{[y_0 + y_n] + 4[y_1 + y_3 + y_5 + \ldots + y_{(n-1)}] + 2[y_2 + y_4 + y_6 + \ldots + y_{(n-2)}]\}.$$

Further, the distance of the centre of gravity of the load from one end is given by:

$$\bar{x} = \frac{\Sigma(Ax)}{\Sigma(A)}$$

where $\Sigma(Ax)$ is the summation of moments about that end of the individual parts of the loading diagram and $\Sigma(A)$ is the total area of the diagram.

Simpson's Rule will be applied to the beam shown in Fig. 5.

Tabulating:

$x=0$	2	4	6	8	10	12	14	16	18	20
$y=6{\cdot}25$	6·4	6·45	6·5	6·5	6·4	6·2	5·9	5·2	3·4	0·25
$xy=0$	12·8	25·8	39·0	52·0	64·0	74·4	82·6	83·2	61·2	5·0

$$\Sigma(Ax) = \Sigma(xy)$$
$$= \tfrac{1}{3} \times 2\{(0+5) + 4(12{\cdot}8 + \ldots + 61{\cdot}2) + 2(25{\cdot}8 + \ldots + 83{\cdot}2)\}$$
$$= 1009{\cdot}5.$$

$$\Sigma(A) = \Sigma(y)$$
$$= \tfrac{1}{3} \times 2\{(6{\cdot}25 + 0{\cdot}25) + 4(6{\cdot}4 + \ldots + 3{\cdot}4) + 2(6{\cdot}45 + \ldots + 5{\cdot}2)\}$$
$$= 113{\cdot}0.$$

Hence the distance of the centroid from R_A

$$= \frac{1009{\cdot}5}{113} = 8{\cdot}93 \text{ ft.}$$

Taking moments about the left support A,

$$R_B \times 20 = \Sigma Ax.$$

Whence $$R_B = \frac{1009{\cdot}5}{20} = 50{\cdot}475 \text{ tons}$$

and $$R_A = 113 - 50{\cdot}475 = 62{\cdot}525 \text{ tons.}$$

S.F. and B.M. Diagrams

Having stated the basic principles relating to simply supported beams, two examples will be given, the first utilising the arithmetical approach and the second the algebraic approach to analysis.

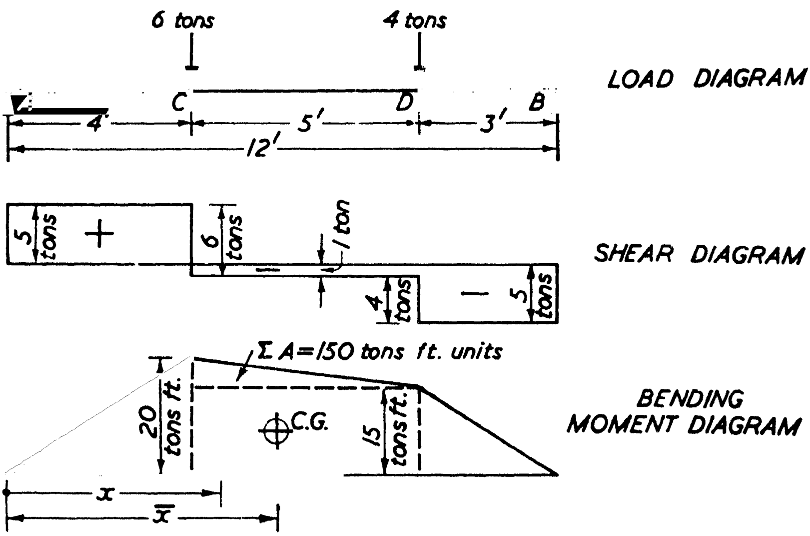


FIG. 6

Example 1. It is required to draw the S.F. and B.M. diagrams for the beam shown in Fig. 6 and to calculate the area and the position of the centre of gravity of the B.M. diagram.

Taking moments about B,

$$R_A \times 12 = (6 \times 8) + (4 \times 3)$$

and $$R_A = \frac{60}{12} = 5 \text{ tons.}$$

The total load = 10 tons. Hence:

$$R_B = 10 - 5 = 5 \text{ tons.}$$

The S.F. diagram can now be drawn as shown. It will be observed that the curve crosses the base line under the 6-ton point load, i.e. at C, and consequently this will be the point at which the B.M. has its maximum value.

The $$\text{B.M. at } C = M_C = R_A \times 4 = 5 \times 4$$
$$= 20 \text{ tons ft.}$$

Similarly, $$M_D = R_A \times 9 - 6 \times 5$$
$$= -R_B \times 3$$
$$= 15 \text{ tons ft.}$$

It is useful to note that the B.M. at any point is numerically equal to the area of the S.F. diagram on either side of that point. Considering the S.F. diagram to the left of D,

$$M_D = (+5 \times 4) + (-1 \times 5) = 20 - 5$$
$$= 15 \text{ tons ft., as found before.}$$

The area of the B.M. diagram may be found by dividing it into three triangles and a rectangle as shown.

Thus $$\Sigma A = \frac{20 \times 4}{2} + (15 \times 5) + \frac{5 \times 5}{2} + \frac{15 \times 3}{2}$$
$$= 40 + 75 + 12{\cdot}5 + 22{\cdot}5$$
$$= 150 \text{ tons ft. units.}$$

The moment of the area of the B.M. diagram about the left support A

$$= \Sigma Ax = \left(40 \times \frac{2 \times 4}{3}\right) + \left[75 \times \left(4 + \frac{5}{2}\right)\right] + \left[12{\cdot}5 \times \left(4 + \frac{5}{3}\right)\right] + \left[22{\cdot}5 \times \left(4 + 5 + \frac{3}{3}\right)\right]$$
$$= 106{\cdot}67 + 487{\cdot}5 + 70{\cdot}83 + 225$$
$$= 890.$$

Therefore, the distance to the C.G. of the B.M. diagram from the left support A:

$$\bar{x} = \frac{\Sigma Ax}{\Sigma A} = \frac{890}{150} = 5{\cdot}933 \text{ ft.}$$

Example 2. It is required to draw the S.F. and B.M. diagrams for the beam shown in Fig. 7 and to calculate the area and the position of the C.G. of the B.M. diagram.

The beam carries a load of 25 tons varying at a constant rate from 1 ton per foot at A to 4 tons per foot at B. Hence the load may be considered as being of two components, one a uniformly distributed load (U.D.L.) of 1 ton per foot and the other a triangular-shaped load varying from zero at A to 3 tons per foot at B, the total weight of each of the components being

respectively 10 and 15 tons. The C.G. of the U.D.L. is in the middle of the beam while that of the triangular load occurs one-third of the distance from B to A.

Taking moments about B,

$$R_A \times 10 = \left(10 \times \frac{10}{2}\right) + \left(15 \times \frac{10}{3}\right)$$

and

$$R_A = 10 \text{ tons.}$$

$$R_B = (10 + 15 - 10)$$
$$= 15 \text{ tons.}$$

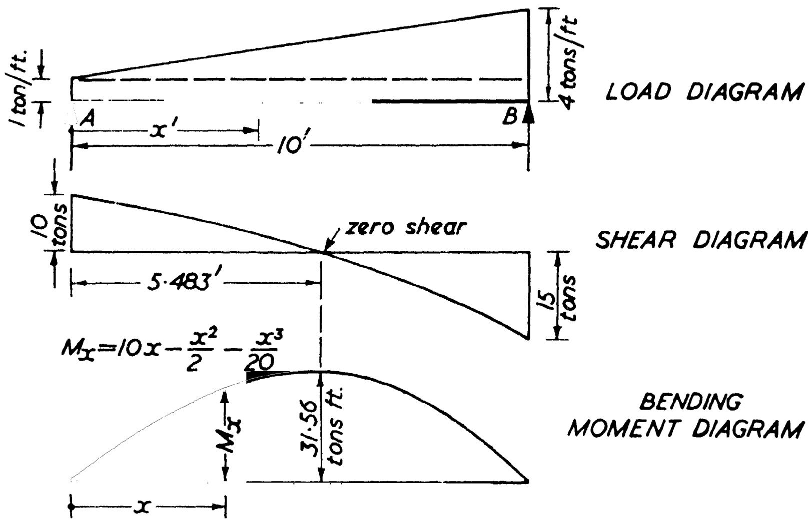


FIG. 7

Now the load to any point x ft. from A towards B

$$= \left(\frac{10}{L} \times x\right) + \left(\frac{3x}{L} \times \frac{x}{2}\right)$$

$$= x + \frac{3x^2}{20}.$$

Then the point of zero S.F. and maximum B.M. occurs where

$$R_A - \left(x + \frac{3x^2}{20}\right) = 0$$

or where

$$\frac{3x^2}{20} + x - 10 = 0,$$

i.e. 5·483 ft. from A.

Alternatively, elementary differentiation may be used to find the point of maximum B.M.:

$$M_x = R_A \,.\, x - \frac{x \times x}{2} - \left(\frac{3x^2}{20} \times \frac{x}{3}\right)$$

$$= 10x - \frac{x^2}{2} - \frac{x^3}{20}$$

Maximum B.M. occurs when $\frac{dM_x}{dx}=0,$

i.e. when $10-x-\frac{3x^2}{20}=0$

or $\frac{3x^2}{20}+x-10=0.$

Whence $x=5{\cdot}483$ ft., as before.

The S.F. diagram may now be drawn.

$$\text{The maximum B.M.}=R_A\,.\,x-\frac{x^2}{2}-\frac{x^3}{20}$$

$$=(10\times5{\cdot}483)-\frac{5{\cdot}483^2}{2}-\frac{5{\cdot}483^3}{20}$$

$$=54{\cdot}83-15{\cdot}03-8{\cdot}24$$

$$=31{\cdot}56 \text{ tons ft.}$$

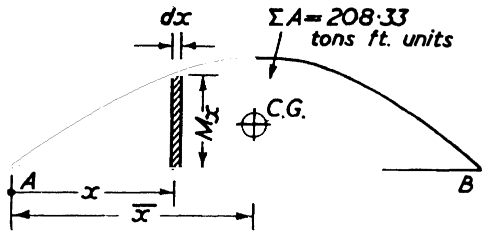


FIG. 8

Consider Fig. 8.

The area of the B.M. diagram

$$=\Sigma A=\int M\,.\,dx$$

$$=\int_0^{10}\left(10x-\frac{x^2}{2}-\frac{x^3}{20}\right)dx$$

$$=\left[\frac{10x^2}{2}-\frac{x^3}{6}-\frac{x^4}{80}\right]_0^{10}$$

$$=500-166{\cdot}67-125$$

$$=208{\cdot}33 \text{ tons ft. units.}$$

The moment of the area of the B.M. diagram about the support A

$$=\Sigma Ax=\int M\,.\,x\,dx$$

$$=\int_0^{10}\left(10x-\frac{x^2}{2}-\frac{x^3}{20}\right)x\,.\,dx$$

$$=\int_0^{10}\left(10x^2-\frac{x^3}{2}-\frac{x^4}{20}\right)dx$$

$$=\left[\frac{10x^3}{3}-\frac{x^4}{8}-\frac{x^5}{100}\right]_0^{10}$$

$$=3333{\cdot}33-1250-1000=1083{\cdot}33.$$

Hence, the distance from the support A to the C.G. of the B.M. diagram.

$$=\bar{x}=\frac{\Sigma Ax}{\Sigma A}=\frac{1083\cdot33}{208\cdot33}=5\cdot20 \text{ ft.}$$

It should be noted that the B.M. curve is a composite one resulting from the addition of the second-degree parabola associated with a U.D.L. and the third-degree parabola associated with a triangular load. The area under a second-degree parabola or under the third-degree parabola for this particular triangular load $= WL^2/12$. The total W in the example $=25$ tons. Hence, the area under the curve should be $25 \times 10^2/12 = 208\cdot33$ tons ft., which checks the result calculated above.

Examples of Sum Curve

The area relationships between the load intensity, shear and moment curves derived on pages 3 to 6 are useful in the case of a beam continuously supported, such as a stiff foundation carrying imposed point or partially distributed loads, as the examples shown in Figs. 9 and 10 indicate.

Example 1.

It is first necessary to find the net load intensity diagram as shown in Fig. 9.

Then:

Shear at B = area of load curve to left of B

$= 2$ tons/ft. $\times$ 6 ft. $= 12$ tons.

Shear at C = area of load curve to left of C

$= 12$ tons $-$ (3 tons/ft. $\times$ 6 ft.) $= 12 - 18 = -6$ tons,

etc.

Since the load intensity diagram is composed of horizontal straight lines, the shear diagram will consist of sloping straight lines and is drawn as shown.

Points of zero shear occur at G, H and J and at each of these the tangent to the B.M. curve will be horizontal and the B.M. value a maximum or minimum.

Then:

B.M. at B = area of shear curve to left of B

$$=\frac{12 \text{ tons} \times 6 \text{ ft.}}{2}=36 \text{ tons ft.}$$

B.M. at G = area of shear curve to left of G

$$=\frac{12 \text{ tons} \times 10 \text{ ft.}}{2}=60 \text{ tons ft.}$$

$$\text{B.M. at } C = 60-\frac{6\times 2}{2}=54 \text{ tons ft.}$$

$$\text{B.M. at } H = 54-\frac{6\times 3}{2}=45 \text{ tons ft.}$$

The principle of symmetry operates, and the values for the remaining points D, J and E will be the same as those calculated for C, G and B respectively.

Since the shear diagram is composed of sloping straight lines, the B.M. diagram will be composed of second-degree parabolas, and is drawn as shown. Note that the slopes of the parabolas change with the reversal of slopes in the shear diagram. The B.M. diagram will be a smooth curve throughout.

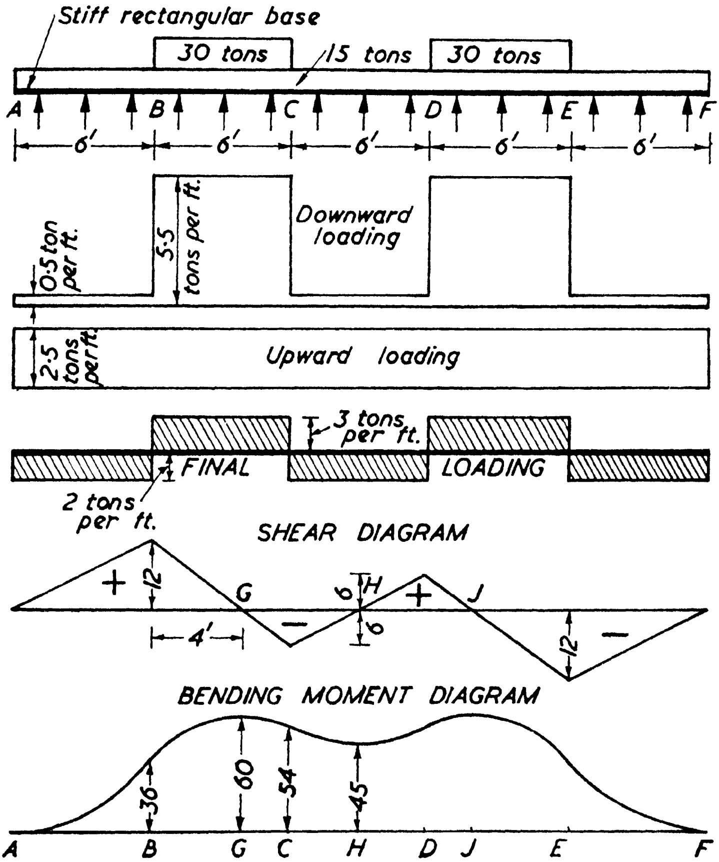


FIG. 9

Example 2.

Consider as an alternative an example with point loads of 30 tons in lieu of the loads distributed over the two lengths of 6 ft. The downward loads are thus two point loads of 30 tons each, and the upward load is 60 tons uniformly distributed. Using the summation of areas as before for the shear due to the uniform load and remembering that there will be a step in the shear diagram at each of the point loads, the S.F. diagram is as shown in Fig. 10.

The B.M. diagram is based upon the areas of the shear diagram and it should be noted that steps in the latter are accompanied by cusps in the former.

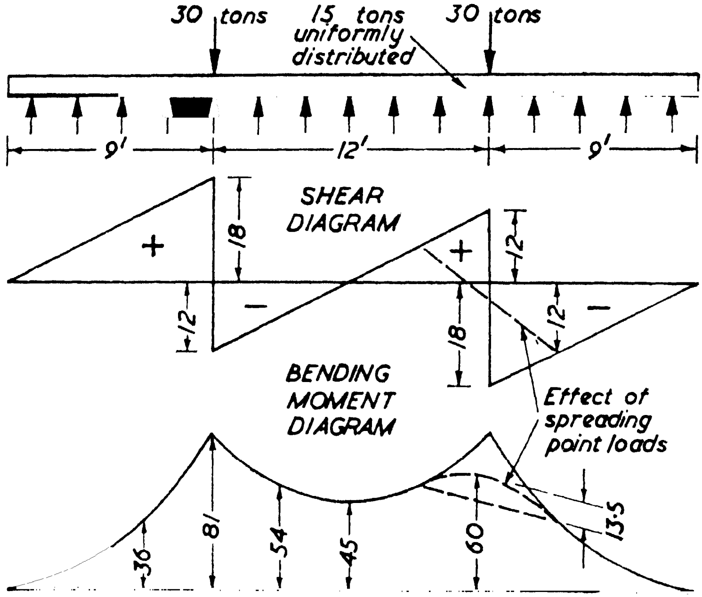
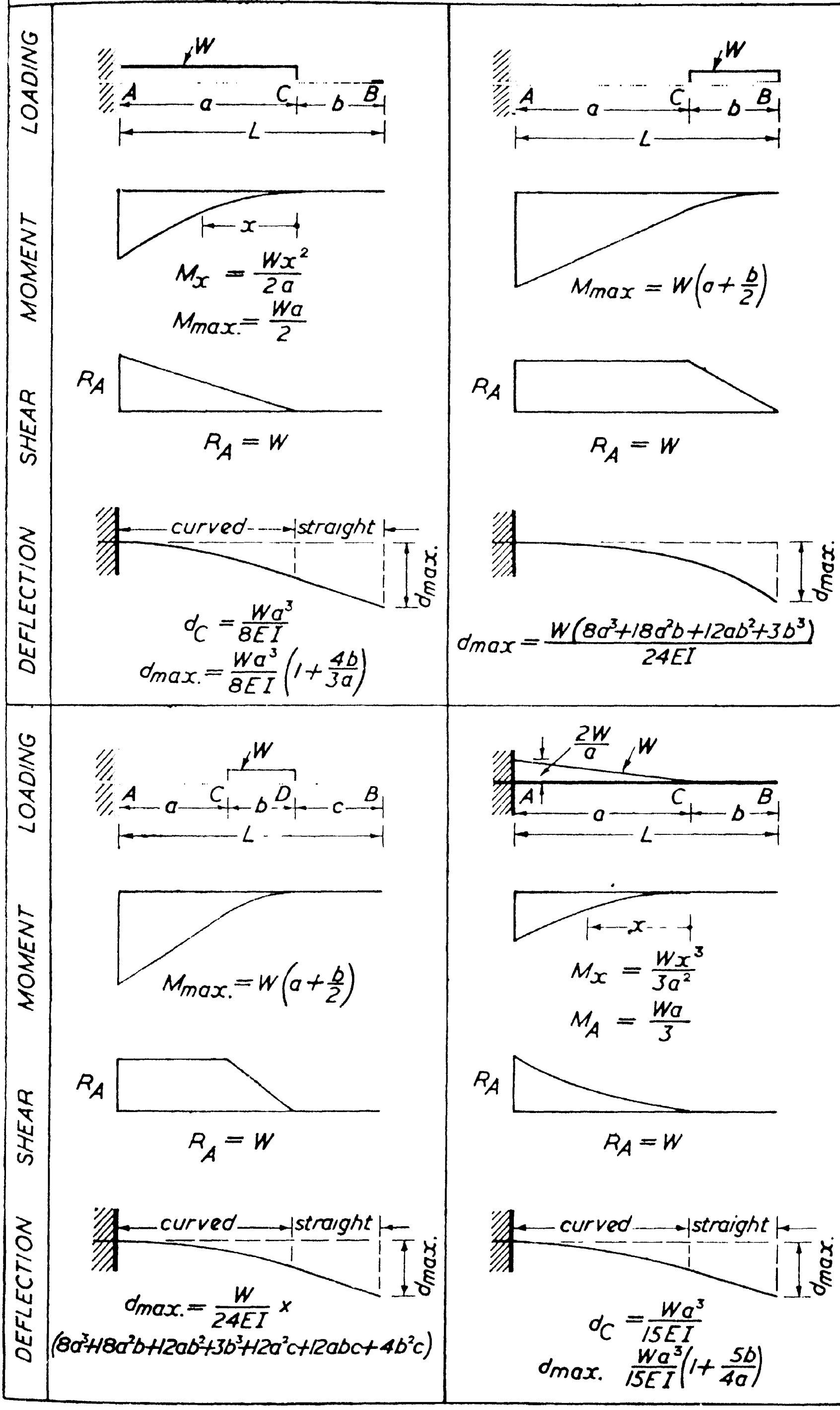


FIG 10

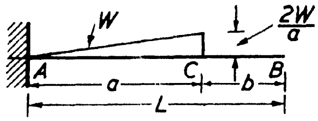
It will be observed that the B.M. diagram in Fig. 9 could be obtained from that in Fig. 10 by calculating the moments at D and E and on the line joining these ordinates constructing a parabola of height $WL/8$ ($=13{\cdot}5$ tons ft.), where $W = 18$ tons, as shown on the shear diagram, and $L = 6$ ft.

CANTILEVERS

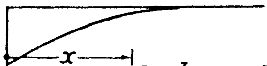


CANTILEVERS

LOADING



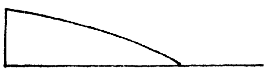
MOMENT



$$M_x = \frac{Wa}{3}\left[\left(\frac{x}{a}\right)^3 - \frac{3x}{a} + 2\right]$$

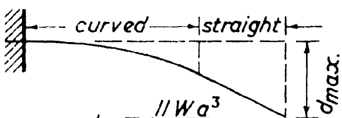
$$M_A = \frac{2Wa}{3}$$

SHEAR



$$R_A = W$$

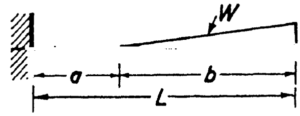
DEFLECTION



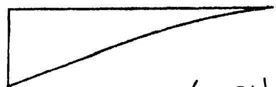
$$d_C = \frac{11Wa^3}{60EI}$$

$$d_{max.} = \frac{11Wa^3}{60EI}\left(1 + \frac{15b}{11a}\right)$$

LOADING



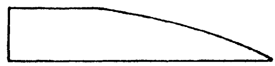
MOMENT



$$M_{max.} = W\left(a + \frac{2b}{3}\right)$$

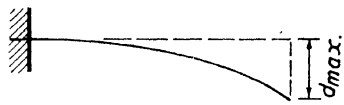
SHEAR

R_A



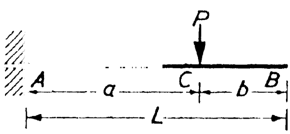
$$R_A = W$$

DEFLECTION

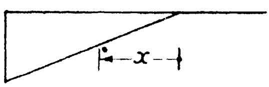


$$d_{max.} = \frac{W(20a^3 + 50a^2b + 40ab^2 + 11b^3)}{60EI}$$

LOADING



MOMENT

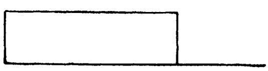


$$M_x = P.x$$

$$M_{max} = P.a$$

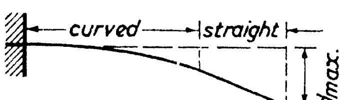
SHEAR

R_A



$$R_A = P$$

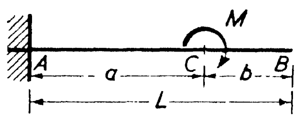
DEFLECTION



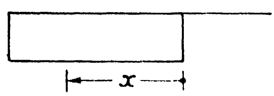
$$d_C = \frac{Pa^3}{3EI}$$

$$d_{max.} = \frac{Pa^3}{3EI}\left(1 + \frac{3b}{2a}\right)$$

LOADING



MOMENT



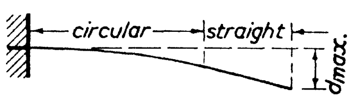
$$M_{max.} = M_x = M_C$$

SHEAR

No shears

N.B. For anti-clockwise moments the deflection is upwards.

DEFLECTION

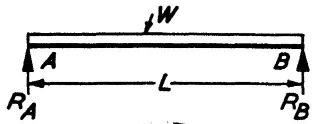


$$d_C = \frac{M.a^2}{2EI}$$

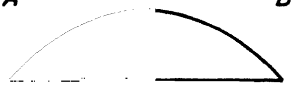
$$d_{max.} = \frac{M.a^2}{2EI}\left(1 + \frac{2b}{a}\right)$$

SIMPLY SUPPORTED BEAMS

LOADING



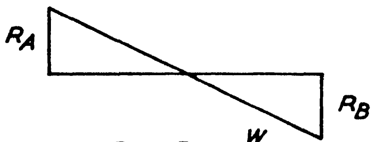
MOMENT



$$M_x = \frac{Wx}{2}\left(1 - \frac{x}{L}\right)$$

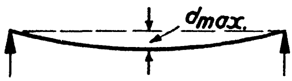
$$M_{max.} = \frac{WL}{8}$$

SHEAR



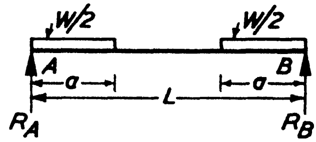
$$R_A = R_B = \frac{W}{2}$$

DEFLECTION



$$d_{max.} = \frac{5}{384} \cdot \frac{WL^3}{EI}$$

LOADING

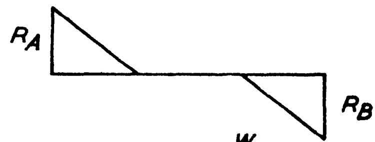


MOMENT



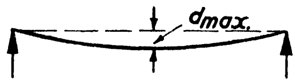
$$M_{max.} = \frac{Wa}{4}$$

SHEAR



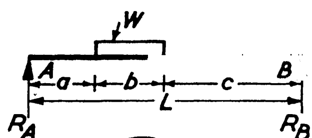
$$R_A = R_B = \frac{W}{2}$$

DEFLECTION

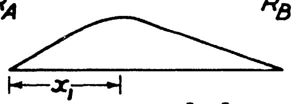


$$d_{max.} = \frac{Wa\,(3L^2 - 2a^2)}{96EI}$$

LOADING



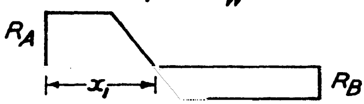
MOMENT



$$M_{max.} = \frac{W}{b}\left(\frac{x_1^2 - a^2}{2}\right)$$

when $x_1 = a + \frac{R_A b}{W}$

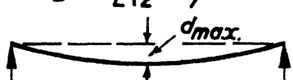
SHEAR



$$R_A = \frac{W}{L}\left(\frac{b}{2} + c\right)$$

$$R_B = \frac{W}{L}\left(\frac{b}{2} + a\right)$$

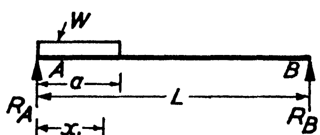
DEFLECTION



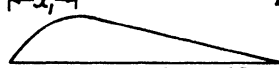
When $a = c$

$$d_{max.} = \frac{W}{384EI}\left(8L^3 - 4Lb^2 + b^3\right)$$

LOADING



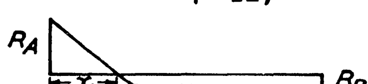
MOMENT



$$M_{max.} = \frac{W}{2}a\left(1 - \frac{a}{2L}\right)^2$$

when $x_1 = a\left(1 - \frac{a}{2L}\right)$

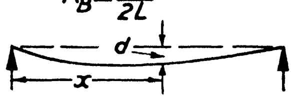
SHEAR



$$R_A = W\left(1 - \frac{a}{2L}\right)$$

$$R_B = \frac{Wa}{2L}$$

DEFLECTION



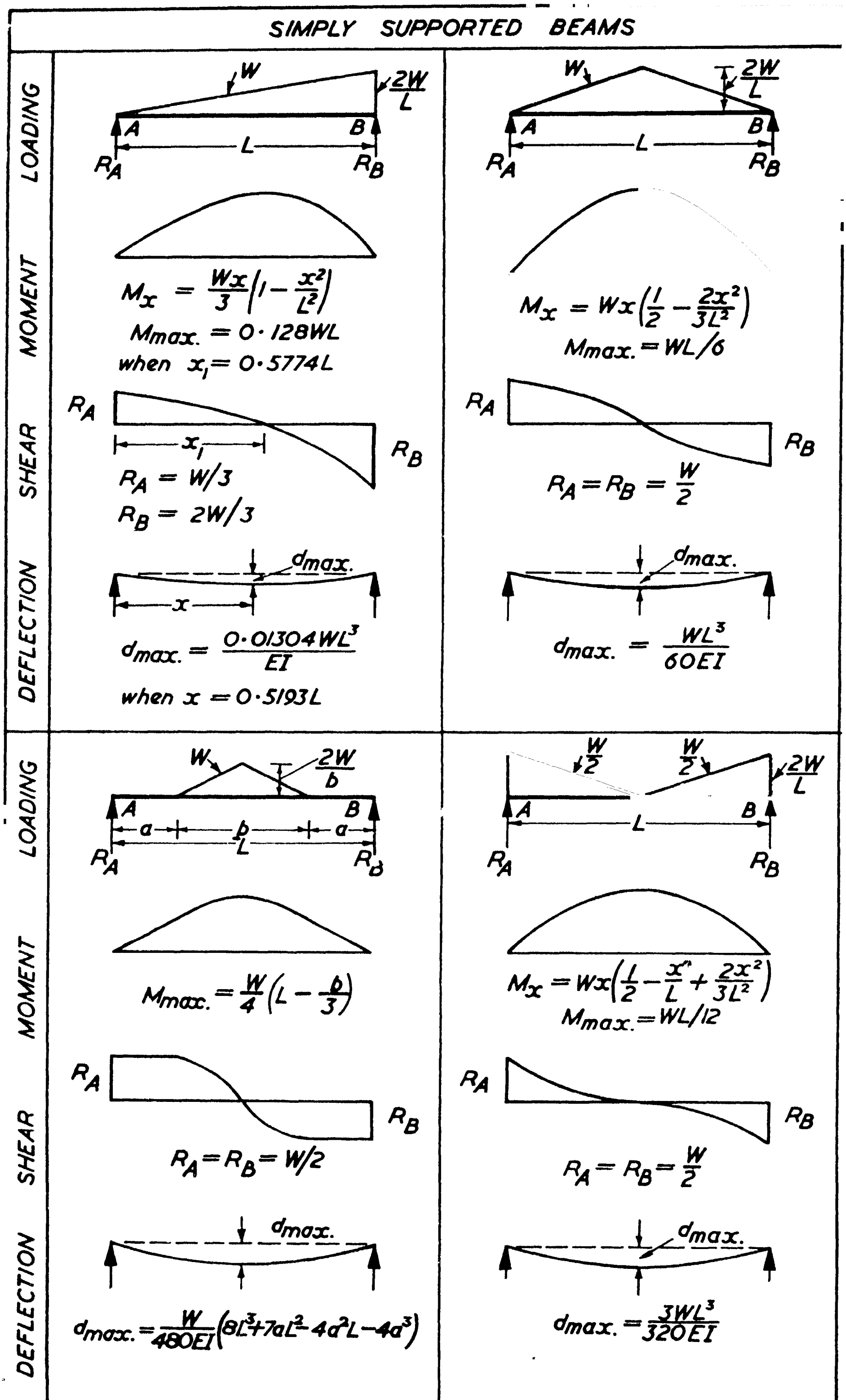
When $x \leq a$,

$$d = \frac{WL^4}{24aEI}\left[m^4 - 2n(2-n)m^3 + n^2(2-n)^2 m\right]$$

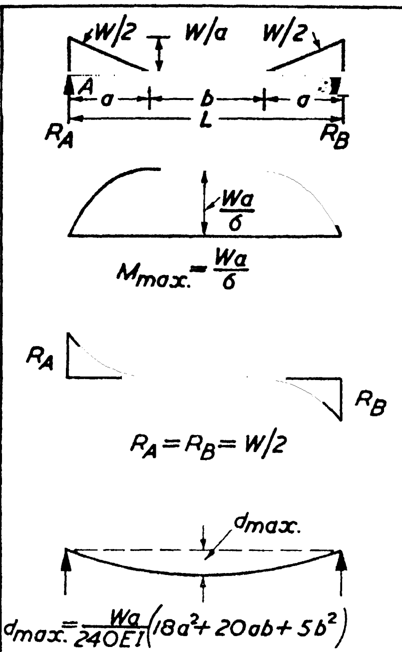
When $x \geq a$,

$$d = \frac{WL^4}{24aEI} \cdot n^2\left[2m^3 - 6m^2 + m(4+n^2) - n^2\right]$$

where $m = x/L$ and $n = a/L$


SIMPLY SUPPORTED BEAMS
LOADING
MOMENT
SHEAR
DEFLECTION
W
2W/L
A
B
L
RA
RB
Mx = Wx/3 (1 − x²/L²)
Mmax. = 0·128WL
when x1 = 0·5774L
x1
RA = W/3
RB = 2W/3
dmax.
x
dmax. = 0·01304WL³/EI
when x = 0·5193L
Mx = Wx(1/2 − 2x²/3L²)
Mmax. = WL/6
RA = RB = W/2
dmax. = WL³/60EI
2W/b
a
b
Mmax. = W/4 (L − b/3)
RA = RB = W/2
dmax. = W/480EI (8L³ + 7aL² − 4a²L − 4a³)
W/2
Mx = Wx(1/2 − x/L + 2x²/3L²)
Mmax. = WL/12
RA = RB = W/2
dmax. = 3WL³/320EI

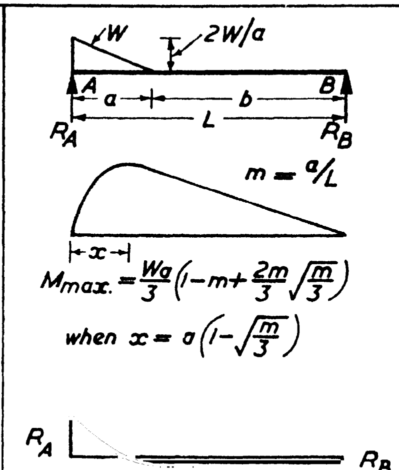
LOADING — MOMENT — SHEAR — DEFLECTION



$M_{max.} = \frac{Wa}{6}$

$R_A = R_B = W/2$

$d_{max.} = \frac{Wa}{240EI}(18a^2 + 20ab + 5b^2)$

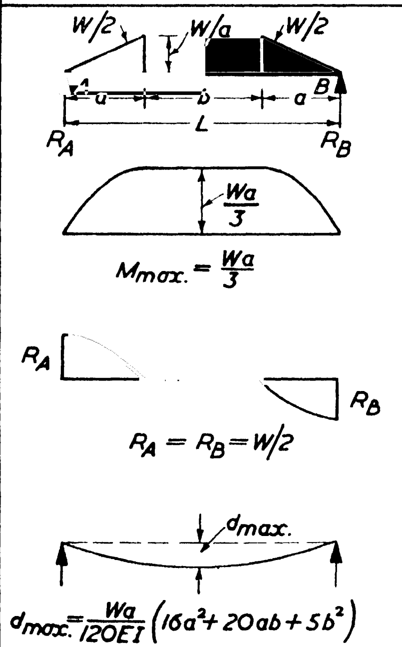


$M_{max.} = \frac{Wa}{3}\left(1 - m + \frac{2m}{3}\sqrt{\frac{m}{3}}\right)$

when $x = a\left(1 - \sqrt{\frac{m}{3}}\right)$

$R_A = W\left(1 - \frac{m}{3}\right)$

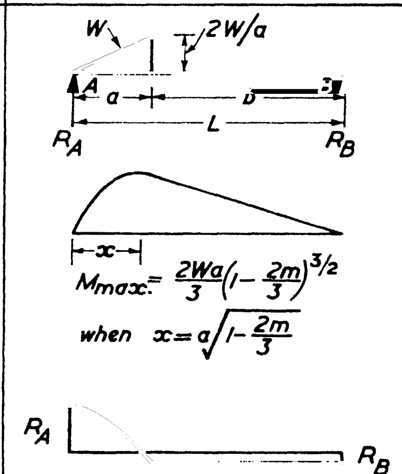
$R_B = \frac{Wm}{3}$



$M_{max.} = \frac{Wa}{3}$

$R_A = R_B = W/2$

$d_{max.} = \frac{Wa}{120EI}(16a^2 + 20ab + 5b^2)$



$M_{max.} = \frac{2Wa}{3}\left(1 - \frac{2m}{3}\right)^{3/2}$

when $x = a\sqrt{1 - \frac{2m}{3}}$

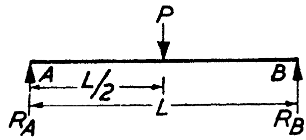
$R_A = W\left(1 - \frac{2m}{3}\right)$

$R_B = \frac{2Wm}{3}$

SIMPLY SUPPORTED BEAMS

Central point load

LOADING

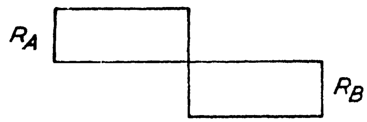


MOMENT



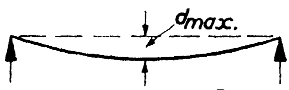
$$M_{max.} = \frac{PL}{4}$$

SHEAR



$$R_A = R_B = \frac{P}{2}$$

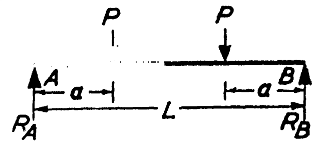
DEFLECTION



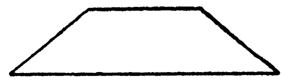
$$d_{max.} = \frac{PL^3}{48EI}$$

Two equal point loads, symmetrically placed

LOADING

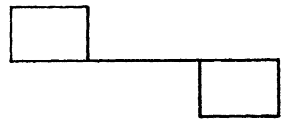


MOMENT



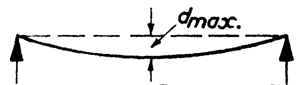
$$M_{max.} = Pa$$

SHEAR



$$R_A = R_B = P$$

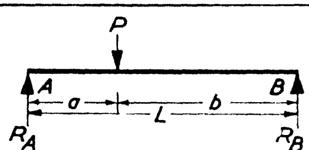
DEFLECTION



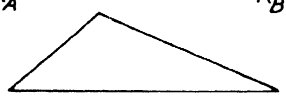
$$d_{max.} = \frac{PL^3}{6EI}\left[\frac{3a}{4L} - \left(\frac{a}{L}\right)^3\right]$$

Eccentric point load

LOADING

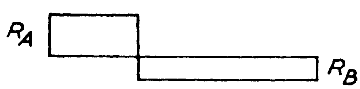


MOMENT



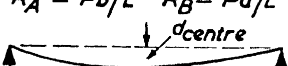
$$M_{max.} = \frac{Pab}{L}$$

SHEAR



$R_A = Pb/L \quad R_B = Pa/L$

DEFLECTION



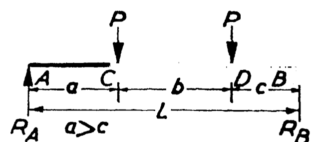
$d_{max.}$ always occurs within 0·0774 L of the centre of the beam. When $b \geqslant a$,

$$d_{centre} = \frac{PL^3}{48EI}\left[\frac{3a}{L} - 4\left(\frac{a}{L}\right)^3\right]$$

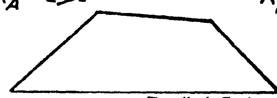
This value is always within 2·5 % of the maximum value.

Two unequal-spaced point loads

LOADING



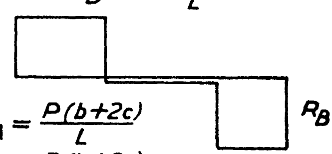
MOMENT



$$M_C = \frac{Pa(b+2c)}{L}$$

$$M_D = \frac{Pc(b+2a)}{L}$$

SHEAR



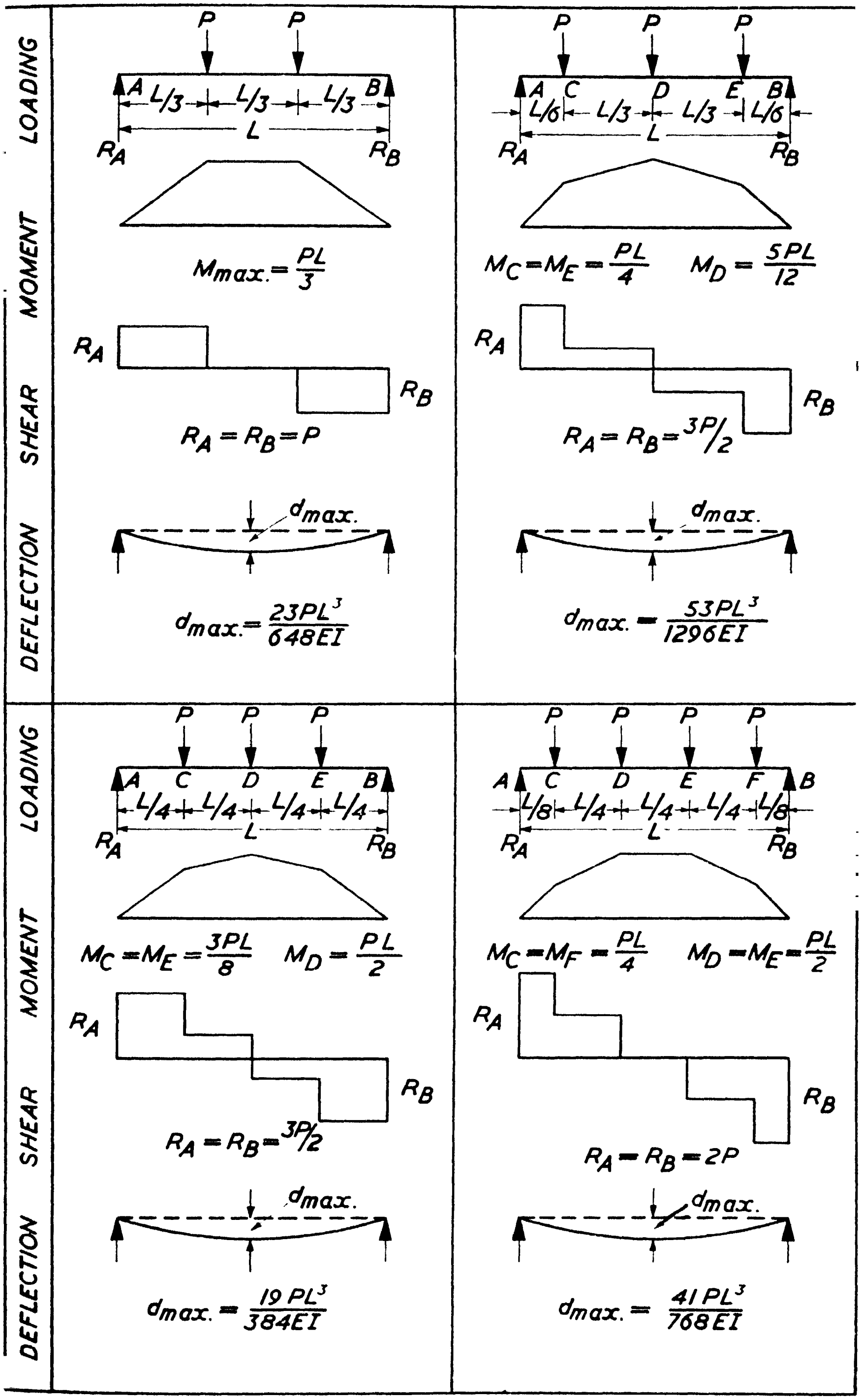
$$R_A = \frac{P(b+2c)}{L}$$

$$R_B = \frac{P(b+2a)}{L}$$

DEFLECTION

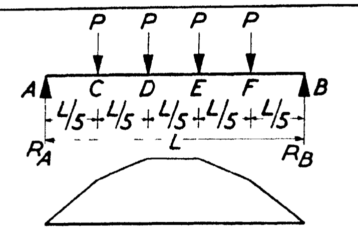
For central deflection add the values for each P derived from the formula in the adjacent diagram.

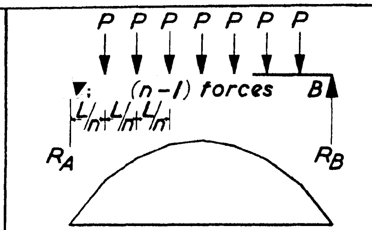
SIMPLY SUPPORTED BEAMS



SIMPLY SUPPORTED BEAMS

LOADING





MOMENT

$M_C = M_F = \frac{2PL}{5}$ $M_D = M_E = \frac{3PL}{5}$

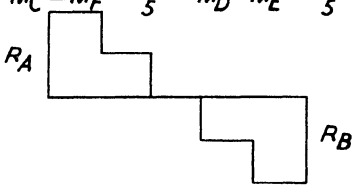
When n is odd,

$M_{max.} = \frac{(n^2-1)\,PL}{8n}$

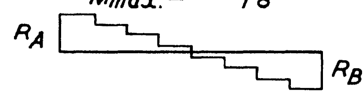
When n is even,

$M_{max.} = n.PL/8$

SHEAR

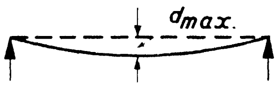


$R_A = R_B = 2P$

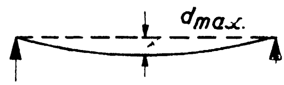


$R_A = R_B = (n-1)P/2$

DEFLECTION



$d_{max.} = \frac{63PL^3}{1000EI}$

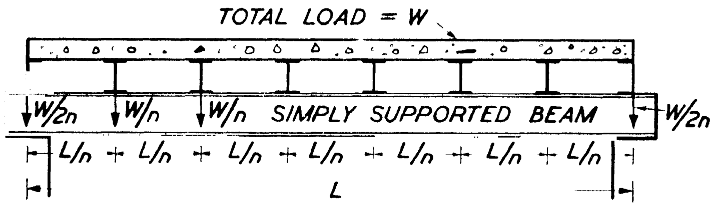


When n is odd

$d_{max} = \frac{PL^3}{192EI}\left[n-\frac{1}{n}\right]\left[3-\frac{1}{2}\left(1-\frac{1}{n^2}\right)\right]$

When n is even

$d_{max} = \frac{PL^3}{192EI}\cdot n\left[3-\frac{1}{2}\left(1+\frac{4}{n^2}\right)\right]$



When $n > 10$, consider the load uniformly distributed

The reaction at the supports = $W/2$, but the maximum S.F.

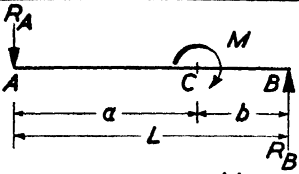
at the ends of the beam = $\frac{W(n-1)}{2n} = A.W$

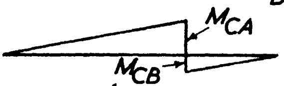
The value of the maximum bending moment $= C.WL$

The value of the deflection at the centre of the span $= k.\frac{WL^3}{EI}$

Value of n	A	C	k
2	0·2500	0·1250	0·0105
3	0·3333	0·1111	0·0118
4	0·3750	0·1250	0·0124
5	0·4000	0·1200	0·0126
6	0·4167	0·1250	0·0127
7	0·4286	0·1224	0·0128
8	0·4375	0·1250	0·0128
9	0·4444	0·1236	0·0129
10	0·4500	0·1250	0·0129

LOADING / MOMENT / SHEAR / DEFLECTION



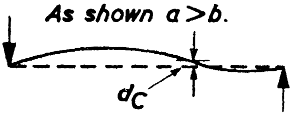


$M_{CA} = M.a/L \quad M_{CB} = M.b/L$



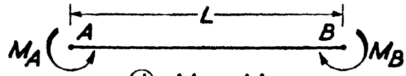
$R_A = R_B = M/L$

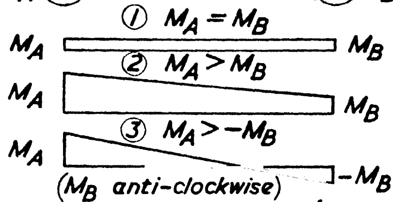
As shown $a > b$.



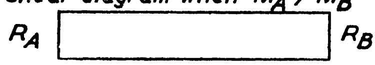
$$d_C = -\frac{M.ab}{3EI}\left(\frac{a}{L} - \frac{b}{L}\right)$$

For anti-clockwise moments the deflections are reversed.

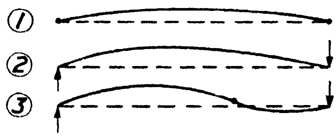




Shear diagram when $M_A \neq M_B$



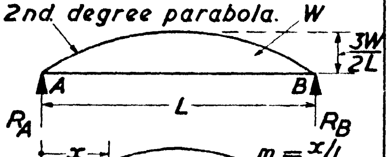
$$R_A = -R_B = \frac{M_A - M_B}{L}$$



When $M_A = M_B$,

$$d_{max} = -\frac{ML^2}{8EI}$$

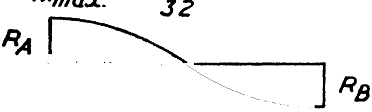
2nd degree parabola.



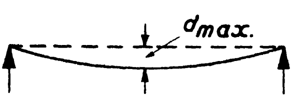


$$M_x = \frac{WL}{2}\left(m^4 - 2m^3 + m\right)$$

$$M_{max.} = \frac{5WL}{32}$$

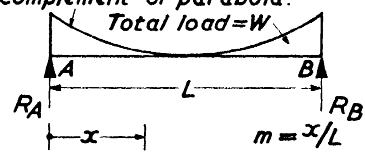


$R_A = R_B = W/2$



$$d_{max.} = \frac{6{\cdot}1WL^3}{384EI}$$

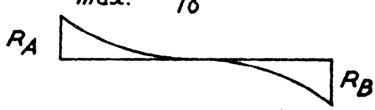
Complement of parabola.



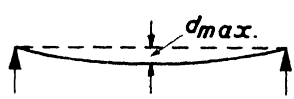


$$M_x = \frac{WL}{2}\left(m - 3m^2 + 4m^3 - 2m^4\right)$$

$$M_{max.} = \frac{WL}{16}$$



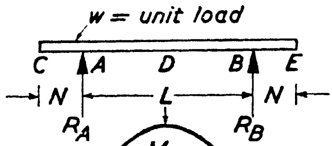
$R_A = R_B = W/2$



$$d_{max.} = \frac{2{\cdot}8WL^3}{384EI}$$

SIMPLY SUPPORTED BEAMS

LOADING

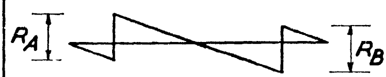


MOMENT



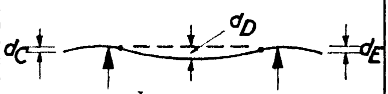
$M_A = M_B = -\frac{wN^2}{2}$ $M_D = \frac{wL^2}{8} + M_A$

SHEAR



$R_A = R_B = w\left(N + \frac{L}{2}\right)$

DEFLECTION

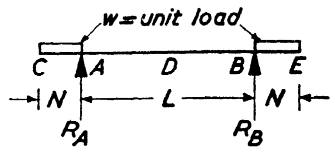


$d_C = d_E = \frac{wL^3N}{24EI}\left(1 - 6n^2 - 3n^3\right)$

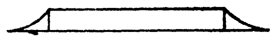
$d_D = \frac{wL^4}{384EI}\left(5 - 24n^2\right)$

Where $n = N/L$

LOADING

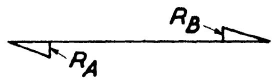


MOMENT



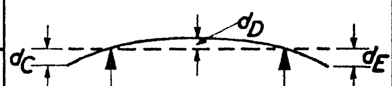
$M_A = M_B = -\frac{wN^2}{2}$

SHEAR



$R_A = R_B = wN$

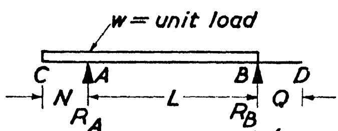
DEFLECTION



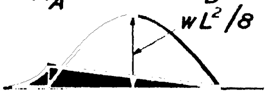
$d_C = d_E = \frac{wLN^3}{8EI}\left(2 + \frac{N}{L}\right)$

$d_D = -\frac{wL^2N^2}{16EI}$

LOADING

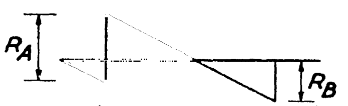


MOMENT



$M_A = -\frac{wN^2}{2}$

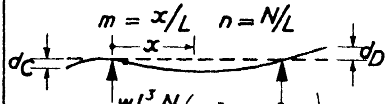
SHEAR



$R_A = \frac{w(N+L)^2}{2L}$ $R_B = \frac{w(L+N)(L-N)}{2L}$

$m = x/L$ $n = N/L$

DEFLECTION

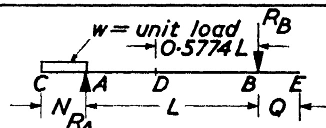


$d_C = \frac{wL^3N}{24EI}\left(3n^3 + 4n^2 - 1\right)$

$d_x = \frac{wL^4}{24EI}\left[m^4 - 2m^3(1-n^2) + m(1-2n^2)\right]$

$d_D = -\frac{wL^3Q}{24EI}\left(2n^2 - 1\right)$

LOADING



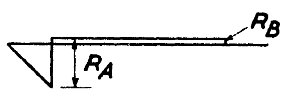
Max. upward deflection is at D.

MOMENT



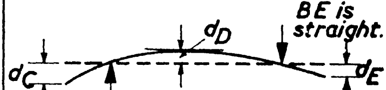
$M_A = -\frac{wN^2}{2}$

SHEAR



$R_A = \frac{wN(2L+N)}{2L}$ $R_B = \frac{wN^2}{2L}$

DEFLECTION



$d_C = \frac{wLN^3}{24EI}\left(4 + 3\frac{N}{L}\right)$

$d_D = -\frac{0{\cdot}032\,wL^2N^2}{EI}$

$d_E = \frac{wLN^2Q}{12EI}$

3. FIXED, BUILT-IN OR ENCASTRE BEAMS

WHEN the ends of a beam are firmly held so that they cannot rotate under the action of the superimposed loads, the beam is known as a fixed, built-in or encastré beam. The B.M. diagram for such a beam is in two parts, viz.: the free or positive B.M. diagram, which would have resulted had the ends been simply supported, i.e. free to rotate, and the fixing or negative B.M. diagram which results from the restraints imposed upon the ends of the beam.

Normally, the supports for built-in beams are on the same level and the ends of the beams are horizontal. This type will be considered first.

Beams with Supports at the same Level

The conditions for solution, derived by Mohr, are two, viz.:

(i) The area of the fixing or negative B.M. diagram is equal to that of the free or positive B.M. diagram.
(ii) The centres of gravity of the two diagrams lie in the same vertical line, i.e. are equidistant from a given end of the beam.

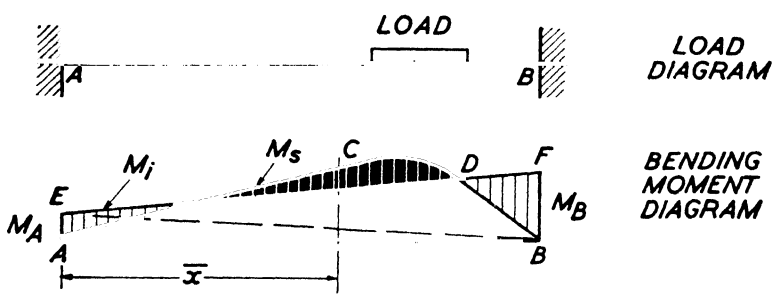


FIG. 11

Fig. 11 shows a typical B.M. diagram for a built-in beam.

ACDB is the diagram of the free moment M_s and the trapezium *AEFB* is the diagram of the fixing moment M_i, the portions shaded representing the final diagram.

Let $\quad A_s$ = the area of the free B.M. diagram
and $\quad A_i$ = the area of the fixing moment diagram.

Then from condition (i) above, $A_s = A_i$, while from condition (ii) their centres of gravity lie in the same vertical line, say, distance $\bar{x}$ from the left-hand support *A*.

Now AE = the fixing moment M_A

and BF = the fixing moment M_B.

Therefore
$$\frac{M_A + M_B}{2} \times L = A_i$$

and
$$M_A + M_B = \frac{2A_i}{L} \quad . \; . \; . \; . \; . \; . \; . \; . \quad \text{(a)}$$

Divide the trapezium $AEFB$ by drawing the diagonal EB and take area moments about the support A.

Then
$$A_i \,.\, \bar{x} = \left(\frac{M_A \times L}{2} \times \frac{L}{3}\right) + \left(\frac{M_B \times L}{2} \times \frac{2L}{3}\right)$$
$$= \frac{L^2}{6}(M_A + 2M_B)$$

$$M_A + 2M_B = \frac{6A_i\bar{x}}{L^2} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad \text{(b)}$$

But
$$M_A + M_B = \frac{2A_i}{L} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad \text{(a)}$$

Also,
$$A_s = A_i$$

Subtracting (a) from (b) and substituting A_s for A_i:

$$M_B = \frac{6A_s\bar{x}}{L^2} - \frac{2A_s}{L}.$$

Similarly
$$M_A = \frac{4A_s}{L} - \frac{6A_s\bar{x}}{L^2}.$$

It will be seen, therefore, that the fixing moments for any built-in beam on level supports can be calculated provided that the area of the free B.M. diagram and the position of its centre of gravity are known.

For point loads, however, the principle of reciprocal moments provides the simplest solution.

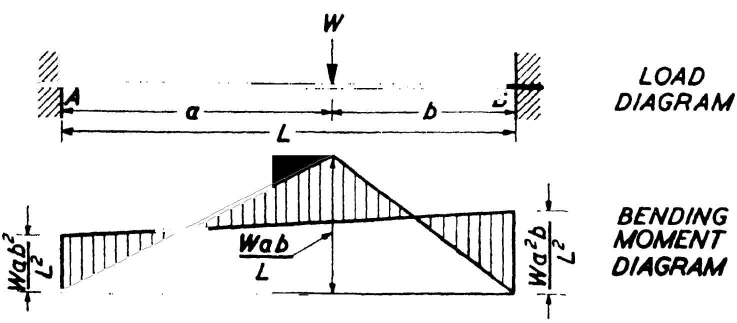


FIG. 12

With reference to Fig. 12,

$$M_A = \frac{Wab}{L} \times \frac{b}{L} = \frac{Wab^2}{L^2}$$

$$M_B = \frac{Wab}{L} \times \frac{a}{L} = \frac{Wa^2b}{L^2},$$

i.e. the fixing moments are in reciprocal proportion to the distances of the ends of the beam from the point load.

In the case of several isolated loads, this principle is applied to each load in turn and the results summated.

It should be noted that appropriate formulæ for built-in beams are given on pp. 29 to 35.

Beams with Supports at Different Levels

The ends are assumed, as before, to be horizontal.

The bent form of the unloaded beam as shown in Fig. 13 is similar to the bent form of two simple cantilevers which can be achieved by cutting the

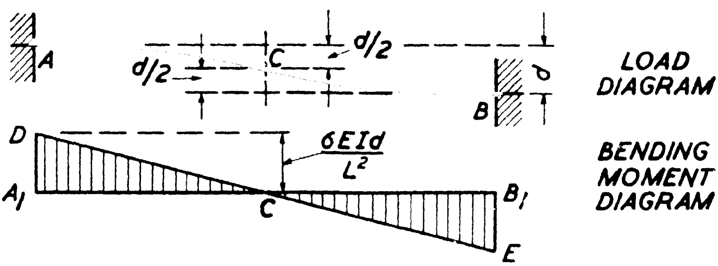


FIG. 13

beam at the centre C, and placing downward and upward loads at the free ends of the cantilevers such that the deflection at the end of each cantilever is $d/2$.

Therefore $$\frac{d}{2}=\frac{P(L/2)^3}{3EI},$$ (being the standard deflection formula)

or $$P=\frac{12EId}{L^3}.$$

This load would cause a B.M. at A or B equal to

$$P\times\frac{L}{2}=\frac{12EId}{L^3}\times\frac{L}{2}=\frac{6EId}{L^2}.$$

The solution in any given case consists of adding to the ordinary diagram of B.M.s, the B.M. diagram A_1DCEB_1.

Shear Forces in Fixed Beams

It must be noted that in the case of fixed beams, it is necessary to evaluate the B.M.s before the S.F.s can be determined. This is the converse of the procedure in the case of simply supported beams.

The S.F. at the ends of a beam is found in the following manner:

$$\text{S.F.}_A=\text{the simple support reaction at } A+\frac{M_A-M_B}{L}$$

$$\text{S.F.}_B=\text{the simple support reaction at } B+\frac{M_B-M_A}{L},$$

where M_A and M_B are the numerical values of the moments at the ends of the beam.

These formulæ must be followed exactly with respect to the signs shown since if M_A is smaller than M_B the signs will adjust themselves.

It will be seen that for symmetrical loads where $M_A = M_B$, the reactions will be the same as for simply supported beams.

As an example, consider Fig. 14.

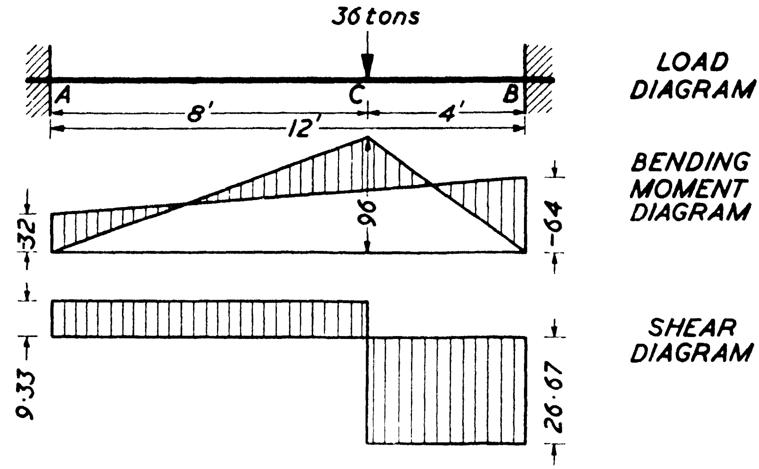


FIG. 14

If AB were simply supported, then the maximum M, at C, would be:

$$\frac{Wab}{L} = \frac{36 \times 8 \times 4}{12} = 96 \text{ tons ft.},$$

while R_A would be 12 tons and R_B 24 tons.

By the principle of reciprocal moments (p. 26)

$$M_A = 32 \text{ tons ft.} \quad \text{and} \quad M_B = 64 \text{ tons ft.}$$

Now M_A could be considered as being caused by a downward force at B equal to $M_A/L = 2{\cdot}67$ tons, which would have to be balanced at A, for the equilibrium of vertical loads, by an upward force of 2·67 tons. Similarly, M_B could be caused by a downward force of 5·33 tons at A, with an upward balancing force of 5·33 tons at B.

Hence, the final shear force at A

$$= \text{the simple support reaction at } A + \left(\frac{M_A}{L} - \frac{M_B}{L}\right)$$

$$= 12 + 2{\cdot}67 - 5{\cdot}33 = 9{\cdot}33 \text{ tons.}$$

Similarly, the final shear force at B

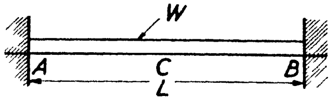
$$= \text{the simple support reaction at } B + \left(\frac{M_B}{L} - \frac{M_A}{L}\right)$$

$$= 24 + 5{\cdot}33 - 2{\cdot}67 = 26{\cdot}67 \text{ tons.}$$

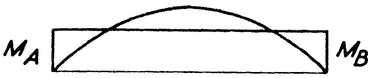
A simple rule to remember is that the shear force is greater than the simple support reaction at the fixed end which has the greater numerical fixing moment.

LOADING — MOMENT — SHEAR — DEFLECTION

LOADING



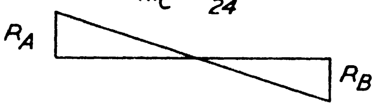
MOMENT



$$M_A = M_B = -\frac{WL}{12}$$

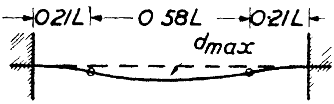
$$M_C = \frac{WL}{24}$$

SHEAR



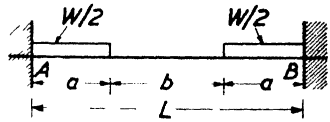
$$R_A = R_B = W/2$$

DEFLECTION

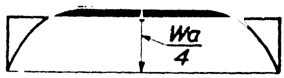


$$d_{max} = \frac{WL^3}{384EI}$$

LOADING

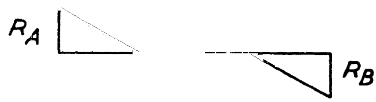


MOMENT



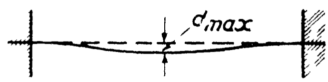
$$M_A = M_B = -\frac{Wa}{12L}(3L-2a)$$

SHEAR



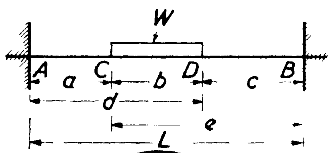
$$R_A = R_B = W/2$$

DEFLECTION

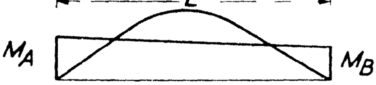


$$d_{max} = \frac{Wa^2}{48EI}(L-a)$$

LOADING



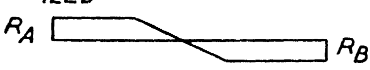
MOMENT



$$M_A = \frac{-W}{12L^2 b}\left[e^3(4L-3e) - c^3(4L-3c)\right]$$

$$M_B = \frac{-W}{12L^2 b}\left[d^3(4L-3d) - a^3(4L-3a)\right]$$

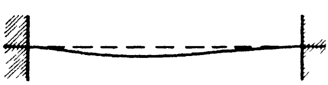
SHEAR



When r is the simple support reaction

$$R_A = r_A + \frac{M_A - M_B}{L} \qquad R_B = r_B + \frac{M_B - M_A}{L}$$

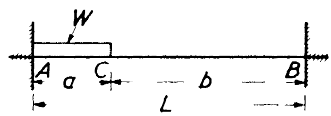
DEFLECTION



When $a = c$ d_{max} =

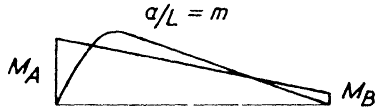
$$\frac{W}{384EI}\left(L^3 + 2La^2 + 4La^2 - 8a^3\right)$$

LOADING



$a/L = m$

MOMENT



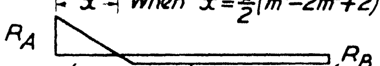
$$M_A = -\frac{WL}{12}\, m\,(3m^2 - 8m + 6)$$

$$M_B = -\frac{WL}{12}\, m^2(4-3m) \qquad +M_{max} =$$

$$\frac{WL}{12} m^2\left(-\frac{3}{2}m^5 + 6m^4 - 6m^3 - 6m^2 + 15m - 8\right)$$

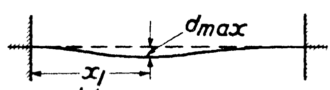
$\vert\leftarrow x \rightarrow\vert$ When $x = \frac{a}{2}(m^3 - 2m^2 + 2)$

SHEAR



$$R_A = \frac{W(m^3 - 2m^2 + 2)}{2} \qquad R_B = \frac{W\, m^3(2-m)}{2m}$$

DEFLECTION



When $a = L/2$ and $x_1 = 0\ 445\,L$

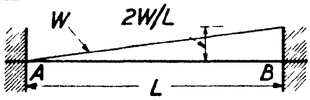
$$d_{max} = \frac{WL^3}{333EI}$$

$$d_C = \frac{WL^3}{384EI}$$

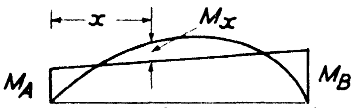
BUILT-IN BEAMS

Case 1

Loading



Moment

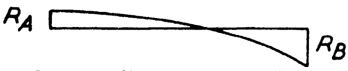


$$M_x = -\frac{WL}{30}\left(\frac{10x^3}{L^3} - \frac{9x}{L} + 2\right)$$

$+M_{max.} = WL/23{\cdot}3$ when $x = 0{\cdot}55L$

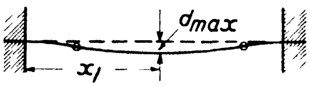
$M_A = -WL/15 \quad M_B = -WL/10$

Shear



$R_A = 0{\cdot}3W \quad R_B = 0{\cdot}7W$

Deflection

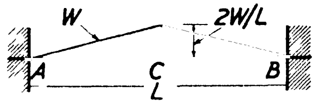


$$d_{max.} = \frac{WL^3}{382EI}$$

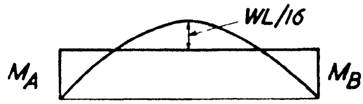
when $x_1 = 0{\cdot}525L$

Case 2

Loading



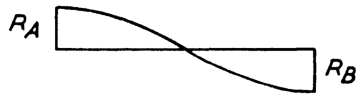
Moment



$$M_A = M_B = -\frac{5WL}{48}$$

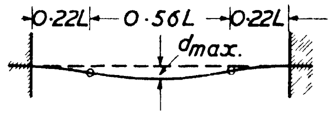
$M_C = WL/16$

Shear



$R_A = R_B = W/2$

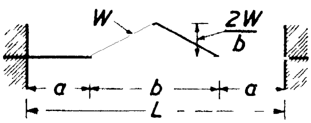
Deflection



$$d_{max} = \frac{1{\cdot}4WL^3}{384EI}$$

Case 3

Loading

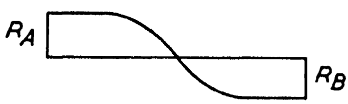


Moment



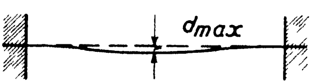
$$M_A = M_B = \frac{-W}{48L}\left(5L^2 + 4aL - 4a^2\right)$$

Shear



$R_A = R_B = W/2$

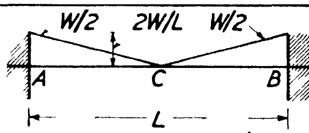
Deflection



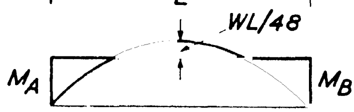
$$d_{max.} = \frac{W}{1920EI}\left(7L^3 + 8aL^2 + 4a^2L - 16a^3\right)$$

Case 4

Loading



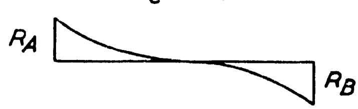
Moment



$M_A = M_B = -WL/16$

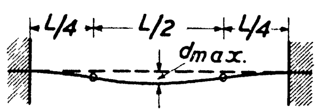
$M_C = WL/48$

Shear



$R_A = R_B = W/2$

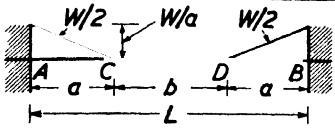
Deflection



$$d_{max.} = \frac{0{\cdot}6WL^3}{384EI}$$

BUILT-IN BEAMS

LOADING



MOMENT



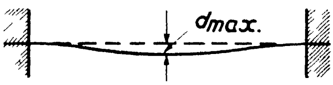
$$M_A = M_B = -\frac{Wa}{12L}(2L-a)$$

SHEAR



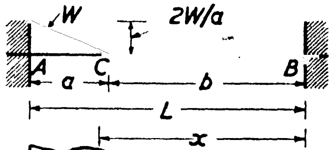
$$R_A = R_B = W/2$$

DEFLECTION

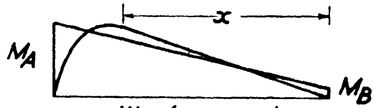


$$d_{max.} = \frac{Wa^2}{480EI}(5L-4a)$$

LOADING



MOMENT



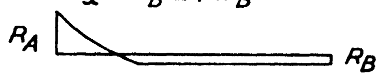
$$M_A = -\frac{Wa}{30L^2}(3a^2+10bL)$$

$$M_B = -\frac{Wa^2}{30L^2}(5L-3a)$$

In AC, $M_x = R_B.x + M_B - \frac{2W(x-b)^3}{6ab}$

In CB, $M_x = R_B.x + M_B$

SHEAR



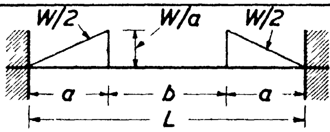
$$R_A = \frac{W}{10L^3}(10L^3-5La^2+2a^3)$$

$$R_B = \frac{Wa^2}{10L^3}(5L-2a)$$

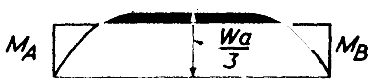
DEFLECTION

———

LOADING



MOMENT



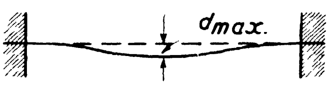
$$M_A = M_B = -\frac{Wa}{12L}(4L-3a)$$

SHEAR



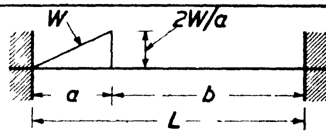
$$R_A = R_B = W/2$$

DEFLECTION

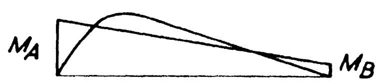


$$d_{max.} = \frac{Wa^2}{480EI}(15L-16a)$$

LOADING



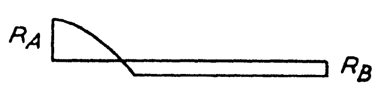
MOMENT



$$M_A = -\frac{Wa}{15L}(10L^2-15aL+6a^2)$$

$$M_B = -\frac{Wa^2}{10L^2}(5L-4a)$$

SHEAR



$$R_A = \frac{W}{10L^3}(10L^3-15La^2+8a^3)$$

$$R_B = \frac{Wa^2}{10L^3}(15L-8a)$$

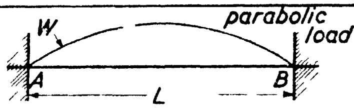
DEFLECTION

———

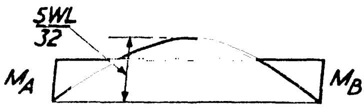
BUILT—IN BEAMS

Parabolic load

LOADING

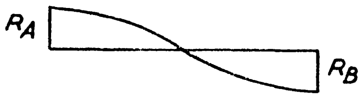


MOMENT



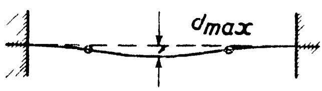
$M_A = M_B = -WL/10$

SHEAR



$R_A = R_B = W/2$

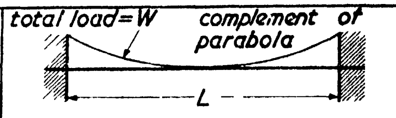
DEFLECTION



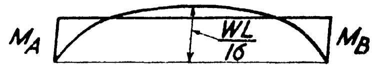
$$d_{max} = \frac{1.3\,WL^3}{384EI}$$

Complement of parabola

LOADING

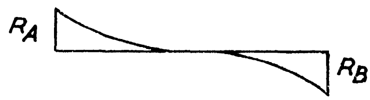


MOMENT



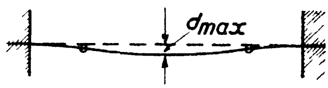
$M_A = M_B = -WL/20$

SHEAR



$R_A = R_B = W/2$

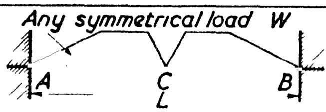
DEFLECTION



$$d_{max} = \frac{0.4\,WL^3}{384EI}$$

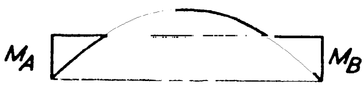
Any symmetrical load

LOADING



MOMENT

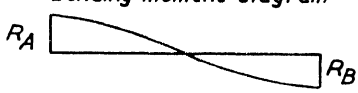
symmetrical diagram



$M_A = M_B = -A_s/L$

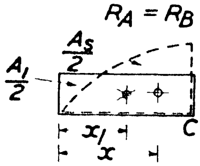
where A_s is the area of the 'free' bending moment diagram

SHEAR



$R_A = R_B = W/2$

DEFLECTION



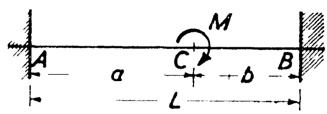
The figure shown is half the bending moment diagram [✱ and ⌽ are C G's]

$$d_{max} \text{ at } C = \frac{A_s x - A_1 x_1}{2EI}$$

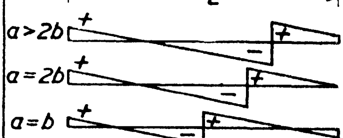
Where A_1 is the area of the fixing moment diagram

Applied moment M

LOADING



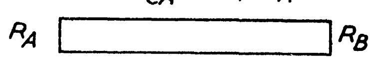
MOMENT



$M_{AC} = M.\frac{b}{L^2}(3a-L)$ $M_{BC} = -M\frac{a}{L^2}(3b-L)$

When $a/L = m$, $M_{CA} = -M(1-m)(1-3m+6m^2)$

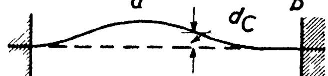
SHEAR



$R_A = R_B =$ slope of moment diagram

$$= \frac{M_{AC}+M_{CA}}{a} = \frac{M_{CB}+M_{BC}}{b}$$

DEFLECTION



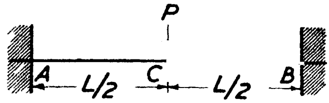
When $a/L = m$,

$$d_C = \frac{M.L^2 m^2(1-m)^2(1-2m)}{2EI}$$

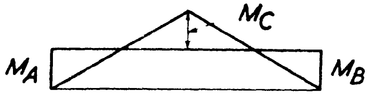
For anticlockwise moments reverse the deflections

BUILT – IN BEAMS

LOADING

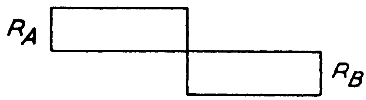


MOMENT



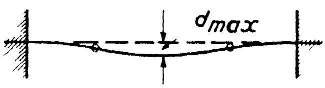
$$-M_A = -M_B = M_C = PL/8$$

SHEAR



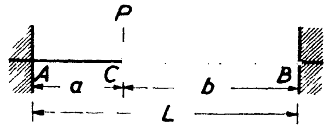
$$R_A = R_B = P/2$$

DEFLECTION

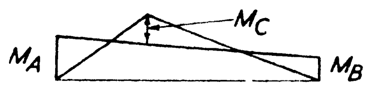


$$d_{max} = \frac{PL^3}{192\,EI}$$

LOADING



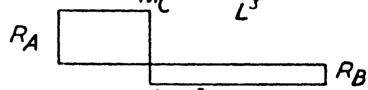
MOMENT



$$M_A = -\frac{Pab^2}{L^2} \qquad M_B = -\frac{Pba^2}{L^2}$$

$$M_C = \frac{2Pa^2b^2}{L^3}$$

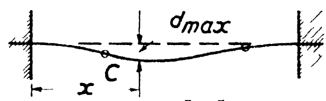
SHEAR



$$R_A = P\left(\frac{b}{L}\right)^2\left(1 + 2\,\frac{a}{L}\right)$$

$$R_B = P\left(\frac{a}{L}\right)^2\left(1 + 2\,\frac{b}{L}\right)$$

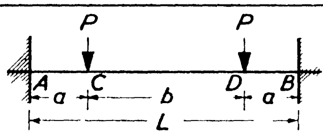
DEFLECTION



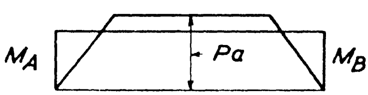
$$d_C = \frac{Pa^3b^3}{3EIL^3}$$

$$d_{max} = \frac{2Pa^2b^3}{3EI(3L-2a)^2} \text{ when } x = \frac{L^2}{3L-2a}$$

LOADING



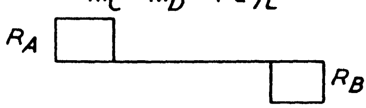
MOMENT



$$M_A = M_B = -\frac{Pa(L-a)}{L}$$

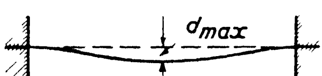
$$M_C = M_D = Pa^2/L$$

SHEAR



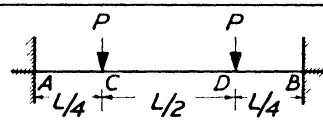
$$R_A = R_B = P$$

DEFLECTION

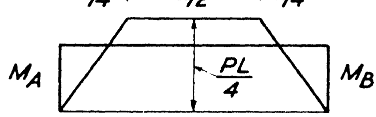


$$d_{max} = \frac{PL^3}{6EI}\left[\frac{3a^2}{4L^2} - \left(\frac{a}{L}\right)^3\right]$$

LOADING



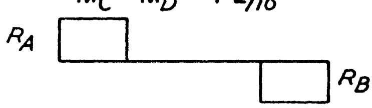
MOMENT



$$M_A = M_B = -3PL/16$$

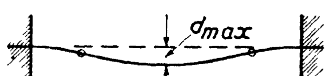
$$M_C = M_D = PL/16$$

SHEAR



$$R_A = R_B = P$$

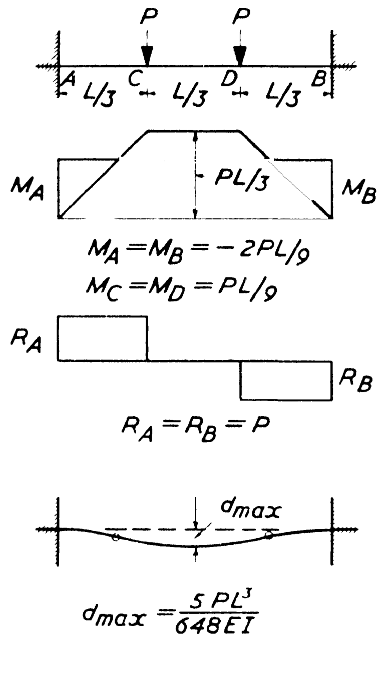
DEFLECTION



$$d_{max} = \frac{PL^3}{192EI}$$

BUILT — IN BEAMS

LOADING — MOMENT — SHEAR — DEFLECTION

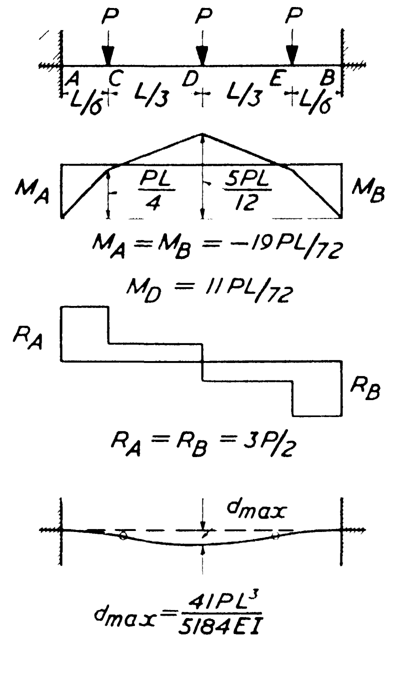


$M_A = M_B = -2PL/9$

$M_C = M_D = PL/9$

$R_A = R_B = P$

$d_{max} = \dfrac{5PL^3}{648EI}$

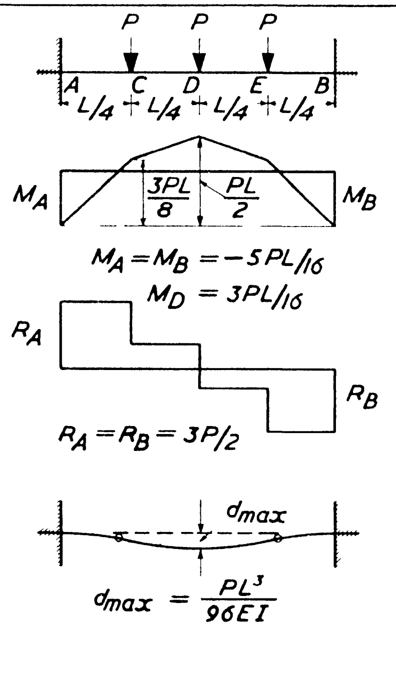


$M_A = M_B = -19PL/72$

$M_D = 11PL/72$

$R_A = R_B = 3P/2$

$d_{max} = \dfrac{41PL^3}{5184EI}$

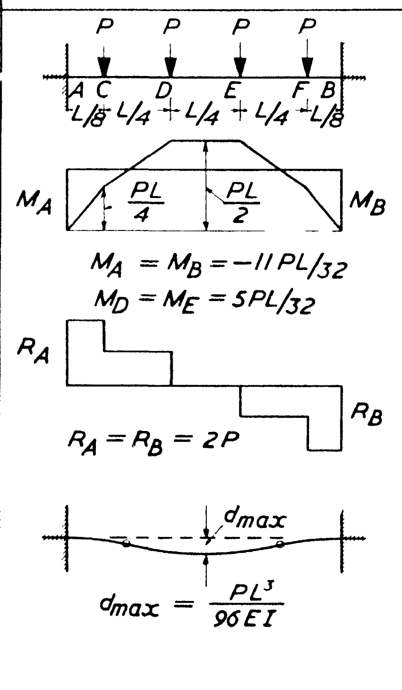


$M_A = M_B = -5PL/16$

$M_D = 3PL/16$

$R_A = R_B = 3P/2$

$d_{max} = \dfrac{PL^3}{96EI}$



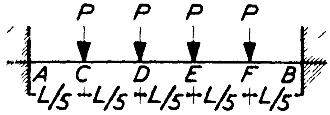
$M_A = M_B = -11PL/32$

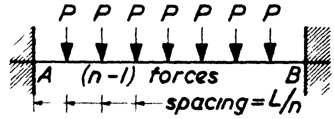
$M_D = M_E = 5PL/32$

$R_A = R_B = 2P$

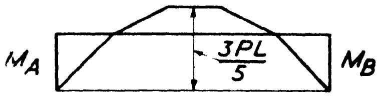
$d_{max} = \dfrac{PL^3}{96EI}$

LOADING



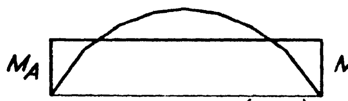


MOMENT



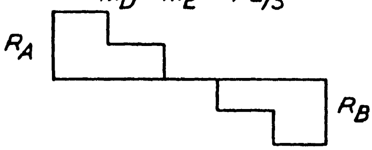
$M_A = M_B = -2PL/5$

$M_D = M_E = PL/5$



$M_A = M_B = -\frac{PL(n^2-1)}{12n}$

SHEAR

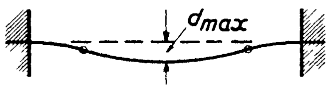


$R_A = R_B = 2P$

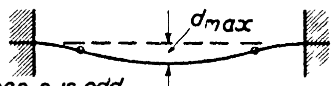


$R_A = R_B = (n-1)\,P/2$

DEFLECTION



$$d_{max} = \frac{13\,PL^3}{1000\,EI}$$

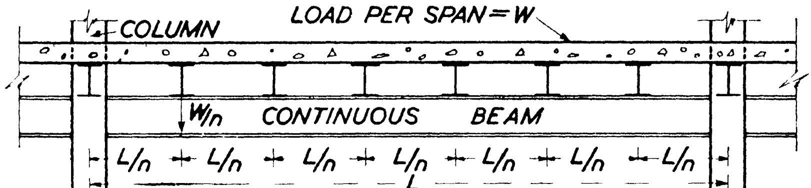


When n is odd,

$$d_{max} = \frac{PL^3}{192EI}\left[n-\frac{1}{n}\right]\left[1-\frac{1}{2}\left(1-\frac{1}{n^2}\right)\right]$$

When n is even,

$$d_{max} = \frac{PL^3}{192EI}\left\{\left[3-\frac{1}{2}\left(1+\frac{4}{n^2}\right)\right]n-2\left(n-\frac{1}{n}\right)\right\}$$



When $n > 10$, consider the load uniformly distributed

The load on the outside stringers is carried directly by the supports

The continuous beam is assumed to be horizontal at each support

The reaction at the supports for each span = $W/2$ but the maximum shear force in any span of the continuous beam = $\frac{W(n-1)}{2n} = A\,W$

The value of the fixing moment at each support = $-\,B\,WL$

The value of the maximum positive moment for each span = $C\,WL$

The value of the maximum deflection for each span $\simeq 0\ 0026\ \frac{WL^3}{EI}$

Value of n	A	B	C
2	0 2500	0 0625	0 0625
3	0 3333	0 0741	0 0370
4	0 3750	0 0781	0 0469
5	0 4000	0 0800	0 0400
6	0 4167	0 0811	0 0439
7	0 4286	0 0816	0 0408
8	0 4375	0 0820	0 0430
9	0 4444	0 0823	0 0413
10	0 4500	0 0825	0 0425

4. CONTINUOUS BEAMS

THE solution of this type of beam consists, in the first instance, of the evaluation of the fixing or negative moments at the supports.

The most general method is the use of Clapeyron's Theorem of Three Moments.

The theorem applies only to any two adjacent spans in a continuous beam and in its simplest form deals with a beam which has all the supports at the same level, and has a constant section throughout its length.

The proof of the theorem results in the following expression:

$$M_A \times L_1 + 2M_B(L_1 + L_2) + M_C \times L_2 = 6\left(\frac{A_1 \times x_1}{L_1} + \frac{A_2 \times x_2}{L_2}\right),$$

where M_A, M_B and M_C are the numerical values of the moments at the supports A, B and C respectively and the remaining terms are as illustrated in Fig. 15.

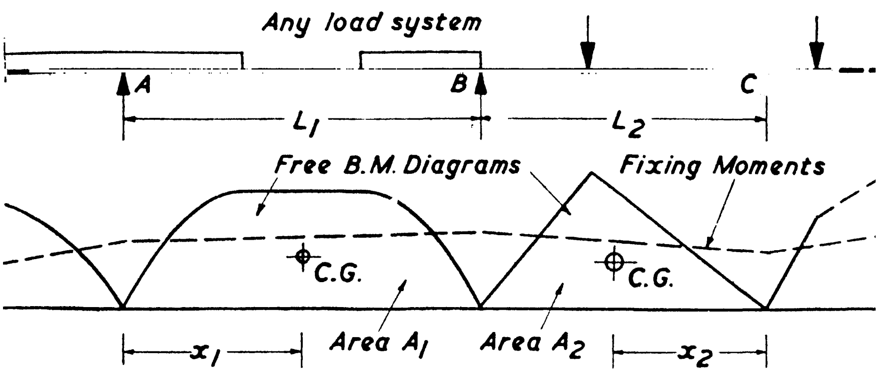


FIG. 15

In a continuous beam the conditions at the end supports are usually known, and these conditions provide starting points for the solution.

The types of end conditions are three in number, viz.:

1. Simply supported.
2. Partially fixed, e.g. a cantilever.
3. Completely fixed, i.e. the end of the beam is horizontal as in the case of a fixed beam.

One example of each of the above types will be worked out in full.

The S.F. at the end of any span is calculated after the support moments have been evaluated, in the same manner as for a fixed beam, each span being treated separately.

It is essential to note the difference between S.F. and reaction at any support, e.g. with reference to Fig. 15 the S.F. at support B due to span AB is a certain amount, while that at support B due to span BC is another amount, but the reaction at the support is the sum of these two amounts.

If the section of the beam is not constant over its whole length, but remains constant for each span, the expression for the moments is rewritten as follows:

$$M_A \times \frac{L_1}{I_1} + 2M_B\left(\frac{L_1}{I_1} + \frac{L_2}{I_2}\right) + M_C \times \frac{L_2}{I_2} = 6\left(\frac{A_1 \times x_1}{L_1 \times I_1} + \frac{A_2 \times x_2}{L_2 \times I_2}\right),$$

in which I_1 = moment of inertia for span L_1 and I_2 = moment of inertia for span L_2

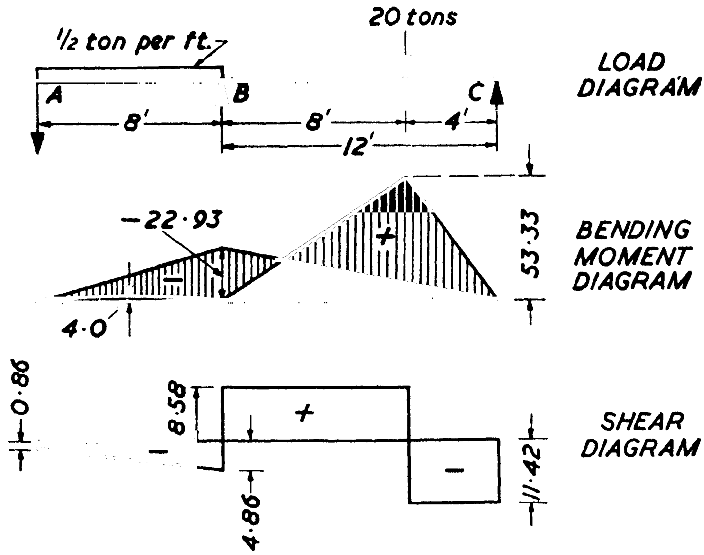


FIG. 16

Example 1. A two-span continuous beam ABC, of constant cross-section, is simply supported at A and C and loaded as shown in Fig. 16.

Applying Clapeyron's theorem,

$$L_1 = 8 \text{ ft.}$$

$$L_2 = 12 \text{ ft.}$$

$$A_1 = \frac{4 \times 8}{8} \times \frac{8 \times 2}{3} = \frac{64}{3} \text{ ton ft. units.}$$

$$A_2 = \frac{20 \times 8 \times 4}{12} \times \frac{12}{2} = 320 \text{ ton ft. units.}$$

$$x_1 = 4 \text{ ft.}$$

$$x_2 = \frac{16}{3} \text{ ft.}$$

Therefore $M_A \times 8 + 2M_B(8+12) + M_C \times 12 = 6\left(\frac{64\times 4}{3\times 8} + \frac{320\times 16}{3\times 12}\right).$

Since A and C are simple supports

$$M_A = M_C = 0.$$

Therefore $$M_B = \frac{6}{40}\left(\frac{32}{3} + \frac{1280}{9}\right) = 22{\cdot}93 \text{ tons ft.}$$

$$\text{S.F.}_A = 2 + \frac{0 - 22{\cdot}93}{8} = 2 - 2{\cdot}86 = -0{\cdot}86 \text{ tons}$$

$$\text{S.F.}_B \text{ for span } AB = 2 + 2{\cdot}86 = 4{\cdot}86 \text{ tons}$$

$$\text{S.F.}_C = \frac{20\times 8}{12} + \frac{0 - 22{\cdot}93}{12} = 13{\cdot}33 - 1{\cdot}91 = 11{\cdot}42 \text{ tons}$$

$$\text{S.F.}_B \text{ for span } BC = \frac{20\times 4}{12} + 1{\cdot}91 = 6{\cdot}67 + 1{\cdot}91 = 8{\cdot}58 \text{ tons.}$$

Note that the negative reaction at A means that the end A will tend to lift off its support and will have to be held down.

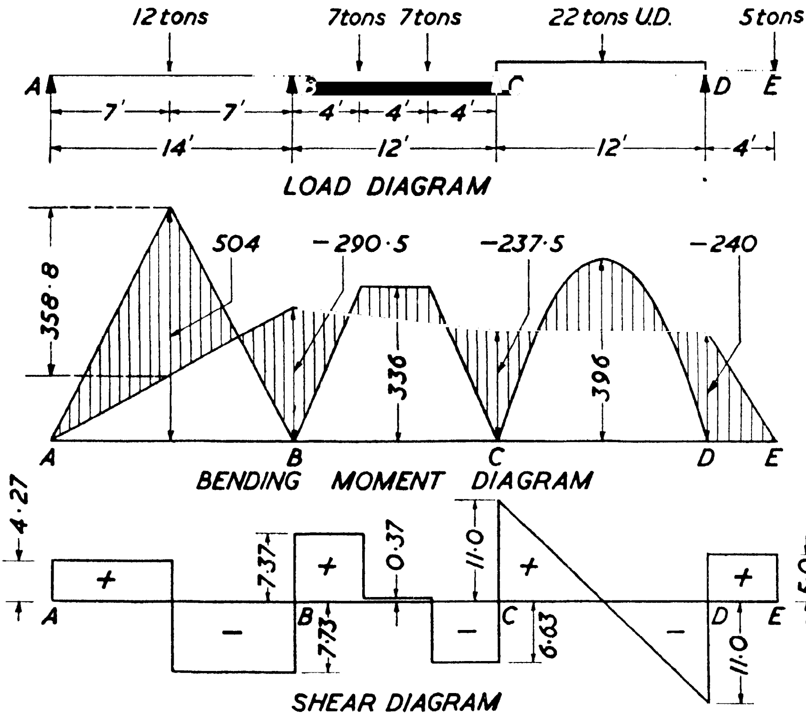


FIG. 17

Example 2. A three-span continuous beam $ABCDE$ is simply supported at A, cantilevered at the end D, and loaded as shown in Fig. 17. The section is not constant but varies as follows:

$$I_{AB} = 205 \text{ in.}^4$$
$$I_{BC} = 146 \text{ in.}^4$$
$$I_{CE} = 122 \text{ in.}^4.$$

Applying Clapeyron's theorem and noting that since the moments of inertia are given in inch units the B.M.s and spans must also be given in inch units.

$$M_A = 0$$

$$M_D = 240 \text{ tons in.}$$

$$A_1 = \frac{12 \times 168}{4} \times \frac{168}{2} = 42{,}336 \text{ tons in. units}$$

$$A_2 = 7 \times 48 \times \frac{144 \times 2}{3} = 32{,}256 \text{ tons in. units}$$

$$A_3 = \frac{22 \times 144}{8} \times \frac{144 \times 2}{3} = 38{,}016 \text{ tons in. units.}$$

First considering spans AB and BC.

$$2M_B\left(\frac{168}{205} + \frac{144}{146}\right) + M_C \times \frac{144}{146} = 6\left(\frac{42{,}336 \times 84}{168 \times 205} + \frac{32{,}256 \times 72}{144 \times 146}\right)$$

Therefore $\quad 3{\cdot}6112M_B + 0{\cdot}9862M_C = 1282{\cdot}3 \quad \ldots\ldots\ldots \quad$ (i)

Next considering spans BC and CD.

$$M_B \times \frac{144}{146} + 2M_C\left(\frac{144}{146} + \frac{144}{122}\right) + 240 \times \frac{144}{122} = 6\left(\frac{32{,}256 \times 72}{144 \times 146} + \frac{38{,}016 \times 72}{144 \times 122}\right)$$

Therefore $\quad 0{\cdot}9862M_B + 4{\cdot}3324M_C = 1314{\cdot}3 \quad \ldots\ldots \quad$ (ii)

From equations (i) and (ii):

$$M_B = 290{\cdot}5 \text{ tons in.}$$

$$M_C = 237{\cdot}5 \text{ tons in.}$$

$$\text{S.F.}_A \qquad = 6{\cdot}0 + \frac{0 - 290{\cdot}5}{168} = 6{\cdot}0 - 1{\cdot}73 = 4{\cdot}27 \text{ tons}$$

$$\text{S.F.}_B \text{ for span } AB = 6{\cdot}0 + 1{\cdot}73 = 7{\cdot}73 \text{ tons}$$

$$\text{S.F.}_B \text{ for span } BC = 7{\cdot}0 + \frac{290{\cdot}5 - 237{\cdot}5}{144} = 7{\cdot}0 + 0{\cdot}37 = 7{\cdot}37 \text{ tons}$$

$$\text{S.F.}_C \text{ for span } BC = 7{\cdot}0 - 0{\cdot}37 = 6{\cdot}63 \text{ tons.}$$

Since M_C and M_D are almost equal, the S.F.s for span CD are the same as the simply supported reactions, i.e. 11·0 tons at each end.

Example 3. A two-span continuous beam ABC, of constant cross-section, is built-in horizontally at A and C, supported at the same level at B, and loaded as shown in Fig. 18.

The effect of a fixed end is the same as that which would be produced if there had been, on the side of the support opposite to the span, another span of equal length to, and carrying the same load as, the span, except that this imaginary span is completely reversed from the actual span.

The effect can best be imagined if mirrors were to be placed at A and C when the span AB would be mirrored into the span AB_1 and the span BC into the span CC_1.

Interpreted, this means that the negative moment at each of the points B_1 and C_1 is equal to the negative moment at B.

Applying Clapeyron's theorem,

A_1 (area of free B.M. diagram on span B_1A) $= \dfrac{20 \times 10}{8} \times \dfrac{10 \times 2}{3}$

$= \dfrac{500}{3}$ tons ft. units.

A_2 (,, ,, ,, AB) $= \dfrac{500}{3}$,, ,,

A_3 (,, ,, ,, BC) $= \dfrac{12 \times 12}{8} \times \dfrac{12 \times 2}{3}$

$= 144$ tons ft. units.

A_4 (,, ,, ,, CC_1) $= 144$,, ,,

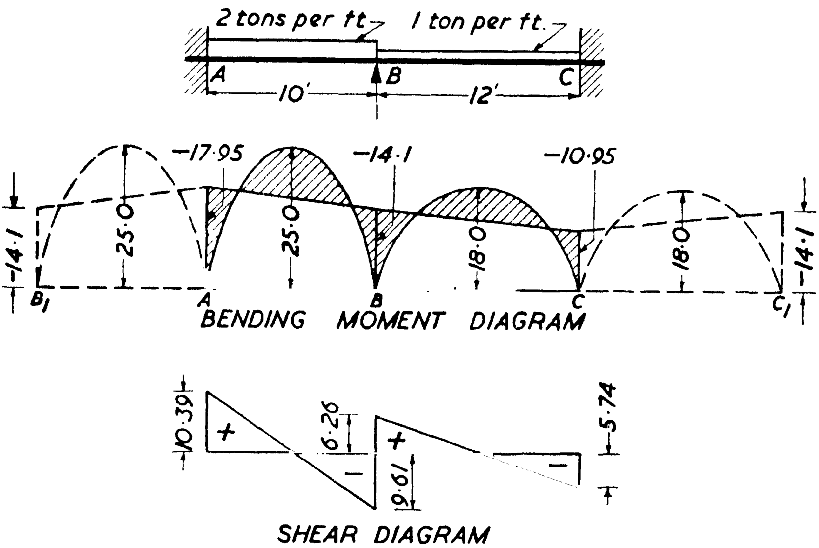


FIG. 18

First considering spans B_1A and AB.

$$M_B \times 10 + 2M_A(10+10) + M_B \times 10 = 6\left(\frac{500 \times 5}{3 \times 10} + \frac{500 \times 5}{3 \times 10}\right)$$

$$M_B + 2M_A = 50 \quad . \; . \; . \; . \; . \; . \; . \; . \quad \text{(i)}$$

Secondly considering spans AB and BC.

$$M_A \times 10 + 2M_B(10+12) + M_C \times 12 = 6\left(\frac{500 \times 5}{3 \times 10} + \frac{144 \times 6}{12}\right)$$

$$M_A + 4{\cdot}4M_B + 1{\cdot}2M_C = 93{\cdot}2 \quad . \; . \; . \; . \; . \; . \; . \quad \text{(ii)}$$

Thirdly considering spans BC and CC_1.

$$M_B \times 12 + 2M_C(12+12) + M_B \times 12 = 6\left(\frac{144 \times 6}{12} + \frac{144 \times 6}{12}\right)$$

$$M_B + 2M_C = 36 \quad . \; . \; . \; . \; . \; . \; . \; . \quad \text{(iii)}$$

From equations (i), (ii) and (iii):

$$M_A = 17{\cdot}95 \text{ tons ft.}$$
$$M_B = 14{\cdot}1 \text{ tons ft.}$$
$$M_C = 10{\cdot}95 \text{ tons ft.}$$

$$\text{S.F.}_A = 10 + \frac{17{\cdot}95 - 14{\cdot}1}{10} = 10 + 0{\cdot}385 = \text{say, } 10{\cdot}39 \text{ tons.}$$

$$\text{S.F.}_B \text{ for span } AB = 10 - 0{\cdot}385 = 9{\cdot}61 \text{ tons.}$$

$$\text{S.F.}_C = 6 + \frac{10{\cdot}95 - 14{\cdot}1}{12} = 6 - 0{\cdot}26 = 5{\cdot}74 \text{ tons.}$$

$$\text{S.F.}_B \text{ for span } BC = 6 + 0{\cdot}26 = 6{\cdot}26 \text{ tons.}$$

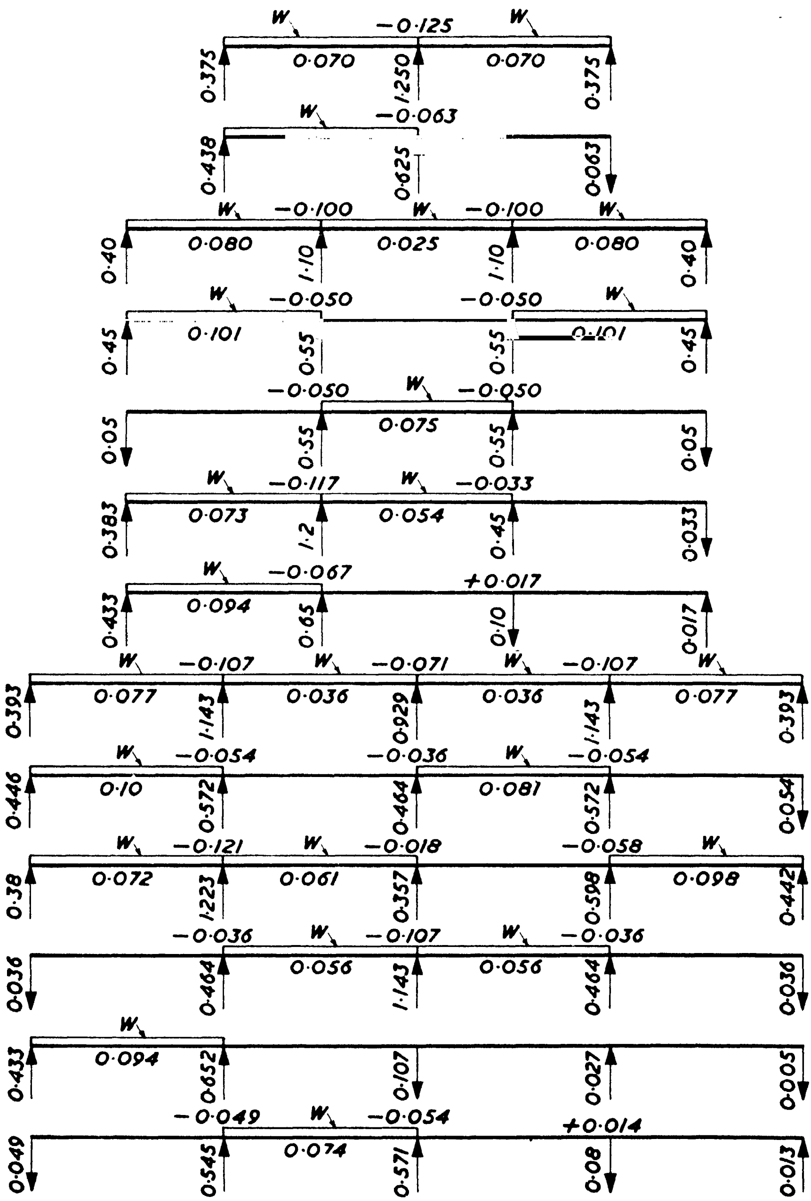
It should be noted that Examples 2 and 3 are solved by Moment Distribution and Slope Deflection, being respectively Examples 2 and 1 by these methods of analysis. See pp. 172, 175, 257 and 258.

Formulæ for continuous beams are given on pp. 42 to 44.

EQUAL SPAN CONTINUOUS BEAMS
UNIFORMLY DISTRIBUTED LOADS

Moment = coefficient x W x L
Reaction = coefficient x W

where W is the U.D.L. on one span only and L is one span

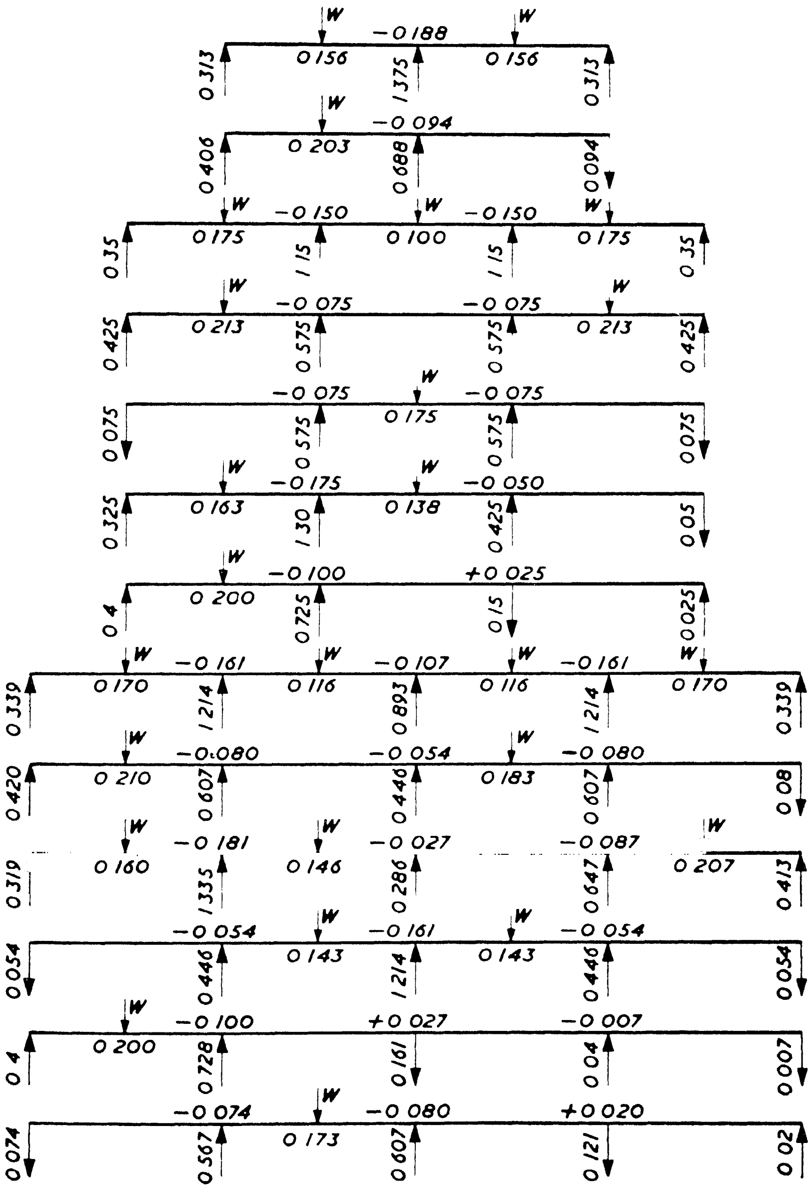


EQUAL SPAN CONTINUOUS BEAMS
CENTRAL POINT LOADS

Moment = coefficient x W x L

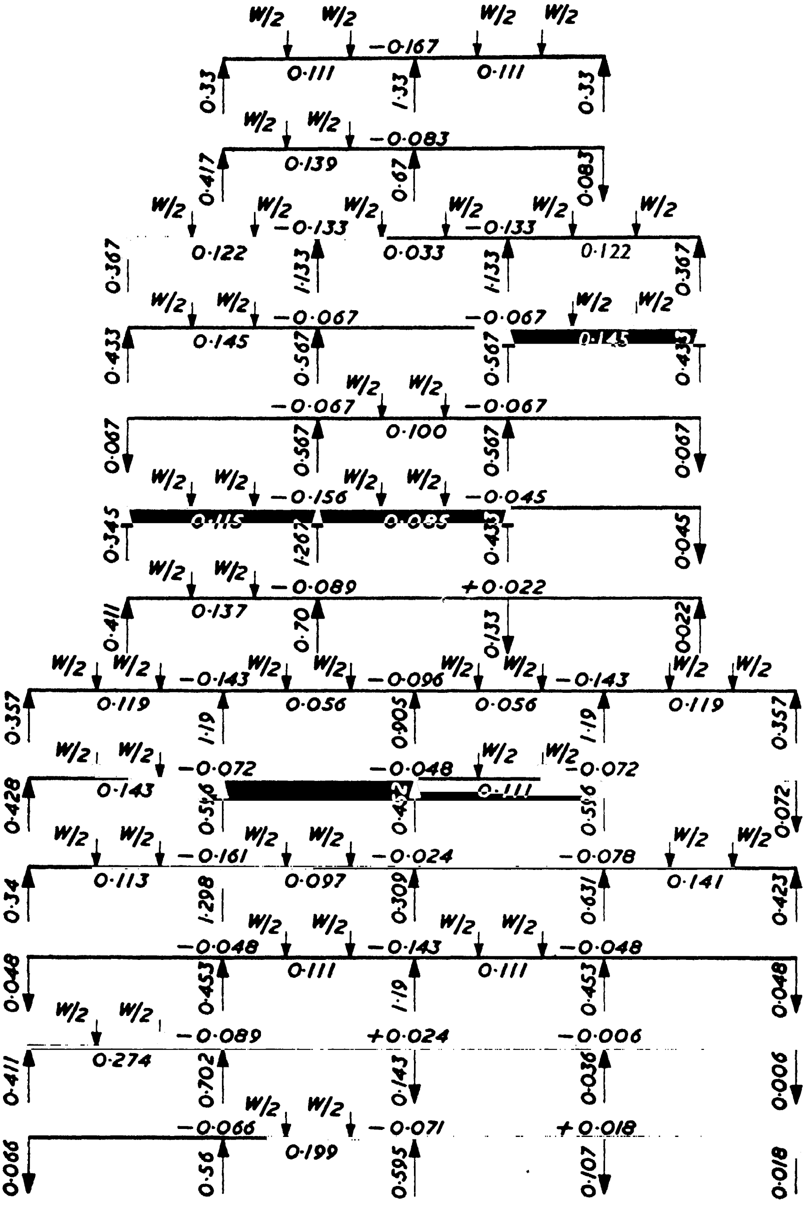
Reaction = coefficient x W

where W is the Load on one span only and L is one span



EQUAL SPAN CONTINUOUS BEAMS
POINT LOADS AT THIRD POINTS OF SPANS

Moment = coefficient x W x L
Reaction = coefficient x W
where W is the __total__ load on one span only & L is one span



5. THE DEFLECTION OF BEAMS

THE calculated deflection of a beam is limited by B.S. 449 to 1/325 of the span. For a simply supported beam which is stressed to the maximum allowable stress of 10 tons per sq. in., this deflection will not be exceeded if the span/depth ratio is less than 19·2.

The most common methods of evaluating the deflection in any given case are three in number, although it is true to say that all these methods are variations of the same root principle.

They are:

(a) A mathematical method commonly called the method of slope-deflection.

(b) The area-moment method, first expounded by Mohr, and commonly called Mohr's theorem.

(c) A graphical method, which is really a graphical interpretation of the area-moment method.

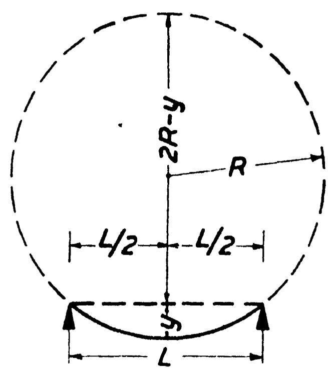


FIG. 19

Before starting to deal with these methods in detail, it is interesting to consider the deflection in the case of circular bending, which it will be remembered is the type of bending assumed in the Ordinary Beam Theory.

The portion *AB* of the beam shown in Fig. 1 bends in a circular arc and by the geometry of the circle shown in Fig. 19.

$$y(2R-y)=\frac{L}{2}\times\frac{L}{2}$$

$$2Ry-y^2=\frac{L^2}{4}$$

y^2 is extremely small compared with the other quantities, and may be neglected.

Therefore $$2Ry = \frac{L^2}{4}$$

and $$y = \frac{L^2}{8R}.$$

But $\frac{1}{R} = \frac{M}{EI}.$

Therefore $$y = \frac{ML^2}{8EI}$$

so that the maximum deflection in circular bending $= ML^2/8EI$.

If the section of the beam is constant, M must be constant for circular bending to occur. If, as is more usual, M varies throughout the span, then I must vary so as to keep the ratio M/I constant. This is not a practical proposition, although it is approached in the case of compound beams and plate girders where the plates are curtailed. A later example will illustrate this point.

The three methods will now be dealt with more fully.

Mathematical Method

It is necessary, first of all, to arrive at a convention of signs.

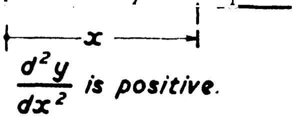


FIG. 20

In Fig. 20, x is positive to the right and y is positive upwards. The slope dy/dx is positive, so that slopes upwards to the right are positive. The curvature shown is such that dy/dx increases as x increases, therefore, d^2y/dx^2 will be positive in this case. The type of bending illustrated in Fig. 19 will have been caused by what are considered positive B.M.s. Therefore positive B.M.s are associated with $+d^2y/dx^2$.

Now the rate of change of the slope, i.e. d^2y/dx^2, is the curvature and in the Beam Theory curvature, i.e. $1/R$, is equal to M/EI. Therefore,

$$\frac{d^2y}{dx^2} = \frac{M}{EI} \quad \text{or} \quad \frac{EId^2y}{dx^2} = M.$$

This method is most easily applied to cases in which the B.M. at any point can be represented by a simple expression.

Example 1. Consider the case of the simply supported beam with a U.D.L. over the whole span, as in Fig. 21.

$$\text{B.M. at } x = \frac{wLx}{2} - \frac{wx^2}{2}$$

$$\frac{EId^2y}{dx^2} = \frac{wLx}{2} - \frac{wx^2}{2}.$$

Integrating,

$$\frac{EIdy}{dx}=\frac{wLx^2}{4}-\frac{wx^3}{6}+C_1.$$

$dy/dx=0$ at midspan, when $x=L/2$. Therefore,

$$0=\frac{wL}{4}\times\left(\frac{L}{2}\right)^2-\frac{w}{6}\left(\frac{L}{2}\right)^3+C_1,$$

when $$C_1=-\frac{wL^3}{24}.$$

Therefore $$\frac{EIdy}{dx}=\frac{wLx^2}{4}-\frac{wx^3}{6}-\frac{wL^3}{24}.$$

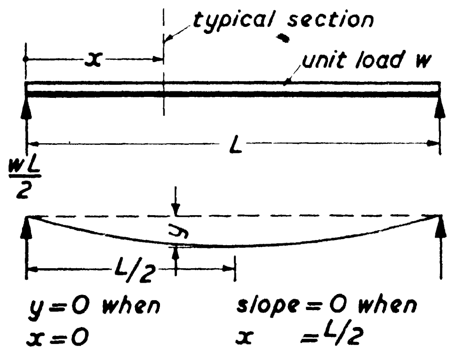


FIG. 21

From this expression the slope, dy/dx, at any point, can be found.

The slopes at the two ends of the beam are found by substituting $x=0$ and $x=L$ in the expression.

In the first case, if $x=0$, then

$$\frac{EIdy}{dx}=-\frac{wL^3}{24},$$

whence $$\frac{dy}{dx}=-\frac{wL^3}{24EI},$$

which by the convention previously referred to, means that the slope is negative or upwards to the left.

In the second case, if $x=L$, then

$$\frac{EIdy}{dx}=\frac{wL^3}{4}-\frac{wL^3}{6}-\frac{wL^3}{24}=+\frac{wL^3}{24},$$

whence $$\frac{dy}{dx}=+\frac{wL^3}{24EI},$$

i.e. the same value as in the first case, because the beam is symmetrically loaded, but of the opposite sign, indicating a slope upwards to the right.

Integrating the main expression again,

$$EIy=\frac{wLx^3}{12}-\frac{wx^4}{24}-\frac{wL^3x}{24}+C_2$$

When $x=0$, $y=0$. Therefore $C_2=0$. Therefore,

$$EIy=\frac{wLx^3}{12}-\frac{wx^4}{24}-\frac{wL^3x}{24},$$

for maximum deflection $x=L/2$. Whence,

$$y=-\frac{5wL^4}{384EI}=-\frac{5WL^3}{384EI},$$

the well-known result, although the minus sign is usually omitted. The diagrams shown on pages 15 to 24, 29 to 35 and 74 to 81 show downward deflections as positive in accordance with normal practice.

The Area-moment Method

This method is easier of application than the first method to cases of complicated loading or indeed to any case of loading which causes a B.M. diagram that can be easily split up into simple geometrical shapes.

The bases of this method are expressed in the following two propositions.

1. When a member is subjected to flexure, the angular change between any two points is equal to the area of the moment diagram between these two points divided by EI, i.e. $\theta=\Sigma A/EI$.
2. When a member is subjected to flexure the deviation of any point measured normal to the original position of the member from a tangent drawn at any other point on the member is equal to the statical moment about the first point of the area of the moment diagram between the two points divided by EI, i.e. $d=\Sigma Ax/EI$.

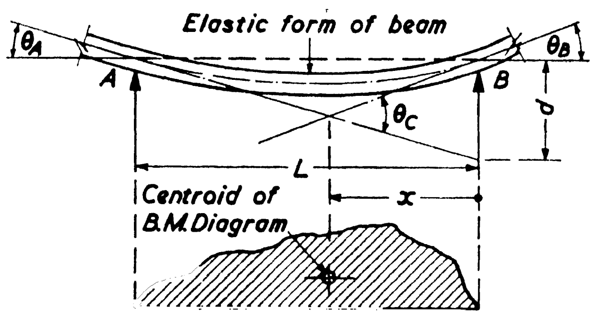


FIG. 22

Fig. 22 will make clear the propositions, θ_C being the angular change between A and B measured in radians, which is equal to the area of the B.M. diagram divided by EI, and d being the deflection, measured normal to the original position of the member, of the point B from a tangent at A, which is equal to the moment of the B.M. diagram about B divided by EI.

Now the angle θ_C is equal to the sum of the angular changes θ_A and θ_B at the ends A and B respectively of the beam. It will be seen that $\theta_A:\theta_B=x:(L-x)$, from which it can be deduced that the angular change at either end of the beam can be obtained by calculating the support reaction at that end when the beam is loaded with its B.M. diagram divided by EI. As an

example, consider Fig. 23, which shows a simple beam AB of constant cross-section, at the end B of which is applied an anticlockwise moment M. Then the B.M. diagram is triangular in shape, as shown, the distance x from B to its centroid being $L/3$ and its area being equal to $ML/2$ units.

Therefore, the angular changes at the ends A and B are:

$$\theta_A = \frac{1}{3} \times \frac{ML}{2} \times \frac{1}{EI} = \frac{ML}{6EI}$$

$$\theta_B = \frac{2}{3} \times \frac{ML}{2} \times \frac{1}{EI} = \frac{ML}{3EI}.$$

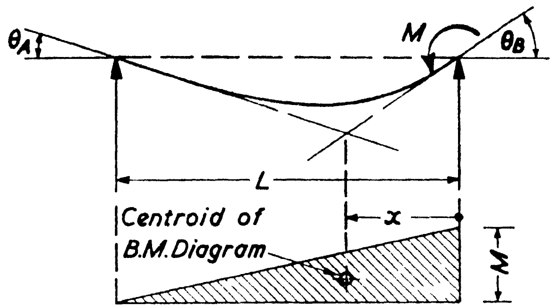


FIG. 23

If the expression for θ_B is rearranged,

$$M = \frac{3EI\theta_B}{L}.$$

This is Equation (iii) employed in the derivation of the Principles of Moment Distribution (see p. 171).

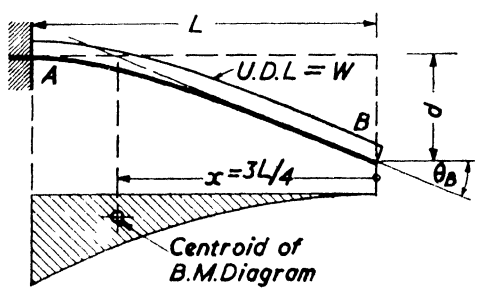


FIG. 24

Fig. 24 demonstrates the application of Proposition 1 to a cantilever loaded over its whole length with a U.D.L. of W.

Then
$$\theta_B = \frac{\Sigma A}{EI} = \frac{WL}{2} \times \frac{L}{3} \times \frac{1}{EI}$$
$$= \frac{WL^2}{6EI}.$$

From Proposition 2, the deflection at B with regard to A is:

$$d=\frac{\Sigma Ax}{EI}=\frac{WL^2}{6EI}\times\frac{3L}{4}$$

$$=\frac{WL^3}{8EI}$$

the well-known formula.

In cases of members of non-prismatic section the area plotted must be the M/EI diagram instead of the moment diagram, but it is probable that in such cases the graphical method of calculation is more easily applied.

From the above propositions it can be readily deduced that the deflection with reference to its supports of any point in a beam is equal to the moment which would occur at the point if the beam were simply supported and loaded with its moment diagram divided by EI.

Since in the expression $EI\times d=$ secondary B.M., the arm of the resultant moment $EI\times d$ is measured downwards, it is consequently negative. Accordingly, the expression should be written:

$$EI\times -d=\text{Secondary B.M.}$$

or

$$d=-\frac{\text{Secondary B.M.}}{EI}.$$

Example 2. A 9-in. × 4-in. × 21-lb. B.S.B. is loaded with a single point load as shown in Fig. 25. $E=13{,}000$ tons per sq. in. $I=81\cdot1$ in.^{4}. It is

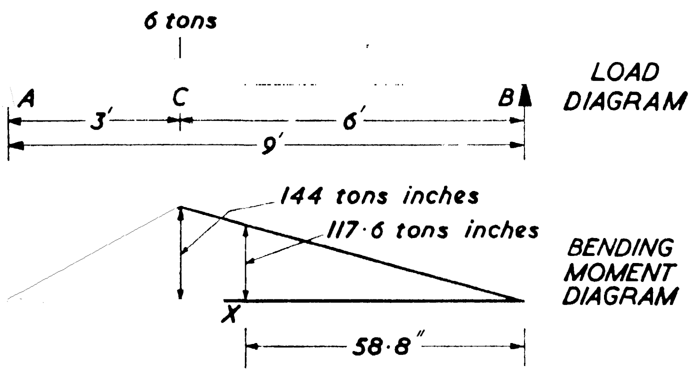


FIG. 25

required to find the deflection under the load, the maximum deflection and the deflection at the centre of the span.

$$\text{B.M. at } C=\frac{Wab}{L}=\frac{6\times3\times6}{9}=12 \text{ tons ft.}=144 \text{ tons in.}$$

Considering the B.M. diagram as a load, taking moments about A and rearranging,

$$R_B=\frac{1}{108}\left(\frac{144\times72}{2}\times60+\frac{144\times36}{2}\times24\right)=3456 \text{ tons in. units.}$$

Deflection at centre of span:

$$\text{B.M. at centre} = \frac{4{\cdot}5}{6} \times 144 = 108 \text{ tons in.}$$

$$\text{Secondary B.M.} = (3456 \times 54) - \left(\frac{108 \times 54}{2} \times \frac{54}{3}\right)$$

$$= 186{,}600 - 52{,}500 = 134{,}100 \text{ in. units.}$$

Therefore $$\text{deflection} = -\frac{134{,}100}{13{,}000 \times 81{\cdot}1} = -0{\cdot}127 \text{ in.}$$

Deflection under the load:

$$\text{Secondary B.M.} = (3456 \times 72) - \left(\frac{144 \times 72}{2} \times \frac{72}{3}\right)$$

$$= 248{,}800 - 124{,}400 = 124{,}400 \text{ in. units.}$$

Therefore $$\text{deflection} = -\frac{124{,}400}{13{,}000 \times 81{\cdot}1} = -0{\cdot}118 \text{ in.}$$

Maximum deflection:

It is first necessary to find the position of maximum deflection.

Since

$$\text{deflection} = -\frac{\text{Secondary } M}{E \times I}$$

it follows that the maximum deflection occurs where the secondary M is a maximum. As in all other cases of loading, the maximum B.M. occurs at the point of zero shear.

The point of zero shear will always occur between the load and the centre of the span.

Let x = the distance of the point X of zero secondary shear from B. Then:

$$\text{B.M. at } X = \frac{x}{72} \times 144 = 2x.$$

Therefore $$2x \times \frac{x}{2} = 3456 \quad \text{whence} \quad x = 58{\cdot}8 \text{ in.}$$

$$\text{Secondary B.M.} = 3456 \times 58{\cdot}8 - 3456 \times \frac{58{\cdot}8}{3}$$

$$= \frac{3456 \times 58{\cdot}8 \times 2}{3} = 135{,}500 \text{ tons in.}$$

$$\text{Maximum deflection} = -\frac{135{,}500}{13{,}000 \times 81{\cdot}1} = -0{\cdot}129 \text{ in.}$$

It will be seen that the maximum variation between all three values is $8\frac{1}{2}$ per cent of the maximum deflection.

It can be proved that the maximum deflection, under any system of loading whatever, never occurs further than $0{\cdot}0774L$ from the centre of the span, provided that the beam rests on simple supports.

If the slope at any point is required, use is made of the first proposition previously referred to. It is necessary to have a reference point from which all slopes can be measured, and this obviously is the point of zero

slope, i.e. where the beam is horizontal. This is the point of maximum deflection.

The slopes at the ends of the beam will now be determined.

Slope at end B:

Slope at $X=0$

$$\text{B.M. at } X=\frac{58\cdot8}{72}\times144=117\cdot6 \text{ tons in.}$$

Area of B.M. diagram between X and B

$$=\frac{117\cdot6\times58\cdot8}{2}=3456 \text{ tons in. units.}$$

This is confirmed by the fact that this area must be equal to the reaction at B when the B.M. diagram is considered as a load. Therefore:

$$\text{Slope at } B=+\frac{3456}{EI}=+\frac{3456}{13{,}000\times81\cdot1}=+0\cdot00328 \text{ radian.}$$

The slope is positive because the area is measured to the right of X and is therefore positive.

Slope at A:

The total area of the B.M. diagram

$$=\frac{144\times108}{2}$$

$$=7776 \text{ tons in. units.}$$

Consequently the area of the B.M. diagram between X and A

$$=7776-3456=4320 \text{ tons in. units.}$$

$$\text{Therefore slope at } A=-\frac{4320}{EI}=-\frac{4320}{13{,}000\times81\cdot1}=-0\cdot0041 \text{ radian.}$$

The slope is negative because the area is measured to the left of X.

Slope at C:

The area of the B.M. diagram between X and C

$$=\frac{144+117\cdot6}{2}\times(72-58\cdot8)$$

$$=1727 \text{ tons in. units.}$$

C is to the left of X. Hence,

$$\text{Slope at } C=-\frac{1727}{EI}=-\frac{1727}{13{,}000\times81\cdot1}=-0\cdot00164 \text{ radian.}$$

Graphical Method

Example 3. A compound girder consisting of a 24-in. $\times$ 7½-in. $\times$ 95-lb. B.S.B. with plates making 12 in. $\times$ 1½ in. on each flange is loaded as shown

in Fig. 26. The plates are made up in ½-in. thicknesses and are curtailed as shown. The nett moments of inertia, taken from a standard section book, are as follows:

R.S.J. + 12-in. × ½-in. plates top and bottom = 3950 in.^{4}
R.S.J. + 12-in. × 1-in. ,, ,, ,, = 5749 in.^{4}
R.S.J. + 12-in. × 1½-in. ,, ,, ,, = 7691 in.^{4}.

It is required to find the maximum deflection.

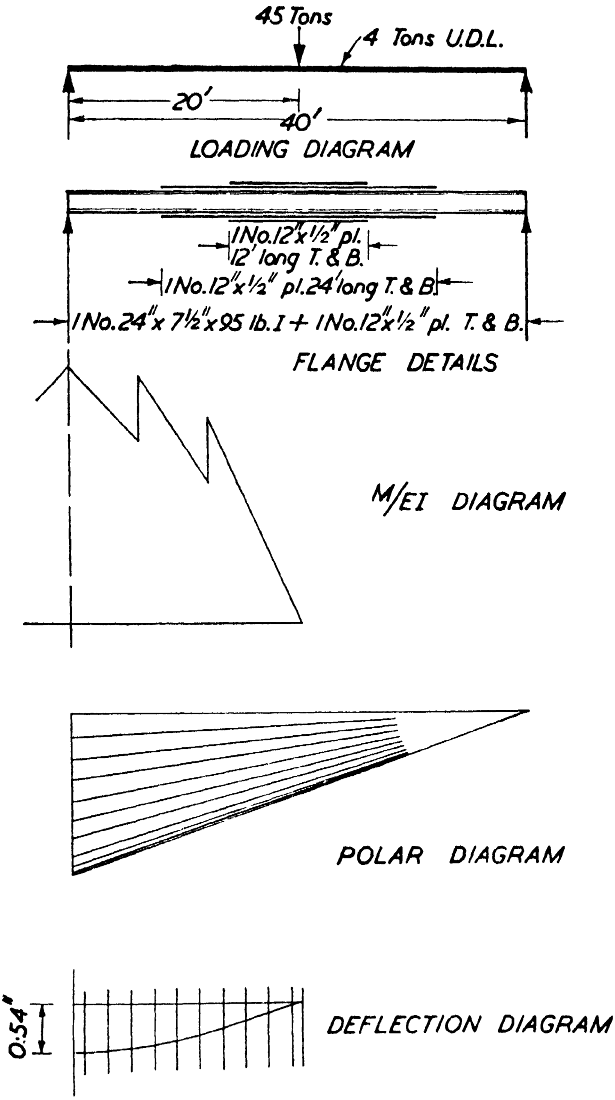


Fig. 26

Since the girder is symmetrically loaded, the maximum deflection will occur at the centre and consequently only one-half of the construction is shown in the diagram.

The *BM*/*EI* diagram is shown and below it are the polar diagram and funicular polygon. The latter is the deflection diagram to the correct scale.

The scale of the deflection diagram is made up as follows:

Space scale × *BM*/*EI* diagram scale × polar distance.

The diagram was originally drawn to the following scales:

Space scale 1 in. = 8 ft. = 96 in.

$\frac{BM}{EI}$ scale 1 in. $= \frac{1}{200}$ tons in. units.

Polar distance = 5 in.

The deflection scale is therefore:

$$1 \text{ in.} = 96 \times \frac{1}{200} \times 5 = 2{\cdot}4 \text{ in.}$$

The maximum ordinate measures 0·54 in. and so the maximum downward deflection

$$= -0{\cdot}54 \times 2{\cdot}4 = -1{\cdot}296 \text{ in.}$$

By comparison assuming circular bending and using maximum B.M. and maximum I:

$$-\frac{ML^2}{8EI} = -\frac{420 \times 12 \times 480 \times 480}{8 \times 13{,}000 \times 7691} = -1{\cdot}45 \text{ in.}$$

Again, if the deflection is calculated on the ordinarily accepted formulæ for central point loads and U.D.L.s, using maximum B.M. and maximum I:

$$\text{Maximum deflection} = -\left(\frac{5 \times 4 \times 40^3 \times 1728}{384 \times 13000 \times 7691} + \frac{1 \times 45 \times 40^3 \times 1728}{48 \times 13000 \times 7691}\right)$$

$$= -(0{\cdot}056 + 1{\cdot}04) = -1{\cdot}096 \text{ in.}$$

It is to be expected that the actual deflection lies between these two calculated figures.

It will be noted that allowance was made for rivet holes in computing the moments of inertia of the various sections of the compound beam. In practice, it is found that the actual deflections obtained are more nearly related to the gross sections, i.e. without allowance being made for rivet holes. However, the deflection calculated above is the maximum possible value.

A few further examples will now be worked out to illustrate the application of the principles of the theory of deflection.

Simple Beams

Example 4. To find the deflection at C of the cantilevered beam shown in Fig. 27. $E = 13{,}000$ tons per sq. in. I is constant and equals 81 in.^{4}.

The first step is to find the slope at B of the span AB assuming that there is no load on the cantilevered portion.

$$\text{Slope} = \frac{\text{Area of half B.M. diagram}}{EI} = \frac{\frac{1}{2} \times 8 \times 12 \times 16 \times 12 \times \frac{2}{3}}{13{,}000 \times 81}$$

$$= 0{\cdot}00584 \text{ radian.}$$

This slope is positive as will be seen from Fig. 20.

If there were no load on the cantilevered portion BC, then the slope at B would be continued along BC at a constant value. Since, in the case of the relatively small angles of slope, the tangent of the angle is, for all practical purposes, equal to the angle, then the slope would cause an upward deflection at C equal to

$$0{\cdot}00584 \times 72 = 0{\cdot}42 \text{ in.}$$

The load on the cantilever causes the beam to rotate at B so that the deflection at C, with respect to B, is composed of two parts, viz. that due to the rotation and that due to the load.

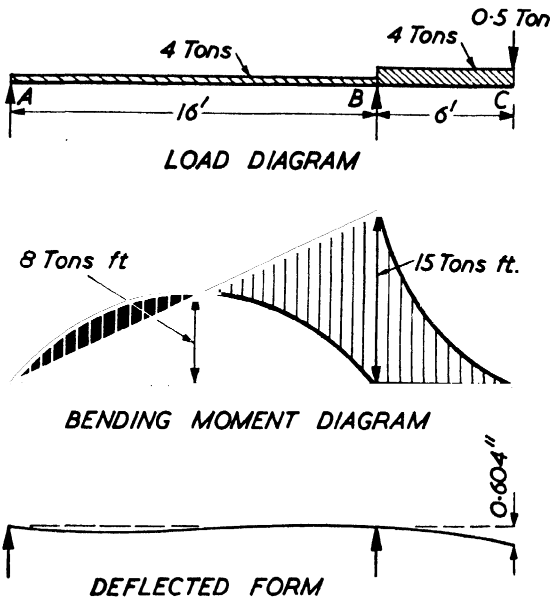


FIG. 27

The rotation at B due to the load on the cantilever has been shown on p. 49 to be $ML/3EI$, where M is the B.M. at B due to the load on the cantilever, and L is the length of the span AB. Therefore:

$$\text{Slope at } B = \frac{15 \times 12 \times 16 \times 12}{3EI} = \frac{15 \times 12 \times 16 \times 12}{3 \times 13{,}000 \times 81} = 0{\cdot}01096 \text{ radian.}$$

This slope is negative.

The downward deflection at C due to this slope

$$= 0{\cdot}01096 \times 72$$
$$= 0{\cdot}788 \text{ in.}$$

The downward deflection at C, with respect to B, due to the load on the cantilever

$$= \frac{1}{8} \times \frac{4 \times 6^3 \times 12^3}{13{,}000 \times 81} + \frac{1}{3} \times \frac{0{\cdot}5 \times 6^3 \times 12^3}{13{,}000 \times 81}$$
$$= 0{\cdot}177 + 0{\cdot}059$$
$$= 0{\cdot}236 \text{ in.}$$

The net result is a downward deflection at C

$$=0{\cdot}42-0{\cdot}788-0{\cdot}236=-0{\cdot}604 \text{ in.}$$

Fixed Beams

The next example deals with a beam fixed at both ends, but before working the example it will be as well to consider the application of the area-moment theory to the case of fixed beams.

The deflection at any point in a fixed beam is given by the difference between the downward deflection due to the load had the beam been simply supported and the upward deflection due to the fixing moments.

Since, in any fixed beam, the C.G.s of the areas of the free moment and fixing moment diagrams lie in the same vertical line, it follows that when these areas are considered as loads, the reactions to the areas must be equal and opposite. Therefore, when the areas are considered as loads, there are no resultant reactions, and the maximum deflection will occur at the point where the area of the free moment diagram equals the area of the fixing moment diagram, working from the same end of the beam.

Example 5. As an illustration, consider the simple case of a fixed beam of span L carrying a total U.D.L. of W as shown in Fig. 28.

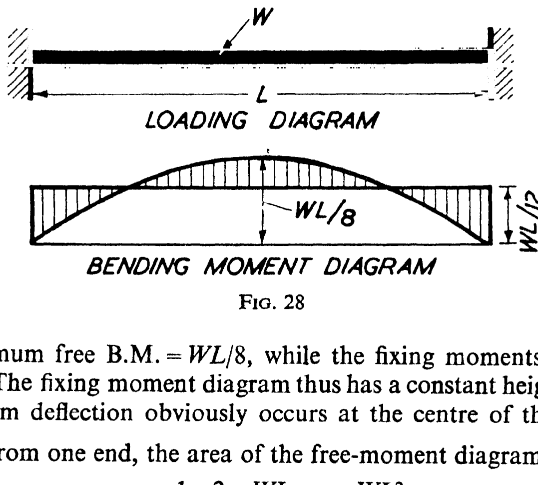


FIG. 28

The maximum free B.M. $= WL/8$, while the fixing moments at each end $= WL/12$. The fixing moment diagram thus has a constant height of $WL/12$. The maximum deflection obviously occurs at the centre of the span.

Working from one end, the area of the free-moment diagram to this point

$$=\frac{1}{2}\times\frac{2}{3}\times\frac{WL}{8}\times L=\frac{WL^2}{24}$$

while the area of the fixing moment diagram

$$=\frac{WL}{12}\times\frac{L}{2}=\frac{WL^2}{24}$$

to the same point.

The maximum deflection in an encastré beam is given by the difference between the moments of the areas to one side of, and taken about, the point of maximum deflection, in this case the centre.

It must be remembered, however, that since, in all cases, the moment is equal to Reaction Moment minus Load Moment, where the Reaction and

the Load refer to the areas of the appropriate B.M. diagrams, and as the reactions cancel out as previously shown, then the maximum deflection is given, numerically and with the correct sign, by deducting from the moment of the free moment diagram, the moment of the fixing moment diagram both taken about the point of maximum deflection.

Thus, in the case under consideration, the maximum deflection

$$=\frac{1}{EI}\left\{\left(\frac{WL^2}{24}\times\frac{3L}{16}\right)-\left(\frac{WL^2}{24}\times\frac{L}{4}\right)\right\}=-\frac{WL^3}{384EI}.$$

In this particular case, and indeed for any case of symmetrical loading, the result could also have been arrived at in the following manner.

The central deflection due to the free moment diagram only

$$=-\frac{5}{384}\times\frac{WL^3}{EI},$$

while the deflection due to the fixing moment diagram (circular bending since M is constant) $=ML^2/8EI$, where $M=WL/12$, giving a deflection of $WL^3/96EI$, and the net deflection

$$=-\left(\frac{5}{384}\times\frac{WL^3}{EI}\right)+\left(\frac{1}{96}\times\frac{WL^3}{EI}\right)=-\frac{WL^3}{384EI}.$$

Example 6. To find the position and value of the maximum deflection of the beam loaded as shown in Fig. 29.

$E=13{,}000$ tons per sq. in. $I=56$ in. units.

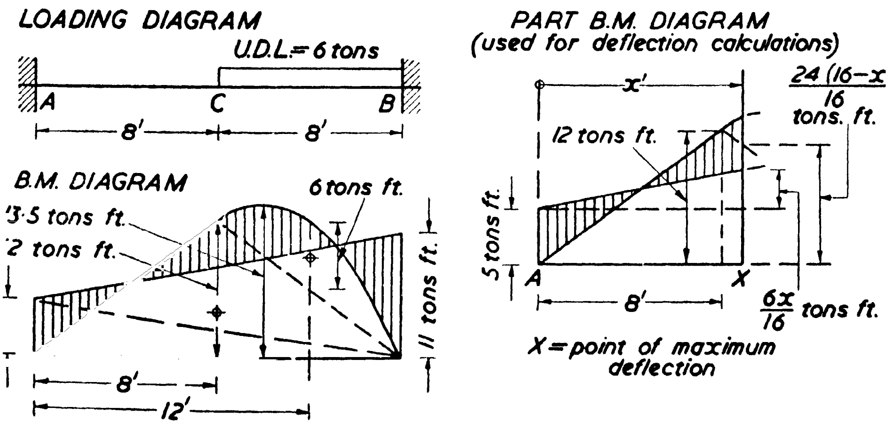


FIG. 29

The free moment diagram can be divided into an isoceles triangle of height 12 tons ft. at C, on which is superimposed over the length CB a parabola, of maximum height 6 tons ft.

The area of the free moment diagram

$$=\frac{12\times16}{2}+\frac{2}{3}\times6\times8$$

$$=96+32=128 \text{ tons ft. units.}$$

Therefore $$\frac{M_A + M_B}{2} \times 16 = 128,$$

whence $$M_A + M_B = 16 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Taking area moments about A, the moment of the fixing moment diagram, which can be divided into two triangles as shown, must equal the moment of the free moment diagram.

Hence $$\left(M_A \times \frac{L}{2} \times \frac{L}{3}\right) + \left(M_B \times \frac{L}{2} \times \frac{2L}{3}\right) = (96 \times 8) + (32 \times 12),$$

whence $$M_A + 2M_B = 27 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

From the two equations $M_A = 5$ tons-ft. and $M_B = 11$ tons ft.

The maximum deflection will occur at X, at a distance x from A such that the areas of the free moment and fixing moment diagrams above the length x are equal. (See Fig. 29.)

At this point it will be necessary to digress for a while in order to establish certain properties of a part of a parabola. See Fig. 30.

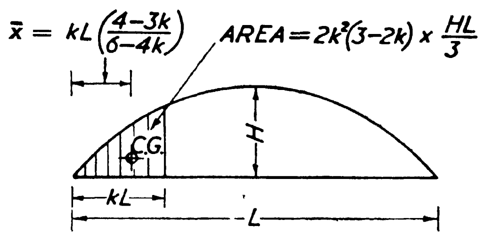


FIG. 30

If a parabola of height H is constructed on a base L, then the area over a length kL measured from one end

$$= 2k^2(3 - 2k) \times \frac{HL}{3},$$

while the C.G. of this portion of area from the same end ($\bar{x}$ in the figure)

$$= kL\left[\frac{4 - 3k}{6 - 4k}\right]$$

Reverting to the parabola in the example,

$H = 6$ tons ft.,

$L = 8$ ft. (the half span on which the parabola is constructed)

and $$k8 = x - \frac{16}{2} \quad \text{or} \quad k = \frac{x}{8} - 1.$$

The area of the fixing moment diagram on the length x

$$= 5x + \frac{6x^2}{32},$$

while the area of the free moment diagram on the same length

$$=\frac{12\times 8}{2}+\left[\frac{12+\frac{24(16-x)}{16}}{2}(x-8)\right]+2\left(\frac{x}{8}-1\right)^2\left[3-2\left(\frac{x}{8}-1\right)\right]\times\frac{6\times 8}{3}$$

$$=-\frac{x^3}{8}+\frac{15x^2}{4}-24x+64.$$

The two areas must be equal, so that

$$5x+\frac{6x^2}{32}=-\frac{x^3}{8}+\frac{15x^2}{4}-24x+64,$$

whence $x=8{\cdot}9$ ft.

The values of the individual small portions of areas are:

Free moment diagram	(a) triangle	=48·0	tons ft. units
	(b) trapezium	=10·2	,, ,,
	(c) parabola	= 1·15	,, ,,
		59·35	,, ,,
Fixing moment diagram	(a) rectangle	=44·5	,, ,,
	(b) triangle	=14·85	,, ,,
		59·35	,, ,,

Taking moments about X, the maximum deflection

$$=-\frac{1728}{13{,}000\times 56}(48\times 3{\cdot}57+10{\cdot}2\times 0{\cdot}46+1{\cdot}15\times 0{\cdot}31-44{\cdot}5\times 4{\cdot}45-14{\cdot}852\times{\cdot}97)$$

$$=-0{\cdot}156 \text{ in.}$$

Example 7. A post is fixed at the base and hinged at the top. It carries an applied B.M. at an intermediate point. See Fig. 31. Find the maximum deflection in the portion AB, the deflection at B, and the maximum deflection in the portion BC.

$E=13{,}000$ tons per sq. in. $I=205$ in. units.

The first step is to find the reaction at A.

Suppose the reaction at A to be removed, then the B.M. diagram would appear as shown in Fig. 31 (b).

The deflection at A due to this B.M. diagram is given by

$$\frac{\text{Moment of B.M. diagram about } A}{EI}=\frac{20\times 12\times 14}{EI}=\frac{3360}{EI}.$$

If the reaction at A is now replaced, the deflection at A due to R_A

$$=\frac{1}{3}\times\frac{R_A\,.\,L^3}{EI}=\frac{1}{3}\times\frac{R_A\times 8000}{EI}.$$

Therefore for there to be no resultant deflection at A:

$$\frac{3360}{EI}=\frac{1}{3}\times\frac{R_A\times 8000}{EI},$$

whence $R_A=1{\cdot}26$ tons.

The final B.M. diagram is shown in Fig. 31 (c).

The B.M. diagram is now considered as a load and taking moments about C, the secondary reaction at A

$$=\frac{\left(5{\cdot}2\times\frac{4{\cdot}13}{2}\times\frac{4{\cdot}13\times 1}{3}\right)-\left(9{\cdot}92\times\frac{7{\cdot}87}{2}\times 9{\cdot}38\right)+\left(10{\cdot}08\times\frac{8}{2}\times 14{\cdot}67\right)}{20}$$

$=12{\cdot}01$ tons ft. units.

The maximum deflection between A and B is given at a distance x from A such that the area of the B.M. diagram on the length x is equal to the reaction at A.

Hence $$\frac{10{\cdot}08x}{8}\times\frac{x}{2}=12{\cdot}01, \quad \text{whence} \quad x=4{\cdot}37 \text{ ft.}$$

and the maximum deflection

$$=-\frac{1728}{13{,}000\times 205}\left[\left(12{\cdot}01\times 4{\cdot}37\right)-\left(12{\cdot}01\times\frac{4{\cdot}37}{3}\right)\right]=-0{\cdot}023 \text{ in.,}$$

i.e. a deflection to the right.

It is desirable to differentiate, by signs, between deflections to the right or to the left of the original vertical line of the post. If the notation set out on page 50 is adhered to, it will be seen that, if the post is viewed from the right, the secondary B.M. is of the type that corresponds to negative deflection. Accordingly, deflection to the right is considered negative and, conversely, that to the left is considered positive.

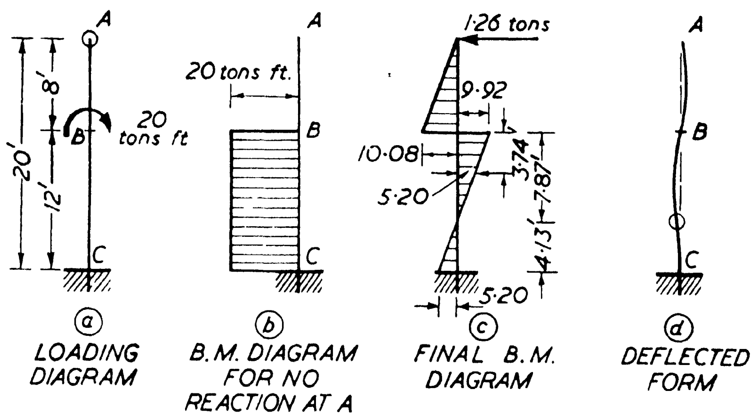


FIG. 31

The deflection at B

$$=-\frac{1728}{13{,}000\times 205}\left[\left(12{\cdot}01\times 8{\cdot}0\right)-\left(10{\cdot}08\times\frac{8}{2}\times\frac{8}{3}\right)\right]=+0{\cdot}0074 \text{ in.,}$$

i.e. a deflection to the left.

The maximum deflection between B and C occurs at a distance x_1 below B such that the area of the positive B.M. diagram on AB is equal to the sum of the reaction and the area of the negative B.M. diagram on the length x_1.

Hence $$\frac{10{\cdot}08\times 8}{2}=12{\cdot}01+\frac{x_1}{2}\left[9{\cdot}92+9{\cdot}92\left(\frac{7{\cdot}87-x_1}{7{\cdot}87}\right)\right],$$

whence $x_1^2 - 15{\cdot}75x_1 + 44{\cdot}9 = 0$ and $x_1 = 3{\cdot}74$ ft.

and the maximum deflection

$$= -\frac{1728}{13{,}000 \times 205}\left[\left(12{\cdot}01 \times 11{\cdot}74\right) - \left(10{\cdot}08 \times \frac{8}{2} \times 6{\cdot}41\right) + \left(\frac{5{\cdot}20 \times 3{\cdot}74^2}{2}\right) + (9{\cdot}92 - 5{\cdot}20) \times \frac{3{\cdot}74^2}{3}\right]$$

$= +0{\cdot}038$ in.

This deflection is also to the left.

Continuous Beams

Example 8. To find the positions and values of the maximum deflections in the spans of a continuous beam. The beam shown in Fig. 17 in the section on the Continuous Beam Theory will be used for this example, being reproduced as Fig. 32. $E = 13{,}000$ tons per sq. in.

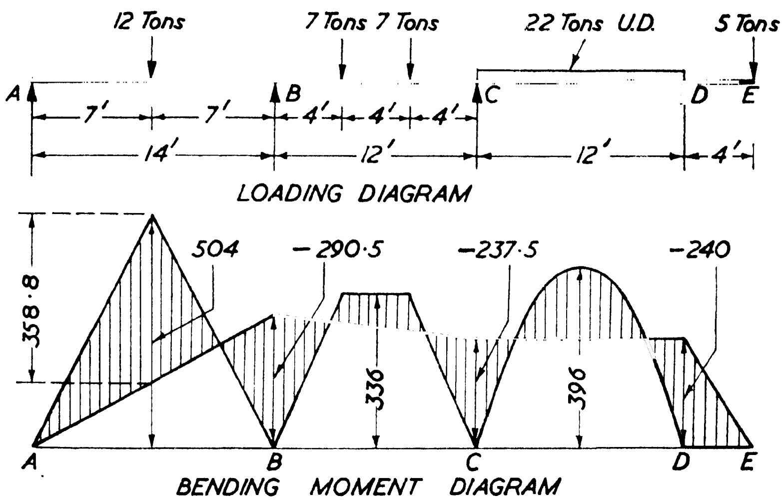


FIG. 32

The slopes and deflections in any span of a continuous beam can be readily determined by the use of the area-moment method, each span being treated separately after the negative moments at the points of support have been found.

Span AB

There are two B.M. diagrams in the span, one of which is positive and the other negative.

Considering the B.M. diagrams as loads, the secondary reaction at A

$$= \left(\frac{1}{2} \times \frac{504 \times 168}{2}\right) - \left(\frac{1}{3} \times \frac{290{\cdot}5 \times 168}{2}\right) = 13{,}034 \text{ tons ins. units.}$$

The maximum deflection occurs at the point of maximum secondary B.M.,

which occurs at X at a distance x from A such that the net area of the B.M. diagram on the length x equals the secondary reaction.

Therefore $$13{,}034 = \left(\frac{504}{84} \times \frac{x^2}{2}\right) - \left(\frac{290\cdot5}{168} \times \frac{x^2}{2}\right),$$

whence $$x = 78\cdot1 \text{ in.}$$

The maximum deflection is found by the application of the principles previously set out and

$$= -\frac{1}{EI}\left[\left(13{,}034 \times 78\cdot1\right) - \left(13{,}034 \times \frac{78\cdot1}{3}\right)\right] = -\frac{13{,}034 \times 78\cdot1 \times 2}{13{,}000 \times 205 \times 3}$$

$$= -0\cdot255 \text{ in.}$$

Span BC

Considering the B.M. diagrams as loads, the secondary reaction at B

$$= \left(\frac{336 \times 96}{2}\right) - \left(\frac{237\cdot5 \times 144}{2}\right) - \left(\frac{53 \times 144}{2} \times \frac{2}{3}\right) = -3516 \text{ tons in. units,}$$

while the secondary reaction at C

$$= \left(\frac{336 \times 96}{2}\right) - \left(\frac{237\cdot5 \times 144}{2}\right) - \left(\frac{53 \times 144}{2} \times \frac{1}{3}\right) = -2244 \text{ tons in. units}$$

The fact that both secondary reactions are negative quantities means that the beam is hogging at each end, so that there will either be one point of maximum upward deflection or three points of maximum deflection: two upwards towards the ends of the span, and one downwards towards the centre. The latter circumstance will apply in the case under consideration.

The point of maximum deflection nearest to B will occur at a point X at a distance x from B such that

$$3516 + \left(\frac{336}{48} \times \frac{x^2}{2}\right) = x\left(\frac{290\cdot5 + 237\cdot5 + \frac{144 - x}{144} \times 53}{2}\right)$$

$$3516 + 3\cdot5x^2 = x\left(290\cdot5 - \frac{53x}{288}\right),$$

therefore $$\frac{1061x^2}{288} - 290\cdot5x + 3516 = 0,$$

whence $$x = 14\cdot93 \text{ in.}$$

The corresponding maximum upward deflection

$$= -\frac{1}{13{,}000 \times 146}\left[\left(-3516 \times 14\cdot93\right) - \left(\frac{336}{48} \times \frac{14\cdot93^3}{6}\right) + \left(285 \times \frac{14\cdot93^2}{2}\right) + \left(5\cdot5 \times \frac{14\cdot93^2}{2} \times \frac{2}{3}\right)\right]$$

$$= +0\cdot01275 \text{ in.}$$

The point of maximum deflection nearest to C will occur at a point X_1 at a distance x_1 from C such that

$$2244 + \frac{336}{48} \times \frac{x_1^2}{2} = x_1\left(\frac{237\cdot5 + 237\cdot5 + \frac{53x_1}{144}}{2}\right)$$

$$2244 + 3\cdot5x_1^2 = x_1\left(237\cdot5 + \frac{53x_1}{288}\right),$$

therefore $\frac{955x_1^2}{288} - 237{\cdot}5x_1 + 2244 = 0,$

whence $x_1 = 11{\cdot}2$ in.

The corresponding maximum upward deflection

$$= -\frac{1}{13{,}000 \times 146}\left[\left(-2244 \times 11{\cdot}2\right) - \left(\frac{336}{48} \times \frac{11{\cdot}2^3}{6}\right) + \left(237{\cdot}5 \times \frac{11{\cdot}2^2}{2}\right) + \left(4{\cdot}12 \times \frac{11{\cdot}2^2}{2} \times \frac{1}{3}\right)\right]$$

$= +0{\cdot}00621$ in.

The point of maximum deflection nearest the centre is more easily found by working from the end C and will occur at a point X_2 at a distance x_2 from C such that

$$2244 + \frac{336 \times 48}{2} \times 336(x_2 - 48) = x_2\left(\frac{237{\cdot}5 + 237{\cdot}5 + \frac{53x_2}{144}}{2}\right)$$

$$2244 + 8064 - 16{,}128 + 336x_2 = x_2\left(237{\cdot}5 + \frac{53x_2}{288}\right),$$

therefore $\frac{53x_2^2}{288} - 98{\cdot}5x_2 + 5820 = 0,$

whence $x_2 = 67{\cdot}66$ in.

The corresponding maximum downward deflection

$$= -\frac{1}{13{,}000 \times 146}\left[\left(-2244 \times 67{\cdot}66\right) - \left(\frac{336 \times 48}{2} \times 35{\cdot}66\right) - \left(\frac{336 \times 19{\cdot}66^2}{2}\right) + \left(\frac{237{\cdot}5 \times 67{\cdot}66^2}{2}\right) + \left(24{\cdot}9 \times \frac{67{\cdot}66^2}{2} \times \frac{1}{3}\right)\right]$$

$= -0{\cdot}0307$ in.

Span CD

Since the negative moments at the ends of the span are very nearly equal, it can be assumed, for all practical purposes, that an average negative moment of 238·75 tons ins. obtains throughout the length of the span and that the maximum deflection occurs at the centre.

If that assumption is made, the maximum deflection

$$= -\left(\frac{5}{384} \times \frac{WL^3}{EI}\right) + \left(\frac{ML^2}{8EI}\right)$$

$$= \frac{1}{13{,}000 \times 122}\left(-\frac{5 \times 22 \times 144^3}{384} + \frac{238{\cdot}75 \times 144^2}{8}\right)$$

$$= -0{\cdot}149 \text{ in.}$$

6. THE DEFLECTION OF COMPOUND GIRDERS WITH CURTAILED FLANGE PLATES

THE formula for the maximum deflection of a simply supported beam of prismatic section is

$$d=k\,.\,\frac{WL^3}{EI}$$

where k is a constant depending on the form which the load W takes. When the centre of a beam is reinforced with curtailed flange plates the deflection is naturally not so great, but the reduction of deflection due to the extra plates requires special calculations. It can be shown that the appropriate formula for the deflection of a beam provided with one curtailed plate per flange as in Fig. 33, is

$$d=\frac{WL^3}{EI_0}\left[k-\frac{(n-1)}{n}\,.f(m)\right]$$

where $n=I_1/I_0$ and $m=a/L$, I_0 being the original moment of inertia of the beam and I_1 being the total moment of inertia when the curtailed flange plates are added, while the function of m, $f(m)$, varies with the type of loading, k being the constant as above.

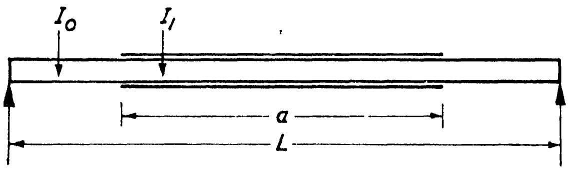


FIG. 33

Suppose that the beam of Fig. 33 is provided with a central point load W. Then the B.M. diagram, I diagram and M/I diagram appropriate to the beam are shown in Fig. 34, E being assumed to be constant and therefore ignored.

Considering the left half of the M/I diagram and taking moments about the left-hand support,

$$EId=\left(\frac{WL}{4}\times\frac{L}{4}\times\frac{L}{3}\right)-\frac{WL}{4}\left(\frac{n-1}{n}\right)\left(\frac{mL}{4}\right)\left(\frac{1}{2}-\frac{m}{6}\right)L$$

$$-\frac{WL}{4}(1-m)\left(\frac{n-1}{n}\right)\left(\frac{mL}{4}\right)\left(\frac{1}{2}-\frac{m}{3}\right)L$$

$$=\frac{WL^3}{48}-\frac{n-1}{n}\left(\frac{WL^3}{16}\right)m\left(\frac{1}{2}-\frac{m}{6}\right)-\frac{n-1}{1}\left(\frac{WL^3}{16}\right)\left(1-m\right)m\left(\frac{1}{2}-\frac{m}{3}\right)$$

$$=WL^3\left[\frac{1}{48}-\frac{n-1}{n}\cdot\frac{1}{16}\left(\frac{m}{2}-\frac{m^2}{6}+\frac{m}{2}-\frac{5m^2}{6}+\frac{m^3}{3}\right)\right]$$

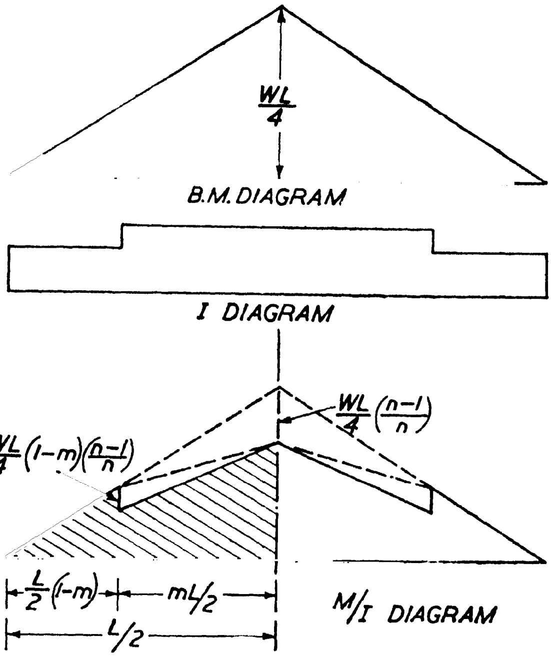


FIG. 34

Hence,

$$d=\frac{WL^3}{EI}\left[\frac{1}{48}-\frac{n-1}{n}\;\frac{1}{16}\left(m-m^2+\frac{m^3}{3}\right)\right].$$

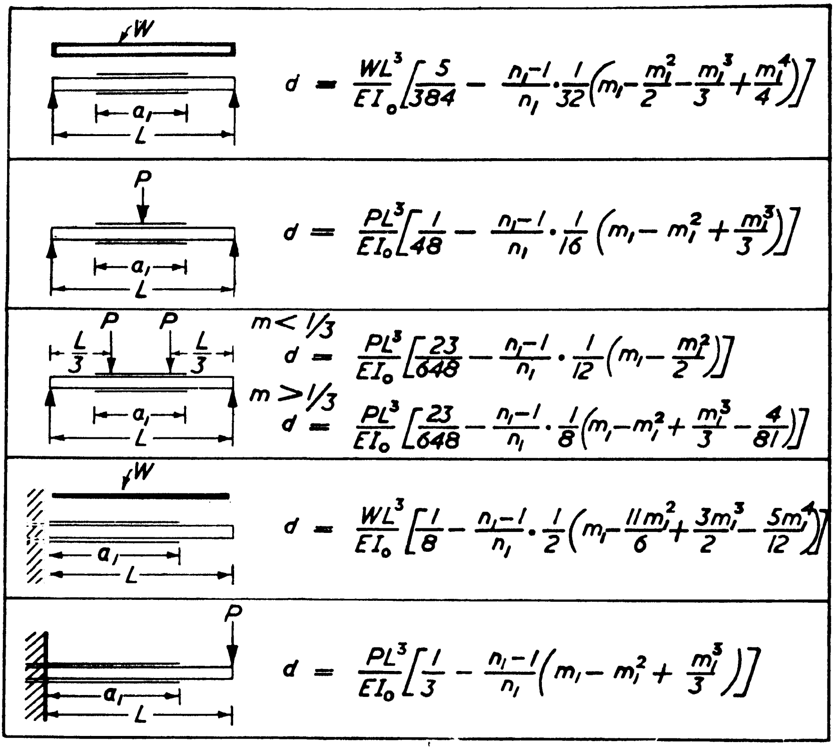
In a similar manner formulæ for other loadings may be calculated. Five examples are shown in Fig. 35, each having one curtailed plate on each flange.

When an additional curtailed plate is added on each flange the amount of deflection is still further reduced and the appropriate formula can be expressed as:

$$d=\frac{WL^3}{EI_0}\left[k-\phi\cdot\frac{n-1}{n}\cdot f(m)\right].$$

It is desirable to make a small table as shown in Fig. 36 to calculate the value of *d*. In this table the values of *n* and *m* are functions of the beam. The values of *D*, *E* and *F* are calculated from the appropriate formulæ for f(m) given in Fig. 35. The values *G*, *H* and *J* are the deflection reduction coefficients for *each* individual plate.

It will be appreciated when considering the second plate that the first plate contributes to the value of I_2 and that the value *G* for the first plate has already been calculated. Allowance for this is made in calculating the value of $\phi(n-1)/n$. Thus the value for the second plate is $B-A$. Similarly, for the third plate the value is $C-B$, and so on.



$$d = \frac{WL^3}{EI_0}\left[\frac{5}{384} - \frac{n_1-1}{n_1}\cdot\frac{1}{32}\left(m_1 - \frac{m_1^2}{2} - \frac{m_1^3}{3} + \frac{m_1^4}{4}\right)\right]$$

$$d = \frac{PL^3}{EI_0}\left[\frac{1}{48} - \frac{n_1-1}{n_1}\cdot\frac{1}{16}\left(m_1 - m_1^2 + \frac{m_1^3}{3}\right)\right]$$

$m < 1/3$
$$d = \frac{PL^3}{EI_0}\left[\frac{23}{648} - \frac{n_1-1}{n_1}\cdot\frac{1}{12}\left(m_1 - \frac{m_1^2}{2}\right)\right]$$

$m > 1/3$
$$d = \frac{PL^3}{EI_0}\left[\frac{23}{648} - \frac{n_1-1}{n_1}\cdot\frac{1}{8}\left(m_1 - m_1^2 + \frac{m_1^3}{3} - \frac{4}{81}\right)\right]$$

$$d = \frac{WL^3}{EI_0}\left[\frac{1}{8} - \frac{n_1-1}{n_1}\cdot\frac{1}{2}\left(m_1 - \frac{11m_1^2}{6} + \frac{3m_1^3}{2} - \frac{5m_1^4}{12}\right)\right]$$

$$d = \frac{PL^3}{EI_0}\left[\frac{1}{3} - \frac{n_1-1}{n_1}\left(m_1 - m_1^2 + \frac{m_1^3}{3}\right)\right]$$

FIG. 35

Plate No.	n	$\frac{n-1}{n}$	$\phi\cdot\frac{n-1}{n}$	m	$f(m)$	$\phi\cdot\frac{n-1}{n}\cdot f(m)$
1	I_1/I_0	A	A	a_1/L	D	$G = A \times D$
2	I_2/I_0	B	$B - A$	a_2/L	E	$H = E(B-A)$
3	I_3/I_0	C	$C - B$	a_3/L	F	$J = F(C-B)$

$$d = \frac{WL^3}{EI_0}\left[k - (G + H + J)\right].$$

FIG. 36

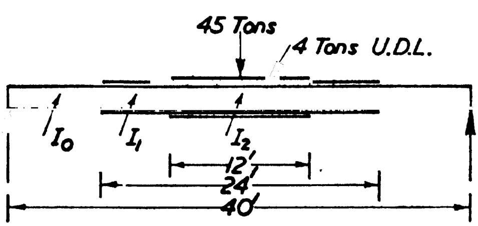


FIG. 37

As an example, the maximum deflection of the beam shown in Fig. 37 will be calculated.

This deflection has already been calculated by graphical means as shown in Fig. 26.

Allowing for rivet holes, the properties of the beam are as follows:

$$I_0 = 3950 \text{ in.}^4. \quad I_1 = 5749 \text{ in.}^4. \quad I_2 = 7691 \text{ in.}^4.$$
$$a_1 = 24 \text{ ft.} \quad a_2 = 12 \text{ ft.}$$

Hence
$$n_1 = I_1/I_0 = 1{\cdot}455 \quad \text{and} \quad n_2 = I_2/I_0 = 1{\cdot}947$$
$$m_1 = a_1/L = 0{\cdot}6 \quad \text{and} \quad m_2 = a_2/L = 0{\cdot}3.$$

The beam carries two loads, viz.: its own, assumed to be 4 tons uniformly distributed, and a central point load of 45 tons. Now the central deflection of a prismatic beam with a uniformly distributed load is $5WL^3/384EI$, while that for a central point load is $8WL^3/384EI$. Hence, as the uniformly distributed load is small compared with the point load, it could be considered with little inaccuracy to be concentrated at the centre of the beam, when the equivalent load would be $4 \times \frac{5}{8} = 2{\cdot}5$ tons. Therefore, the beam will be analysed assuming that it carries only a central point load of 47·5 tons.

The appropriate calculations are tabulated in Fig. 38.

Plate No.	n	$\frac{n-1}{n}$	$\phi \cdot \frac{n-1}{n}$	m	$f(m)$	$\phi \cdot \frac{n-1}{n} \cdot f(m)$
1	1·455	0·313	0·313	0·6	0·0195	0·0061
2	1·947	0·486	0·173	0·3	0·0137	0·0024
					$\Sigma\phi \cdot \frac{n-1}{n} \cdot f(m) =$	0·0085

FIG. 38

The values of f(m) for each plate is obtained from the appropriate formula in Fig. 35, viz: $(1/16)[m_1 - m_1^2 + m_1^3/3]$.

$$d = \frac{WL^3}{EI_0}\left[\frac{1}{48} - 0{\cdot}0085\right] = \frac{47{\cdot}5 \times 480^3}{13{,}000 \times 3950}(0{\cdot}02083 - 0{\cdot}0085)$$
$$= 1{\cdot}26 \text{ in.}$$

It will be observed that the final deflection of 1·26 in. agrees closely with that obtained graphically, viz.: 1·296 in.

7. PROPPED CANTILEVERS

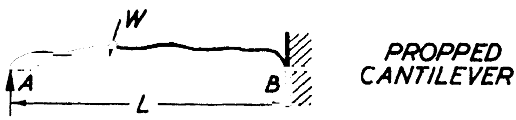
BEAMS which are built-in at one end and simply supported at the other are known as propped cantilevers. Normally, the ends of the beams are on the same level, in which case bending moments and reactions may be derived in two ways, viz.: by employing the Theorem of Three Moments or by deflection formulæ.

Solution by Theorem of Three Moments

Consider the propped cantilever AB in Fig. 39.

The bending moment at B may be found by using the Theorem of Three Moments, and assuming that AB is one span of a two-span continuous beam ABC which is symmetrical in every way about B.

Then the loads on AB and BC will produce free B.M.s whose areas are A_1 and A_2 respectively, the C.G.s of the areas being distances x_1 and x_2 from A and C respectively.



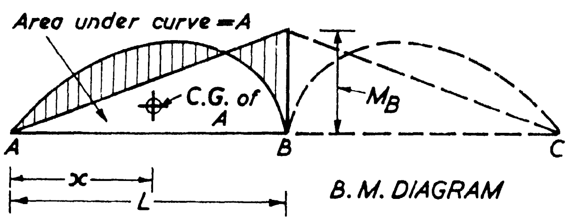


FIG. 39

Now
$$M_A L_1 + 2M_B(L_1 + L_2) + M_C L_2 = 6\left(\frac{A_1 x_1}{L_1} + \frac{A_2 x_2}{L_2}\right).$$

But
$$L_1 = L_2 = \text{say, } L,$$
$$M_A = M_C = 0,$$
$$A_1 = A_2 = \text{say, } A$$

and
$$x_1 = x_2 = \text{say, } x.$$

Hence,
$$2M_B(2L) = 6 \times 2\left(\frac{Ax}{L}\right).$$

and
$$M_B = \frac{3Ax}{L^2}.$$

Therefore the moment at the fixed end of a propped cantilever $= 3Ax/L^2$,

where A = the area of the free B.M. diagram, AB being considered as a simply-supported beam,

x = the distance from the prop to the C.G. of the free B.M. diagram

and L = the span.

The reactions at each support may be found by employing a modified form of the formula used for beams built-in at both ends, viz.:

$$\text{S.F.}_A = \text{the simple support reaction at } A - \frac{M_B}{L}$$

$$\text{S.F.}_B = \text{the simple support reaction at } B + \frac{M_B}{L}$$

where, A is the propped end and B is built-in.

Example 1. The beam AB shown in Fig. 40 is propped at A and built-in at B. It is required to draw the B.M. and S.F. diagrams for the symmetrical triangular load of 10 tons.

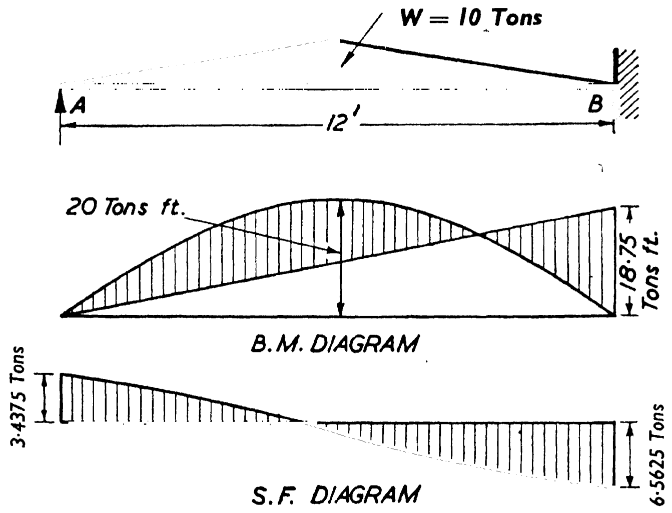


FIG. 40

Consider the free B.M. diagram. At any point x ft. from A between A and the centre of the beam,

$$M_x = W\left(\frac{x}{2} - \frac{2x^3}{3L^2}\right).$$

Hence, the maximum B.M., at the centre,

$$= WL/6 = 10 \times 12/6 = 20 \text{ tons ft.}$$

The area A of the free B.M. diagram

$$=2\int_0^{L/2} W\left(\frac{x}{2}-\frac{2x^3}{3L^2}\right)dx$$

$$=2W\left[\frac{x^2}{4}-\frac{2x^4}{12\times 12^2}\right]_0^6=20(9-1{\cdot}5)$$

$$=150 \text{ tons ft. units.}$$

The C.G. of the free B.M. diagram is in the centre of the beam, i.e. 6 ft. from A.

Hence,
$$M_B=\frac{3Ax}{L^2}=\frac{3\times 150\times 6}{144}$$
$$=18{\cdot}75 \text{ tons ft.}$$

Following normal convention this value is negative:

$$\text{S.F.}_A=\frac{W}{2}-\frac{M_B}{L}=5-\frac{18{\cdot}75}{12}=3{\cdot}4375 \text{ tons.}$$

$$\text{S.F.}_B=10-\text{S.F.}_A=6{\cdot}5625 \text{ tons.}$$

The B.M. and S.F. diagrams may now be drawn.

Solution by Deflection Formulæ

When this method of solution is employed the propping force is found first and the B.M.s follow. The method is best explained by an example.

Example 2. Consider the propped cantilever shown in Fig. 41 which carries a uniformly distributed load of 4 tons.

Let the force in the prop be F.

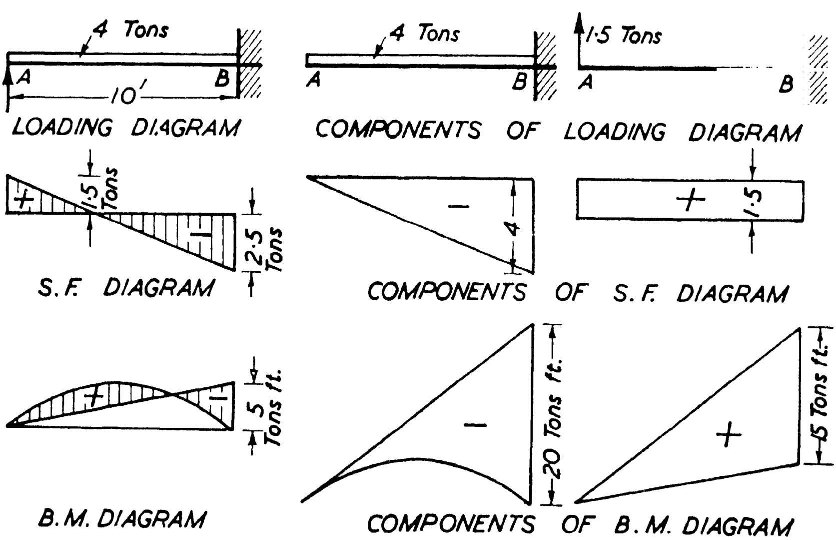


FIG. 41

Assume that this prop is removed. Then the deflection at A will equal $WL^3/8EI$, this being the standard deflection formula for a U.D.L. Now if the prop is assumed to be back in position then the upward deflection due to the prop will be $FL^3/3EI$.

Since A is on the same level as B,

$$\frac{FL^3}{3EI}=\frac{WL^3}{8EI},$$

whence $$F=\text{S.F.}_A=\frac{3W}{8}=\frac{12}{8}=1{\cdot}5 \text{ tons.}$$

and $$\text{S.F.}_B=\frac{5W}{8}=\frac{20}{8}=2{\cdot}5 \text{ tons.}$$

Without the prop the moment at B due to the load is $-WL/2$, whilst the moment at B due to the prop alone is $FL=3WL/8$.

Hence the final moment at B

$$=M_B=WL\left(-\frac{1}{2}+\frac{3}{8}\right)=-\frac{WL}{8}=-\frac{4\times 10}{8}=-5 \text{ tons ft.}$$

The S.F. and B.M. diagrams may now be drawn.

Encastré Beams and Propped Cantilevers

There is one further method in which the fixing moment may be determined in a propped cantilever, i.e. from a knowledge of the fixing moments in a similar encastré beam, as will be shown in the following example.

Example 3. Fig. 42 shows the B.M. diagram for an encastré beam AB, 10 ft. long, carrying a point load of 25 tons 4 ft. from A.

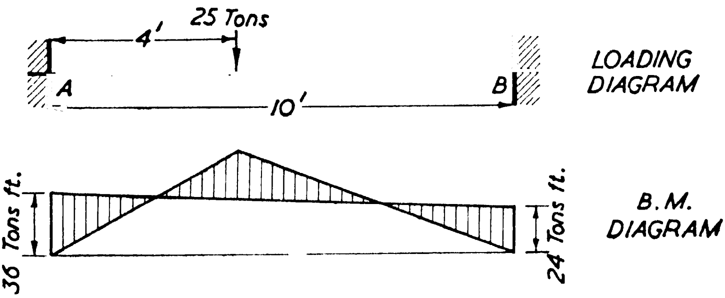


FIG. 42

Now if the fixing moment M_A is released, then it may be shown that the fixing moment M_B is increased by half the numerical value of M_A.

Therefore, for the propped cantilever shown in Fig. 43,

$$M_B=24+\frac{36}{2}=42 \text{ tons ft.}$$

It will be appreciated that for symmetrical loading, the single fixing moment for a propped cantilever is one and a half times as great as the fixing moment at either end of a similar encastré beam.

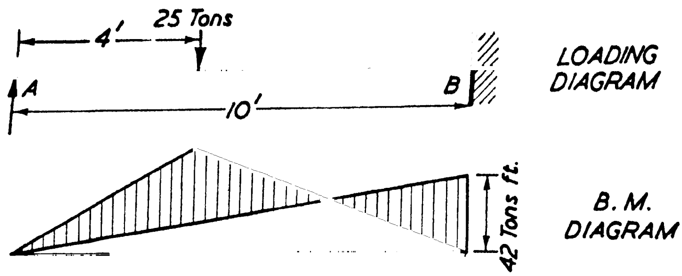


FIG. 43

It should be noted that formulæ for various propped cantilevers are given on pp. 74–81.

Sinking of Supports

When the supports for a loaded propped cantilever do not maintain the same relative levels as in the unloaded condition, the B.M.s and S.F.s may be obtained by using the deflection method of Example 2. When the prop sinks the load which it takes is reduced, while the fixing moment at the other end is increased. Two special cases arise: the first when the prop sinks so much that no load is taken by the prop, and the second when the built-in end sinks so much that the fixing moment is reduced to zero, i.e. the cantilever resembles a simply supported beam.

The Deflection of Propped Cantilevers

The value of the deflection at any point along a propped cantilever can be found by the methods explained and illustrated in the section entitled 'The Deflection of Beams'. However, it will be of interest to consider one example here.

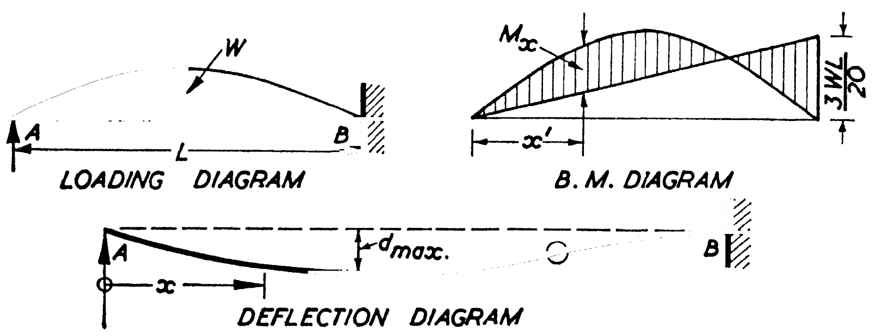


FIG. 44

Example 4. It is required to find the position and amount of the maximum deflection in the propped cantilever shown in Fig. 44.

The load W, which is distributed in the form of a second-degree parabola, produces the B.M. diagram illustrated.

Now $M_B = -3WL/20$ while the free B.M. at any point x ft. from A, considering AB to be a simply supported beam,

$$= M_x = \frac{WL}{2}\left(\frac{x^4}{L^4} - \frac{2x^3}{L^3} + \frac{x}{L}\right).$$

Hence, for the propped cantilever,

$$M_x = \frac{WL}{2}\left(\frac{x^4}{L^4} - \frac{2x^3}{L^3} + \frac{x}{L}\right) - \left(\frac{3WL}{20} \times \frac{x}{L}\right)$$

$$= \frac{WL}{20}\left(\frac{10x^4}{L^4} - \frac{20x^3}{L^3} + \frac{7x}{L}\right).$$

Let $x/L = m$.

Now $$EI\frac{d^2y}{dx^2} = M_x.$$

Hence, $$EI\frac{dy}{dx} = \frac{WL^2}{20}\left(\frac{10m^5}{5} - \frac{20m^4}{4} + \frac{7m^2}{2} + C\right).$$

When $x = L$,

the slope $\frac{dy}{dx} = 0$ and $m = 1$.

Therefore, $$0 = (2 - 5 + 3\tfrac{1}{2} + C),$$

whence $$C = -\tfrac{1}{2}.$$

The deflection is a maximum when the slope is zero. Hence,

$$0 = 2m^5 - 5m^4 + \frac{7m^2}{2} - \frac{1}{2}.$$

This equation is satisfied when $m = 0{\cdot}4268$ or 1.

Hence, the maximum deflection occurs at a point $0{\cdot}4268\,L$ from the prop A.

Now $$EIy = \frac{WL^3}{20}\left(\frac{2m^6}{6} - \frac{5m^5}{5} + \frac{7m^3}{6} - \frac{m}{2} + D\right)$$

When $x = 0$,

$y = 0$ and therefore $D = 0$.

Hence $$y = d = \frac{WL^3}{20EI}\left(\frac{m^6}{3} - m^5 + \frac{7m^3}{6} - \frac{m}{2}\right).$$

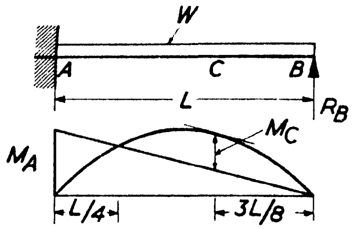
For maximum deflection, $m = 0{\cdot}4268$. Therefore,

$$d_{\max} = -\frac{0{\cdot}006742\,WL^3}{EI}.$$

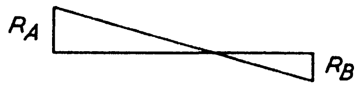
PROPPED CANTILEVERS

(Rows: LOADING · MOMENT · SHEAR · DEFLECTION)

Uniformly distributed load W over full span

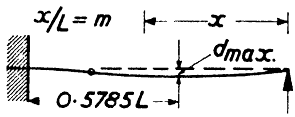


$$M_A = -\frac{WL}{8} \qquad M_C = \frac{9WL}{128}$$



$$R_A = \frac{5W}{8} \qquad R_B = \frac{3W}{8}$$

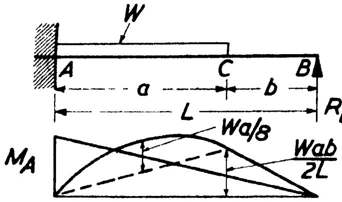
$x/L = m$



$$d = \frac{WL^3}{48EI}(m - 3m^3 + 2m^4)$$

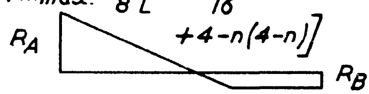
$$d_{max.} = \frac{WL^3}{185EI}$$

Uniformly distributed load W over length a from the fixed end



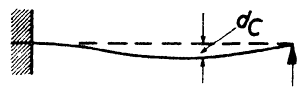
$$M_A = -\frac{Wa}{8}(2-n)^2 \text{ where } a/L = n$$

$$+M_{max.} = \frac{Wa}{8}\left[-\frac{[8-n^2(4-n)]^2}{16} + 4 - n(4-n)\right]$$



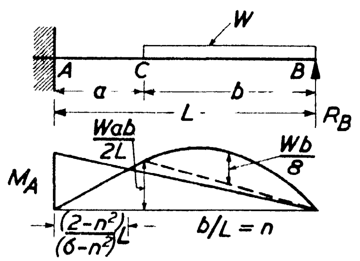
$$R_A = \frac{W}{8}\left[8 - n^2(4-n)\right]$$

$$R_B = \frac{Wn^2}{8}(4-n)$$



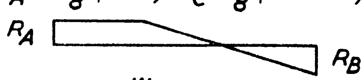
$$d_C = \frac{Wa^3}{48EI}(6 - 12n + 7n^2 - n^3)$$

Uniformly distributed load W over length b adjacent to the prop



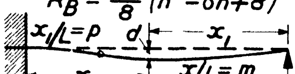
$b/L = n$

$$M_A = -\frac{Wb}{8}(2-n^2) \qquad M_C = \frac{Wb}{8}(6n - n^3 - 4)$$



$$R_A = \frac{Wn}{8}(6-n^2)$$

$$R_B = \frac{W}{8}(n^3 - 6n + 8)$$

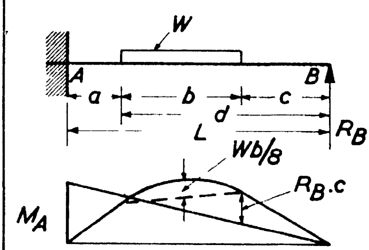


When $x \leqslant a$, $d = \frac{WbL^2}{48EI}\left[(n^2-6)m^3 - (3n^2-6)m^2\right]$

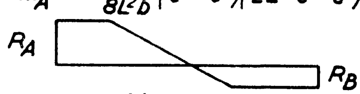
When $x \geqslant a$,

$$d = \frac{WL^4}{48bEI}\left[2p^4 - p^3 n(n^3 - 6n + 8) + pn^2(3n^2 - 8n + 6)\right]$$

Uniformly distributed load W over length b within the span



$$M_A = -\frac{W}{8L^2b}(d^2 - c^2)(2L^2 - c^2 - d^2)$$

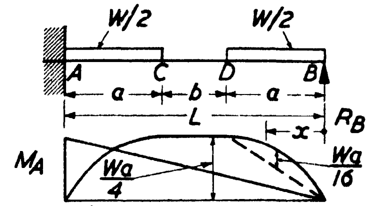


$$R_A = r_A + \frac{M_A}{L} \qquad R_B = r_B - \frac{M_A}{L}$$

Where r_A and r_B are the simple support reactions for the beam (M_A being considered positive)

PROPPED CANTILEVERS

LOADING — MOMENT — SHEAR — DEFLECTION

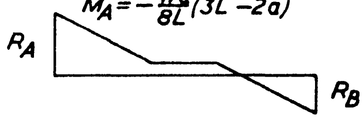


If $m = a/L$, then between B and D.

$M_x = \frac{W}{8a}\left[-2x^2 + xa(4-3m+2m^2)\right]$

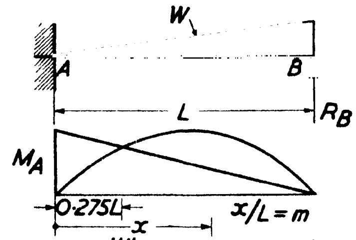
$+M_{max.}$ when $x = \frac{a}{4}(4-3m+2m^2)$

$M_A = -\frac{Wa}{8L}(3L-2a)$



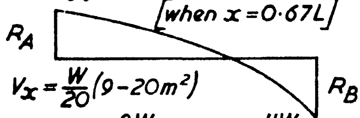
$R_A = \frac{W}{4L^2}(2L^2+3aL-4a^2)$

$R_B = \frac{W}{4L}(2L^2-3aL+4a^2)$



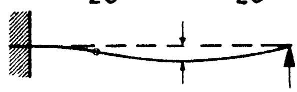
$M_x = -\frac{WL}{60}(20m^2-27m+7)$

$M_A = -\frac{7WL}{60}$ $\quad +M_{max.} = 0{\cdot}0846WL$ [when $x = 0{\cdot}67L$]



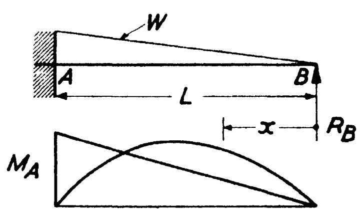
$V_x = \frac{W}{20}(9-20m^2)$

$R_A = \frac{9W}{20}$ $\quad R_B = \frac{11W}{20}$



$d_{max.} = \frac{0{\cdot}0061WL^3}{EI}$

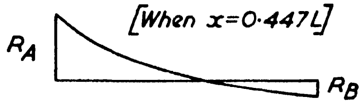
When $x = 0{\cdot}598L$



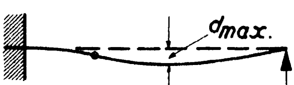
$M_A = -\frac{2WL}{15}$

$+M_{max.} = 0{\cdot}0596WL$

[When $x = 0{\cdot}447L$]

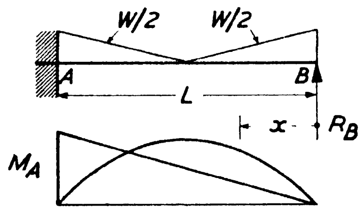


$R_A = \frac{4W}{5}$ $\quad R_B = \frac{W}{5}$



$d_{max.} = \frac{0{\cdot}0047WL^3}{EI}$

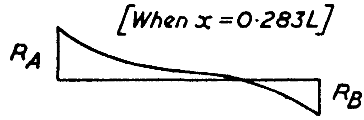
When $x = 0{\cdot}447L$



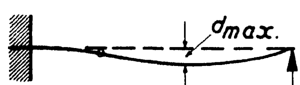
$M_A = -\frac{3WL}{32}$

$+M_{max.} = 0{\cdot}0454WL$

[When $x = 0{\cdot}283L$]



$R_A = \frac{19W}{32}$ $\quad R_B = \frac{13W}{32}$



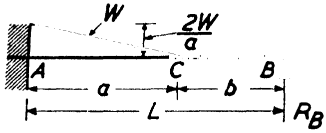
$d_{max.} = \frac{0{\cdot}0037WL^3}{EI}$

When $x = 0{\cdot}404L$

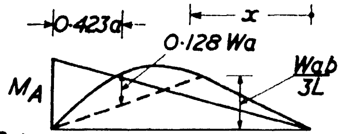
PROPPED CANTILEVERS

Case 1

LOADING



MOMENT



Between C and A, $M_x = R_B . x - \frac{W}{3a^2}(x-b)^3$

$$M_A = -\frac{Wa}{60L^2}(3a^2 - 15aL + 20L^2)$$

$+M_{max}$. when $x = b + \frac{a^2}{2L}\sqrt{1 - \frac{a}{5L}}$

SHEAR

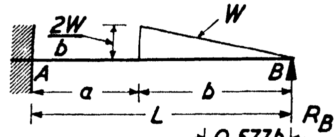


$$R_B = \frac{Wa^2}{20L^3}(5L - a)$$

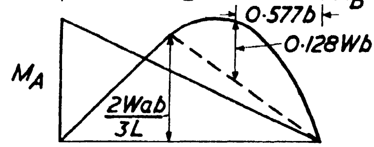
$$R_A = W - R_B$$

Case 2

LOADING



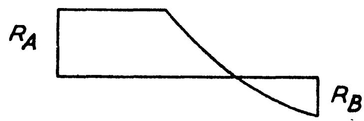
MOMENT



$$M_x = R_B . x - \frac{Wx^3}{3b^2}$$

$$M_A = -\frac{Wb}{15L^2}(5L^2 - 3b^2)$$

SHEAR

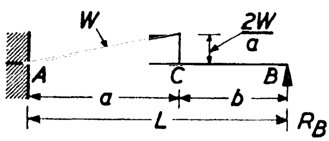


$$R_A = \frac{Wb}{5L^3}(5L^2 - b^2)$$

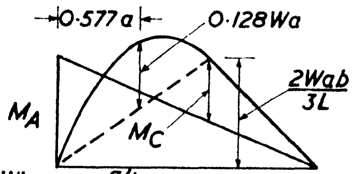
$$R_B = \frac{W}{5L^3}(b^3 + 5aL^2)$$

Case 3

LOADING



MOMENT



When $m = a/L$

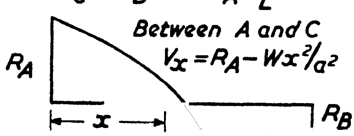
$$M_A = -Wa\left(\frac{m^2}{5} - \frac{3m}{4} + \frac{2}{3}\right)$$

$$M_C = R_B . b + M_A . \frac{b}{L}$$

SHEAR

Between A and C

$V_x = R_A - Wx^2/a^2$

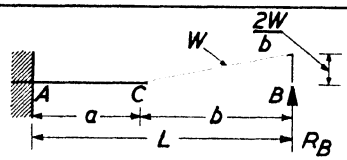


$$R_B = \frac{Wa^2}{20L^3}(15L - 4a)$$

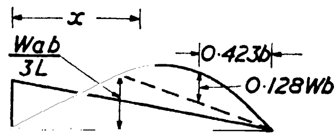
$$R_A = W - R_B$$

Case 4

LOADING



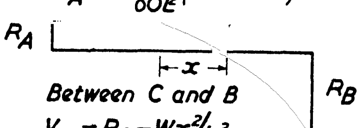
MOMENT



$$M_x = R_A . x + M_A - \frac{W}{3b^2}(x-a)^3$$

$$M_A = -\frac{Wb}{60L^2}(10L^2 - 3b^2)$$

SHEAR



Between C and B

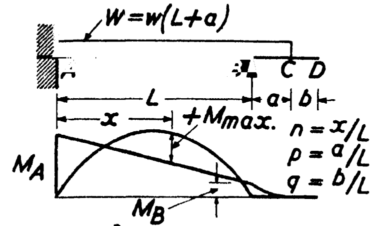
$V_x = R_A - Wx^2/b^2$

$$R_B = \frac{W}{20b^2L^3}\left[L^4(11L - 15a) + a^4(5L - a)\right]$$

$$R_A = W - R_B$$

PROPPED CANTILEVERS

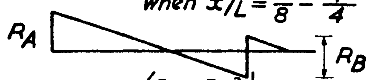
LOADING / MOMENT / SHEAR / DEFLECTION



$M_B = -\frac{wa^2}{2}$ $M_A = -\frac{w}{8}(L^2 - 2a^2)$

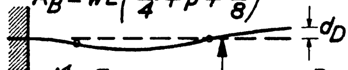
$+M_{max.} = \frac{wL^2}{128}(36p^4 - 28p^2 + 9)$

when $x/L = \frac{5}{8} - \frac{3p^2}{4}$



$R_A = wL\left(\frac{5}{8} - \frac{3p^2}{4}\right)$

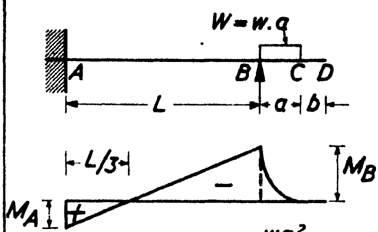
$R_B = wL\left(\frac{3p^2}{4} + p + \frac{3}{8}\right)$



$d_D = \frac{wL^4}{48EI}\left[(8p^3 + 6p^2 - 1)(p+q) - 2p^4\right]$

$d_x = \frac{wL^4}{48EI}\left[2n^4 + (6p^2 - 5)n^3 - (6p^2 - 3)n^2\right]$

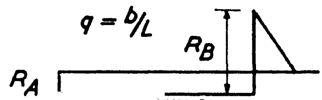
$d_{max.}$ when $x/L = \frac{1}{16}\left[15 - 18p^2 - \sqrt{324p^4 - 156p^2 + 33}\right]$



$M_B = -2M_A = -\frac{wa^2}{2}$

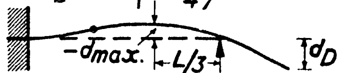
$p = a/L$

$q = b/L$



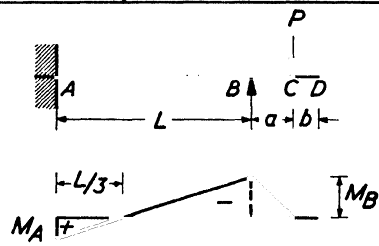
$R_A = -\frac{wap}{4}$

$R_B = wa\left(1 + \frac{3p}{4}\right)$



$d_D = \frac{wL^4}{48EI}\left[p^2(8p+6)q + 6p(p+1)\right]$

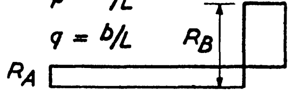
$-d_{max.} = -\frac{wL^4p^2}{54EI}$



$M_B = -2M_A = -Pa$

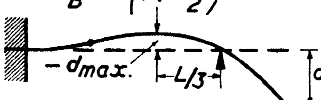
$p = a/L$

$q = b/L$



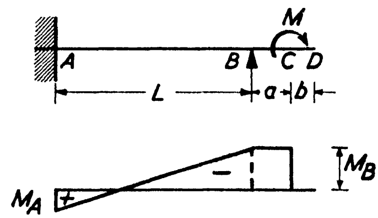
$R_A = -\frac{3Pp}{2}$

$R_B = P\left(1 + \frac{3p}{2}\right)$

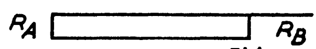


$d_D = \frac{PL^3p}{12EI}(4p^2 + 6pq + 3p + 3q)$

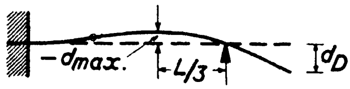
$-d_{max.} = -\frac{PL^3p}{27EI}$



$M_B = -2M_A = -M$



$R_A = -R_B = -\frac{3M}{2L}$



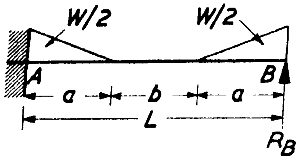
$d_D = \frac{M}{4EI}\left[L(a+b) + a^2\left(2 + \frac{4b}{a}\right)\right]$

$-d_{max.} = -\frac{ML^2}{27EI}$

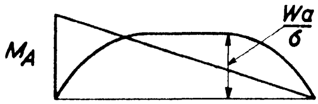
PROPPED CANTILEVERS

Case 1

LOADING



MOMENT



$$M_A = -\frac{Wa}{8L}(2L-a)$$

SHEAR

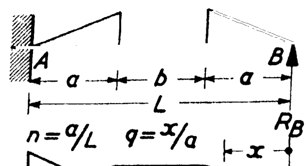


$$R_A = \frac{W}{8L^2}(4L^2+2aL-a^2)$$

$$R_B = W - R_A$$

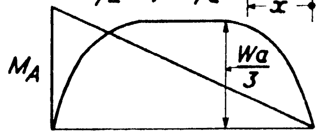
Case 2

LOADING



$n = a/L \quad q = x/a$

MOMENT



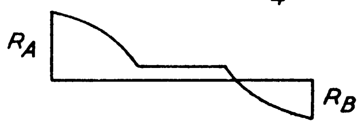
$$M_A = -\frac{Wa}{8L}(4L-3a)$$

When $x < a$,

$$M_x = \frac{W}{24}(9n^2x - 12nx + 12x - 4xq^2)$$

$+M_{max.}$ occurs when $q = \sqrt{\frac{3n^2}{4} - n + 1}$

SHEAR

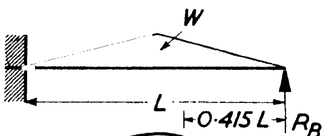


$$R_A = \frac{W}{8L^2}(4L^2+4aL-3a^2)$$

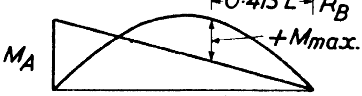
$$R_B = W - R_A$$

Case 3

LOADING



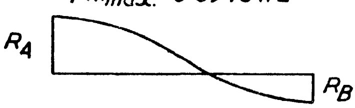
MOMENT



$$M_A = -\frac{5WL}{32}$$

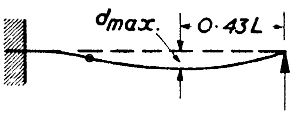
$$+M_{max.} = 0{\cdot}0948\,WL$$

SHEAR



$$R_A = \frac{21W}{32} \qquad R_B = \frac{11W}{32}$$

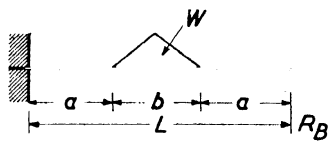
DEFLECTION



$$d_{max.} = 0{\cdot}00727\,\frac{WL^3}{EI}$$

Case 4

LOADING

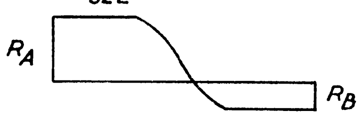


MOMENT



$$M_A = \frac{W}{32L}(5L^2+4aL-4a^2)$$

SHEAR

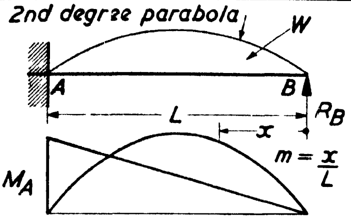


$$R_A = \frac{W}{32L^2}(21L^2+4aL-4a^2)$$

$$R_B = W - R_A$$

PROPPED CANTILEVERS

LOADING — MOMENT

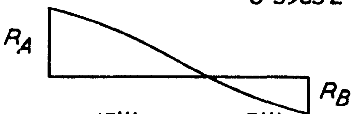


$$M_A = -\frac{3WL}{20}$$

$$M_x = \frac{WL}{20}(10m^4 - 20m^3 + 7m)$$

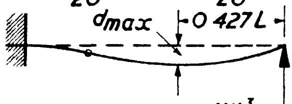
$+M_{max} = 0.0888\,WL$, when $x = 0.3985\,L$

SHEAR



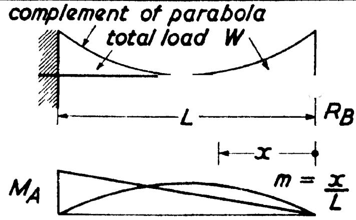
$R_A = \frac{13W}{20}$ $\quad R_B = \frac{7W}{20}$

DEFLECTION



$$d_{max} = 0.00674\,\frac{WL^3}{EI}$$

LOADING — MOMENT



$$M_A = -\frac{3WL}{40}$$

$$M_x = \frac{WL}{40}(-40m^4 + 80m^3 - 60m^2 + 17m)$$

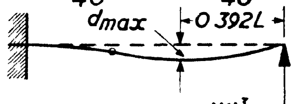
$+M_{max} = 0.0399\,WL$, when $x = 0.2343\,L$

SHEAR



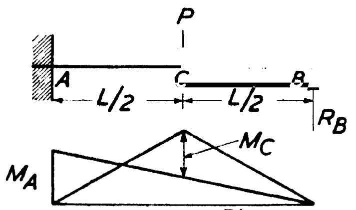
$R_A = \frac{23W}{40}$ $\quad R_B = \frac{17W}{40}$

DEFLECTION



$$d_{max} = 0.00278\,\frac{WL^3}{EI}$$

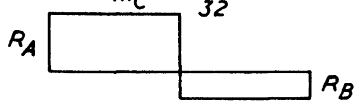
LOADING — MOMENT



$$M_A = -\frac{3PL}{16}$$

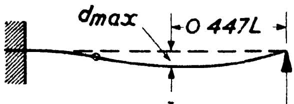
$$M_C = \frac{5PL}{32}$$

SHEAR



$R_A = 11P/16$ $\quad R_B = 5P/16$

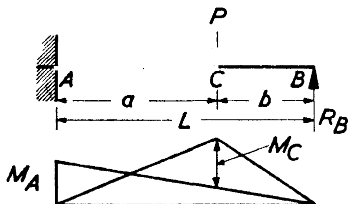
DEFLECTION



$$d_C = \frac{7PL^3}{768\,EI}$$

$$d_{max.} = 0.00932\,\frac{PL^3}{EI}$$

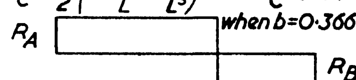
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$M_A = -\frac{Pb(L^2 - b^2)}{2L^2}$ $\quad$ max $M_A = -0.193PL$ when $b = 0.577L$

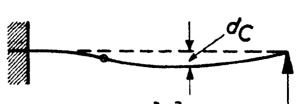
$M_C = \frac{Pb}{2}\left(2 - \frac{3b}{L} + \frac{b^3}{L^3}\right)$ $\quad$ max. $M_C = 0.174PL$ when $b = 0.366L$

SHEAR



$R_B = \frac{Pa^2}{2L^3}(b + 2L)$ $\quad R_A = P - R_B$

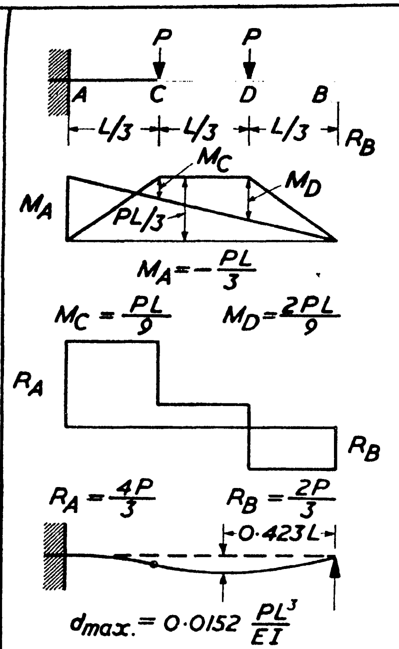
DEFLECTION

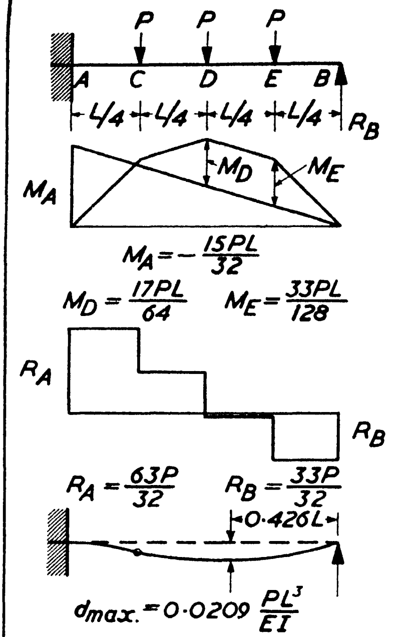


$$d_C = \frac{Pa^3b^2}{12EIL^3}(4L - a)$$

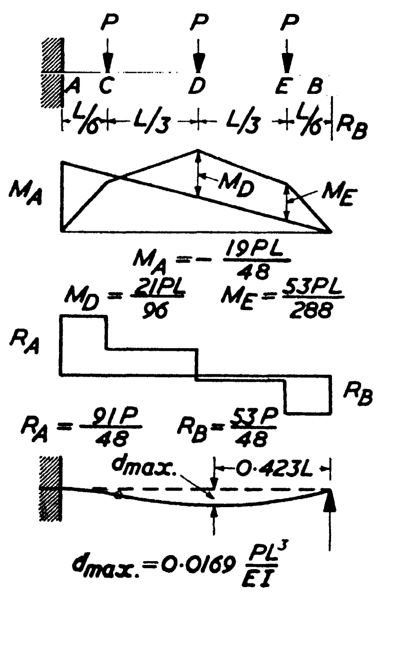
PROPPED CANTILEVERS

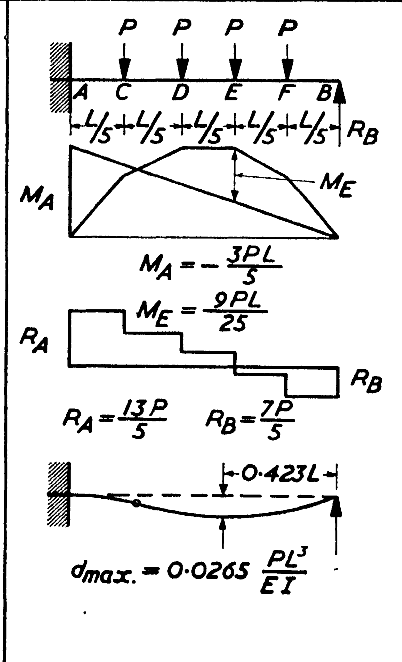
LOADING MOMENT SHEAR DEFLECTION



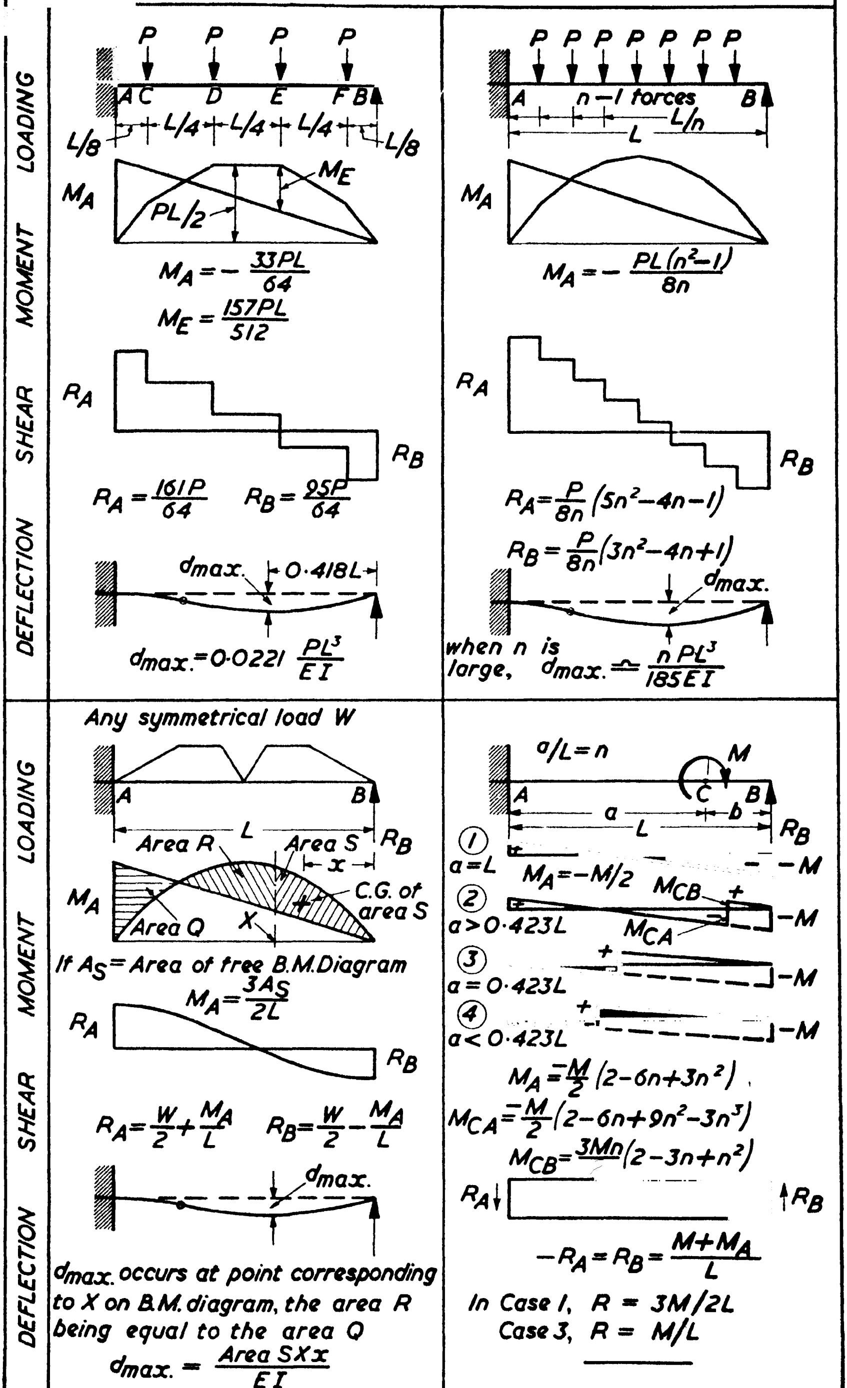


LOADING MOMENT SHEAR DEFLECTION





PROPPED CANTILEVERS



8. ELEMENTS OF GRAPHIC STATICS

GRAPHIC statics is merely the solution on the drawing board of certain arithmetical and algebraic problems relative to force systems.

A force is fully defined when its magnitude, direction and position (or point of application) are known. It can thus conveniently be fully represented by a straight line of finite length, proportional to the magnitude of the force, drawn in the direction of the force and away from the point of

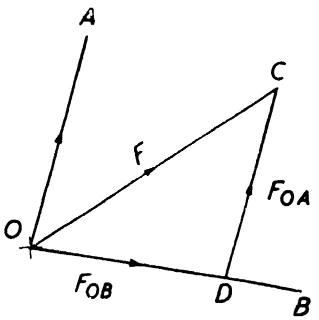


FIG. 45

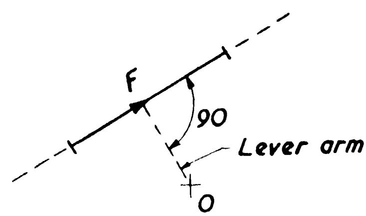


FIG. 46

application. Any line parallel to the original line and of equal length will represent the force in magnitude and direction only.

A number of forces acting together in one plane constitute a coplanar system of forces. Any force acting in a plane can be resolved, at any point in its line of action or line of action produced, into two components in that plane. Each component is represented graphically by the projection of the force on the line of the component as shown in Fig. 45, wherein O is the point on the line of action of force F at which the force is to be resolved into components along OA and OB. Then if OC represents the force F in direction and magnitude, i.e. is drawn to scale, F_{OA}, the component of F in the direction OA, will be represented by the projection CD whilst F_{OB}, the component in the direction OB, will be given by the projection OD.

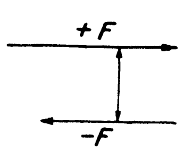


FIG. 47

The tendency of a force to produce rotation about any point is given by the product of the force and the perpendicular distance from its line of action to the point and such a product is called the 'moment' of the force about that point. See Fig. 46.

Two equal, opposite and parallel forces constitute a 'couple', the moment of which is given by the product of either force and the perpendicular distance between their lines of action. Clockwise moments are normally taken as positive, as shown in Fig. 47.

Coplanar force systems may for ease of solution be divided into two groups: those having a common point of application are called concurrent systems, and those which do not have such a common point are called non-concurrent systems. The former is but a particular case of the latter, which is the general case.

Coplanar Concurrent Force Systems

The equilibrium of these systems may be established by the use of the force polygon as demonstrated in Fig. 48. Such a system can only exist either in a state of equilibrium, that is, all the forces form a self-balancing system, or there is a resultant which will be a single force of defined magnitude and direction, acting through the common point of application.

For example, it is desired to investigate the system shown where forces F_1, F_2, F_3 and F_4 act at point O. It is convenient in this solution to use 'Bow's Notation', which consists of delineating a force by letters or numbers

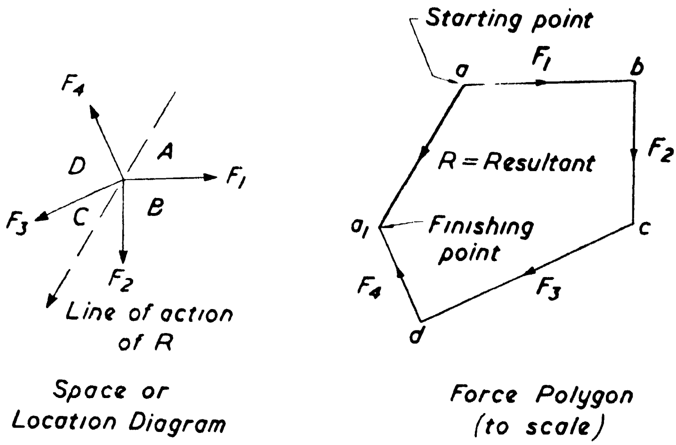


FIG. 48

placed on each side, i.e. force F_1 is known as force AB and in the force polygon is represented by the line ab. This line represents the force in direction, given by drawing the line from a to b and in magnitude by the distance ab. The remaining forces are each set off to the same scale and in their respective directions shown by the lines bc, cd and da_1. The resultant of the system, or the single force having the same effect, is given by the line joining the starting point of the force polygon to the finishing point, i.e. R is given in magnitude and direction by the line aa_1 measured to the scale to which the forces have been set down. The line of action of R is parallel thereto and passes through the common point of application.

In order to have equilibrium it is obvious that a force equal and opposite to the resultant would require to be supplied, and if such a system is in equilibrium the resultant is zero and a 'closed' force polygon is formed.

In Fig. 49 the four forces F_1, F_2, F_3 and F_4 are in equilibrium, giving a closed polygon, and it will be noted that the order in which the forces are set down will not affect the result, although it will be found convenient to set them down in the clockwise order shown.

If each of the forces be resolved into components in two directions at right angles, for convenience taken as vertical and horizontal, it will be seen that the sums of these components V_1, V_2, etc., and H_1, H_2, etc., are both algebraically equal to zero. Thus the algebraic conditions for equilibrium are

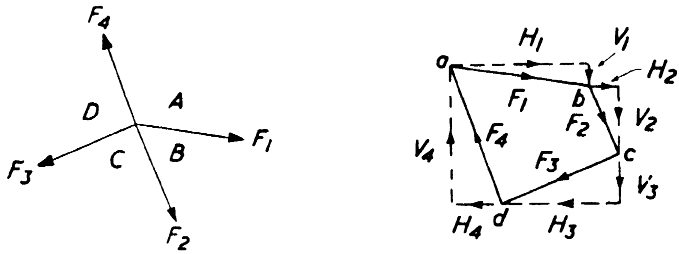


FIG. 49

that the sums of components in two directions at right angles must each be zero, i.e. $\Sigma V=0$ and $\Sigma H=0$, represented graphically by the closed force polygon.

If ΣV and/or ΣH be not zero, then the magnitude of R is given by $\sqrt{(V^2+H^2)}$ and the inclination θ of its line of action to the horizontal is given by $\tan\theta=V/H$. To determine algebraically the directions in which the resultant acts due consideration must be given to the sign of the components

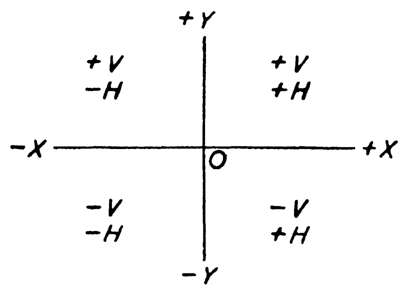


FIG. 50

V and H, and the normal convention is that forces acting upwards and to the right of the point of application are taken as positive. These signs will then indicate in which quadrant the resultant acts. See Fig. 50.

The force polygon may be used to find not more than two unknown quantities in any coplanar concurrent force system, e.g. the amount and direction of one force, the amounts of two forces in known directions or the lines of action of two forces of known magnitudes.

Coplanar Non-concurrent Force Systems

The equilibrium of such systems cannot be established by the use of the force polygon alone since the resultant, if any, may be a single force or a couple, and solution by graphics involves the use of the 'equilibrium polygon' based on the following principles:

(a) Any system of coplanar non-concurrent forces may be replaced by two single forces, the lines of action of these being given by the first and last segments of the equilibrium polygon.
(b) When the first and last segments of the equilibrium polygon intersect, the resultant is a single force.
(c) When the first and last segments of the equilibrium polygon are equal, act along the same straight line and in opposite directions, the forces are in equilibrium.
(d) When the first and last segments of the equilibrium polygon are equal, act along parallel lines and in opposite directions, the resultant is a couple.

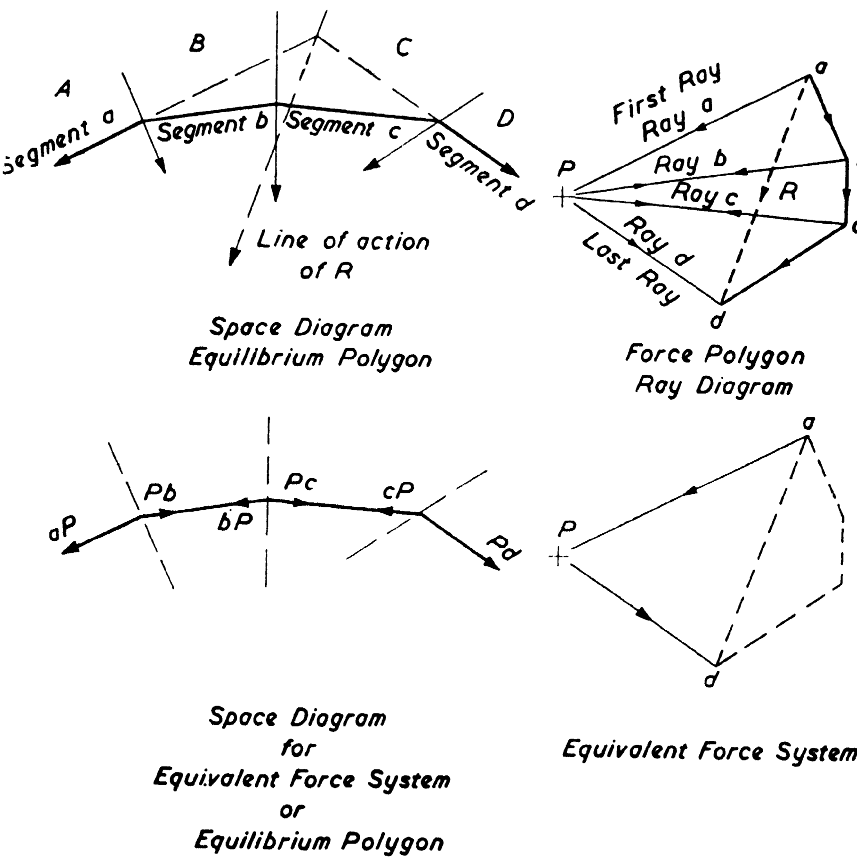


FIG. 51

Fig. 51 demonstrates statements (a) and (b). The resultant of the three forces *AB*, *BC* and *CD* is given as to magnitude and direction by the line *ad* in the force polygon *abcd*. Take any 'pole' *P* and draw the rays *Pa*, *Pb*, *Pc* and *Pd*; then force *ab* may be resolved at any point on its line of action into two forces *aP* and *Pb*, the magnitudes of which are given by the lengths of the rays. Similarly force *bc* may be resolved along *bP* and *Pc* and force *cd* resolved along *cP* and *Pd*. The forces in the lines *Pb* and *Pc*, being each equal and opposite, are self-cancelling and consequently disappear, leaving the original system *ab*, *bc* and *cd* replaced by two forces *aP* and *Pd*, being the first and last rays of the ray diagram, corresponding to the first and last points in the force polygon.

If now at any point on the line of action of force *AB* this force is resolved into the directions *aP* and *Pb*, their lines of action will be shown on the space diagram by the lines marked Segment *a* and Segment *b* drawn respectively parallel to rays *a* and *b* in the ray diagram. This process is continued, and all the original forces are resolved into components in the lines of the rays until the original system is reduced to the forces *aP*, *Pb*, *bP*, *Pc*, *cP* and *Pd* acting as shown in the equivalent force system—this figure is the 'equilibrium polygon'. Four of the forces are self-cancelling, leaving the original system represented by forces *aP* and *Pd* the lines of action of which are given by the equilibrium polygon and the magnitudes and direction given by the corresponding rays in the ray diagram. These two segments, the first and last, intersect at a point on the line of action of the resultant *R*, the magnitude of which is obtained from the force polygon.

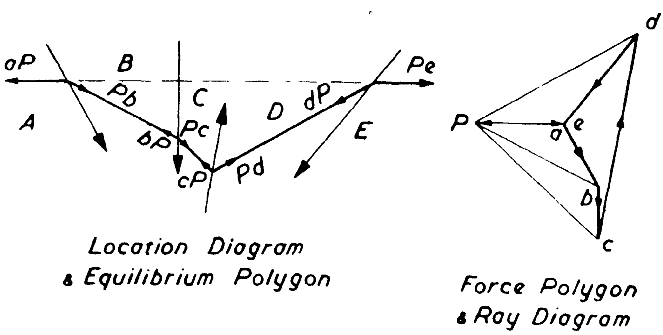


FIG. 52

Turning now to case (c) this is exemplified by Fig. 52. The force polygon closes, but this alone does not indicate equilibrium, it merely shows that there is no unbalanced single force. The first point in the force polygon is *a* and the last is *e* and they are coincident; therefore the first ray in the ray diagram is given by *aP* and the last by *Pe*, that is to say they are equal and opposite. The equilibrium polygon is drawn in the usual manner remembering that the first segment parallel to ray *aP* is drawn across the corresponding space *A* in the location diagram. Completing the equilibrium polygon it will be found that the last segment *Pe* is in line with the first segment *aP* and

all the segments in the equilibrium polygon are self-cancelling, indicating that there is no resultant force or moment.

The algebraic equivalent to the closure of the force polygon is, as before, $\Sigma V=0$ and $\Sigma H=0$, whilst the cancelling of the segments of the equilibrium polygon indicates that $\Sigma M=0$.

An example of case (d) is shown in Fig. 53. Here the force polygon closes, indicating that there is no single resultant, but when the equilibrium polygon is drawn the first and last segments are equal, opposite and on parallel lines, and, since the forces on these lines are equivalent to the original system, the product of one of them and the distance between their lines of action gives the value of the resultant moment which is equivalent to the original force system. The magnitude of the forces is given by the appropriate ray in the ray diagram (measured to the force scale), and the distance between them scaled from the location diagram (measured to the space scale). The sense of the couple, clockwise or anticlockwise, is self-evident.

Since there are three conditions for equilibrium of coplanar non-concurrent force systems it follows that in any problem involving these there cannot be more than three unknowns if the solution is to be provided by simple statics.

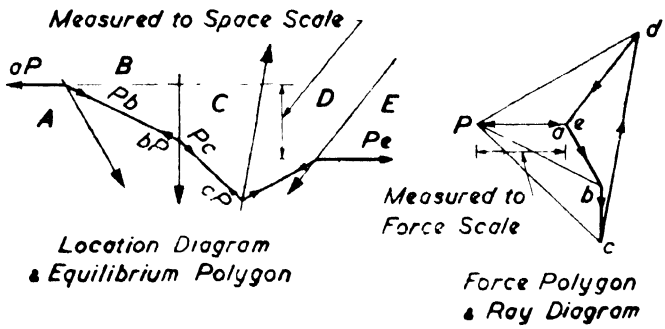


FIG. 53

Loads and Reactions

The loads acting on any plane structure together with the reactions at the points of support must constitute a system of forces in equilibrium and it is necessary to determine these external forces before proceeding to evaluate the internal forces. A structure is said to be 'statically determinate' with respect to the external forces acting on it when the reactions are so arranged that they may be determined by simple statics, that is to say, by the use of the force and equilibrium polygons or by the use of the equations $\Sigma V=0$, $\Sigma H=0$ and $\Sigma M=0$. In order that this can be possible there must be not more than two reactions of which the line of action of one and a point on the line of action of the other are known. There are then three unknowns to be determined, namely the magnitude of both reactions and the direction of one. The problem therefore reduces to the equilibrium of three forces,

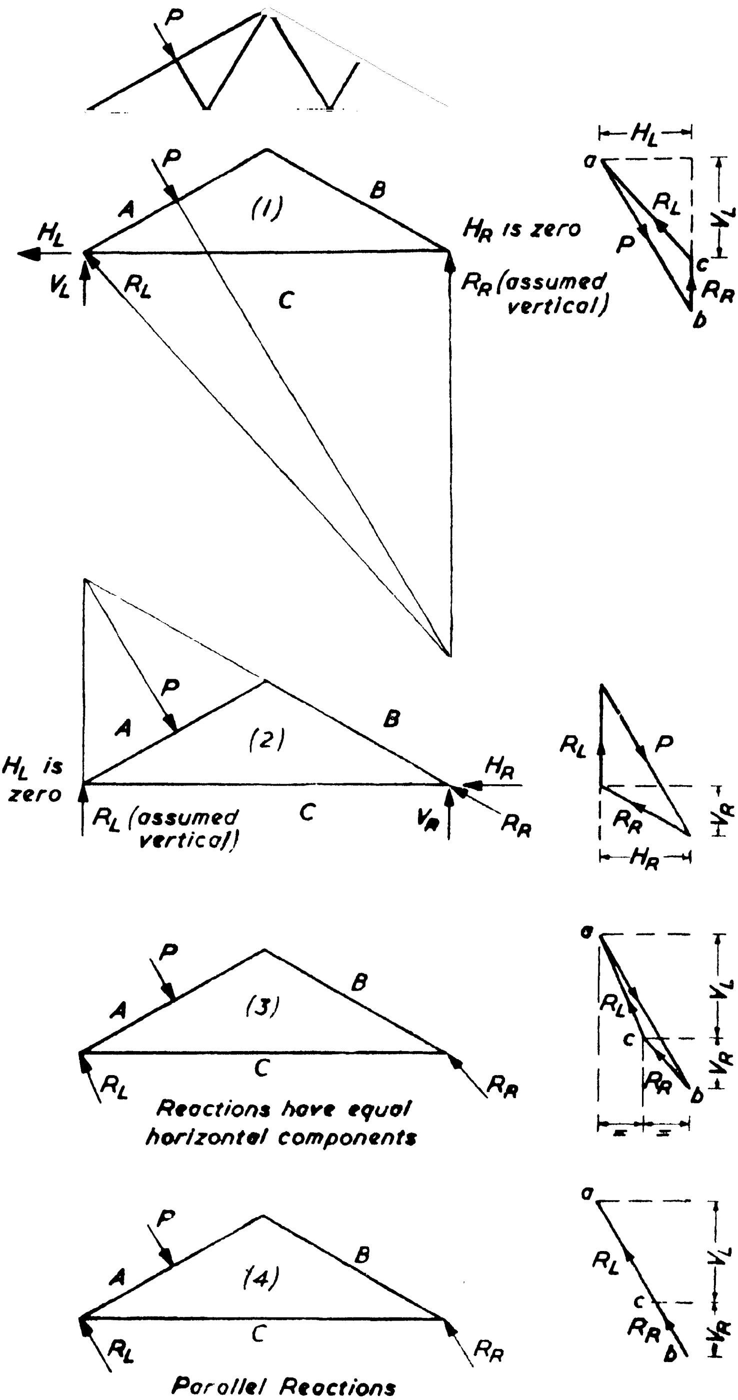

P
P
A
(1)
B
H_L
H_R is zero
V_L
R_L
C
R_R (assumed vertical)
H_L
a
R_L
V_L
P
c
R_R
b
P
A
(2)
B
H_L is zero
H_R
R_L (assumed vertical)
C
V_R
R_R
R_L
P
R_R
V_R
H_R
P
A
(3)
B
C
R_L
R_R
Reactions have equal horizontal components
a
R_L
V_L
c
V_R
R_R
b
P
A
(4)
B
C
R_L
R_R
Parallel Reactions
a
R_L
V_L
c
R_R
V_R
b

FIG. 54

two reactions and the resultant load, and three forces can only be in equilibrium if they are (a) parallel or (b) have a common point in their lines of action.

Consider the simple roof truss shown in Fig. 54 under the action of an inclined load P, and simply supported at the shoes. Since the load has components in two directions at right angles, P_V, P_H, it follows that as a general case, each reaction will have vertical and horizontal components, that is to say, there will be four unknown quantities. The problem can be made statically determinate by any arbitrary assumption as to the disposition of the horizontal reactions; this will not affect the magnitudes of the vertical components of the reactions in this case, since the supports are on the same horizontal line. The figure shows four different assumptions frequently made. It should be noted that it is only the horizontal components of the reactions which are altered by the various assumptions: the vertical components V_L and V_R are the same throughout. It is perhaps hardly necessary to point out that the solution which is adopted must be carried through when detailing.

Reactions by the Use of the Equilibrium Polygon

It is possible to find the reactions of any statically determinate structure supported at two points by the following procedure without finding the resultant load.

Fig. 55 sets out a typical example; a plane frame, simply supported at its extremities X and Y, carries two inclined loads, and the right-hand reaction

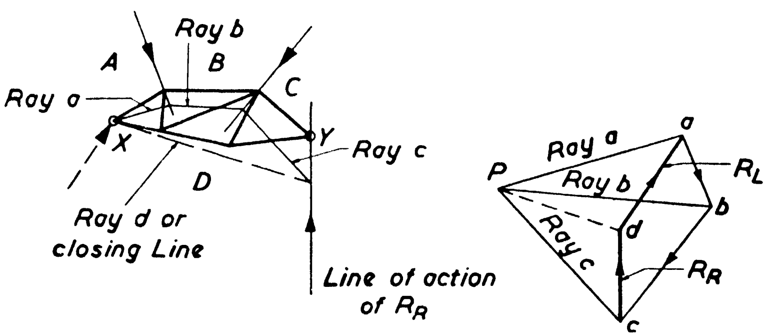


FIG. 55

is postulated vertical. The load line ab, bc is first drawn, and on this a ray diagram is constructed with any pole P. The corresponding equilibrium polygon is then drawn, noting that ray a must pass through the point X which is the only known point on the line of action of R_L, and that ray c will end on the vertical through Y, the line of action of R_R. Complete the equilibrium polygon by drawing ray d across space D, this line is known as the 'closing line'. Draw the corresponding ray, Pd, in the ray diagram to intersect the vertical through c and then complete the force polygon by the line da. Then the lines cd and da in the force polygon represent the reactions

R_R and R_L in magnitude and direction. It will be seen that the force polygon closes, i.e. ΣV and ΣH are each zero, and in addition the equilibrium polgyon closes, showing that ΣM also is zero. The reactions so determined are the only two forces which will meet the conditions imposed.

S.F. and B.M. Diagrams, Graphical Calculations

Consider the beam shown in Fig. 56, a horizontal beam cantilevered past its two supports and carrying vertical loads. The load line *ae* is drawn, then the ray diagram from pole *P* and the equilibrium polygon with its closing line across space *O* to give point *O* on the load line. This determines the reactions from which the shear diagram may be constructed by horizontal and vertical projection as shown.

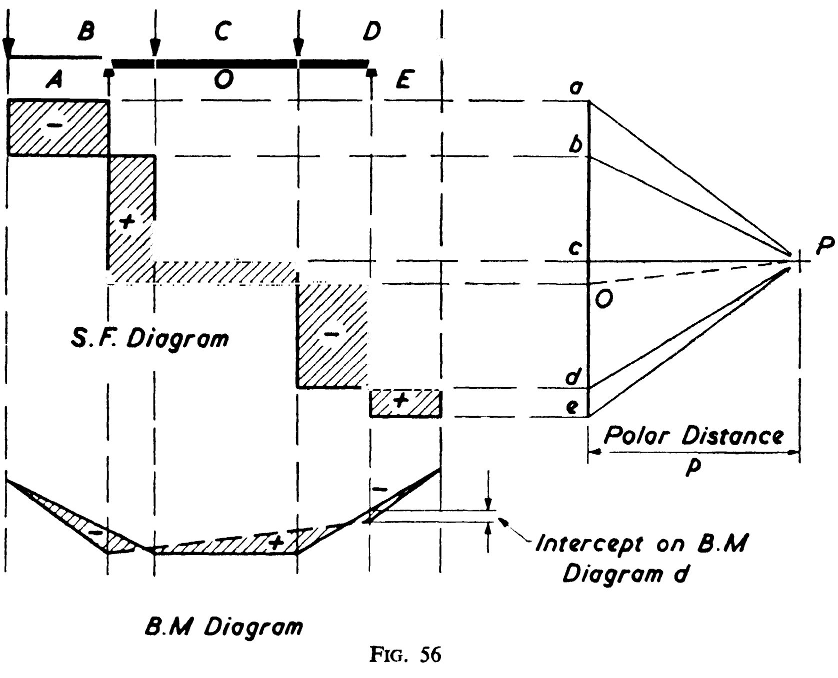


FIG. 56

The equilibrium polygon is also the B.M. diagram for the beam. To calculate the moment at any point the following measurements are made: the polar distance *p* is measured in load units from the load line to the pole *P*, and the vertical intercept, between the segments of the equilibrium polygon at the point at which the B.M. is required, is measured in space units. Then the B.M. at this point $= p \times d$. It should be noted that *p* and *d* must be at right angles. For ease of calculation, therefore, the polar distance should be taken as a convenient number of force units. The proof of this construction can be found in elementary text-books on mechanics. Should there be inclined loads on the beam it is usual to measure the polar distance at right angles to the resultant load line, and the intercept to the B.M. diagram at right angles to the polar distance.

General Notes on the Equilibrium Polygon

For any given set of coplanar forces an infinite number of equilibrium polygons may be drawn since an infinite number of poles may be selected, but it is occasionally necessary to draw the equilibrium polygon to pass through selected points. It will be clear that, if for each of the segments of

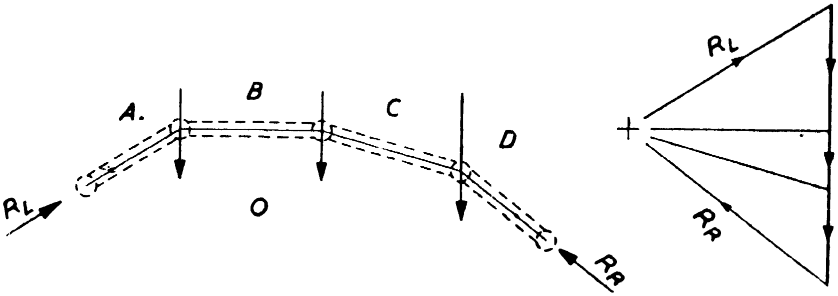


FIG. 57

the equilibrium polygon there is substituted a rigid member with frictionless hinged ends, the resulting structure will be in equilibrium under the given set of loads, and the forces in each member will be given by the corresponding line in the ray diagram as shown in Fig. 57.

Any segment of the equilibrium polygon can be drawn through a given

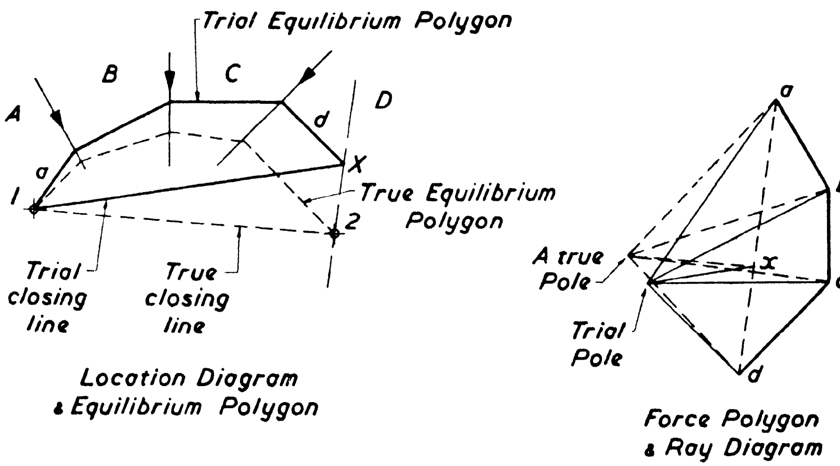


FIG. 58

point by first drawing this segment and completing the polygon on each side. Since the pole may be taken anywhere it follows that an infinite number of equilibrium polygons may be drawn through one point.

Should it be required to draw an equilibrium polygon through two points, the procedure is as shown in Fig. 58.

The force system is as shown and it is required that the equilibrium polygon should pass through points 1 and 2. The force polygon *abcd* is set down, giving the resultant load *ad* in magnitude and direction. A trial pole is selected and a trial ray diagram and equilibrium polygon are drawn to correspond, segment *a* passing through point 1, whilst segment *d* is ended at point *X* on a line through point 2 drawn parallel to the resultant load line, *ad*. The trial closing line 1*X* is drawn and the corresponding ray inserted in the ray diagram to give point *x* on the resultant load line. In order, however, for the equilibrium polygon to pass through points 1 and 2, the true closing line must be that joining these two points. If then a line is drawn from *x* parallel to line 1–2, a pole on this line will produce the appropriate equilibrium polygon. Since the pole may be taken anywhere on this line it follows that an infinite number of equilibrium polygons may be drawn to pass through two points.

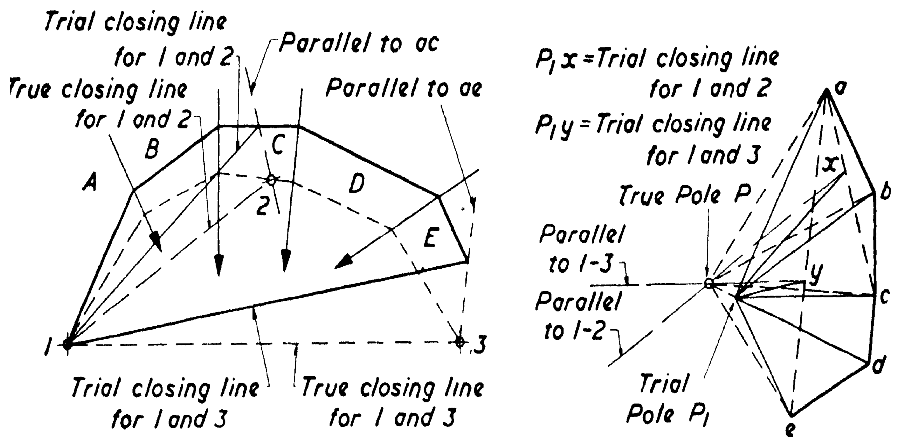


FIG. 59

The procedure for drawing the polygon through three points 1, 2 and 3 is exemplified in Fig. 59. The preceding construction is repeated twice; first for forces *AB*, *BC* between points 1 and 2 and then for all the forces between points 1 and 3, ignoring point 2. Points *x* and *y* on resultant load lines *ac* and *ae* are found from the trial closing lines in the trial polygon, and from these points lines are drawn parallel to the closing lines required 1–2 and 1–3. The intersection of these lines is the only possible position for a pole which will give an equilibrium polygon to pass through points 1, 2 and 3. It is therefore evident that only one equilibrium polygon can be drawn to pass through three points, and further that three points are the maximum number through which an equilibrium polygon can be drawn.

9. FORCES IN PLANE FRAMES

A PERFECT frame is composed of straight bars between node points, assumed to be frictionless pin joints, arranged in such a manner as to give direct forces only in the several bars. This result will be achieved if the frame consists of an assemblage of triangles formed by the bars, so that the total number of bars is $2N-3$, where N is the number of nodes, when all the external forces in the members of such a frame can be determined by simple statics in the following alternative ways:

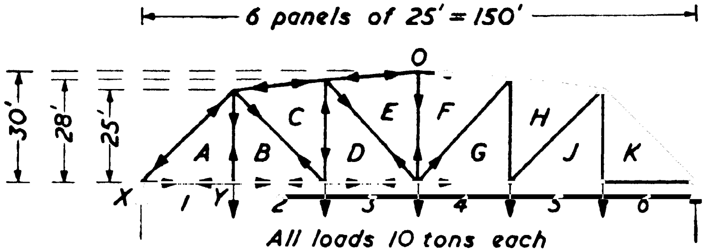
(a) graphically by the use of the reciprocal diagram;
(b) by the method of sections or moments;
(c) by trigonometry or joint resolution;
(d) from B.M. and S.F. diagrams;
(e) by resolution;
(f) from the equilibrium polygon.

It is self-evident that the external forces acting on any frame, that is to say the loads and reactions, must form a system of forces in equilibrium. Further, since direct forces only are to be induced in the frame members all external forces must act at node points in the frame, and all the forces, internal and external, at each node also constitute separately systems of forces in equilibrium.

(*a*) *Graphical Solution*

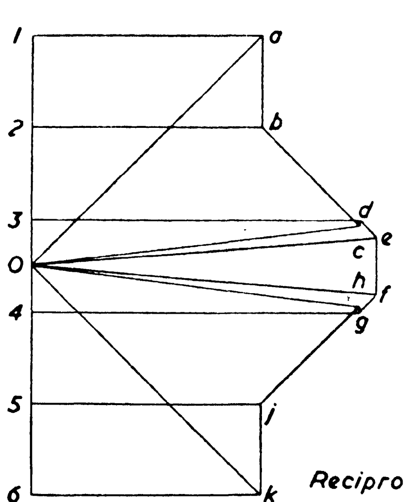
Two figures consisting of lines and points lying in a plane are said to be reciprocal when: (1) to any node of one figure at which a given number of lines meet there is a corresponding polygon in the other figure bounded by the same number of sides, (2) to every line in one figure there corresponds a parallel line in the other figure, and (3) to every line in one figure joining the nodes there corresponds a line in the other figure separating the polygons corresponding to those nodes. Then, if the first figure represents a framework with loads acting on it, the second or reciprocal figure will give the forces acting on the frame and the forces in the members of the frame. The reciprocal figure is, in fact, an assembly of force polygons for the various nodes combined into one diagram, and is commonly, though incorrectly, called a stress diagram. Each line in the figure, taken in opposite directions, represents two forces: one in each of two separate force polygons. Consider the truss shown in Fig. 60. The reactions at the supports, by the principle of symmetry, are 25 tons each, whence the load line is set down 1–6 and point O determined. Points in the force diagram are then found in the order a, b, c, d, e and f. The nature of the forces in the members is determined from the force polygons for the joints. For example, the force polygon for joint X is $O1$, $1a$, aO, and the arrows indicate the

direction of the forces for this joint, being determined from the known force $O1$ acting upwards. Similarly for joint Y the force polygon is $a1$, 12, $2b$, ab, the known directions being $a1$, which is $1a$ reversed, and 12.



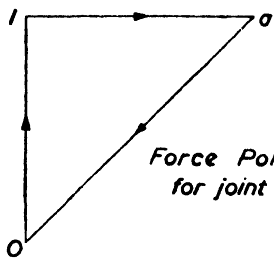
All loads 10 tons each

LOADED FRAME

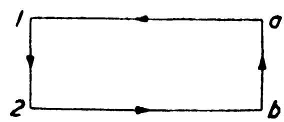


Members	Load in tons	Nature
1a, 5j, 2b, 6k.	25	T
3d, 4g.	35·7	T
Oa, Ok.	35·4	C
Oc, Oh.	36	C
Oe, Of.	37·8	C
ab, jk	10	T
bc, hj.	15·1	T
cd, gh.	0·7	C
de, fg.	2·9	T
ef	6	T

Reciprocal Figure or Stress Diagram



Force Polygon for joint X



Force Polygon for joint Y

FIG. 60

It should be noted that no more than two unknowns, corresponding to the two conditions for equilibrium of a coplanar concurrent system of forces, can exist at any one node if the solution is to be by simple

statics. Each line in the diagram, e.g. 1*a*, is used twice, once for each polygon and in opposite directions.

(*b*) *Solution by Method of Sections or Moments*

The basis of this method is the general case of equilibrium of non-concurrent forces. In Fig. 61 if *ABCDE* represents a portion of the

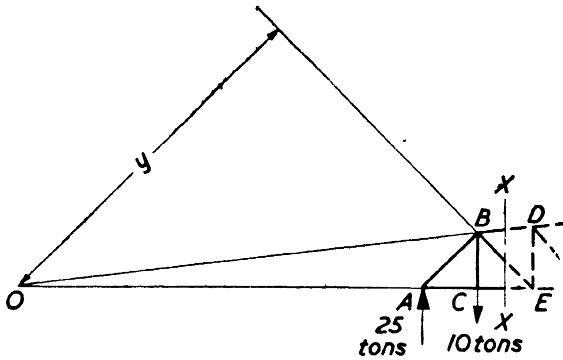


FIG. 61

frame shown in Fig. 60 cut by an imaginary section *XX*, then the forces in the bars cut by this section must hold in equilibrium all forces to one side of the section. All such sections must be drawn to cut three

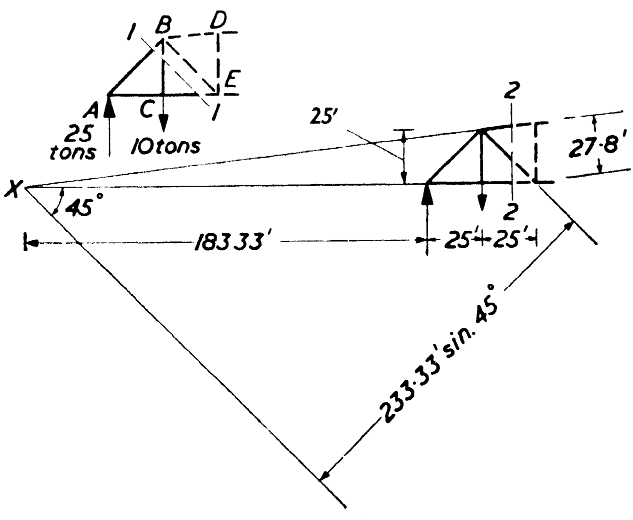


FIG. 62

bars in conformity with the three conditions for equilibrium, $\Sigma V=0$, $\Sigma H=0$ and $\Sigma M=0$. that is, the summation of the vertical components in the bars *BD*, *BE* and *CE* must be equal and opposite to the vertical force at the section; the horizontal components must summate to zero

and the moments of the forces in the bars taken about any convenient point must balance the moment of the external forces to one side of the section taken about the same point. Thus by taking moments about a point where two of the bars meet, e.g. point O, the moment of the force in the remaining bar BE also taken about O, $F_{BE} \times y$ must equal the moment about that point of the external forces at A and C.

Consider Fig. 62.

Section	*Member*	*Moments about*	*Moments*
1–1	AB	C (Intersection of BC and CE)	$+25$ tons $\times 25$ ft. $-F_{AB} \times 0{\cdot}707 \times 25$ft. $=0$. $F_{AB}=25 \times 1{\cdot}414=35{\cdot}4$ tons acting towards A, i.e. F_{AB} is compression.
	BC	A (Intersection of AB and CE)	-10 tons $\times 25$ ft. $+F_{BC} \times 25$ ft. $=0$. $F_{BC}=10$ tons acting away from C, i.e. F_{BC} is tension.
	CE	B (Intersection of AB and BC)	$+25$ tons $\times 25$ ft. $-F_{CE} \times 25$ ft. $=0$. $F_{CE}=25$ tons acting away from C, i.e. F_{CE} is tension.
2–2	BD	E (Intersection of BE and CE)	$+25$ tons $\times 50$ ft. -10 tons $\times 25$ ft. $+F_{BD} \times 27{\cdot}8$ ft. $=0$. $F_{BD}=\frac{1000}{27{\cdot}8}=36{\cdot}0$ tons compression.
	BE	X (Intersection of BD and CE)	-25 tons $\times$ $183{\cdot}3$ ft. $+$ 10 tons $\times$ $208{\cdot}3$ ft. $-F_{BE} \times 0{\cdot}707 \times 233{\cdot}3$ ft. $=0$. $F_{BE}=15{\cdot}1$ tons tension.

The remaining members may be calculated in a similar manner, but when dealing with the centre vertical it will be found impossible to draw a section other than to cut four bars, and in this case the loads in the other three bars must be known before the load in the centre vertical can be evaluated.

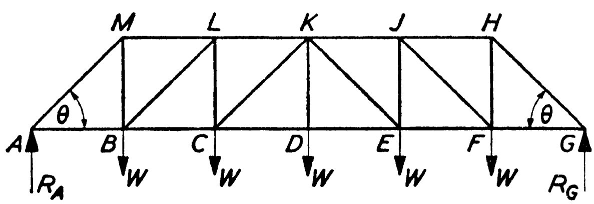


FIG. 63

(*c*) *Solution by Trigonometry or Joint Resolution*

This method is particularly useful for the solution of girders with parallel booms where the web members are inclined at constant angles. Consider the girder shown in Fig. 63. At joint A the known force is R_A, whence, for equilibrium, the force in AB, $F_{AB}=F_{FG}=R_A \cot \theta$,

whilst the force in AM is $F_{AM} = F_{HG} = R_A \operatorname{cosec} \theta$. Similarly, at joint M the known force is $F_{AM} = R_A \operatorname{cosec} \theta$, whence for equilibrium $F_{ML} = F_{JH} = R_A \cot \theta$, whilst $F_{BM} = F_{HF} = R_A$. At joint B there are two known forces F_{AB} and F_{BM}, whence the two unknown forces may be determined. The remaining joints are treated in a similar fashion and it will be noted that the method corresponds to the graphical construction in that the equilibrium is calculated separately for each joint.

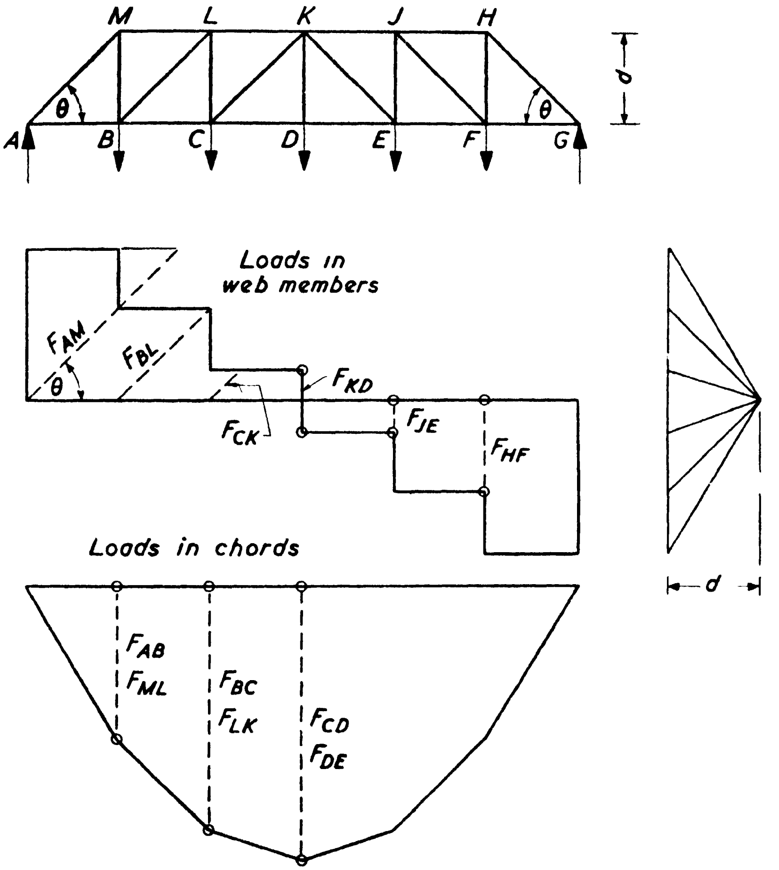


FIG 64

(*d*) *Solution by B.M. and S.F. Diagrams*

This is an alternative method useful for the solution of parallel boom girders. Consider the girder shown in Fig. 64, then, by the method of sections, F_{CD} = B.M. at K divided by the depth between the booms whilst F_{LK} = B.M. at C divided by the depth. Hence the B.M. diagram to some scale will represent the boom forces or, if the B.M. diagram is drawn with a polar distance equal to the depth it will give the chord forces to the load scale.

The shear in any panel is resisted wholly by the web members since the chords are horizontal, thus the loads in the verticals are read directly from the S.F. diagram whilst the loads in the diagonals are found by resolving the shearing force in the direction of the diagonals.

(*e*) *Solution by Resolution*

This method consists of resolving the resultant force acting on any section into components in the directions of the three bars cut by the

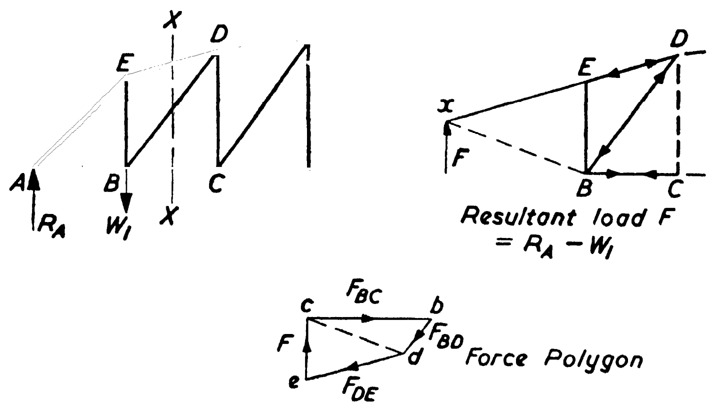


FIG. 65

sections and is particularly useful in finding stresses in web members in girders with non-parallel flanges subjected to varying loads.

Consider Section *XX* in Fig. 65. The resultant load on this section is $R_A - W_1$ acting where shown: the resultant of F_{DE}, F_{BD} and F_{BC} must be in equilibrium with this load. Produce *DE* to cut the line of action of *F* at point *x* and join this to *B*, the point of intersection of the other two bars cut by the section. Draw *ec* to represent *F*, *ed* parallel to *DE*, *cd* parallel to *Bx* to give point *d*. Then draw *bd* parallel to *BD* and *bc* parallel to *BC*, which gives the force polygon for the four forces concerned.

(*f*) *Solution from the Equilibrium Polygon*

This method is the application of the method of sections, but applied to the forces delineated by the equilibrium polygon in lieu of the original loads and reactions and in consequence is of particular application to three-hinged arch trusses. An example is illustrated in Fig. 66, where *ABCDE* is part of a frame and F_1 and F_2 are the forces in the segments of the equilibrium polygon.

Then

$$F_{AB} = F_1 \times y/z \text{ (moments about } E\text{).}$$

$$F_{DE} = F_2 \times w/x \text{ (moments about } B\text{).}$$

$$F_{BE} = F_2 \times u/v \text{ (moments about } X\text{).}$$

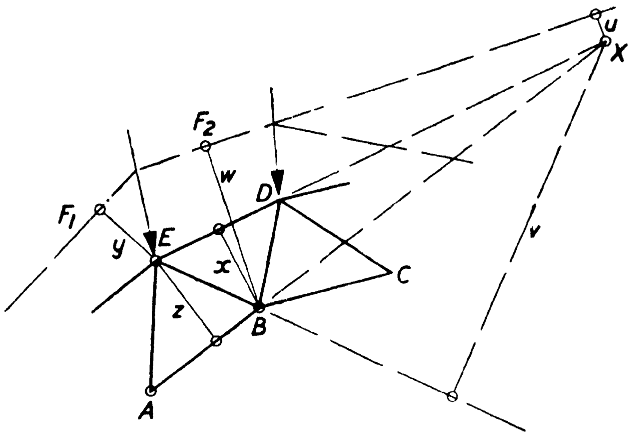


FIG 66

Forces in Knee-braced Frames

Fig. 67 gives sections through a knee-braced shed (a) with columns hinged at their bases, (b) with columns fixed at their bases, with exaggerated diagrams of the deformed shape under side-wind loading with the resulting B.M. diagram on the columns, both knee-braces being assumed to act.

It is necessary first to find the reactions, and, considering case (a), there are four unknowns to be found, a vertical and a horizontal force at each hinge, so that the external forces are an indeterminate system. The problem can only be rendered statically determinate by a reduction of the unknowns to three or less, and the usual practice in this matter is to assume a relationship between the horizontal forces at the hinges. These are usually proportioned according to the stiffnesses of the columns, and hence, if of equal section, length and moment of inertia, as is usually the case, the columns will share the horizontal load equally between them and hence the horizontal reactions are known, and the vertical reactions can be found by simple statics. Turning now to case (b), a further assumption is necessary regarding the position of the points of contraflexure, normally taken as mid-way between the base of the column and the knee-brace. These points having been determined, the portion of the frame above them reduces to case (a).

In both cases the columns are subjected to direct and shear forces and bending moments, and therefore cannot be evaluated by simple statics. The method of calculating these values and the forces in the members of the frame is best explained by the examination of a typical problem, as shown in Fig. 68 (a).

To make the example as comprehensive as possible, the wind loads used will be those given in B.S. Code of Practice CP3—Chapter V (1952). If it is assumed that $p = 15$ lb. per sq. ft., and allowance is made for internal pressure, the following figures result, forming the basis of Fig. 68 (b).

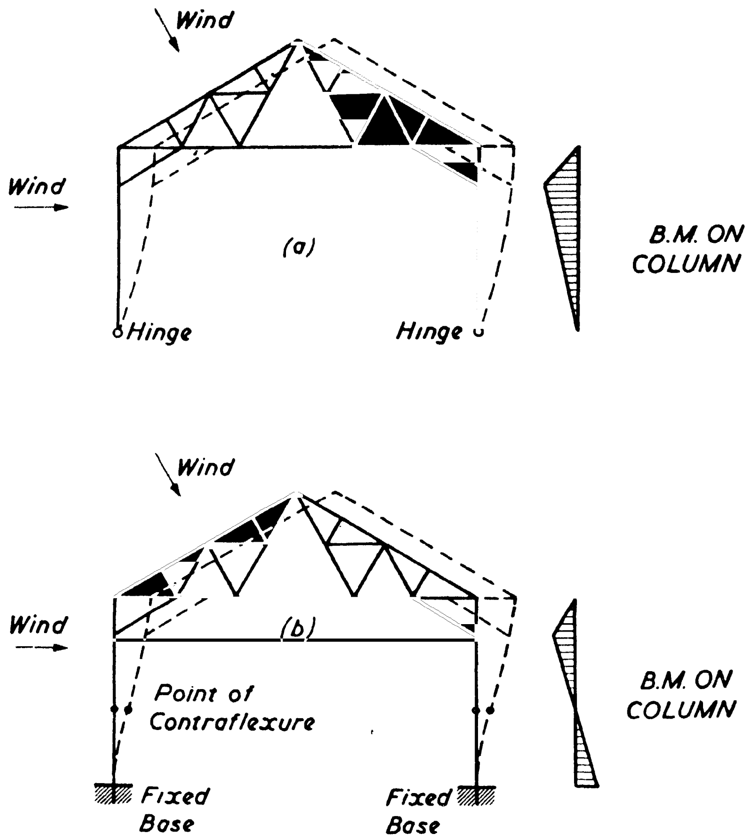


FIG. 67

$W_1 = +0{\cdot}5p - 0{\cdot}2p = +0{\cdot}3p$
$= 0{\cdot}3 \times 15 \times 12{\cdot}5$ ft. $\times 15$ ft. $= 844$ lb. = say $0{\cdot}38$ ton.

$W_2 = -0{\cdot}1p - 0{\cdot}2p = -0{\cdot}3p$
$= 0{\cdot}3 \times 15 \times 12{\cdot}5$ ft. $\times 28{\cdot}86$ ft. $= 1623$ lb. = say $0{\cdot}73$ ton.

$W_3 = -0{\cdot}45p - 0{\cdot}2p = -0{\cdot}65p$
$= 0{\cdot}65 \times 15 \times 12{\cdot}5$ ft. $\times 28{\cdot}86$ ft. $= 3517$ lb. = say $1{\cdot}57$ tons.

$W_4 = +0{\cdot}5p + 0{\cdot}2p = +0{\cdot}7p$
$= 0{\cdot}7 \times 15 \times 12{\cdot}5$ ft. $\times 15$ ft. $= 1968$ lb. = say $0{\cdot}88$ ton.

Total horizontal load, left to right,

$$= 0{\cdot}38 + 0{\cdot}5(1{\cdot}57 - 0{\cdot}73) + 0{\cdot}88$$
$$= 1{\cdot}68 \text{ tons}$$

Whence $H_L = H_R = 0{\cdot}84$ ton . . . . (a)

Total vertical load, upwards,

$$= 0{\cdot}866(0{\cdot}73 + 1{\cdot}57)$$
$$= 1{\cdot}99 \text{ tons.}$$

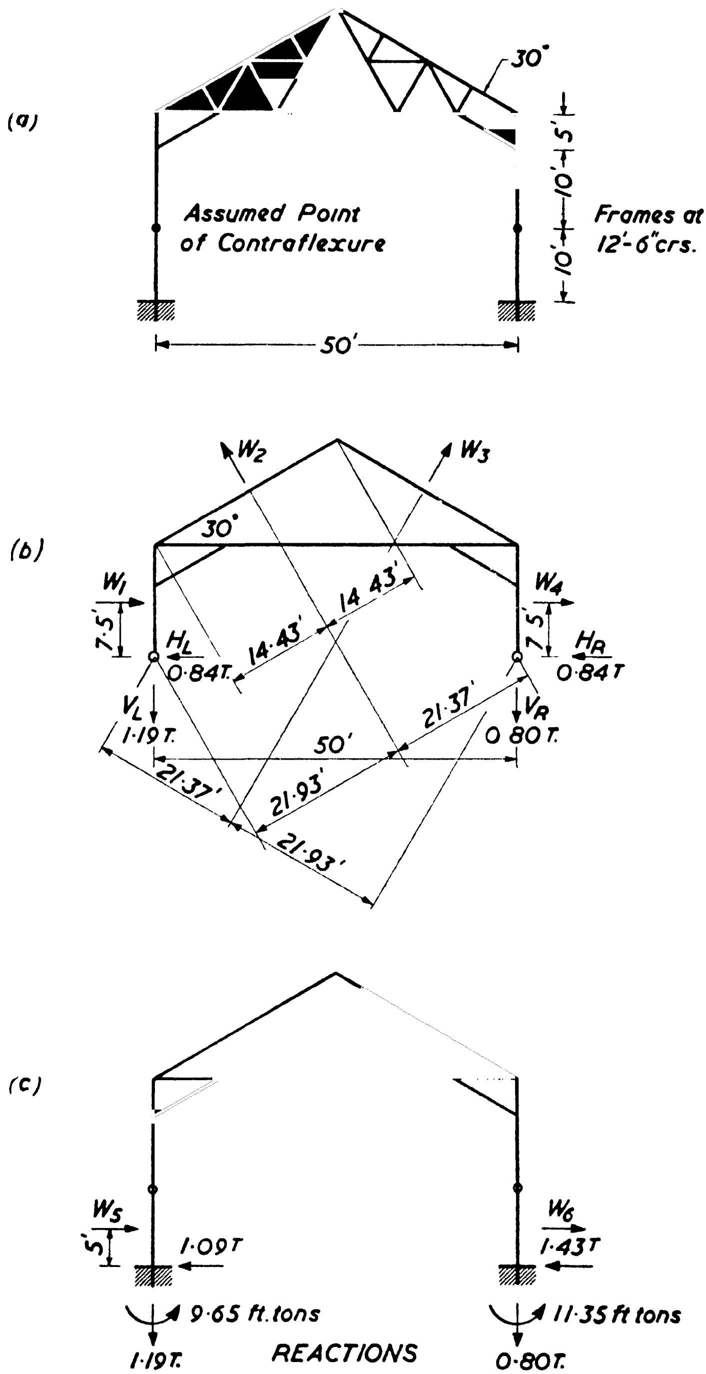


FIG. 68 (*a*) (*b*) and (*c*)

Moment about left hinge,

$$V_R = \frac{(0{\cdot}38 + 0{\cdot}88) \times 7{\cdot}5 \text{ ft.} - 0{\cdot}73 \times 21{\cdot}93 \text{ ft.} - 1{\cdot}6 \times 21{\cdot}37 \text{ ft.}}{50}$$

$= 0{\cdot}80$ ton (downwards) . . . . (b)

Moment about right hinge,

$$V_L = \frac{(0{\cdot}38 + 0{\cdot}88) \times 7{\cdot}5 \text{ ft.} + 0{\cdot}73 \times 21{\cdot}37 \text{ ft.} + 1{\cdot}6 \times 21{\cdot}93 \text{ ft.}}{50}$$

$= 1{\cdot}19$ tons (downwards) . . . . (c)

These values (a), (b) and (c) are reactions at the points of contra-flexure.

The additional loads on the columns below these points are:

$W_5 = 0{\cdot}3 \times 15 \times 12{\cdot}5 \times 10 = 562$ lb. $= 0{\cdot}25$ ton

$W_6 = 0{\cdot}7 \times 15 \times 12{\cdot}5 \times 10 = 1313$ lb. = say $0{\cdot}59$ ton.

Taking these into consideration the reactions at the fixed bases will be:

Windward Column

Vertical reaction $= -1{\cdot}19$ tons.
Horizontal reaction $= -0{\cdot}84 - 0{\cdot}25 = -1{\cdot}09$ tons.
Fixing moment $= -(0{\cdot}84 \times 10 + 0{\cdot}25 \times 5) = -9{\cdot}65$ tons ft.

Leeward Column

Vertical reaction $= -0{\cdot}8$ ton.
Horizontal reaction $= -0{\cdot}84 - 0{\cdot}59 = -1{\cdot}43$ tons.
Fixing moment $= -(0{\cdot}84 \times 10 + 0{\cdot}59 \times 5) = -11{\cdot}35$ tons ft.

In order to draw the stress diagram for the roof truss, it is necessary to make allowance for the B.M.s in the columns. This can be done in two ways: (a) by removal of the columns and substitution in their place of systems of forces applied at nodes, whose effect will be the same as that of the columns, and (b) by the addition of temporary framework arranged so that moments in the columns cause direct stresses only in the auxiliary frames.

The calculations for method (a) are shown in Fig. 68 (d) and (e). Considering the windward column, the total side wind load on this, which is $W_1 = 0{\cdot}38$ ton, is divided proportionally to the column lengths, as to 0·06 ton at the eaves, 0·19 ton at the knee-brace level, and 0·13 ton at the hinge. These forces are then combined with the reactions at the point of contra-flexure, resulting in final forces at the hinge of 1·19 tons vertically downwards and 0·71 ton horizontally. This last force must be transferred to the frame, and its effect about the knee-brace level is a horizontal shear of 0·71 ton and a moment of 7·1 tons ft.; the two forces shown of 1·42 tons at eaves and 2·13 tons at knee-brace level, will reproduce both the shear and the moment. To these forces are then added algebraically the appropriate wind loads, and the final truss reactions are shown on the extreme right of the diagram in Fig. 68 (d). Similarly, the forces on the leeward column are evaluated and the final truss reactions shown in Fig. 68 (e).

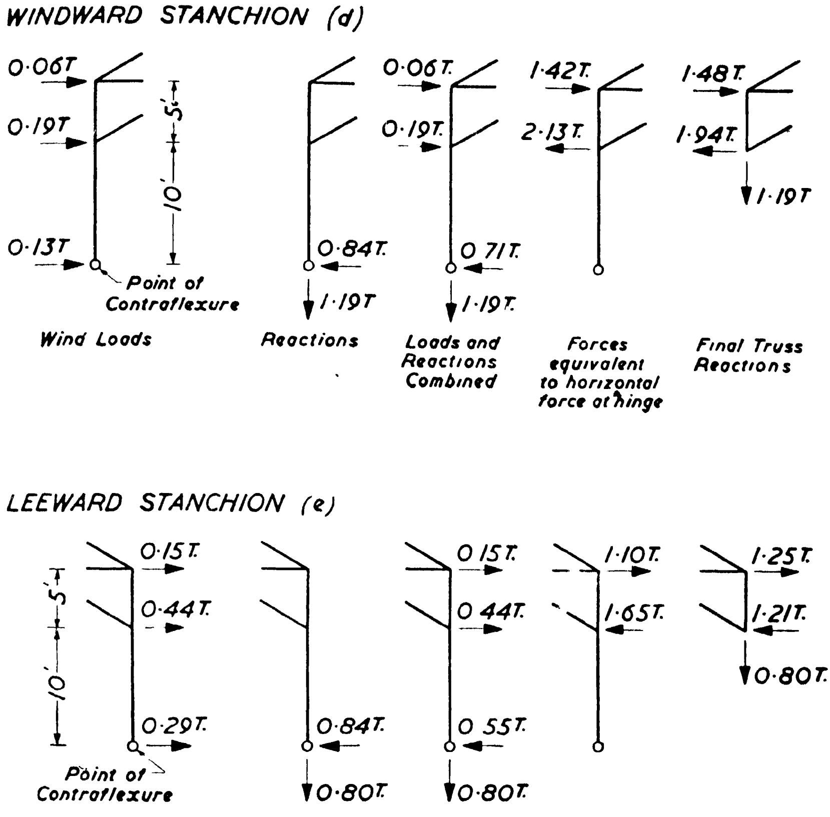


FIG. 68 (*d*) and (*e*)

These six forces hold the truss in equilibrium against the applied wind loads, and the stress diagram is reproduced in Fig. 69.

The direct forces in the columns above the knee-brace are given directly by the stress diagram; the vertical forces below this are 1·19 tons and 0·80 tons tension (windward and leeward respectively), the shears are 0·84 ton each. In addition, there is a moment of 7·1 tons ft. at knee-brace level and a moment of opposite sign of 9·65 tons ft. at the base for the windward column, with corresponding figures 5·5 tons ft. and 11·35 tons ft. for the leeward column.

The solution for method (b) is shown in Fig. 70. Any convenient framework arrangement can be used as indicated by dotted lines: in this case the truss rafter has been prolonged to meet the horizontal through the foot of the knee-brace, to enable the wind loads previously calculated to be used in conjunction with the previously determined reactions at the points of contraflexure. The stress diagram is then drawn in the usual manner.

The stresses given in the diagram are correct for all members other than the columns, for these have been affected by the addition of the auxiliary

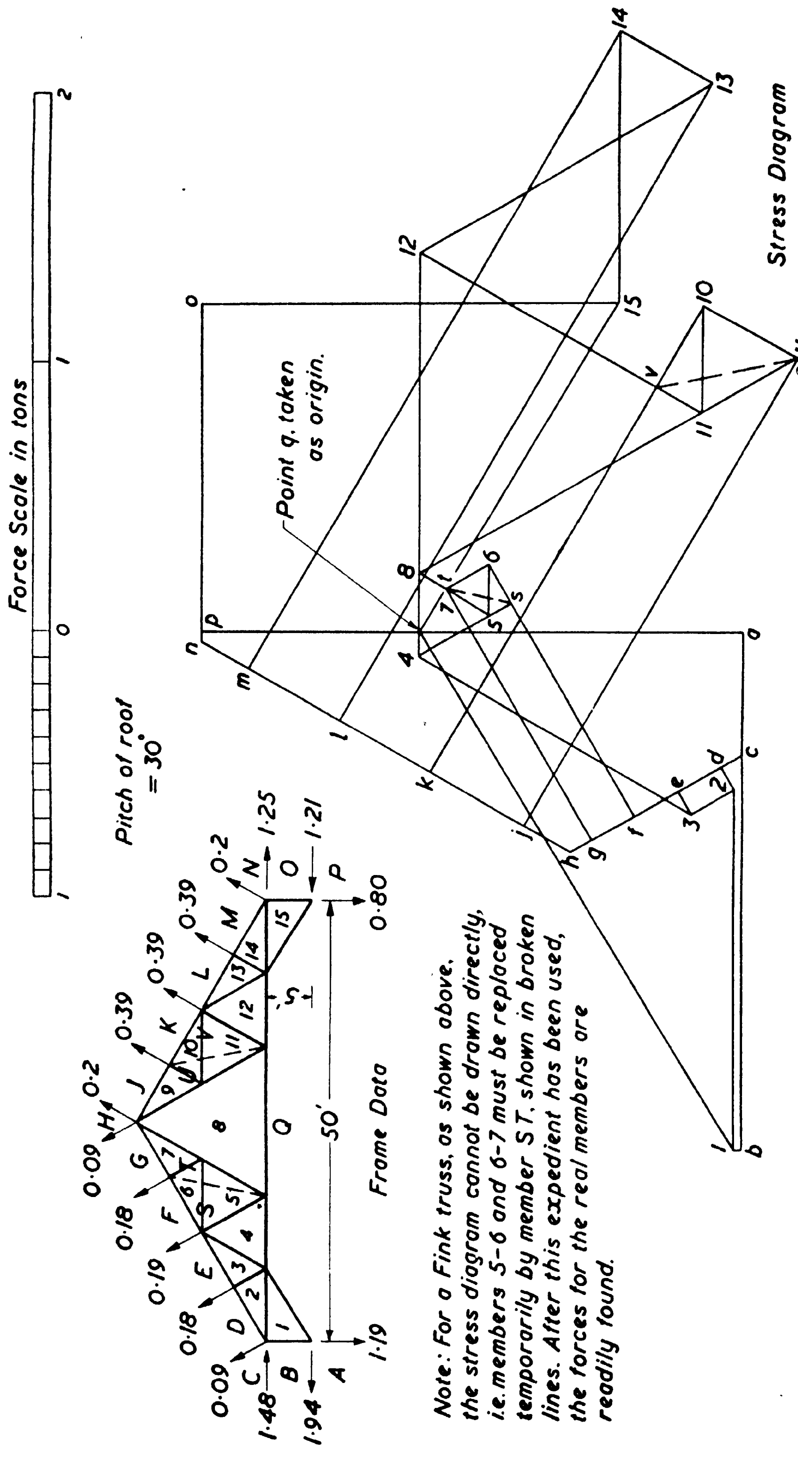

Force Scale in tons
1
0
1
2
Pitch of roof
= 30°
Frame Data
50'
1·48
1·94
0·09
0·18
0·19
0·18
0·09
0·2
0·39
0·39
0·39
0·39
0·2
1·25
1·21
1·19
0·80
A
B
C
D
E
F
G
H
J
K
L
M
N
O
P
Q
S
T
U
V
Point q. taken
as origin.
Stress Diagram
Note: For a Fink truss, as shown above,
the stress diagram cannot be drawn directly,
i.e. members 5-6 and 6-7 must be replaced
temporarily by member ST, shown in broken
lines. After this expedient has been used,
the forces for the real members are
readily found.

FIG. 69

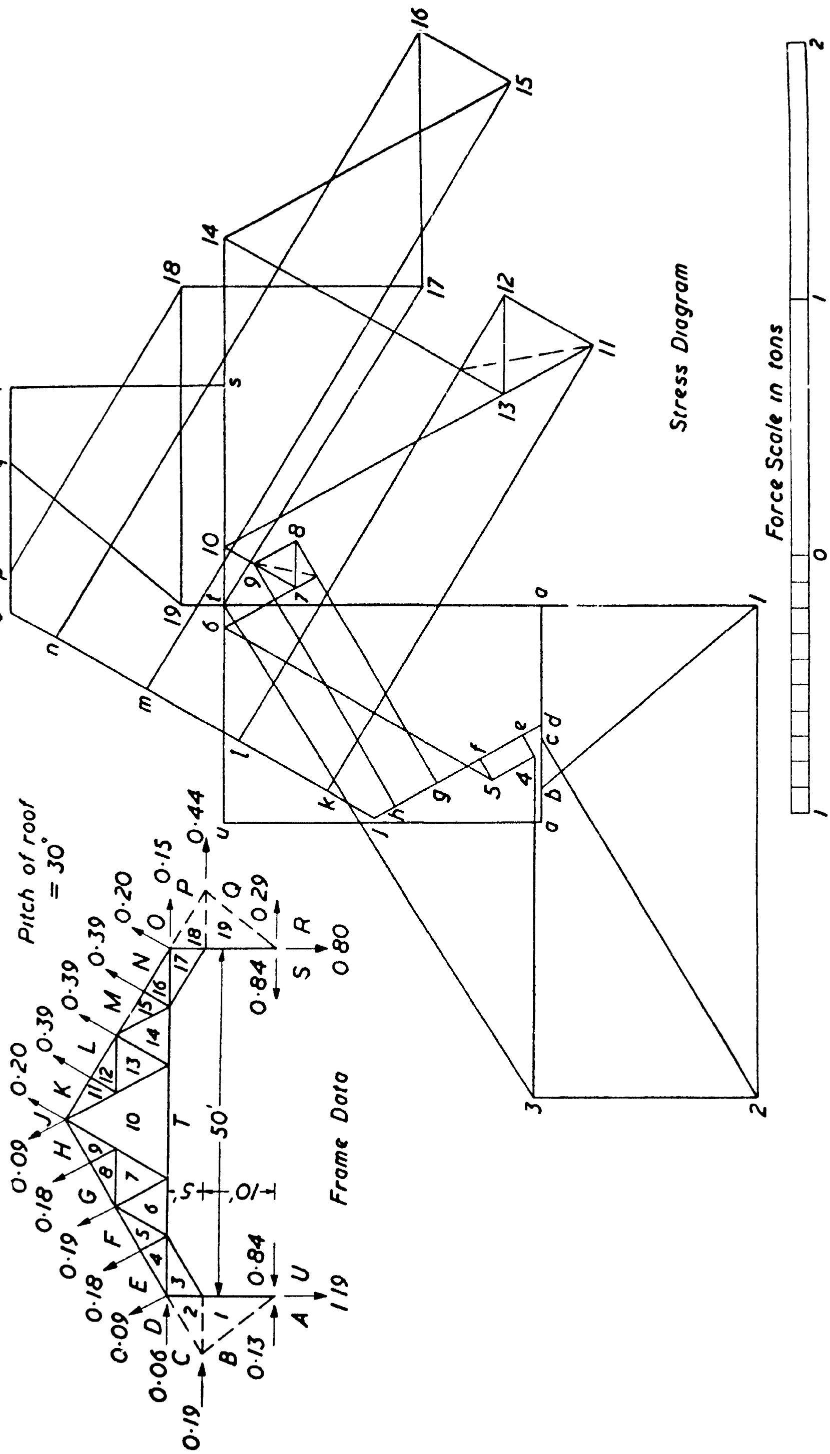

Stress Diagram
Force Scale in tons
Frame Data
Pitch of roof
= 30°

FIG. 70

framework. To determine the stresses in the columns, the auxiliary frames are removed and each column considered separately. Taking the windward column, then it is seen that below the knee-brace the direct force must equal the reaction, whilst in addition there is a moment of maximum value 7·1 tons ft. at knee-brace level. For the upper portion of the column the direct force must be such that it will hold in equilibrium the vertical reaction and the vertical component of the force in the knee-brace.

It will be found that the stress diagram for method (a) can be constructed more easily than that for method (b), since there are four fewer joints to be considered and also since the diagram is more compact; moreover, the first method gives column forces directly, whereas the second does not. Both methods, of course, will give the same final answers.

The method of treating knee-braced frames described here is approximate, but it has been found satisfactory in practice. A more rigorous and accurate analysis would involve a consideration of the theory associated with portal frames.

The Three-hinged Arch

The three-hinged arch or portal is the only arch construction in which the horizontal reactions are statically determinate since there are three known points on the equilibrium polygon, or as it is more usually known in the case of an arch, the line of pressure.

The reactions and line of pressure can be found in three ways: (i) by drawing the equilibrium polygon to pass through the three hinges, (ii) by calculation of vertical reactions and horizontal thrust and constructing the ray diagram accordingly, or (iii) by superimposing upon the arch the free B.M. diagram caused by the loads acting as on a beam spanning between the two outer hinges, the scale being such that the B.M. diagram passes through the crown hinge.

The arch, XYZ, chosen for the examples is 60 ft. in span and carries 5 ton loads at 10 ft. intervals. The left hinge, X, is 10 ft. higher than the right hinge, Z.

Method (i) (see Fig. 71)

The load line *abcdef* is set down, a trial pole taken and the construction explained on page 92 is then followed, resulting in the pole of the ray diagram being fixed at point j in the force polygon, giving the reactions shown in Fig. 71, namely $V_L = 10$ tons, $V_R = 15$ tons and $H = 15$ tons. The hatched diagram, i.e. the difference between the equilibrium polygon (or line of pressure) and the line representing the arch axis, is the B.M. diagram for the arch, and the B.M. under load CD is given by the product of the polar distance jc of 15 tons and the vertical intercept between the line of pressure and arch axis at that point, namely 2 ft. 6 in., giving a B.M. of 37·5 tons ft.

It should be noted that the polar distance jc represents the horizontal thrust H, and consequently the B.M. at any point in the arch is given by the product of the horizontal thrust and the vertical intercept between the line of pressure and the arch axis at the point.

Method (ii)

The B.M. on the arch is zero at the hinge points X, Y and Z. Consequently, the following equations may be evolved by taking moments to the left of Y and Z respectively:

$$\Sigma M_Y = V_L \times 20 \text{ ft.} - 5 \text{ tons} \times 10 \text{ ft.} - H \times 10 \text{ ft.} = 0$$

$$\Sigma M_Z = V_L \times 60 \text{ ft.} - 25 \text{ tons} \times 30 \text{ ft.} + H \times 10 \text{ ft.} = 0$$

whence $V_L = 10$ tons, $H = 15$ tons and $V_R = 15$ tons.

With these figures known the ray diagram *abcdefgjka* can be constructed and the line of pressure drawn. The B.M.s on the arch can be found as for

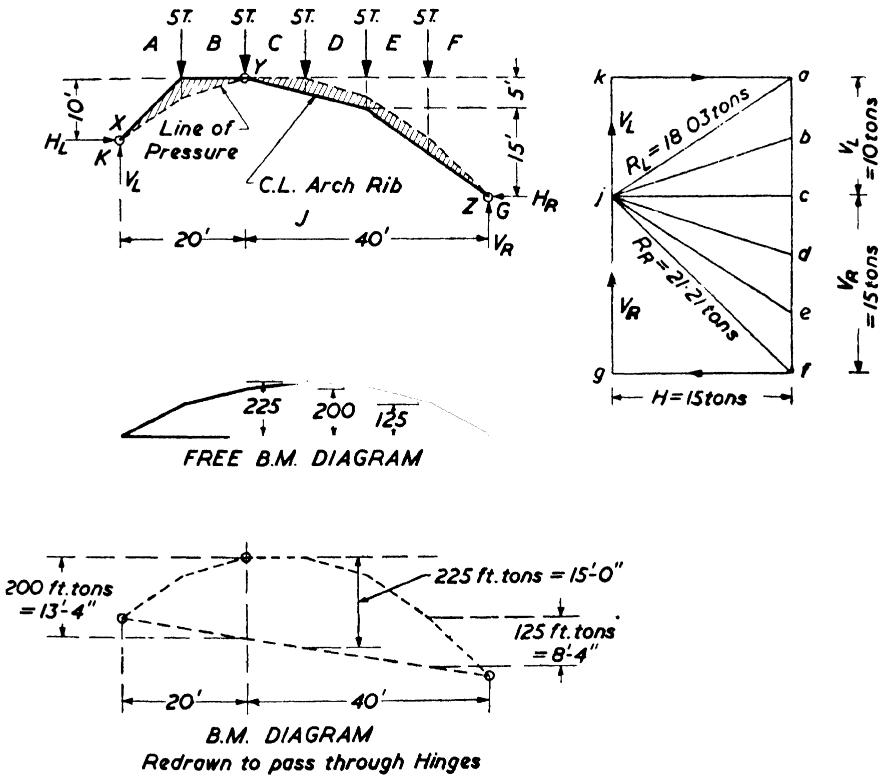


FIG. 71

method (i) or may be obtained directly by calculation, e.g. taking moments of all forces to the right of load CD we have

$$\begin{aligned} M_{CD} &= 15 \text{ tons} \times 30 \text{ ft.} - 15 \text{ tons} \times 17{\cdot}5 \text{ ft.} - (5 \text{ tons} \times 10 \text{ ft.} + 5 \text{ tons} \times 20 \text{ ft.}) \\ &= -37{\cdot}5 \text{ tons ft.} \end{aligned}$$

Method (iii)

The free B.M. diagram, or B.M. diagram for the five loads on a span of 60 feet, is as shown, and the B.M. at the crown hinge is 200 tons ft. For

the B.M. diagram to pass through the crown hinge the moment of 200 tons ft. must be represented by the vertical distance of 13 ft. 4 in. shown in the figure, that is, the space scale for moments is 15 tons ft. per foot on the scale layout of the arch. The B.M. diagram is then redrawn to this scale on the sloping base line between the two end hinges and superimposed on the line representing the arch axis, giving the same result as before.

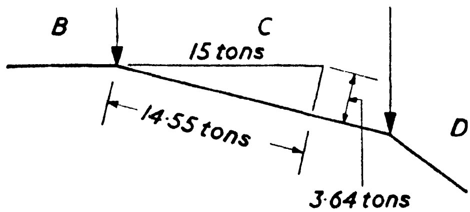


Fig. 72

The direct compression and shear in the rib at any point can be obtained by resolution from the line of pressure, as, for example, between *BC* and *CD* the force is 15 tons which gives 14·55 tons rib compression and 3·64 tons shear as shown in Fig. 72. With regard to the B.M. it should be noted that tension on the underside of the arch occurs when the line of pressure is above

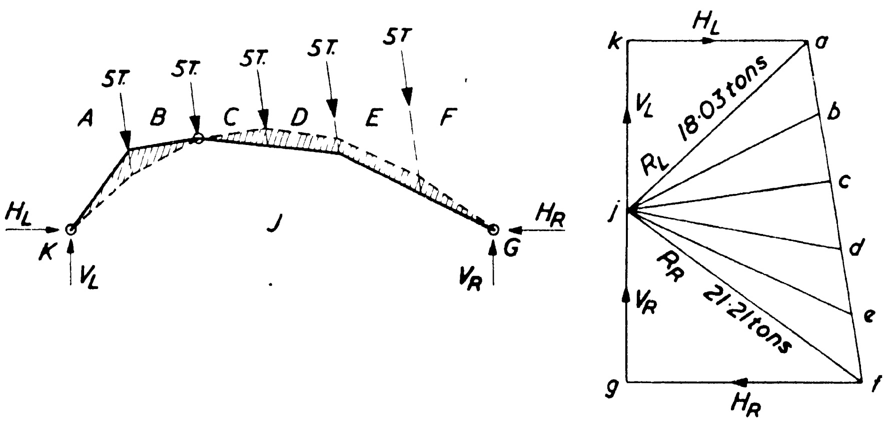


Fig. 73

the arch axis and, conversely, tension occurs on the top of the arch when the line is below the axis.

It will be noted that the reactions at the supports are compounded of the horizontal thrust, equal at both ends, and the vertical reactions. The horizontal thrusts at supports are in this case equal and opposite since there are no loads other than vertical loads on the arch; should there be horizontal loads or inclined loads having horizontal components then the horizontal reactions will not be equal and opposite and the horizontal thrust will not be constant across the arch. This can be shown if the arch previously con-

sidered is rotated so that the end hinges are in a horizontal line, all dimensions remaining unaltered as shown in Fig. 73. The magnitudes of the reactions will be unaltered, since the layout of the force system has not changed, but their vertical and horizontal components alter as indicated.

To deal with lattice or braced three-hinged arches, it is only necessary to calculate the reactions by any of the methods indicated, and, this having been done, the forces in the members of the frame can be found either graphically or analytically.

10. DEFLECTION OF FRAMED STRUCTURES

Mathematical Method

When a structure is loaded, deflection will occur, and each of the loads on the structure will move through a certain distance thereby doing a certain amount of *external* work. Each of the members will become strained and in so doing will absorb a certain amount of *internal* work. According to the Principle of Work, the external work done on a structure must be equal to the internal work absorbed in straining the structure, and by applying this principle the deflection of the structure can be determined.

For the purpose of dealing with statically determinate plane frames in which all the loads are applied at node points and in which the supports are assumed not to move in the direction of the applied load, the mathematical treatment is relatively simple, since the internal work is done only in elongating and shortening the members of the frame.

Fig. 74 shows a simple bracket consisting of two members AB and BC carrying a load W. The supports A and C are fixed in position.

Under the load W, AB will be in tension while BC will be in compression, and the point B will deflect by an amount d, the frame taking up the position shown by the dotted lines.

Let L_1 = length of member AB,

A_1 = the cross-sectional area of member AB,

F_1 = force in member AB (tension considered positive),

x_1 = extension of member AB (positive),

L_2 = length of member BC,

A_2 = cross-sectional area of member BC,

F_2 = force in member BC (compression considered negative),

and x_2 = shortening of member BC (negative).

Then, if E is constant for both members,

$$x_1 = \frac{F_1 L_1}{A_1 E}$$

and

$$x_2 = -\frac{F_2 L_2}{A_2 E}.$$

Now if the load is applied so as to increase gradually from zero to the value W the work done will be the average force acting × the distance moved through and therefore the total work done in straining the members

$$= \tfrac{1}{2}F_1 x_1 + \tfrac{1}{2}F_2 x_2$$

$$= \frac{1}{2} \cdot \frac{F_1^2 L_1}{A_1 E} + \frac{1}{2} \cdot \frac{F_2^2 L_2}{A_2 E}.$$

The plus sign is used because both F_2 and x_2 are negative.

Therefore, for any number of members, the total internal work done

$$=\frac{1}{2}\sum\frac{F^2L}{AE}.$$

At the same time the external work done will be $\frac{1}{2}W.d.$

Therefore, in the example,

$$d=\frac{F_1^2L_1}{WA_1E}+\frac{F_2^2L_2}{WA_2E}$$

Now F_1 and F_2 will be proportional to W, and if U_1 and U_2 represent the

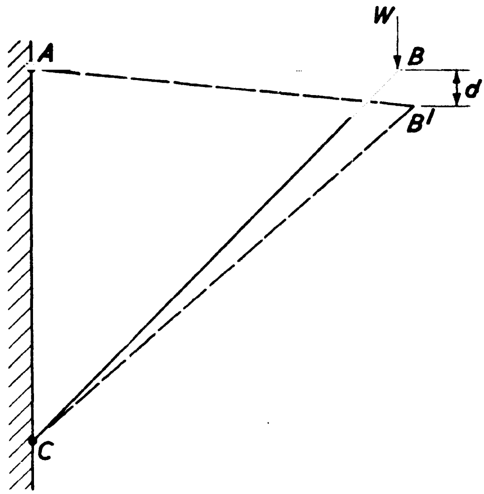


FIG. 74

forces caused in AB and BC respectively when a unit load is placed at B, then $U_1=F_1/W$ and $U_2=F_2/W$, consequently

$$d=W\left(\frac{U_1^2L_1}{A_1E}+\frac{U_2^2L_2}{A_2E}\right)$$

or in the general case

$$d=W\sum\frac{U^2L}{AE}.$$

Fig. 75 represents a more general case.

It is required to find the deflection of the point P in the direction C.

Consider the member 1 (AB) and imagine that, for the moment, all the other members are rigid, i.e. suffer no deformation.

Then, if a unit load is applied at P in the direction C and causes a deflection d_1 at P, the external work done

$$=\tfrac{1}{2}\times 1\times d_1.$$

The internal work done $=\frac{1}{2}U_1x_1$ where

$U_1=$ force in member 1 due to unit load at P

and $x_1=$ strain in member 1 due to unit load at P.

Therefore,

$$\tfrac{1}{2}\times 1\times d_1=\tfrac{1}{2}U_1x_1$$

or

$$d_1=U_1x_1.$$

If, instead of placing unit load at P, the member 1 had been strained by an amount x_1, the deflection at P would have been d_1.

Accordingly the following rule is derived:

> The deflection of any point in a framed structure in a given direction due to an extension x in any one member of the structure is equal to the load in that member caused by unit load at the given point, acting in the given direction, multiplied by the extension x.

This rule can now be extended to cover all the members in the frame and consequently the total deflection $=\Sigma Ux$.

In a loaded structure x will be the extension in any member due to the

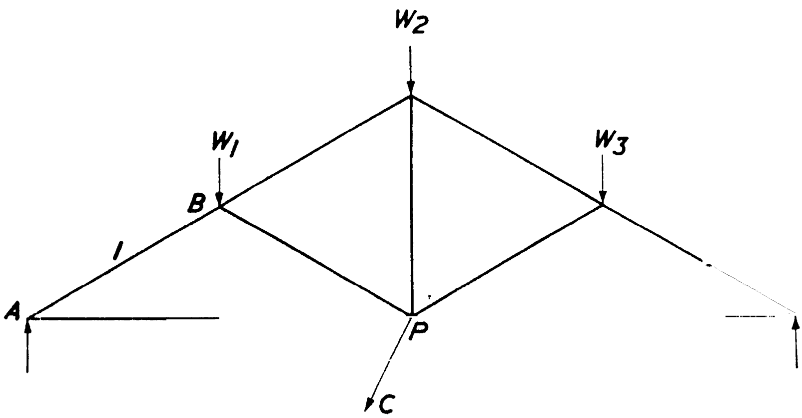


FIG. 75

force F in the member caused by the loading and $x=FL/AE$, where A is the cross-sectional area of the member.

Therefore the total deflection

$$d=\sum\frac{UFL}{AE}.$$

A complete determination of the movements of all node points in a girder requires two deflection calculations for each joint, vertical and horizontal. The graphical method referred to later gives a single displacement diagram from which the resultant movement of all the joints can be determined at the same time. The algebraic solution is exact while the graphical solution, although limited by the size of the drawing and the inherent inaccuracies in all graphical work, is a great saver of time.

Deflections due to temperature changes may be calculated by treating the change in length due to variation in temperature in a similar manner to strain caused by stress.

Deflections due to the forced distortion of members may be calculated by replacing FL/AE by the estimated or known change in length, increase in length being positive and decrease negative.

Example 1. Calculate the vertical deflection of the central point P of the lower boom of the Warren girder loaded symmetrically as shown in Fig. 76 (a). $E=13{,}000$ tons per sq. in.

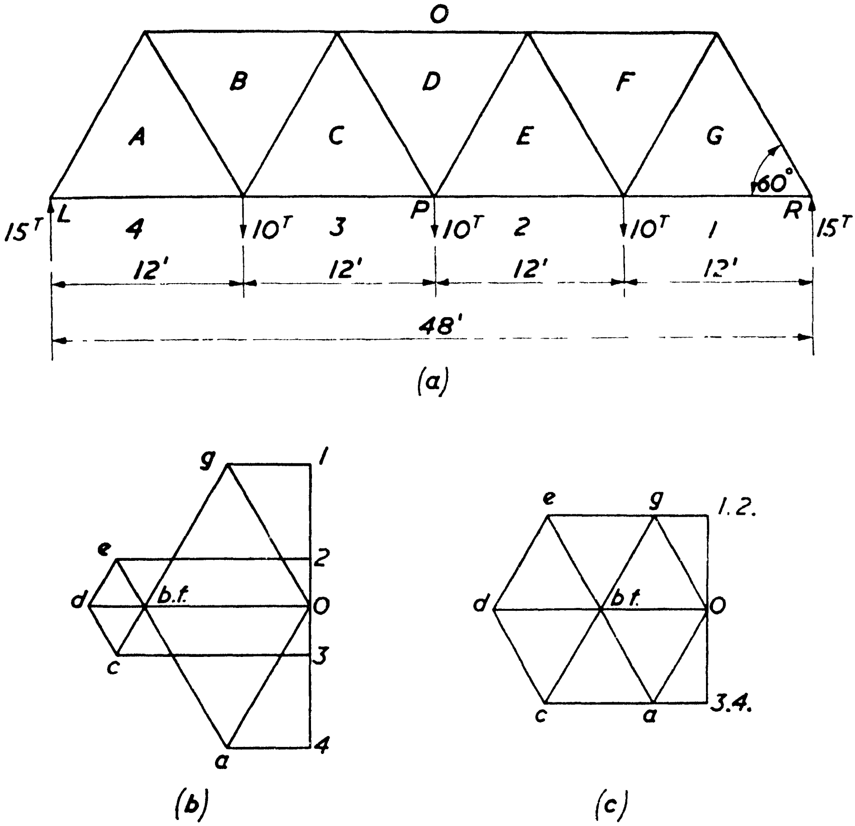


FIG. 76

Fig. 76 (b) shows the force diagram for all the external loads, while Fig. 76 (c) shows the force diagram due to unit vertical load at P.

In Fig. 77, for the purpose of this Example the members have not been designed in detail, but their areas have been assumed on the basis of using stresses of 9 and 6 tons per sq. in. in the tension and compression members respectively, and, since the girder is symmetrically loaded, members having similar lengths and forces have been taken together, the length of each being multiplied by 2.

Therefore the vertical deflection at P

$$\sum \frac{UFL}{AE} = \frac{10{,}000}{13{,}000} = 0{\cdot}769 \text{ in.}$$

MEMBER	FORCE F (TONS)	AREA A (Sq. In.)	LENGTH L (Ins.)	UNIT FORCE U (TONS)	$\frac{U \times F \times L}{A}$
4A, 1G	$\frac{15}{\sqrt{3}}$	0·96	2 X 144	$\frac{1}{2\sqrt{3}}$	750
3C, 2E	$\frac{35}{\sqrt{3}}$	2·24	2 X 144	$\frac{3}{2\sqrt{3}}$	2250
OA, OG	$-\frac{30}{\sqrt{3}}$	2·88	2 X 144	$-\frac{1}{\sqrt{3}}$	1000
OB, OF	$-\frac{30}{\sqrt{3}}$	2·88	2 X 144	$-\frac{1}{\sqrt{3}}$	1000
OD	$-\frac{40}{\sqrt{3}}$	3·84	144	$-\frac{2}{\sqrt{3}}$	1000
AB, FG	$\frac{30}{\sqrt{3}}$	1·92	2 X 144	$\frac{1}{\sqrt{3}}$	1500
BC, EF	$-\frac{10}{\sqrt{3}}$	0·96	2 X 144	$-\frac{1}{\sqrt{3}}$	1000
CD, DE	$\frac{10}{\sqrt{3}}$	0·64	2 X 144	$\frac{1}{\sqrt{3}}$	1500
				TOTAL	= 10000

+ INDICATES TENSION, — INDICATES COMPRESSION

FIG. 77

Example 2. Calculate the horizontal deflection of the central point *P* of the lower boom of the Warren girder used in Example 1 and loaded as shown in Example 1. $E = 13{,}000$ tons per sq. in.

Bearing in mind that the girder is symmetrical and symmetrically loaded and that the point at which the deflection is required is the central point of the lower flange, it will be obvious that, in relation to the centre line of the girder, this point will have no deflection in a horizontal direction.

There will, however, be some resultant horizontal movement between this point and the ends of the girder *L* and *R*, and it is really this horizontal movement that is required.

Accordingly, if it is required to find the resultant horizontal movement between *L* and *P*, the point *L* can be assumed to be fixed in position. In this case all the horizontal reaction will be supplied at *L* and it will be obvious that with a horizontal unit load at *P* and the reaction to this load wholly supplied at *L*, and therefore equal to unity, the only members of the truss subjected to forces are the two members *A*4 and *C*3. If the unit load *U* is

applied at P in the direction towards L, i.e. towards the left, then the forces in $A4$ and $C3$ under this unit load will be compressive.

The table corresponding to this unit loading is given below.

Member	*Force F* (*tons*)	*Area A* (*sq. in.*)	*Length L* (*in.*)	*Unit Load* (*tons*)	$\frac{UFL}{A}$
A4	$15/\sqrt{3}$	0·96	144	-1·0	-1,300
C3	$35/\sqrt{3}$	2·24	144	-1·0	-1,300
				Total	-2,600

Therefore, the horizontal deflection at $P = -\frac{2{,}600}{13{,}000} = -0{\cdot}2$ in.

This means that the point tends to move, in relation to L, in the opposite direction to the applied load, i.e. the point P tends to move away from L, or conversely the point L tends to move away from the point P. The value of the calculated movement is confirmed by the measurement of the horizontal distance between a and e in Fig. 82 which equals 0·2 in.

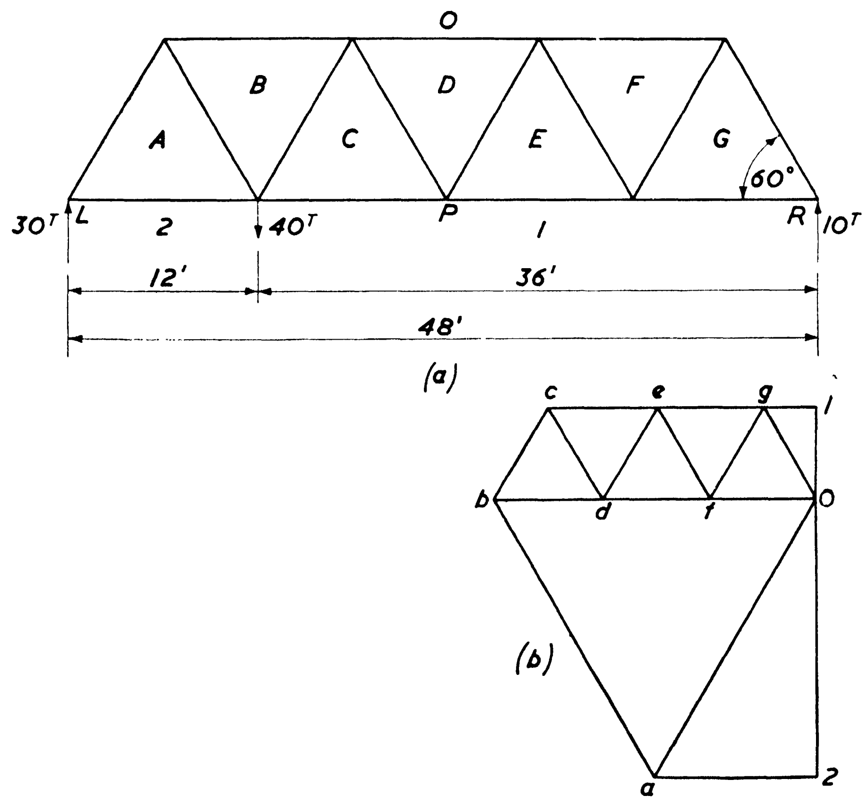


FIG. 78

MEMBER	FORCE F (TONS)	AREA A (Sq. In.)	LENGTH L (Ins.)	UNIT FORCE U (TONS)	$\frac{U \times F \times L}{A}$
2A	$\frac{30}{\sqrt{3}}$	0·96	144	$\frac{1}{2\sqrt{3}}$	750
1C	$\frac{50}{\sqrt{3}}$	2·24	144	$\frac{3}{2\sqrt{3}}$	1607
1E	$\frac{30}{\sqrt{3}}$	2·24	144	$\frac{3}{2\sqrt{3}}$	964
1G	$\frac{10}{\sqrt{3}}$	0·96	144	$\frac{1}{2\sqrt{3}}$	250
OA	$-\frac{60}{\sqrt{3}}$	2·88	144	$-\frac{1}{\sqrt{3}}$	1000
OB	$-\frac{60}{\sqrt{3}}$	2·88	144	$-\frac{1}{\sqrt{3}}$	1000
OD	$-\frac{40}{\sqrt{3}}$	3·84	144	$-\frac{2}{\sqrt{3}}$	1000
OF	$-\frac{20}{\sqrt{3}}$	2·88	144	$-\frac{1}{\sqrt{3}}$	333
OG	$-\frac{20}{\sqrt{3}}$	2·88	144	$-\frac{1}{\sqrt{3}}$	333
AB	$\frac{60}{\sqrt{3}}$	1·92	144	$\frac{1}{\sqrt{3}}$	1500
BC	$\frac{20}{\sqrt{3}}$	0·96	144	$-\frac{1}{\sqrt{3}}$	−1000
CD	$-\frac{20}{\sqrt{3}}$	0·64	144	$\frac{1}{\sqrt{3}}$	−1500
DE	$\frac{20}{\sqrt{3}}$	0·64	144	$\frac{1}{\sqrt{3}}$	1500
EF	$-\frac{20}{\sqrt{3}}$	0·96	144	$-\frac{1}{\sqrt{3}}$	1000
FG	$\frac{20}{\sqrt{3}}$	1·92	144	$\frac{1}{\sqrt{3}}$	500
				TOTAL	= 9237

+ INDICATES TENSION, − INDICATES COMPRESSION

Fig. 79

Example 3. Calculate the vertical deflection of the central point P of the lower boom of the Warren girder used in Example 1 but loaded asymmetrically as shown in Fig. 78 (a). $E = 13{,}000$ tons per sq. in.

It should be noted that, as the areas of the members are the same as in Example 1 the stresses in the members under this alternate load will be varied.

Fig. 78 (b) shows the force diagram for the external load. The force diagram for unit vertical load at P is exactly as in Example 1 and consequently need not be redrawn. Fig. 79 gives the summation UFL/A.

Therefore the vertical deflection at $P = \frac{9{,}237}{13{,}000} = 0{\cdot}71$ in.

Graphical Method

The deflection of a framed structure may also be found by a graphical method which consists of plotting, to an enlarged scale, the movements of the joints of the structure. These diagrams are commonly referred to as Williott diagrams.

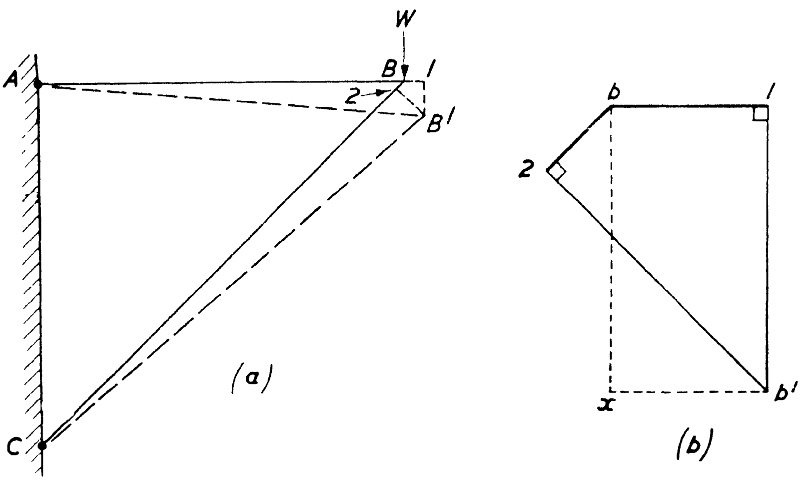


FIG. 80

As an illustration of the method, consider the simple bracket of Fig. 74, which is redrawn in Fig. 80 (a).

Under the load W the member AB stretches by an amount $B1$, while the member BC contracts by an amount $B2$.

Assuming that the member AB can rotate about A, and that member BC can rotate about C, then with centres A and C and radii $A1$ and $C2$ respectively, arcs can be drawn to intersect at B^1, which is the position taken up by the point B when the load W is applied.

In reality, the distances $B1$ and $B2$ are extremely small so that the arcs $1B^1$ and $2B^1$ can be replaced by straight lines at right angles to $A1$ and $C2$ respectively. The following construction, which is illustrated in Fig. 80 (b), will describe the method to be used.

Set out, to a much enlarged scale, the lengths $b1$ and $b2$ parallel to their respective members AB and CB to represent the strains in those members, in

value and direction From 1 and 2 draw perpendiculars to $b1$ and $b2$ to intersect in b^1. b^1 will be the displaced position of the point b and if bx is vertical and b^1x is horizontal, then they represent, respectively, the vertical and horizontal deflections of the point B.

The method will now be extended to deal with a Warren girder and the three previous examples will be re-worked by the graphical method.

Example 4. The data is the same as in Example 1.

For the graphical construction it is required to know the strains in each member, and therefore another figure is required to arrive at these strains. See Fig. 81.

MEMBER	LENGTH L (Ins.)	AREA A (Sq. In.)	FORCE F (TONS)	STRAIN $= \frac{FL}{AE}$ (Ins.)
2, 14	144	0·96	$\frac{15}{\sqrt{3}}$	0·100
6, 10	144	2·24	$\frac{35}{\sqrt{3}}$	0·100
1, 15	144	2·88	$-\frac{30}{\sqrt{3}}$	−0·067
4, 12	144	2·88	$-\frac{30}{\sqrt{3}}$	−0·067
8	144	3·84	$-\frac{40}{\sqrt{3}}$	−0·067
3, 13	144	1·92	$\frac{30}{\sqrt{3}}$	0·100
5, 11	144	0·96	$-\frac{10}{\sqrt{3}}$	−0·067
7, 9	144	0·64	$\frac{10}{\sqrt{3}}$	0·100

+ INDICATES TENSION, − INDICATES COMPRESSION

FIG. 81

It is more convenient, when dealing with the graphical method, to letter the points of the girder and to number the members. For this reason the notation used in Fig. 82 (a) is different from that used in Fig. 76 (a).

The displacement diagram is shown in Fig. 82 (b).

Before commencing to draw the displacement diagram, it is necessary to relate all the displacements to one point on a member of the frame. This reference point must be a node point and it is assumed to be fixed in position.

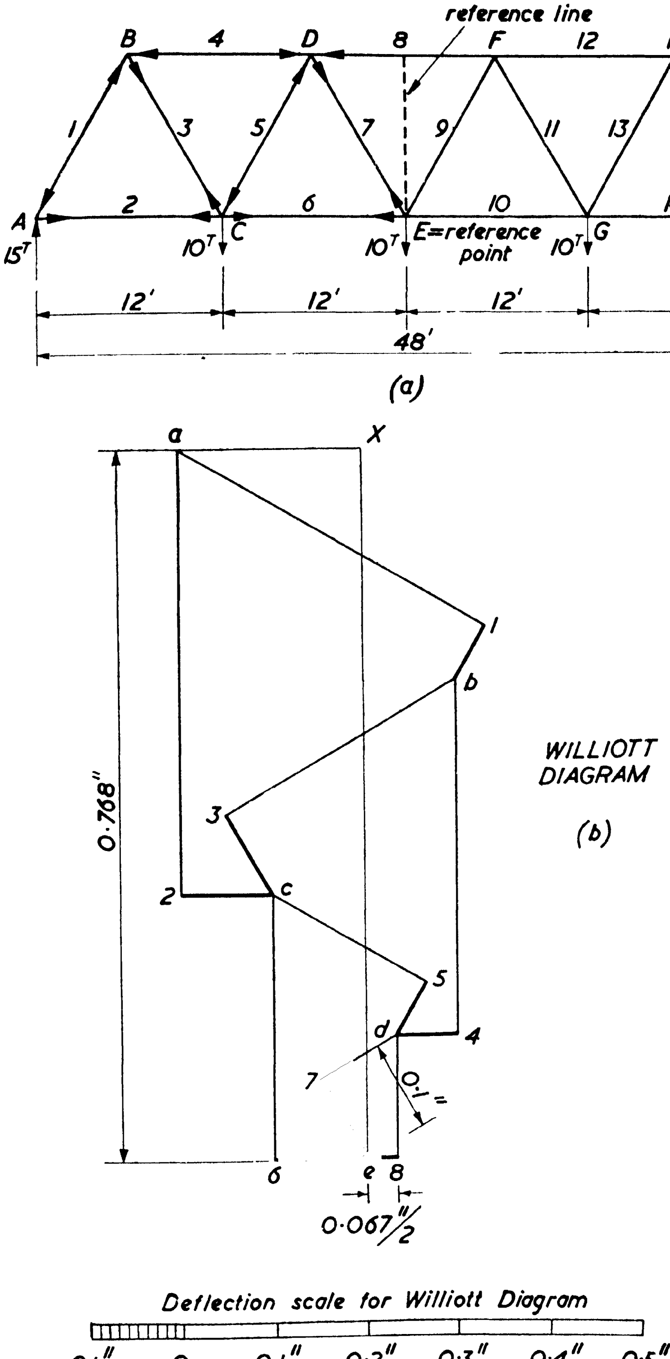

reference line
B
4
D
8
F
12
H
1
3
5
7
9
11
13
15
A
2
C
6
E=reference point
10
G
14
60°
J
15^T
10^T
10^T
10^T
15^T
12'
12'
12'
12'
48'
(a)
a
X
1
b
WILLIOTT DIAGRAM
(b)
3
2
c
5
d
4
7
0·1"
0·768"
6
e
8
0·067"/2
Deflection scale for Williott Diagram
0·1"
0
0·1"
0·2"
0·3"
0·4"
0·5"

FIG. 82

In addition this point must lie on a line which is assumed not to rotate. This line may or may not be a member of the frame.

For the purpose of this Example, which is for a symmetrical frame, symmetrically loaded, the point E is a particularly convenient reference point and the line which is assumed not to rotate is the line joining E to the mid-point of member 8. In this Example the principle of symmetry applies and it will only be necessary to deal with one half of the girder.

Accordingly the point e in Fig. 82 (b) is the starting point and the first operation is to find the displacement of the point D with respect to the point E. The analogy of the simple bracket will be apparent, because this displacement is concerned only with the strains in the members 7 and 8, and, more precisely, with only half the strain in member 8 since this member is bisected by the reference line.

Set out $e8$ parallel to member 8 and equal to half the strain in member 8, and set out $e7$ parallel to member 7 and equal to the strain in member 7. The values of the strains are taken from Fig. 81. Draw $8d$ and $7d$ at right angles to $e8$ and $e7$ respectively to meet at d. Then d is the displaced position of the point D with respect to the point E, and $d8$ will represent the vertical deflection of D with respect to E.

The only difficulty concerns the directions in which the lines $e8$ and $e7$ are to be drawn and the following simple rule will govern this matter.

Arrows are placed on the frame diagram, as shown in Fig. 82 (a), to represent the forces in the members. If the deflection of D with respect to E is required, the two members concerned are 7 and 8, and the directions of the forces in these members at the point D are considered. The lines $e8$ and $e7$ on Fig. 82 (b) are then drawn in the opposite directions to the respective arrows.

In other words, since member 8 is a strut, it will tend to shorten and consequently the point D will tend to move towards the right in relation to the point E, while since member 7 is a tie, the point D will tend to move away from the point E.

Having found the displaced position of D, it is now possible to find the displacement of C with respect to D. Draw $d5$ and $e6$ on Fig. 82 (b) parallel to members 5 and 6 respectively. The directions of $d5$ and $e6$ are governed by the arrows at C. Draw $5c$ and $6c$ at right angles to $d5$ and $e6$ respectively to meet in c, which is the displaced position of the point C.

By similar means the points b and a, which are the displaced positions of the points B and A respectively, are found.

The final vertical deflection of the point A with respect to the point E is the vertical distance between a and e and scales 0·768 in. which compares very closely with the figure of 0·769 in. obtained by calculation in Example 1. Similarly, the horizontal distance between a and $e = 0{\cdot}2$ in., as calculated in Example 2.

The scale used for the construction of the Williott diagram is too large to be applied directly to the truss drawing, but if a smaller scale is used the displacement may be indicated, although in an exaggerated manner, by the dotted lines in Fig. 83.

This outline represents the shape of the deformed truss on the basis that

point E is fixed, i.e. the same basis on which the Williott diagram was drawn. It is obvious, however, that since points A and J remain fixed in elevation (although not laterally) a correction must be made by moving the dotted outline downwards. The corresponding correction can be made on the Williott diagram by taking as reference point the point X, which lies on the horizontal line through a and the vertical line through e. All displacements,

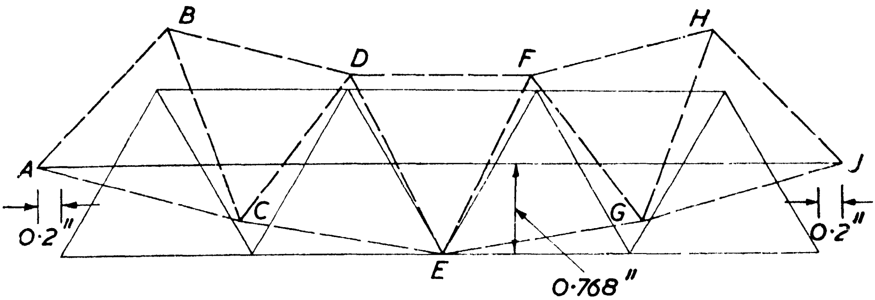


FIG. 83

both horizontal and vertical, are measured from X and thus all joints can be located from the one diagram.

Example 5. The data is the same as in Example 3.

In this case the loading is asymmetrical and accordingly the assumption that $E8$ remains vertical is not true. Therefore, the Williott diagram does not represent the true conditions and a suitable correction diagram must be devised.

Fig. 85 (a) shows the frame diagram with the arrows representing the forces.

Fig. 85 (b) shows the displacement diagram which is drawn by applying the principles described in Example 4, the strains being obtained from Fig. 84. The point e is the starting point, and the points a and j represent the displaced positions of the points A and J respectively. It will be seen that the vertical distances from a to e and from j to e are not equal, as, of course, they should be if the supports A and J are at the same level.

Fig. 86 shows the deformed shape of the truss on the basis that the point e is fixed and the line $e8$ is vertical. In order that this deformed shape may correctly represent the deflection of the various joints it is necessary to rotate the dotted line figure about E in a clockwise direction until the points A and J are on a horizontal line.

In Fig. 86 let θ be the angle relative to the fixed point E through which the dotted outline must be rotated, then since the truss is a rigid frame all joints of the truss are rotated through the same angle and the amount of displacement of any joint is in direct proportion to the distance of that joint from the reference point. For example, due to rotation about E, points B and F move horizontally to the right, point B moving vertically upwards and point F vertically downwards.

MEMBER	LENGTH L (Ins.)	AREA A (Sq.In.)	FORCE F (Tons)	STRAIN = $\frac{FL}{AE}$ (Ins.)
2	144	0·96	$\frac{30}{\sqrt{3}}$	0·2
6	144	2·24	$\frac{50}{\sqrt{3}}$	0·143
10	144	2·24	$\frac{30}{\sqrt{3}}$	0·086
14	144	0·96	$\frac{10}{\sqrt{3}}$	0·067
1	144	2·88	$-\frac{60}{\sqrt{3}}$	−0·133
4	144	2·88	$-\frac{60}{\sqrt{3}}$	−0·133
8	144	3·84	$-\frac{40}{\sqrt{3}}$	−0·067
12	144	2·88	$-\frac{20}{\sqrt{3}}$	−0·044
15	144	2·88	$-\frac{20}{\sqrt{3}}$	−0·044
3	144	1·92	$\frac{60}{\sqrt{3}}$	0·2
5	144	0·96	$\frac{20}{\sqrt{3}}$	0·133
7	144	0·64	$-\frac{20}{\sqrt{3}}$	−0·2
9	144	0·64	$\frac{20}{\sqrt{3}}$	0·2
11	144	0·96	$-\frac{20}{\sqrt{3}}$	−0·133
13	144	1·92	$\frac{20}{\sqrt{3}}$	0·067

+ INDICATES TENSION. — INDICATES COMPRESSION

FIG. 84

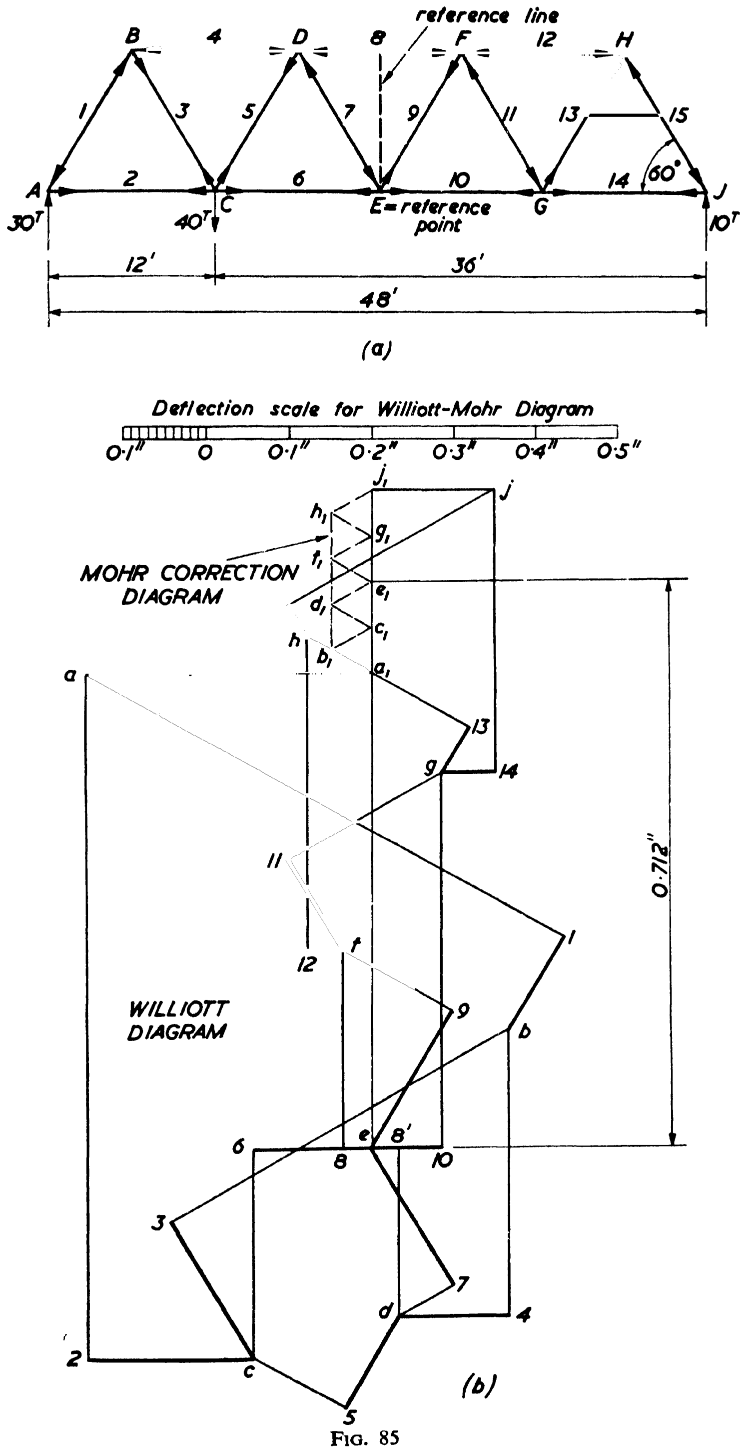

reference line
B
4
D
8
F
12
H
1
3
5
7
9
11
13
15
60°
A
2
6
10
14
J
C
E = reference point
G
30T
40T
10T
12'
36'
48'
(a)
Deflection scale for Williott-Mohr Diagram
0·1"
0
0·1"
0·2"
0·3"
0·4"
0·5"
MOHR CORRECTION DIAGRAM
WILLIOTT DIAGRAM
0·712"
(b)

FIG. 85

The correction for each joint can be made graphically as follows:

Draw a vertical through e in Fig. 85 (b) to cut the horizontals through a and j in a_1 and j_1 respectively. On a_1j_1 draw a replica of the truss to scale, as shown in chain lines. Denote each of the joints in the truss by letters corresponding to the original truss. This outline is a Mohr correction diagram and the final figure is a Williott-Mohr diagram.

The displacement of any joint may then be determined by direct measurement between points which correspond in the Mohr diagram and the Williott diagram, this being most conveniently done by measuring both horizontal and vertical components. For example the vertical deflection of the point E is given by the vertical distance between e_1 and e on the Williott-Mohr diagram and measures 0·712 in., which compares with 0·710 in. obtained by calculation in Example 2. Similarly the horizontal movement of B is the horizontal distance from b_1 to b.

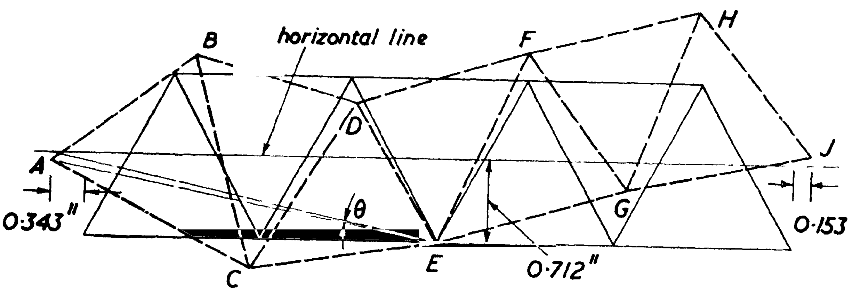


FIG. 86

The principle of the Mohr correction diagram is further exemplified by the two additional Williott-Mohr diagrams for the same truss shown in Figs. 87 and 89.

In Fig. 87 the point A is assumed to be fixed in position and the member AB prevented from rotating (although allowed to shorten).

If the exaggerated distorted frame were drawn, it would be apparent that the frame would have to be rotated in a clockwise direction about A to bring the point J down to the same level. The appropriate Mohr correction diagram is shown in Fig. 87. The deflected frame is shown in Fig. 88.

In Fig. 89 the point E is once again assumed to be fixed, but this time the member EF is prevented from rotating. When the Williott diagram is completed the point a is higher than the point j. Hence, if the distorted frame were drawn it would have to be rotated in an anticlockwise direction about E to bring A and J to the same level. The Mohr diagram now appears on the right-hand side of the vertical through e.

It is interesting to compare the results obtained. The Williott-Mohr diagrams of Figs. 85 (b) and 89 may be compared directly. Thus in Fig. 85 (b) the horizontal displacement of B measures 0·214 in. while in Fig. 89 it measures 0·215 in. As a different reference point was used for Fig. 87 the horizontal deflection of B is compared by subtracting the horizontal component of e_1e from that of b_1b, i.e. $0{\cdot}556 - 0{\cdot}343 = 0{\cdot}213$ in. The

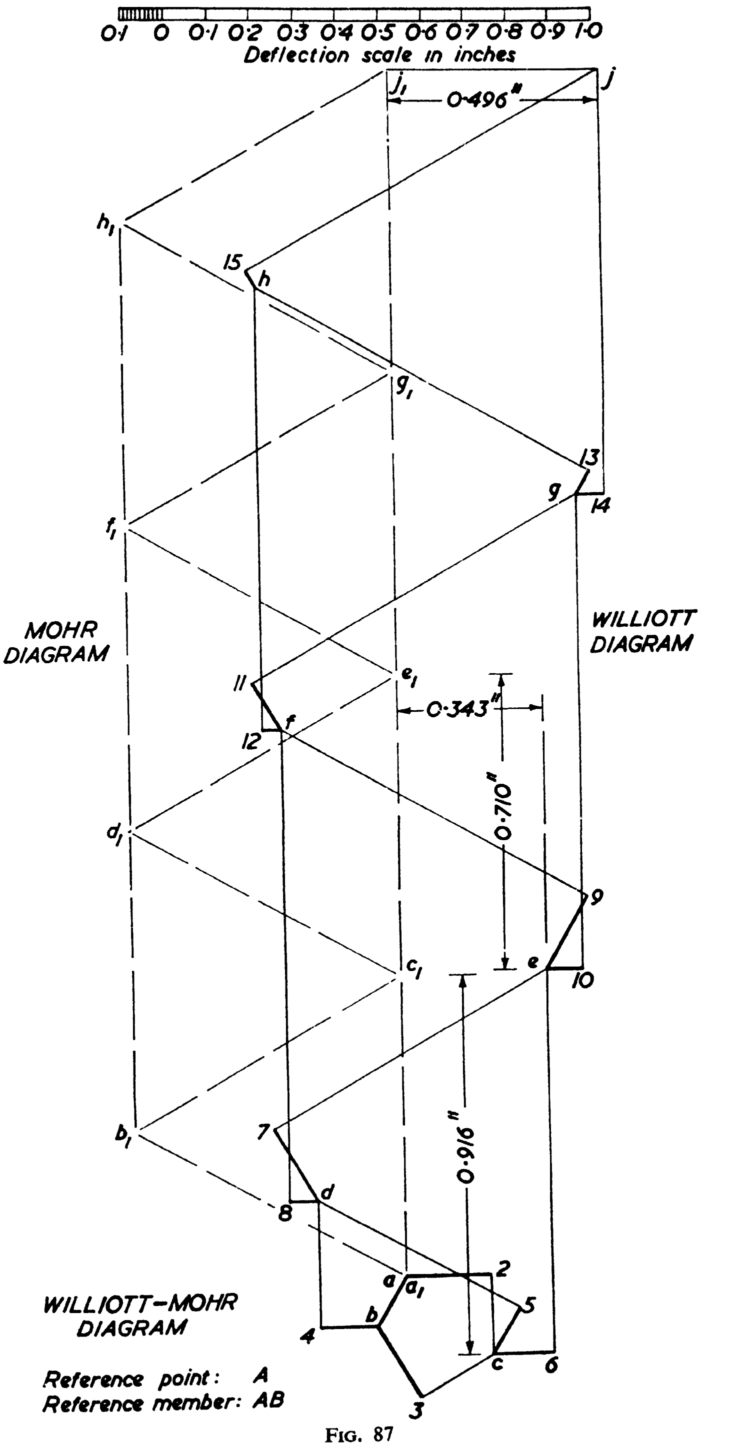

0·1 0 0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0
Deflection scale in inches
0·496"
0·343"
0·710"
0·916"
MOHR DIAGRAM
WILLIOTT DIAGRAM
WILLIOTT-MOHR DIAGRAM
Reference point: A
Reference member: AB

FIG. 87

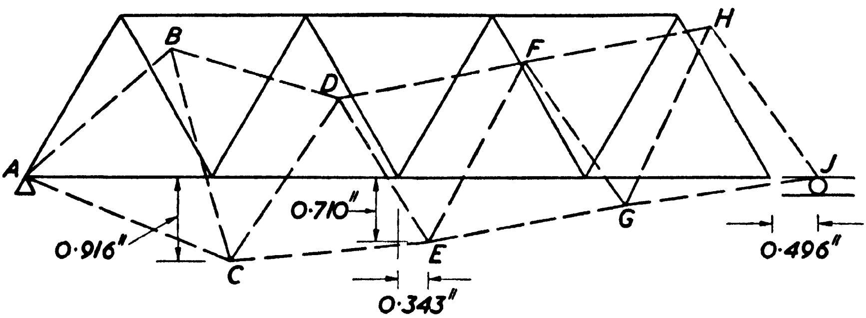


FIG. 88

vertical displacements of E, which can be measured directly on all the diagrams, measure respectively 0·712 in., 0·710 in. and 0·713 in.

It will be observed that a replica truss forming a Mohr correction diagram is placed on the vertical through the reference point, its span and position being fixed by the horizontal intercepts from the points on the Williott diagram which correspond to the bearings.

The Williott-Mohr diagram shown in Fig. 87 is the type of diagram which would normally be used for a truss where one end is fixed in position and the other is allowed to move horizontally.

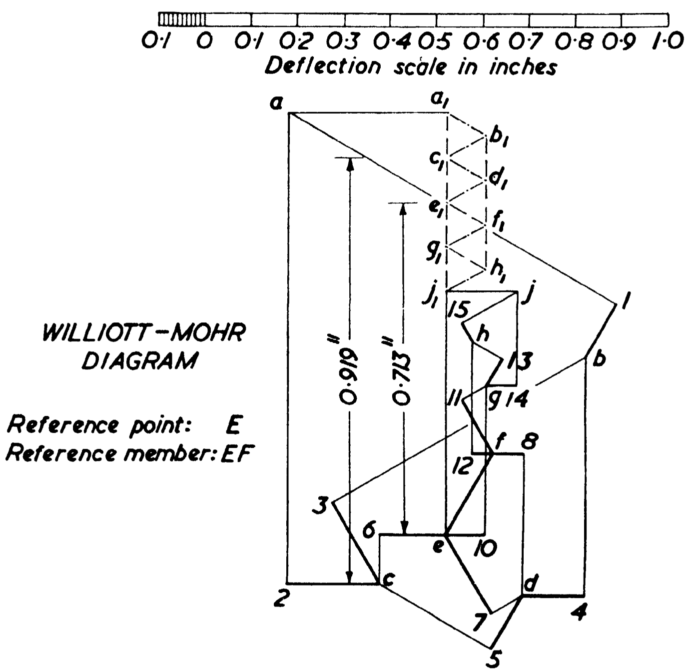


FIG. 89

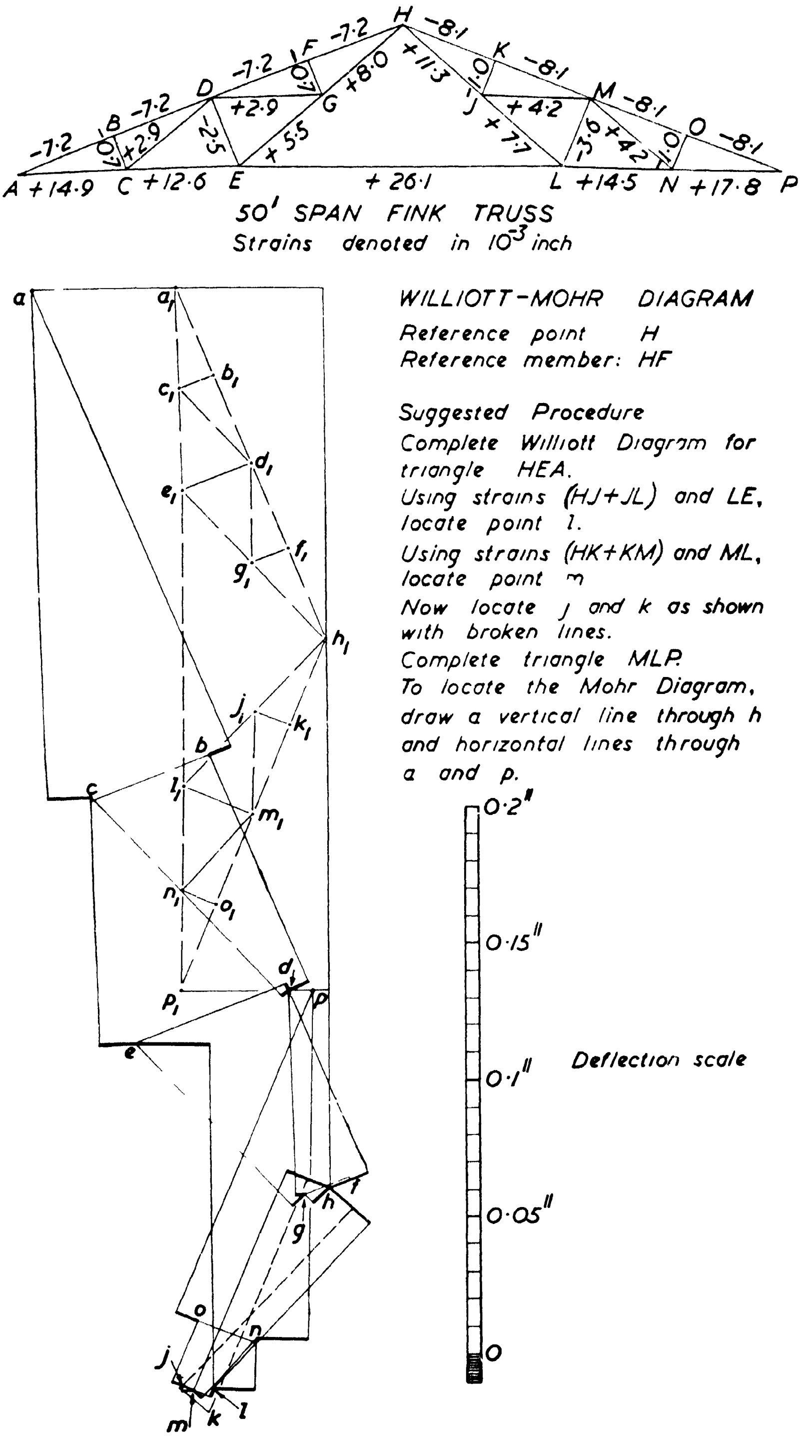

50' SPAN FINK TRUSS
Strains denoted in 10⁻³ inch
A +14·9 C +12·6 E +26·1 L +14·5 N +17·8 P
-7·2 -7·2 -7·2 -7·2
-8·1 -8·1 -8·1 -8·1
+2·9 +2·9 -2·5 +5·5 +8·0 +11·3 +7·7 +4·2 -3·6 +4·2
WILLIOTT-MOHR DIAGRAM
Reference point H
Reference member: HF
Suggested Procedure
Complete Williott Diagram for triangle HEA.
Using strains (HJ+JL) and LE, locate point l.
Using strains (HK+KM) and ML, locate point m
Now locate j and k as shown with broken lines.
Complete triangle MLP.
To locate the Mohr Diagram, draw a vertical line through h and horizontal lines through a and p.
0·2"
0·15"
0·1"
0·05"
0
Deflection scale

FIG. 90

A further truss and an appropriate Williott-Mohr diagram is shown in Fig. 90.

If it is required that any girder should, when fully loaded, have a horizontal bottom chord it is necessary to camber the truss by increasing the length of each compression member and decreasing the length of each tension member by the amount of strain it experiences when loaded. These strains are those given by the elastic calculations or by the Williott diagram and the camber diagram is constructed using the strains but with opposite signs.

11. INFLUENCE LINES

An influence line for any given point P on a structure is such a line that its ordinate at any point Q gives the bending moment, shear force or similar quantity at P when a load is placed at Q.

It is important to note that an influence line is drawn for one point only and that while such a line will give the desired information at that point for any position of a load, it will not necessarily give the maximum value of the B.M., S.F., etc., on the structure as a whole. In any structure or for any system of loading, it may be necessary to construct several influence lines in order to obtain the absolute maxima on the structure. By comparison with an influence line, a B.M. diagram gives the B.M. at all points for one position of a load.

Influence lines can be drawn in three ways, viz.:

1. For unit loads and unit spans.
2. For unit loads and factual spans.
3. For factual loads and factual spans.

Each of these methods will be illustrated with reference to a simply supported beam.

Simply Supported Beams

Fig. 91 (b) and (c) show the B.M. and S.F. influence lines for the point P on a simply supported beam of span AB, drawn for unit load and unit span. Note that when an influence line is drawn for unit span, both a and b are less than unity.

A. Point Loads

1. Bending Moments:

(a) Influence line drawn for unit load and unit span.

If a unit load is placed at Q, Fig. 91, then the B.M. at P is given by the product of the load and the ordinate of the influence line at the point of application of the load, viz.: CQ_1,

$$\text{i.e. B.M. at } P = 1 \times CQ_1.$$

If a series of unit loads are placed at Q, R and S respectively, then the B.M. at P is given by the sum of the products of the loads and their appropriate ordinates, e.g.:

$$\text{B.M. at } P = 1 \times CQ_1 + 1 \times DR_1 + 1 \times ES_1.$$

This particular influence line, i.e. that for unit load and unit span, can readily be used for the case of any loads on any span, but, in this case, the appropriate ordinates must be multiplied by the numerical values of both load and span. Thus if a series of loads W_1, W_2 and W_3

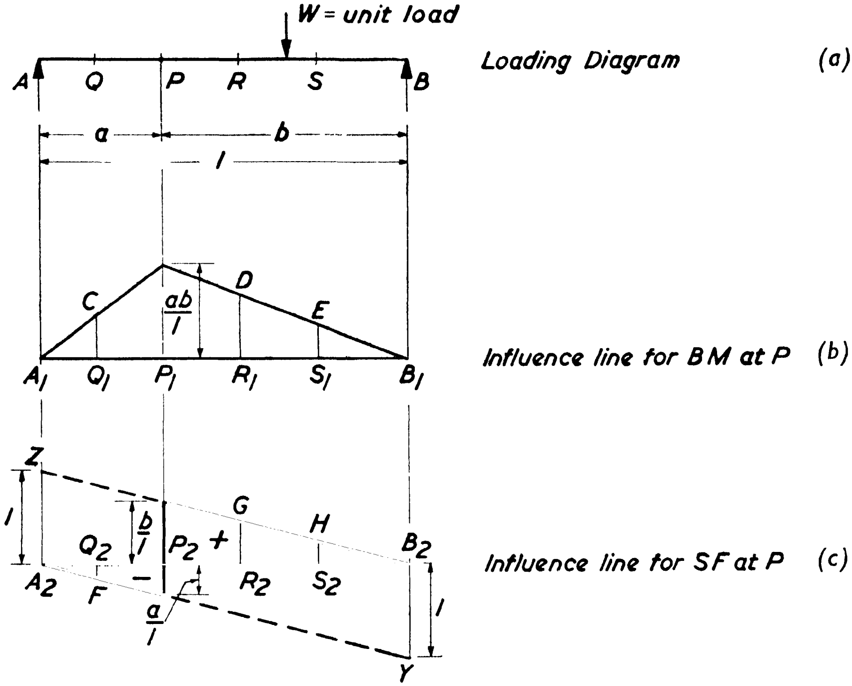


FIG. 91

are placed on a span of length L, at points corresponding to Q, R and S, then

$$\text{B.M. at } P = W_1 \times CQ_1 \times L + W_2 \times DR_1 \times L + W_3 \times ES_1 \times L$$
$$= L\,(W_1 \times CQ_1 + W_2 \times DR_1 + W_3 \times ES_1).$$

(*b*) Influence line drawn for unit load and factual span.

In this case the ordinates will automatically be L times the values given in case 1 (a) as will be seen from the following example.

Suppose AP to be $\frac{1}{3}AB$.

Then in case 1 (a) the ordinate at P would be

$$\frac{\frac{1}{3} \times \frac{2}{3}}{1} = \frac{2}{9},$$

while in case 1 (b) if the span were 60 ft., the ordinate at P would be

$$\frac{20 \times 40}{60} = \frac{40}{3},$$

which is 60 times that in case 1 (a).

In this case, accordingly, if a series of unit loads are placed at Q, R and S respectively then

$$\text{B.M. at } P = 1 \times CQ_1 + 1 \times DR_1 + 1 \times ES_1,$$

and the values can be applied to the case of any loads by multiplying by the numerical values of the loads.

Thus, if loads W_1, W_2 and W_3 are placed at Q, R and S respectively,

$$\text{B.M. at } P = W_1 \times CQ_1 + W_2 \times DR_1 + W_3 \times ES_1.$$

(c) Influence line drawn for factual load and factual span.
In this case the ordinates will automatically be WL times the values given in case 1 (a) and indeed will give directly the value of the B.M. It will be appreciated, however, that case 1 (c) can only be applied to a single load or a series of equal loads.

Summing up the three cases above, it will be seen that for any given loading and span, the B.M. is obtained as follows:

In case 1 (a) by multiplying the ordinates by the appropriate loads and by the span; in case 1 (b) by multiplying the ordinates by the appropriate loads only; and in case 1 (c) by direct measurement of the ordinates.

The three cases will be illustrated by the following example.

Example 1. Find, by the influence-line method, the B.M. at the left-hand quarter point of a span of 100 ft., when a single point load of 10 tons is placed at the other quarter point of the span.

The influence lines for cases 1 (a), (b) and (c) are shown in Fig. 92 (a), (b) and (c) respectively.

In Fig. 92 (a) the ordinate at P (the left-hand quarter point)

$$= \frac{0{\cdot}25 \times 0{\cdot}75}{1{\cdot}0} = 0{\cdot}1875,$$

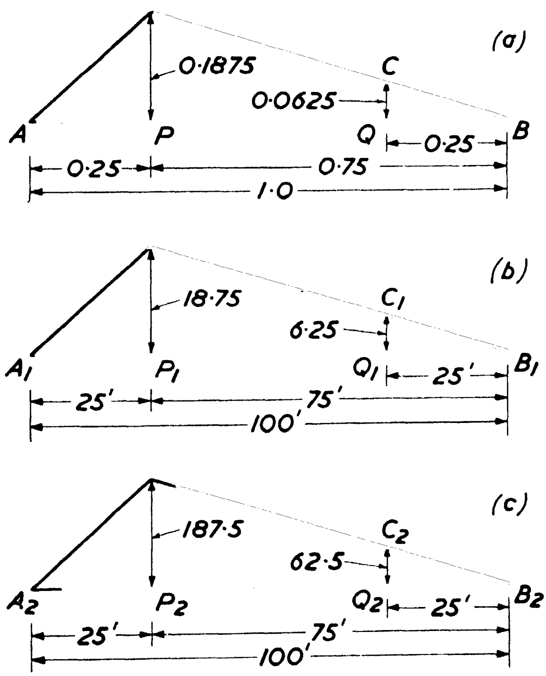


FIG. 92

and the corresponding ordinate at Q (the right-hand quarter point)

$$=\frac{0{\cdot}1875}{3}=0{\cdot}0625 \quad \text{(which is merely a number).}$$

Therefore, if a load of 10 tons is placed at Q and the span is 100 ft.,

$$\text{B.M. at } P=0{\cdot}0625\times 10\times 100=62{\cdot}5 \text{ tons ft.}$$

In Fig. 92 (b) the ordinate at P_1

$$=\frac{25\times 75}{100}=18{\cdot}75 \text{ ft.}$$

and that at $Q_1=6{\cdot}25$ ft.

Therefore, B.M. at P_1 for the same conditions

$$=6{\cdot}25\times 10=62{\cdot}5 \text{ tons ft.}$$

In Fig. 92 (c) the ordinate at $P_2=187{\cdot}5$, and that at $Q_2=62{\cdot}5$ tons ft. Therefore B.M. at P_2 for the same conditions $=62{\cdot}5$ tons ft.

2. *Shear Forces*:

(a) Influence line drawn for unit load and unit span.

If a unit load is placed at Q, Fig. 91, then the S.F. at P is given by the product of the load and the appropriate ordinate FQ_2, i.e.:

$$\text{S.F. at } P=1\times FQ_2.$$

If a series of unit loads are placed at Q, R and S respectively then

$$\text{S.F. at } P=1\times FQ_2+1\times GR_2+1\times HS_2.$$

It will be noted that these quantities must be added *algebraically*, since FQ_2 is negative while the other two are positive.

If a series of loads W_1, W_2 and W_3 are placed at Q, R and S respectively then

$$\text{S.F. at } P=W_1\times FQ_2+W_2\times GR_2+W_3\times HS_2.$$

(b) Influence line drawn for unit load and factual span.

It will be seen from Fig. 91 (c) that, in the case of unit load, if the sloping lines are produced, intercepts of unity result at A and B.

Accordingly, there is no difference in application between cases 2 (a) and 2 (b). In other words, in the case of S.F. the span does not enter into consideration.

(c) Influence line drawn for factual load and factual span.

In this case, as in the case of B.M., the ordinates give directly the values required.

Once again, however, this case can only be applied to either a single load or a series of equal loads.

Summing up the three cases, it will be seen that for any given loading and span the S.F. is obtained as follows:

In cases 2 (a) and 2 (b), by multiplying the ordinates by the appropriate loads, and in case 2 (c) by direct measurement of the ordinates.

The three cases will be illustrated by the following example.

Example 2. Find, by the influence line method, the S.F. at the left-hand quarter point of a span of 100 ft., when a single point load of 10 tons is placed at the other quarter point of the span.

The influence lines for the three cases are shown in Figs. 93 (a), (b) and (c) respectively.

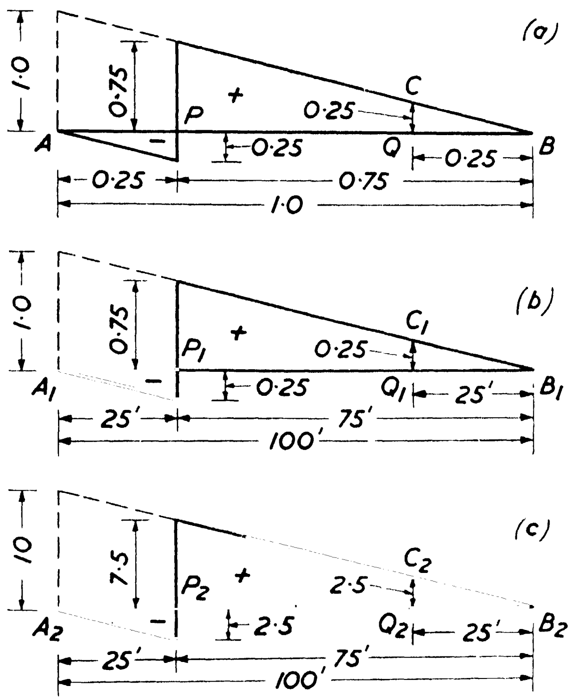


FIG. 93

In Fig. 93 (a) the appropriate ordinate at $P = 0{\cdot}75$ and the corresponding ordinate at $Q = 0{\cdot}25$ (a mere number).

In Fig. 93 (b) it will be seen that, since the intercept of the sloping line at A_1 is 1·0, i.e. exactly as in Fig. 93 (a), the ordinate at Q is also 0·25.

Therefore, irrespective of span, if a 10-ton load is placed at Q,

$$\text{S.F. at } P = 0{\cdot}25 \times 10 = 2{\cdot}5 \text{ tons.}$$

In Fig. 93 (c) the intercept at A_2 is 10 tons, i.e. the value of the load. Consequently the ordinate at $P_2 = 7{\cdot}5$ and that at $Q_2 = 2{\cdot}5$ tons.

Therefore, when 10 tons is placed at Q, the S.F. at $P = 2{\cdot}5$ tons.

The difference in application between B.M. and S.F. will now be clear, i.e. that in the case of B.M., the span plays an important part, while in the case of S.F., it plays no part at all.

B. Uniformly Distributed Loads Shorter than the Span

If a U.D. load is considered to be made up of a large number of small point loads at very small distances apart, it will be appreciated that the

ordinates of the point-load examples become a series of ordinates very close together, and in the limit the ordinates add up to an area.

1. Bending Moments:

(a) Influence line drawn for unit intensity of load and unit span.

If a U.D. load of unit intensity is placed over the length RS, Fig. 91 (a), then the B.M. at P will be given by the area R_1DES_1.

This particular influence line can readily be used for the case of any load on any span, by multiplying the appropriate area by the actual loading intensity and by the square of the span. Thus, if a U.D. load of w per unit length is placed over the portion RS of a span of L units then the B.M. at $P=\text{area } R_1DES_1 \times wL^2$.

It will be noticed that there is a difference between the multipliers in the cases of point and distributed loads. In the former, since the loads are the actual values, the unit ordinates are multiplied by the values of the load and the span, but in the latter, since w is only a load per unit length, the actual load is given in terms of wL and this quantity has still to be multiplied by L, thus giving the multiplier wL^2.

(b) Influence line drawn for unit intensity of load and factual span.

In this case the areas will automatically be L^2 times the values given in case 1 (a).

If a U.D. load of unit intensity is placed over the portion RS then the B.M. at P will be given by the area R_1DES_1, while if the intensity of the loading is w per unit length, then the B.M. at P would be area $R_1DES_1 \times w$.

(c) Influence line drawn for factual load and factual span.

In this case the B.M. is given directly by the appropriate area.

The three cases will be illustrated by the following example.

Example 3. A U.D. load of 2 tons per foot run stands over a length of 25 ft. measured from the right-hand end of a span of 100 ft. Find, by the influence-line method, the B.M. at the left-hand quarter point of the span.

The influence lines for the three cases are shown in Fig. 94 (a), (b) and (c) respectively.

In Fig. 94 (a) the ordinate at P, as previously shown, is 0·1875, while that at Q is 0·0625.

Therefore, the area under the B.M. influence line over the loaded length QB

$$=\frac{0{\cdot}0625 \times 0{\cdot}25}{2}=0{\cdot}0078125 \quad \text{(a mere number).}$$

As the span is 100 ft. and the intensity of loading is 2 tons per ft. run over the length QB, then

$$\text{B.M. at } P=0{\cdot}0078125 \times 2 \times 100^2=156{\cdot}25 \text{ tons ft.}$$

In Fig. 94 (b) the ordinate at P_1 is 18·75 ft., while that at $Q_1=6{\cdot}25$ ft.

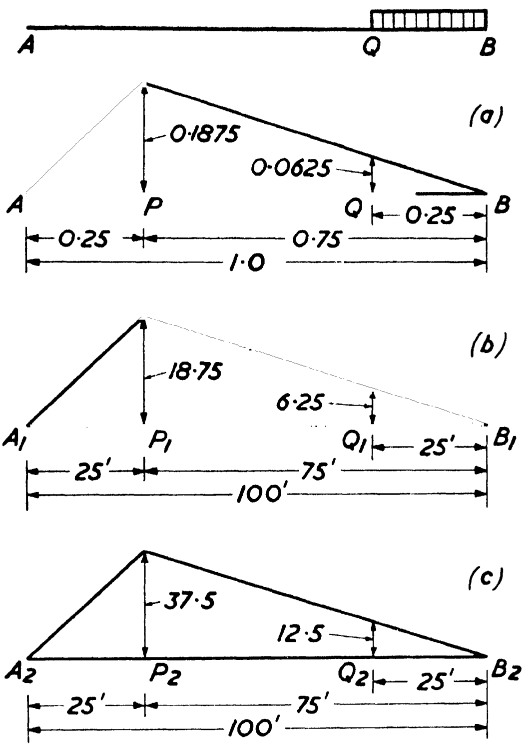


FIG. 94

Therefore, the area under the B.M. influence line over the loaded length QB

$$=\frac{6{\cdot}25\times25}{2}=78{\cdot}125 \text{ sq. ft.,}$$

and with the intensity of loading of 2 tons per ft. run, the B.M. at P, the loaded length being QB

$$=2\times78{\cdot}125=156{\cdot}25 \text{ tons ft.}$$

In Fig. 94 (c) the ordinate at P_2 is 37·5 tons and is found by multiplying the ordinate in case (b) by the intensity of loading. Consequently the ordinate at Q_2 is 12·5, and the area over the loaded length QB

$$=\frac{12{\cdot}5\times25}{2}=156{\cdot}25 \text{ tons ft.}$$

Therefore the B.M. at P due to a load of 2 tons per ft. run over the length $QB=156{\cdot}25$ tons ft.

2. *Shear Forces*:

(a) Influence line drawn for unit intensity of load and unit span.

If a load of unit intensity is placed over the length RS, Fig. 91 (a), then the S.F. at P will be given by the area R_2GHS_2.

This particular influence line can readily be used for the case of any

intensity of loading on any span, by multiplying the appropriate area by the actual loading intensity and by the span.

Thus if a U.D. load of w per unit length is placed over the portion RS of a span of L units, then the S.F. at P = area $R_2GHS_2 \times wL$.

Once again it will be seen that there is a difference between the applications depending on whether the loading is concentrated or U.D.

(b) Influence line drawn for unit intensity of load and factual span.

In this case, if the loading intensity is unity, the S.F. is again given by the area under the S.F. influence line standing over the loaded length.

In applying this case to any intensity of loading, it is only necessary to multiply the appropriate area by the intensity of loading w.

(c) Influence line drawn for factual load and factual span.

Once again, in this case, the S.F. is given directly by the appropriate area.

The three cases will be illustrated by the following example.

Example 4. For the same span and conditions of loading given in Example 3, find, by the influence-line method, the S.F. at the left-hand quarter point of the span.

The influence lines for the three cases are shown in Figs. 95 (a), (b) and (c) respectively.

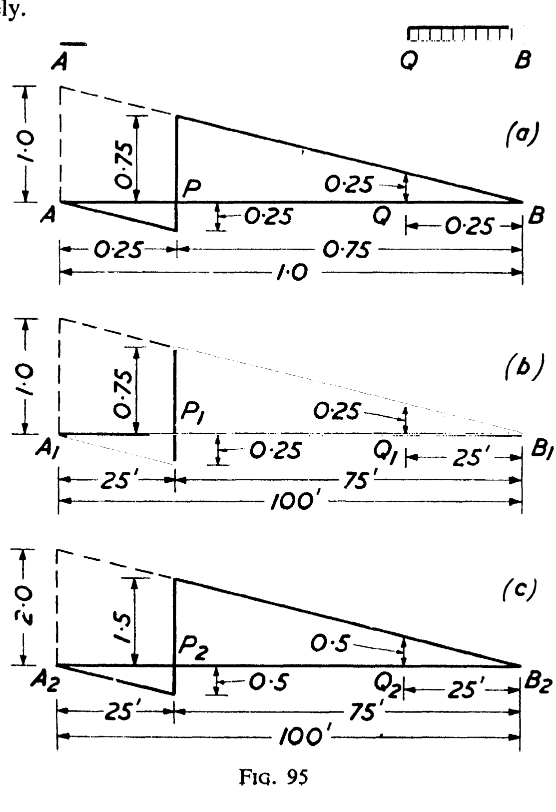


FIG. 95

In Fig. 95 (a) the ordinate at P is 0·75, while that at Q is 0·25. Therefore the area under the S.F. influence line over the loaded length QB

$$=\frac{0\cdot25\times0\cdot25}{2}=0\cdot03125 \quad \text{(a mere number).}$$

As the span is 100 ft., and the intensity of loading is 2 tons per ft. run over the length QB, then the S.F. at P

$$=0\cdot03125\times2\times100=6\cdot25 \text{ tons.}$$

In Fig. 95 (b) the ordinates are exactly as those in Fig. 95 (a). Therefore the area under the S.F. influence line over the loaded length QB

$$=\frac{0\cdot25\times25}{2}=3\cdot125 \text{ ft.}$$

With the intensity of loading of 2 tons per ft. run over the length QB, then the S.F. at P

$$=2\times3\cdot125=6\cdot25 \text{ tons.}$$

In Fig. 95 (c) the ordinate at P is 1·5, while that at Q is 0·5 tons.

Therefore the area under the S.F. influence line over the loaded length QB

$$=\frac{0\cdot5\times25}{2}=6\cdot25 \text{ tons.}$$

Therefore the S.F. at P due to this loading $=6\cdot25$ tons.

In most practical cases, it is required to find the maximum B.M. or S.F. at any point due to the loading system.

In the case under consideration, i.e. that of a loaded length shorter than the span, the maximum B.M. at any point such as P in Fig. 91 (a) will be given when the point P divides the loaded length in the same proportions as it divides the span.

For S.F., two maxima are possible. The maximum positive (+ve) S.F. at P is given when, if the load comes on from the right, the leading edge of the load had just reached P or if the load comes on from the left, the trailing edge of the load has just reached P, while the maximum negative (−ve) S.F. at P is given when, if the load comes on from the right, the trailing edge of the loading has just reached P, or if the load comes on from the left, the leading edge of the load has just reached P. (See Fig. 91 (c).)

C. Uniformly Distributed Loads Longer than the Span

This type of loading could be illustrated by a long train passing over a short span.

The methods of dealing with both B.M. and S.F. are exactly as given under the last heading, i.e. that for U.D. shorter than the span.

It only remains to illustrate the positions for maximum values.

In the case of B.M. it is obvious, by reference to Fig. 91 (b), that the maximum B.M. at P is given when the whole span is covered by the load.

Again in the case of S.F. there are two maxima and these are given when the load is in the positions previously indicated.

The use of influence lines is most helpful in the cases of continuous beams, arches and framed girders, when the values for B.M. and S.F., etc., for varying positions of loads can be much more rapidly evaluated than by any other means.

Influence Lines for Framed Structures

Figs. 96–100 show typical influence lines for direct forces in members in Warren and Pratt or N-type frames.

The main points to be noted are detailed below:

1. The load has been assumed to be transmitted through deck stringers and cross beams so that load is applied to the main girders only at the panel points.

2. Considering Fig. 96 if the force in member *QR* is required, it could be obtained, by the method of sections, by taking moments about the point *D*. Consequently the influence line for the force in *QR* is, to some scale, the influence line for B.M. for the point *D*, with this difference: that, between *Q* and *R* the influence line is a straight line as shown. This truncating of the diagram is peculiar to the Warren-type girder and applies only to the loaded chord. Also, see the influence line for *CD* in Fig. 97. Similarly the influence line for *CD* is, to some scale, the influence line for B.M. for the point *Q*.

3. Considering Fig. 98 the influence line for the member *QR* is, to some scale, the influence line for B.M. for the point *D*. Here it should be noted that the influence line for member *DE* is the influence line for B.M. for the point *R* and is, evidently, the same as that for *QR*. This is substantiated by the fact that, for this type of frame, the forces in *QR* and *DE* are equal.

4. The influence lines for web members are influence lines for shear and are similar to that shown in Fig. 91 (c) with the difference that the line is drawn diagonally across the panel under consideration. It should be noted, that, in the case of the end diagonal in Fig. 96, the slope of the line P_1X_1 depends on the position of *X*, i.e. the length of the approach span *PX*. X_1 is vertically below *X*. It should also be noted that, in this case, and indeed for all cases dealing with web members in the end panel, the shear-force influence line does not cross the base line, i.e. the shear force in the panel at the left-hand end of the girder is always positive. Note also the special case of the end vertical in Fig. 98.

5. If the influence line for any web member, e.g. the influence line for members *QD* and *RD* in Fig. 98, crosses the base line, then it is possible to get both positive and negative shear in the panel *QR*. This means that the member *QD* will, at some stage, be in compression. If the length of the member is such that it would be uneconomical to make it capable of resisting compression, then cross bracing is introduced. This cross bracing, usually comprised of members capable only of taking tension, introduces one or two unusual features in the influence lines for the members affected. For example, if, in Fig. 98, the panel *RSED* is cross-braced and the members *RE* and *SD* are capable only of taking tension, then the influence line for *SD* will be as shown, while, at the same time, the influence line for *RD* will have the unusual form shown, since it will be possible for this member *RD* to take the whole load at the time when the load is at *R*.

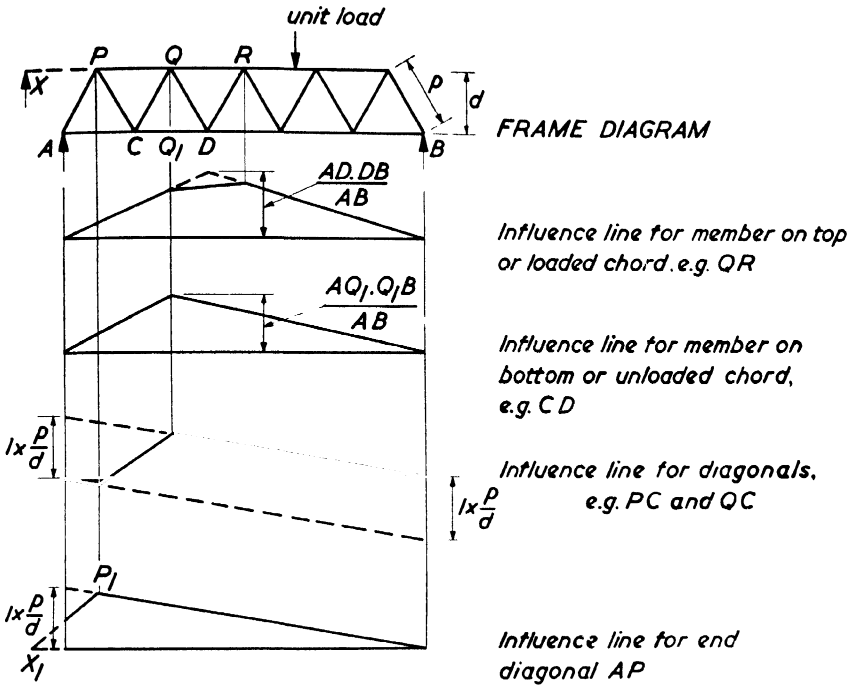


FIG. 96.—Warren girder, load on top chord

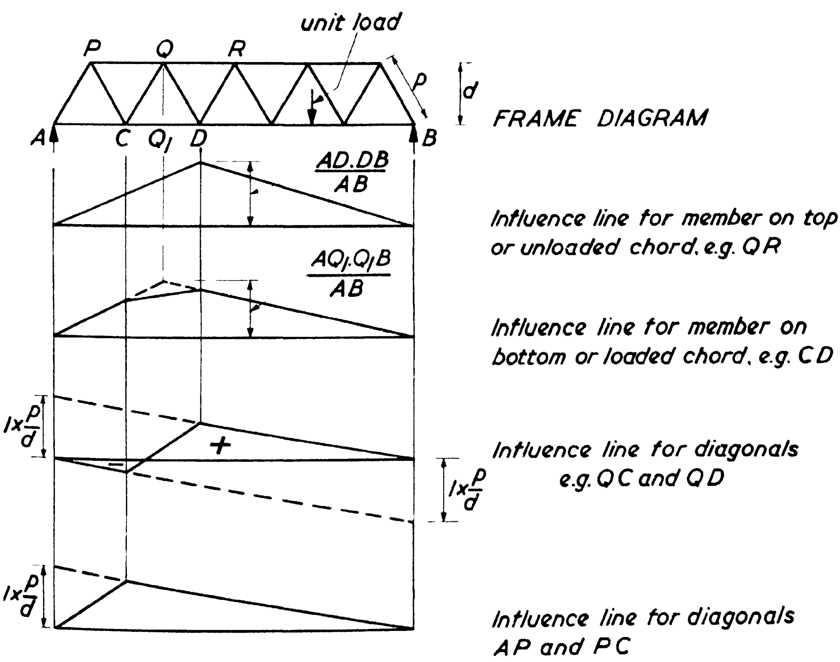


FIG. 97.—Warren girder, load on bottom chord

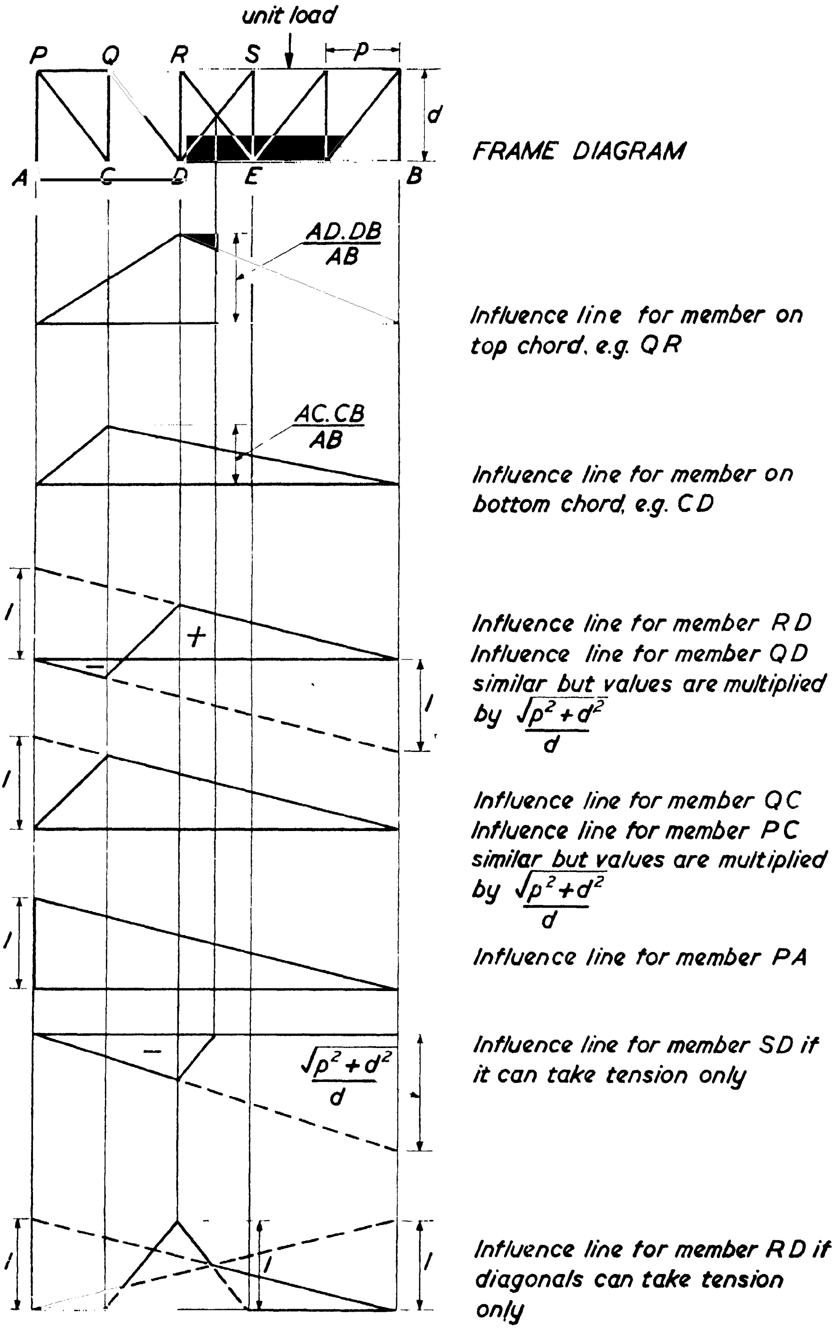


FIG. 98.—Pratt or N-truss, load on top chord

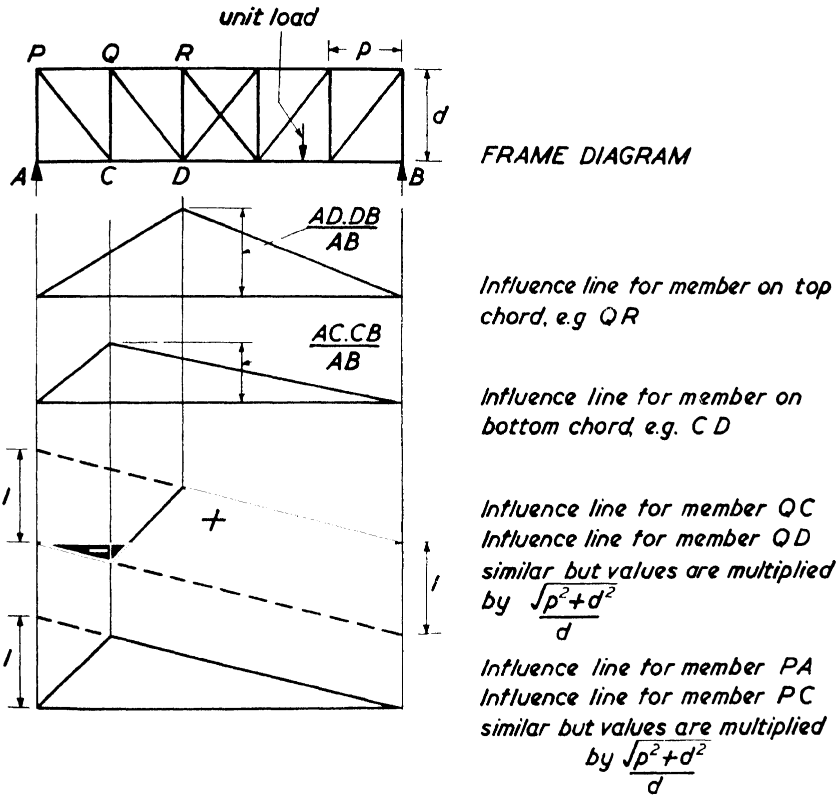


FIG. 99.—Pratt or N-truss, load on bottom chord

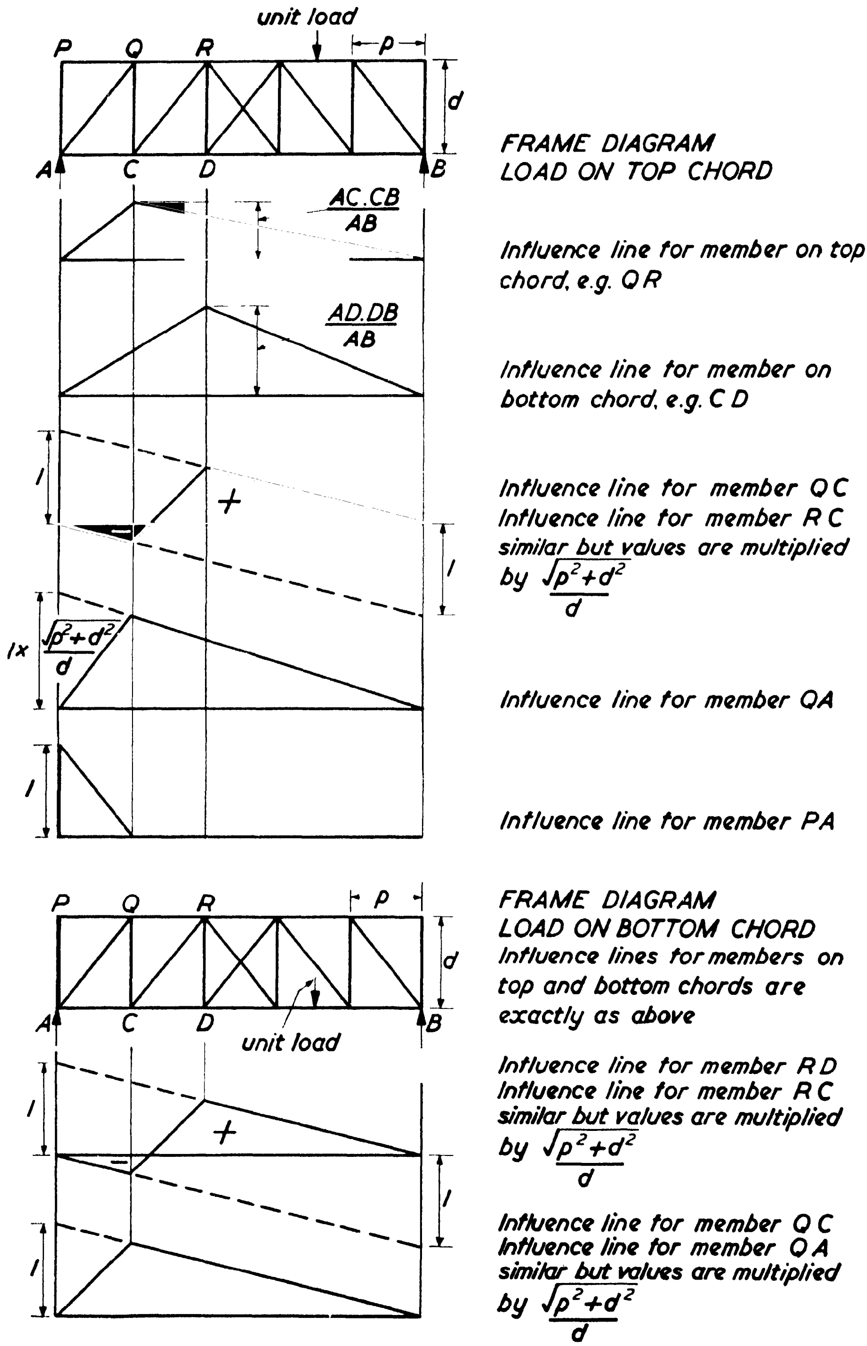


FIG. 100.—Pratt or N-truss with diagonals reversed from Figs. 98 and 99. Load on either top or bottom chord

It will be clear from the foregoing diagrams that for a single point load the maximum B.M. and maximum S.F., both +ve and −ve, are given when the load is placed at clearly defined points.

For a U.D. load longer than the span, it will be clear that for maximum B.M. the whole span must be covered by the load, while for maximum S.F., the load should come up to the point X in Fig. 101.

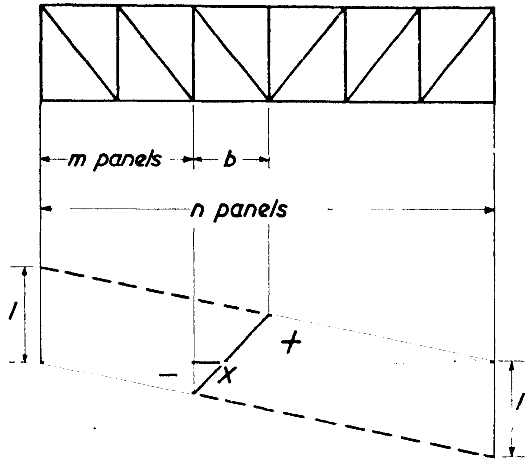


FIG. 101

The point X is found by the following means:

If there are n panels, each of length b, in a span, then the maximum S.F. in the $(m+1)$ panel occurs when $\frac{m}{n-1}$ of that panel is covered by the load, in other words, the point X divides the panel as it divides the span.

Figs. 102 to 106 show influence lines for some of the more uncommon types of frame.

Fig. 102 shows a hog-backed girder.

Consider a unit load moving along the lower chord.

If it is required to draw the influence lines for the members PQ, QC and CD, i.e. those members cut by the line EE, then it should be obvious that the influence line for PQ is similar to that for member QR in Fig. 97, except that the scale is increased in proportion to the ratio of the length of QQ_1, i.e. the vertical height of Q, to the length SC, i.e. the normal from C to PQ, while that for CD is similar to that for member CD in Fig. 97.

Regarding member QC, the force in this member is a measure of the moment of all the external forces to the left of the section taken about the point X, this point being where QP produced and BA produced meet, i.e. Force in $QC = M_X/y$.

The influence line for QC is constructed as follows.

Set up from A_1 a distance x equal to the distance XA. Join B_1 to the point thus found and produce to meet the vertical through X in X_1. The influence line between B and D lies on this line, i.e. B_1D_1. Now join X_1A_1 and pro-

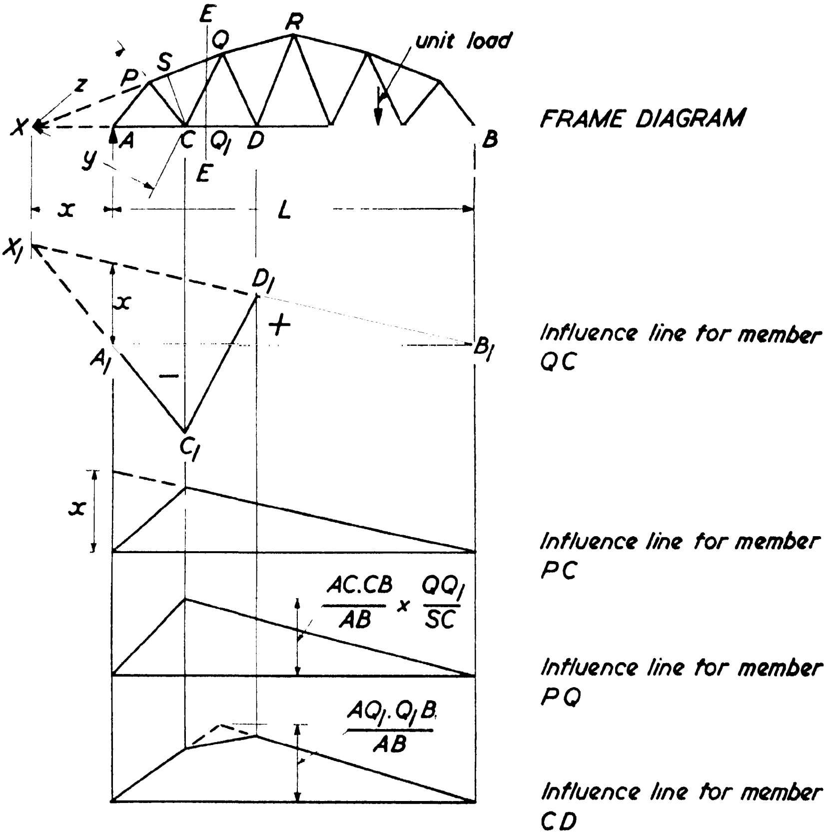


FIG. 102

duce to C_1 which is vertically below C. Join C_1D_1. Then $A_1C_1D_1B_1$ is the complete influence line.

The force in member *PC* is also a measure of the moment about *X*, and consequently the influence line for *PC* is constructed in the same manner as that for *QC*, but, in this instance, the influence line does not cross the base line.

Fig. 103 shows an N-truss with intermediate bracing. This type of bracing is used in long span trusses in order to avoid excessive panel lengths. In the example shown the load is on the top chord, in which case the intermediate bracing is needed only to reduce the unsupported lengths of the top chord in order to minimise or obviate local bending effects.

If the load is on the bottom chord then the bracing should be of the type shown in Fig. 104.

The influence lines for members on the bottom chord are exactly the same as shown in Fig. 98, but those for members in the top chord are affected by the intermediate bracing. For example, the influence line for members

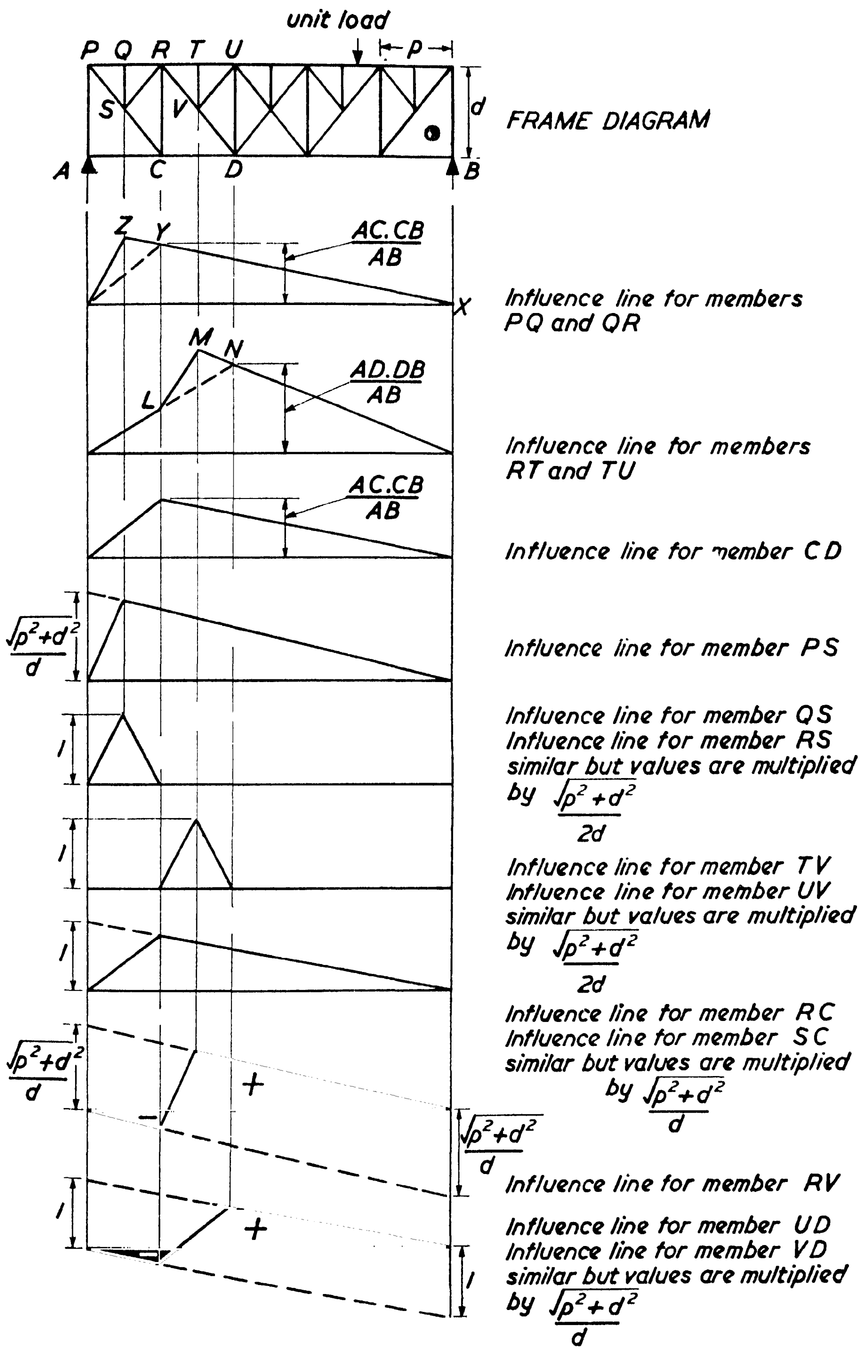


FIG. 103.—Pratt or N-truss with intermediate bracing. Load on top chord

PQ and *QR* is similar to that for the corresponding member *PQ* in Fig. 98, i.e. that for the point *C*, except that in the panel *PR* a small triangle is superimposed on to the basic influence line. This triangle is formed by continuing the line *XY* until it reaches a point *Z* which is vertically under the point *Q* and then completing the influence line as shown.

Similarly the influence line for members *RT* and *TU* is the basic influence line for the point *D* with a similar triangle *LMN* superimposed upon it.

The influence lines for the main web members are of the usual form, except that the main diagonals, e.g. *RD*, have different influence lines for each subdivided part, while the influence lines for the intermediate members, e.g. *TV* and *UV*, have a special form, since no forces are developed in these members except when the load is in the appropriate panel, i.e. *RU*.

Figs. 105 and 106 show the influence lines for lattice girders with K-type bracing. In general, the influence lines follow the usual pattern except that as far as the web members are concerned, any pair of diagonals, e.g. *EQ* and *EC*, share the shear in the panel equally and the actual forces in these members depend on the ratio of their lengths to the half-depth of the truss.

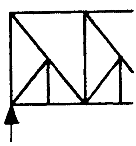


FIG. 104

For the case of loading on the top chord, member *QF* and all other members which form the upper portions of the verticals will have a special form of influence line as shown in Fig. 105. Conversely, when the load is on the bottom chord, member *FC* and all other members which form the lower portions of the verticals will have the special form shown in Fig. 106.

As previously noted in the case of the N-girder, the influence lines for the web members in the end panel do not cross the base line, but in the cases of all other web members, reversals do take place.

Influence Lines for Built-in Beams

Fig. 107 (a) shows the influence lines for the negative B.M.s M_A and M_B at the supports *A* and *B*.

The use of the influence lines in this case is illustrated by the example shown in Fig. 107 (b) and (c).

From the influence lines the fixing moment at *A* due to W_1

$$=0{\cdot}128\ W_1L=0{\cdot}128\times5\times10=6{\cdot}4 \text{ tons ft.,}$$

that due to W_2

$$=0{\cdot}144\ W_2L=0{\cdot}144\times6\times10=8{\cdot}64 \text{ tons ft.}$$

and that due to W_3

$$=0{\cdot}063\ W_3L=0{\cdot}063\times4\times10=2{\cdot}52 \text{ tons ft.}$$

Therefore the total fixing moment at *A*

$$=6{\cdot}4+8{\cdot}64+2{\cdot}52=17{\cdot}56 \text{ tons ft.}$$

Fixing moment at *B* due to W_1

$$=0{\cdot}032\ W_1L=0{\cdot}032\times5\times10=1{\cdot}6 \text{ tons ft.,}$$

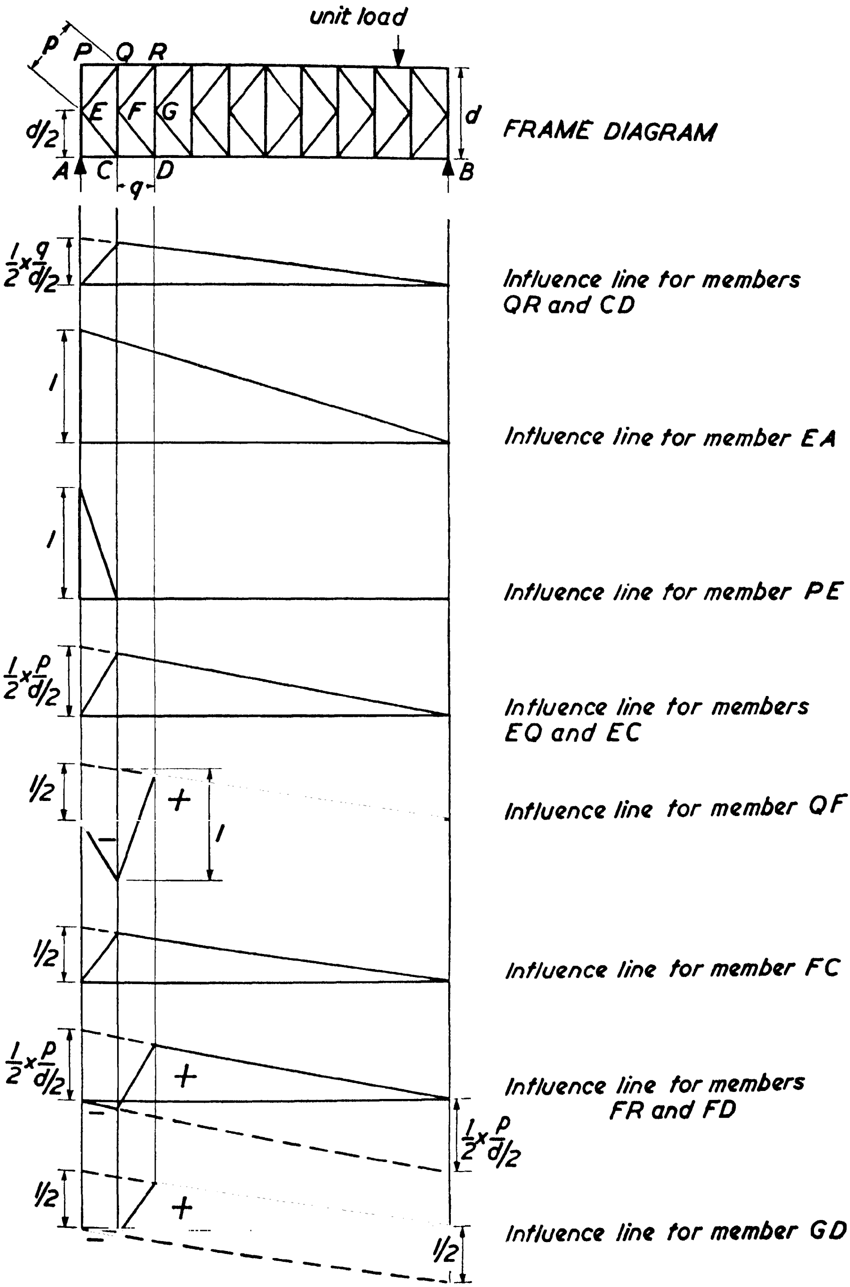


FIG. 105.—Lattice girder with K bracing. Load on top chord.

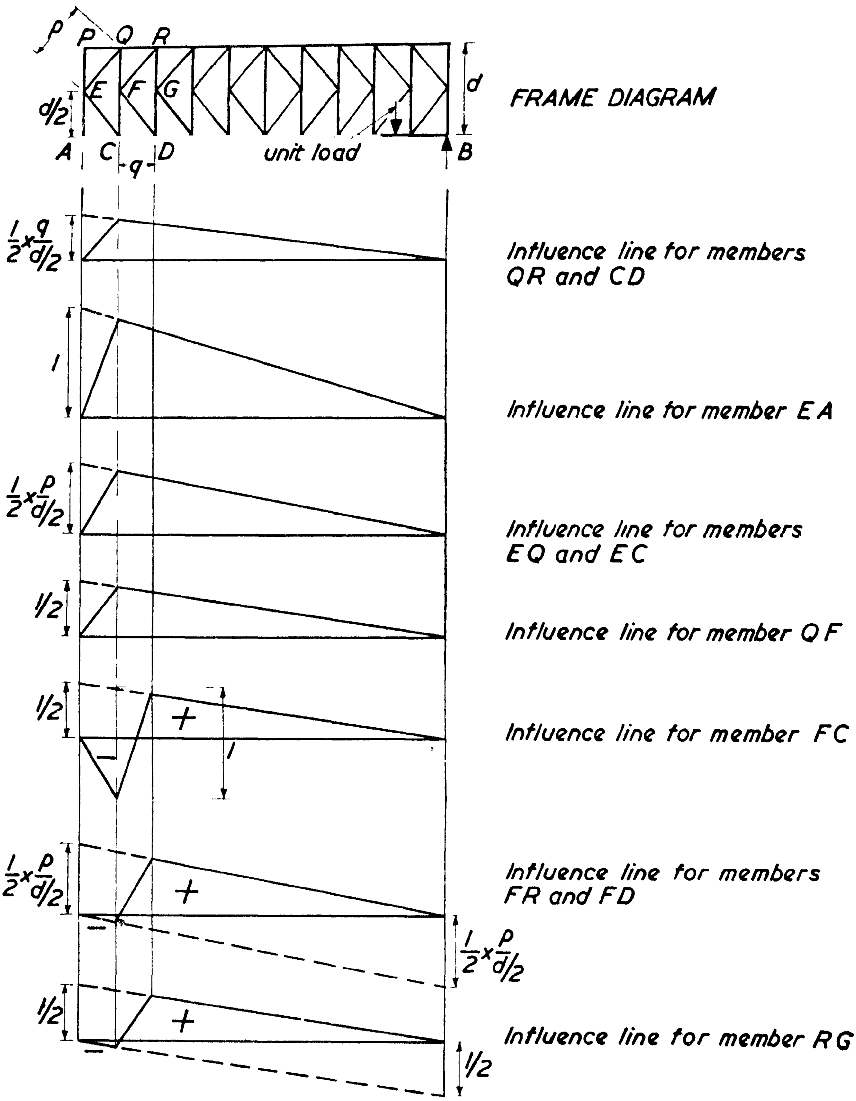


FIG. 106.—Lattice girder with K bracing. Load on bottom chord

that due to W_2

$$=0{\cdot}096\ W_2L=0{\cdot}096\times6\times10=5{\cdot}76 \text{ tons ft.}$$

and that due to W_3

$$=0{\cdot}147\ W_3L=0{\cdot}147\times4\times10=5{\cdot}88 \text{ tons ft.}$$

Therefore the total fixing moment at B

$$=1{\cdot}6+5{\cdot}76+5{\cdot}88=13{\cdot}24 \text{ tons ft.}$$

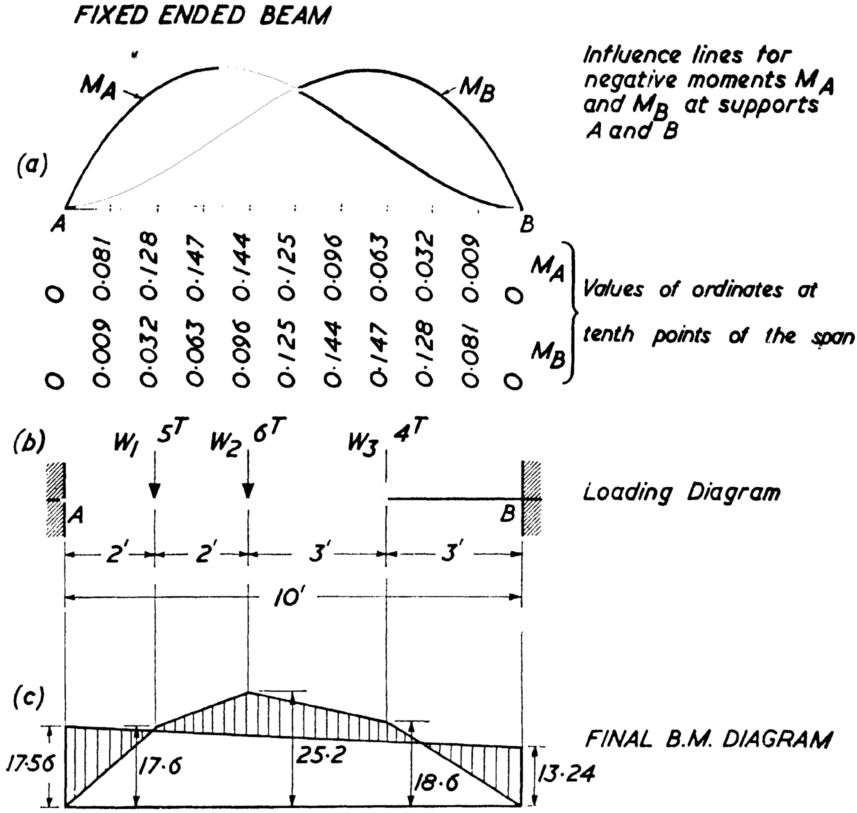


FIG. 107

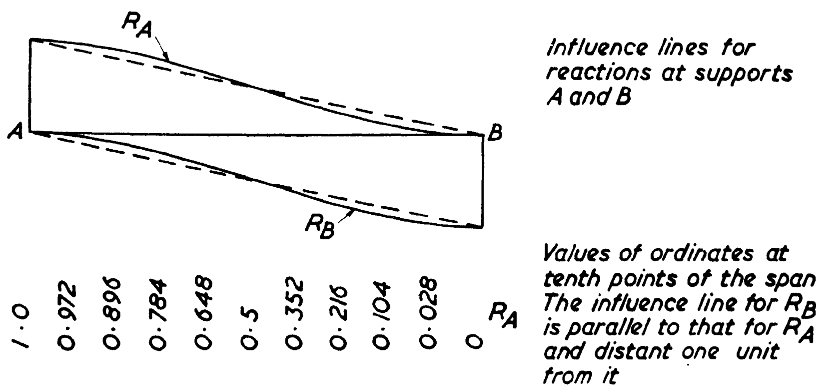


FIG. 108

The results will be checked by the principle of reciprocal moments which was mentioned in the chapter on Fixed Beams. See page 26.

$$M_A=\left(\frac{5\times2\times8}{10}+\frac{8}{10}\right)+\left(\frac{6\times4\times6}{10}\times\frac{6}{10}\right)+\left(\frac{4\times7\times3}{10}\times\frac{3}{10}\right)$$

$$=6{\cdot}4+8{\cdot}64+2{\cdot}52=17{\cdot}56 \text{ tons ft.}$$

$$M_B=\left(\frac{5\times2\times8}{10}\times\frac{2}{10}\right)+\left(\frac{6\times4\times6}{10}\times\frac{4}{10}\right)+\left(\frac{4\times7\times3}{10}\times\frac{7}{10}\right)$$

$$=1{\cdot}6+5{\cdot}76+5{\cdot}88=13{\cdot}24 \text{ tons ft.}$$

The final B.M. diagram is as shown in Fig. 107 (c). Fig. 108 shows the influence lines for the reactions at A and B. The dotted lines represent the corresponding influence lines for a simply supported beam.

Continuous Beams

All influence lines must be constructed from first principles and the following example will illustrate the method of construction.

Bending Moments

It is required to construct the influence line for B.M. at D, the mid-point of the span AB which is one span of a two-span continuous beam ABC, both spans being equal. See Fig. 109.

In any particular case, it is convenient to consider separately the portions of the beam to either side of the point for which the influence line is being drawn, because in these separate portions, the formulæ for the B.M. at the point will take different forms.

Portion A to D. (See Fig. 109 (a).) If a load W is placed at the point X, which divides the span AB into two parts a and $(L-a)$, then the

$$\text{free B.M. at } X=\frac{W\times a\times(L-a)}{L}.$$

If the load is unity,

$$\text{the B.M. at } X=\frac{a(L-a)}{L}.$$

The area of the free B.M. diagram on the span AB

$$=\frac{a(L-a)}{L}\times\frac{L}{2}=\frac{a(L-a)}{2},$$

and the C.G. of this area from A, measured horizontally,

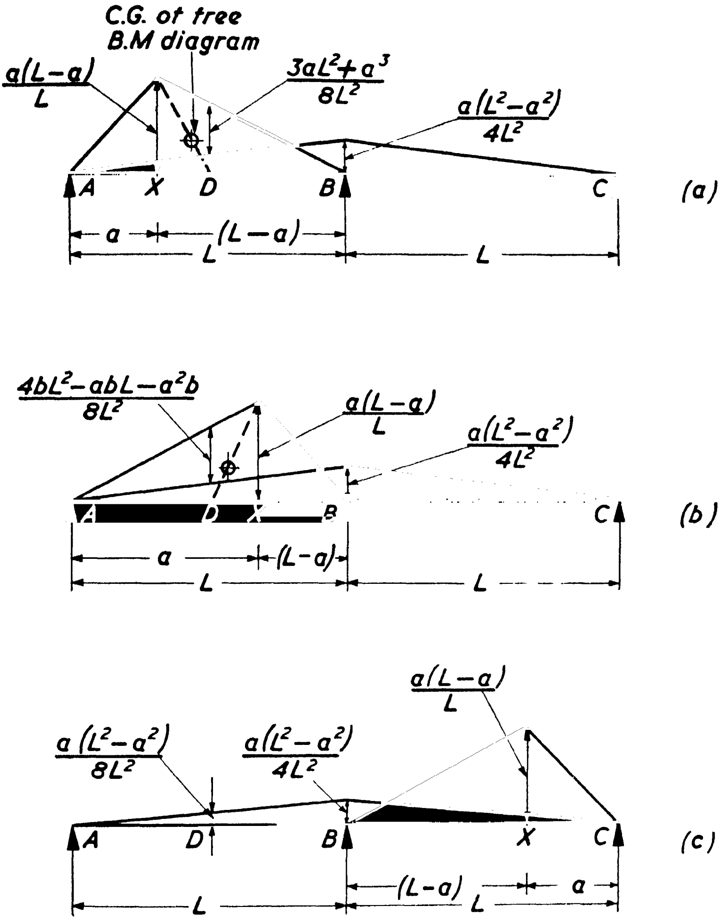
$$=AX+\tfrac{2}{3}XD=a+\frac{2}{3}\left(\frac{L}{2}-a\right)=\frac{L+a}{3}.$$

Applying the Theorem of Three Moments to spans AB and BC, and bearing in mind that $M_A=M_C=0$, that there is no load on the span BC and that the two spans are equal, then

$$2M_B\times2L=6\,.\,\frac{a(L-a)}{2}\times\frac{L+a}{3}\times\frac{1}{L},$$

whence

$$M_B=\frac{a(L^2-a^2)}{4L^2}.$$



ALL THE B.M DIAGRAMS ARE FOR UNIT LOADS

FIG. 109

The height of the free B.M. diagram at D

$$=\frac{a(L-a)}{L}\times\frac{L/2}{L-a}=\frac{a}{2},$$

while the height of the negative B.M. diagram at D

$$=\frac{1}{2}\times\frac{a(L^2-a^2)}{4L^2}=\frac{a(L^2-a^2)}{8L^2},$$

therefore the net B.M. at $D=\frac{a}{2}-\frac{a(L^2-a^2)}{8L^2}=\frac{3aL^2+a^3}{8L^2}$,

and if the span is unity, this expression can be written:

$$\text{net B.M. at } D=\frac{3a+a^3}{8}.$$

From this expression the B.M. influence line for D from A to D can be built up by successive substitution of values for a. This is best done by means of a table as shown below:

a	$3a$	a^3	$3a+a^3$	$\frac{3a+a^3}{8}$
0·1	0·3	0·001	0·301	0·038
0·2	0·6	0·008	0·608	0·076
0·3	0·9	0·027	0·927	0·116
0·4	1·2	0·064	1·264	0·158
0·5	1·5	0·125	1·625	0·203

Portion D to B. (See Fig. 109 (b).) In this portion the horizontal distance from A to the C.G. of the free B.M. diagram is

$$\frac{L}{2}+\frac{1}{3}a-\frac{L}{2}=\frac{L+a}{3},$$

i.e. exactly as in the portion AD, and consequently

$$M_B=\frac{a(L^2-a^2)}{4L^2}\quad\text{also as before.}$$

The height of the free B.M. diagram at D

$$=\frac{a(L-a)}{L}\times\frac{L/2}{a}=\frac{L-a}{2},$$

while the height of the negative B.M. diagram at D

$$=\frac{a(L^2-a^2)}{8L^2}\quad\text{as before.}$$

Therefore the net B.M. at D

$$=\frac{L-a}{2}-\frac{a(L^2-a^2)}{8L^2}=\frac{4L^3-5aL^2+a^3}{8L^2}$$

and again putting $L=1$,

$$\text{net B.M. at } D=\frac{4-5a+a^3}{8}.$$

The corresponding table is given below:

a	$5a$	a^3	$4-5a+a^3$	$\frac{4-5a+a^3}{8}$
0·5	2·5	0·125	1·625	0·203
0·6	3·0	0·216	1·216	0·152
0·7	3·5	0·343	0·843	0·105
0·8	4·0	0·512	0·512	0·064
0·9	4·5	0·729	0·229	0·028
1·0	5·0	1·0	0	0

Portion B to C. (See Fig. 109 (c).) In this portion, if the distance a is measured from C, then the negative B.M. at B will be $[a(L^2-a^2)]/4L^2$ for all values of a, and consequently the height of the B.M. diagram at D will be $[a(L^2-a^2)]/8L^2$.

Putting $L=1$, the B.M. at D will be $(a-a^3)/8$. Note that since there is no positive B.M. in the span AB when the load is on the span BC, all the values of B.M. at D will be negative.

The corresponding table is given below:

a	a^3	$a-a^3$	$\frac{a-a^3}{8}$
0·1	0·001	0·099	0·012
0·2	0·008	0·192	0·024
0·3	0·027	0·273	0·034
0·4	0·064	0·336	0·042
0·5	0·125	0·375	0·047
0·6	0·216	0·384	0·048
0·7	0·343	0·357	0·045
0·8	0·512	0·288	0·036
0·9	0·729	0·171	0·021
1·0	1·000	0	0

The complete influence line is shown in Fig. 111. (I.L. for point 5.)

Figs. 110–12, 114–16 and 118–20 show B.M. influence lines for two-, three- and four-span continuous beams.

The diagrams are drawn to scale, but since it is inadvisable to scale the ordinates from small-scale diagrams the values of the ordinates at the tenth points of each span have been given so that the reader can construct his own influence lines from these ordinates. Influence lines have been drawn for each tenth point of the span and for the internal supports.

The diagrams have been drawn only for the cases of all spans being equal, but they can also be used for varying spans provided that the Moments of Inertia of the beams also vary so that the ratio I/L in each span is constant.

There are certain features of these influence lines to which particular attention must be drawn.

1. It will be seen, from a consideration of Figs. 114 to 116 that, in the case of a three-span beam, at the points for which the influence lines have been drawn, the moments due to loads in the end spans remote from the points under consideration are extremely small. Consequently if a fourth span had been added next to the remote span, the moments due to loads in this fourth span would be even smaller and could be neglected with little loss of accuracy. Moreover, the moments caused by loads in this fourth span would be of opposite sign to those obtaining in a three-span beam, and, accordingly, the discrepancy errs on the side of safety.

It follows that any further addition to the number of spans can make negligible differences to the values given, hence the influence lines for points in the end span of a three-span beam can be taken as applying to the end span of any series of spans exceeding two in number.

2. It will be seen, from Fig. 118, that the B.M. at the 1/5th point of the second span of a four-span beam is zero for all positions of the load

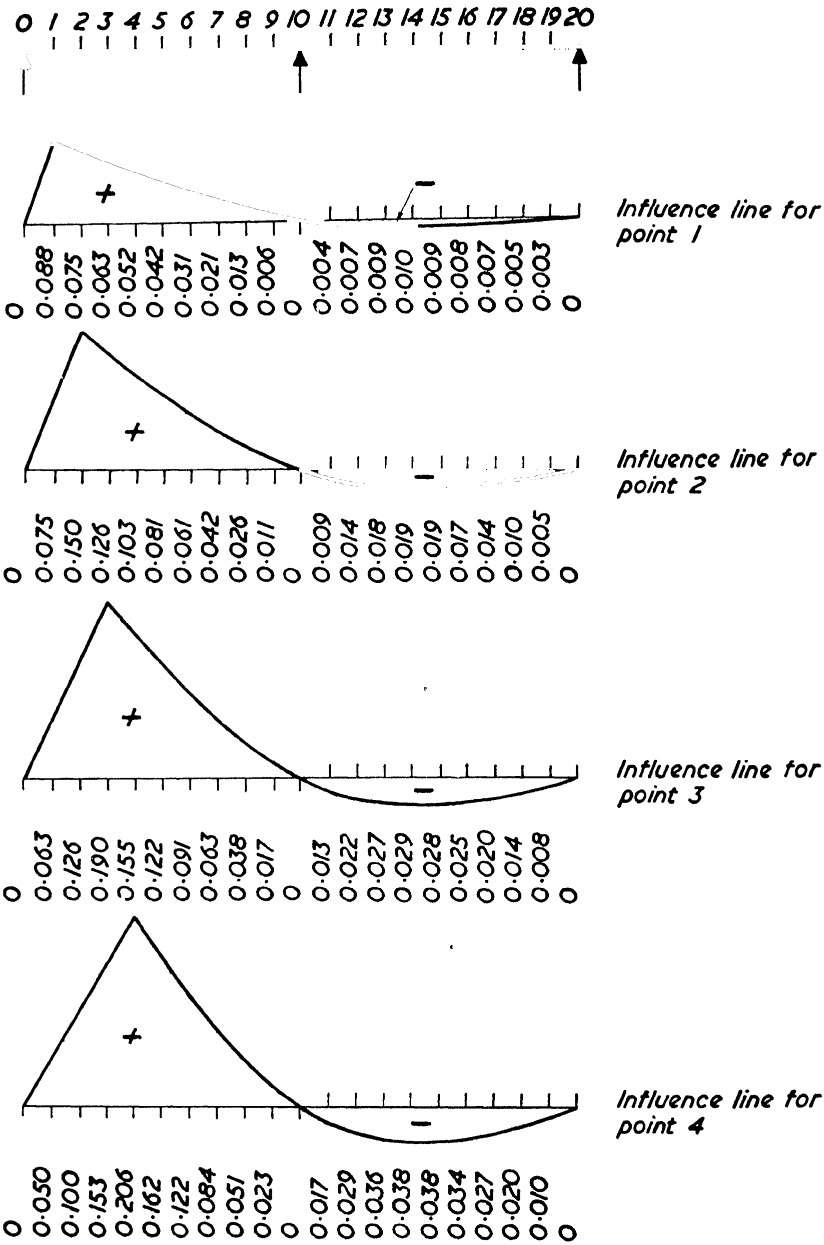


FIG. 110.—Influence lines for bending moments.
Two span beam

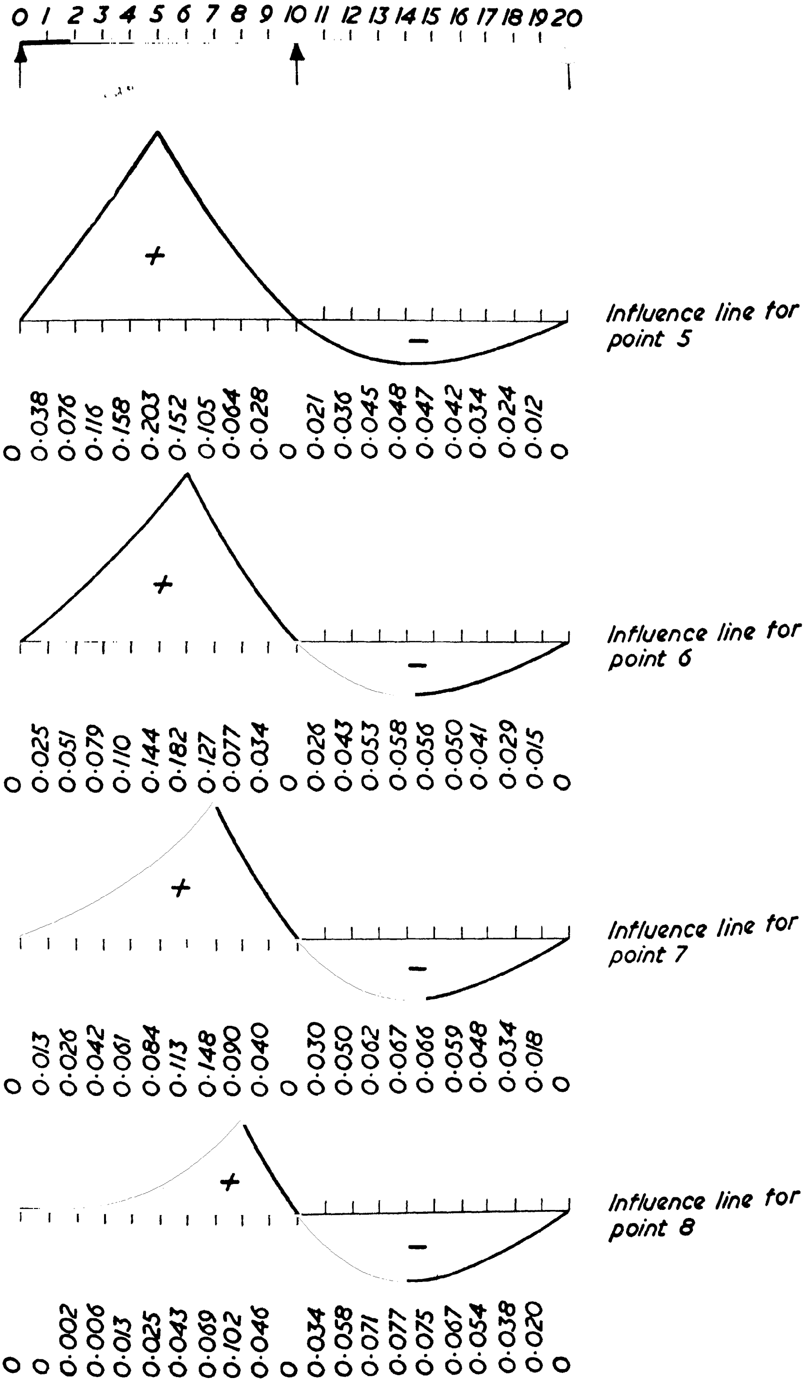


FIG. 111.—Influence lines for bending moments.
Two span beam

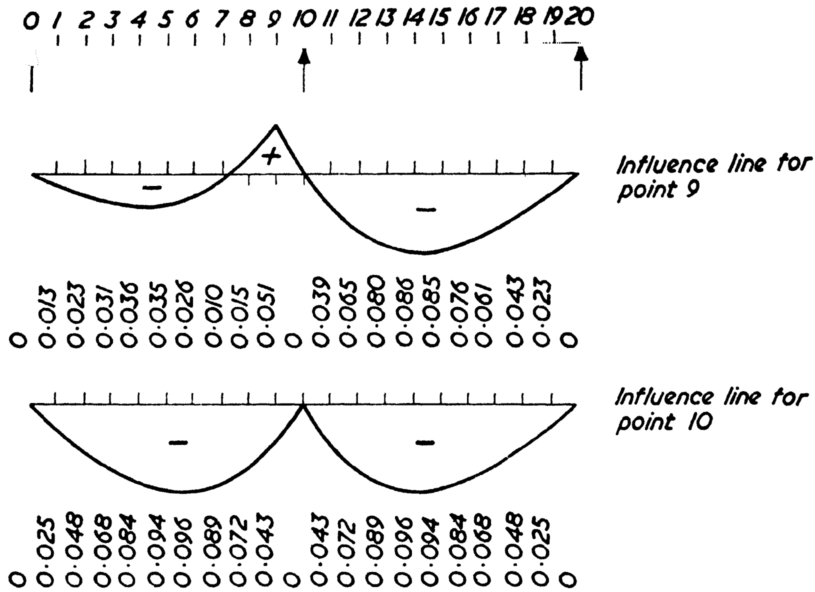


FIG. 112.—Influence lines for bending moments.

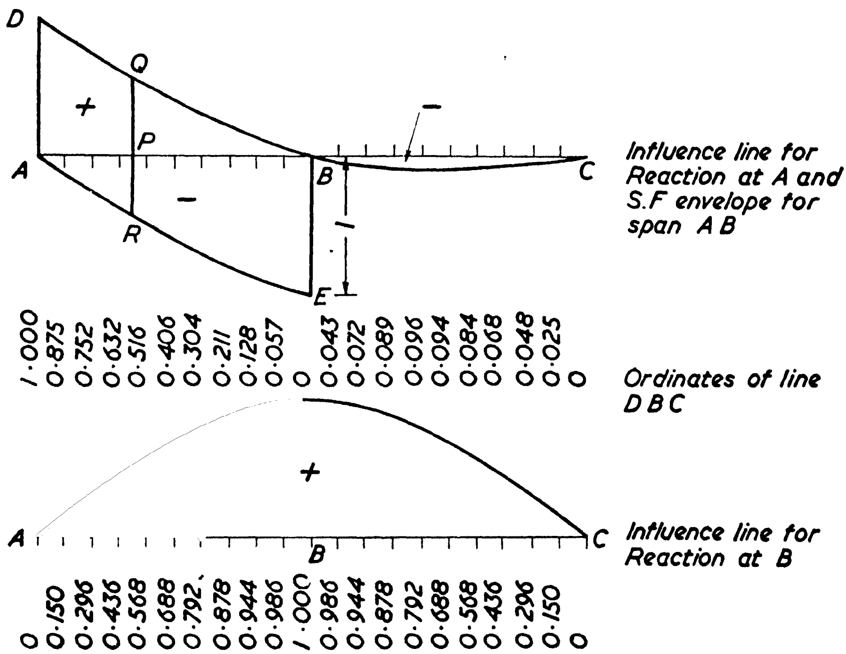


FIG. 113.—Influence lines for reactions and shear forces.
Two span beam

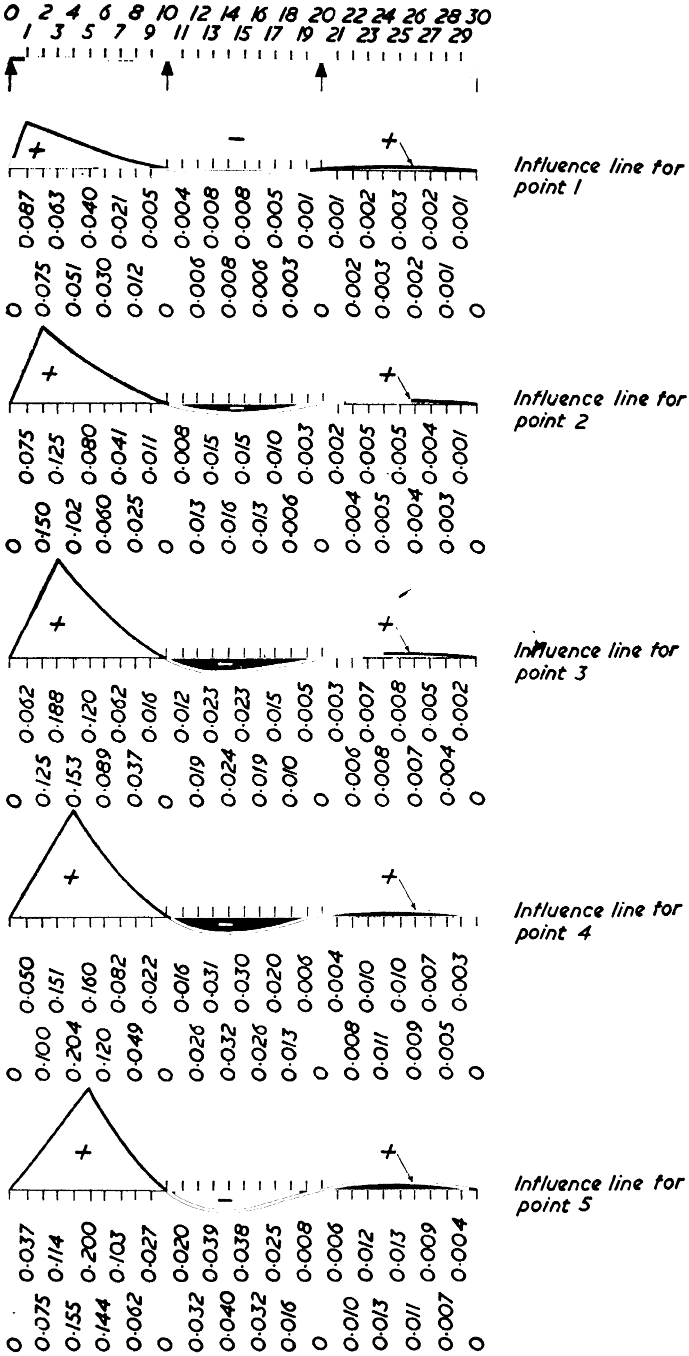


FIG. 114.—Influence lines for bending moments.
Three span beam

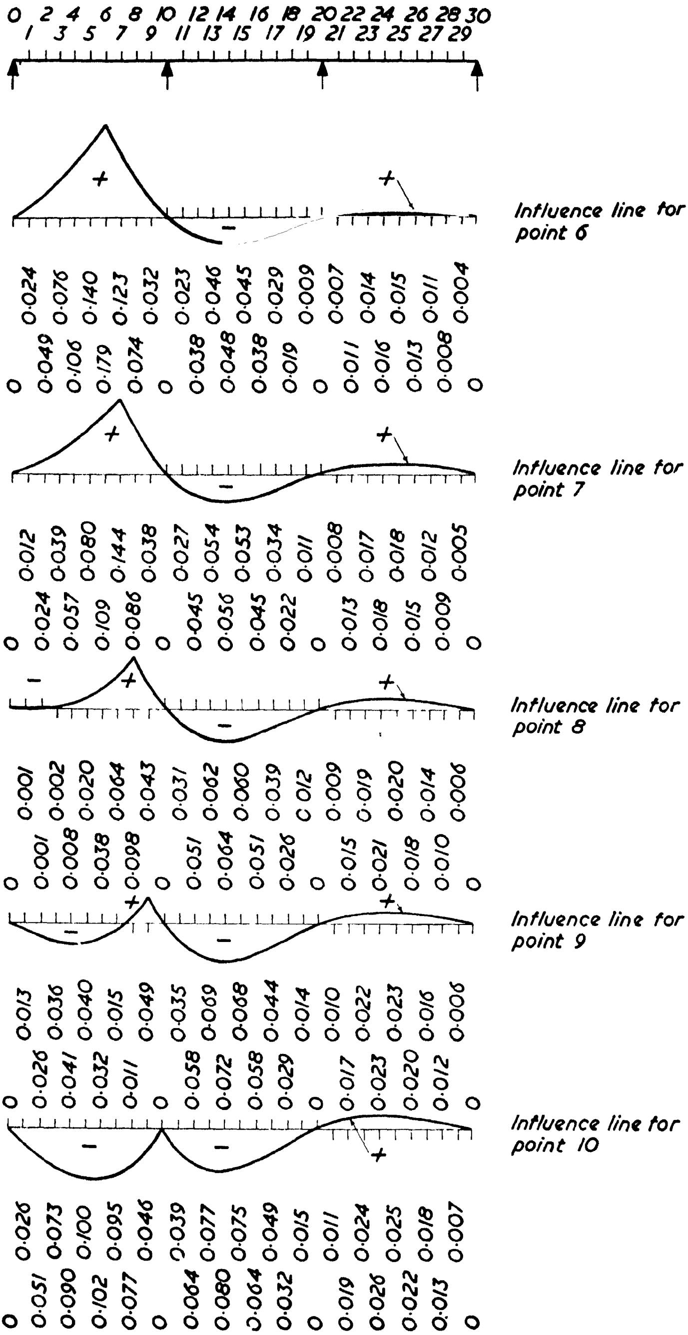


FIG. 115.—Influence lines for bending moments.
Three span beam

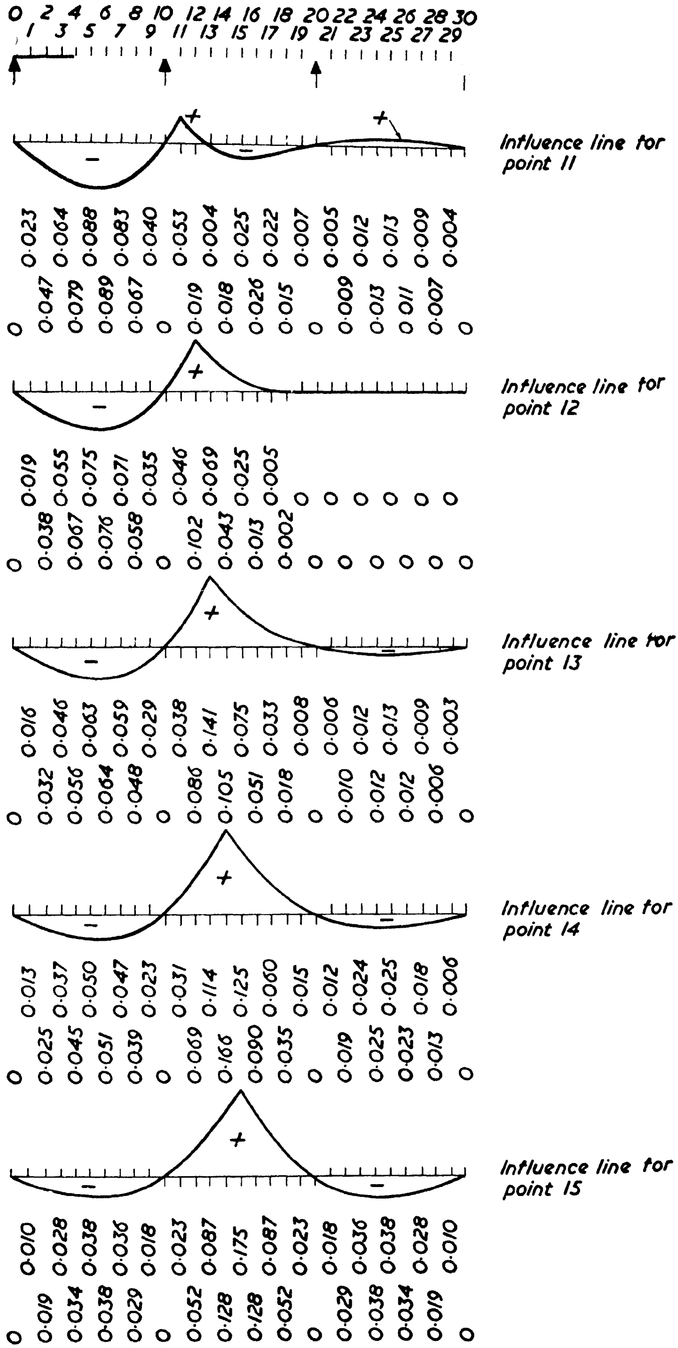


FIG. 116.—Influence lines for bending moments. Three span beam

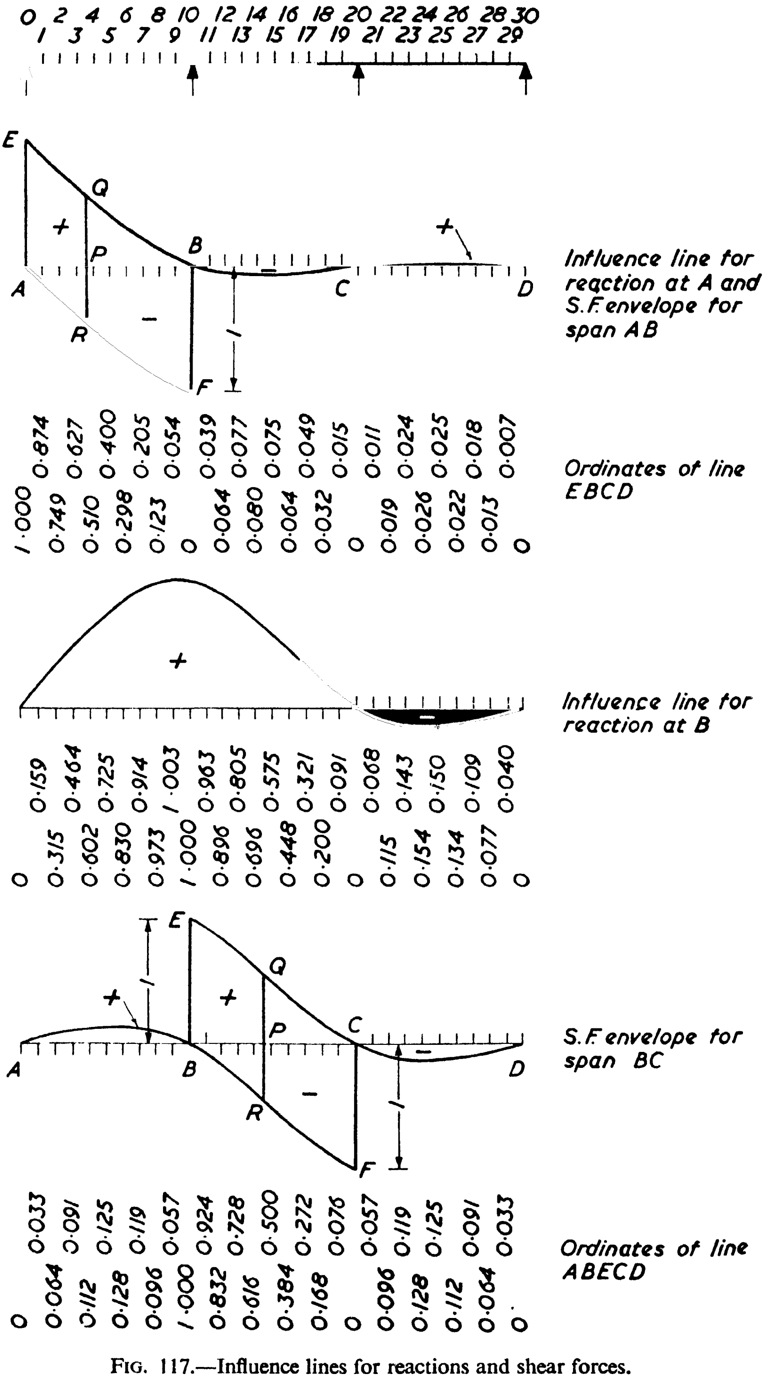


FIG. 117.—Influence lines for reactions and shear forces. Three span beam

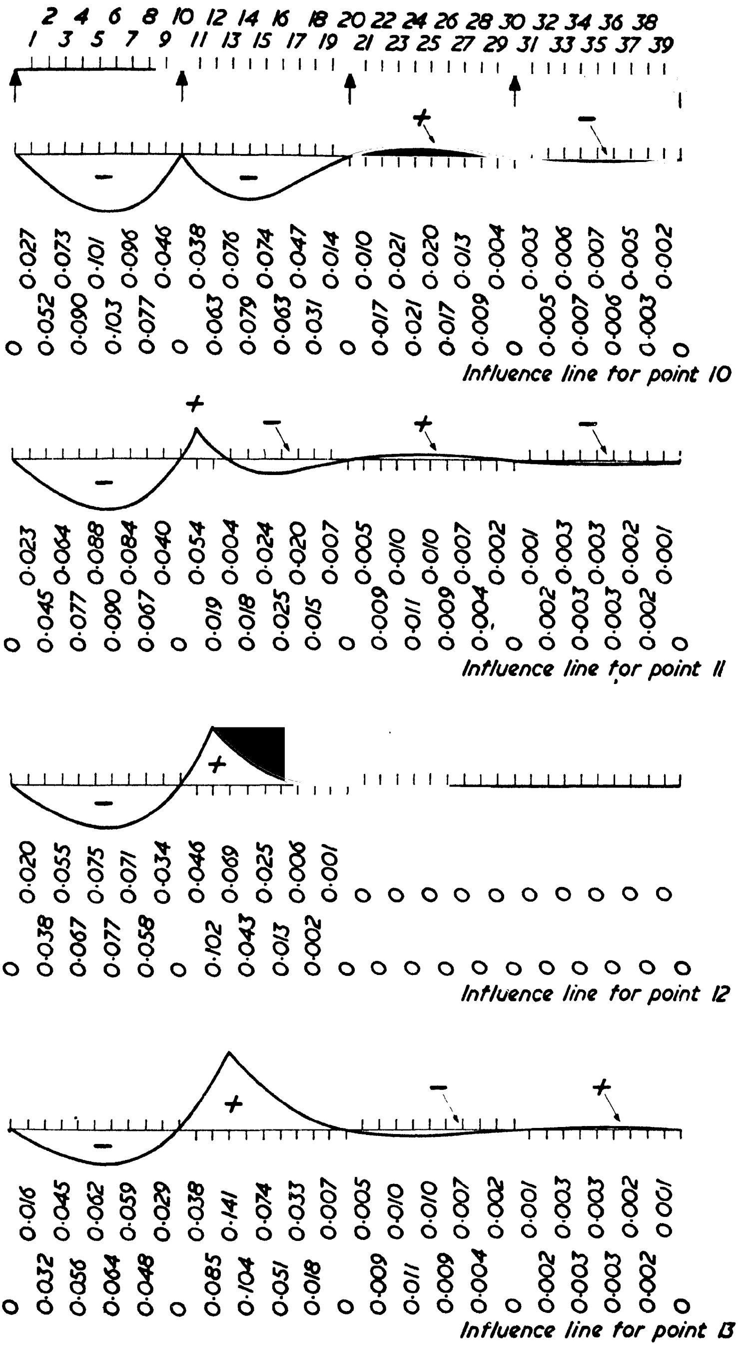


FIG. 118.—Influence lines for bending moments.
Four span beam

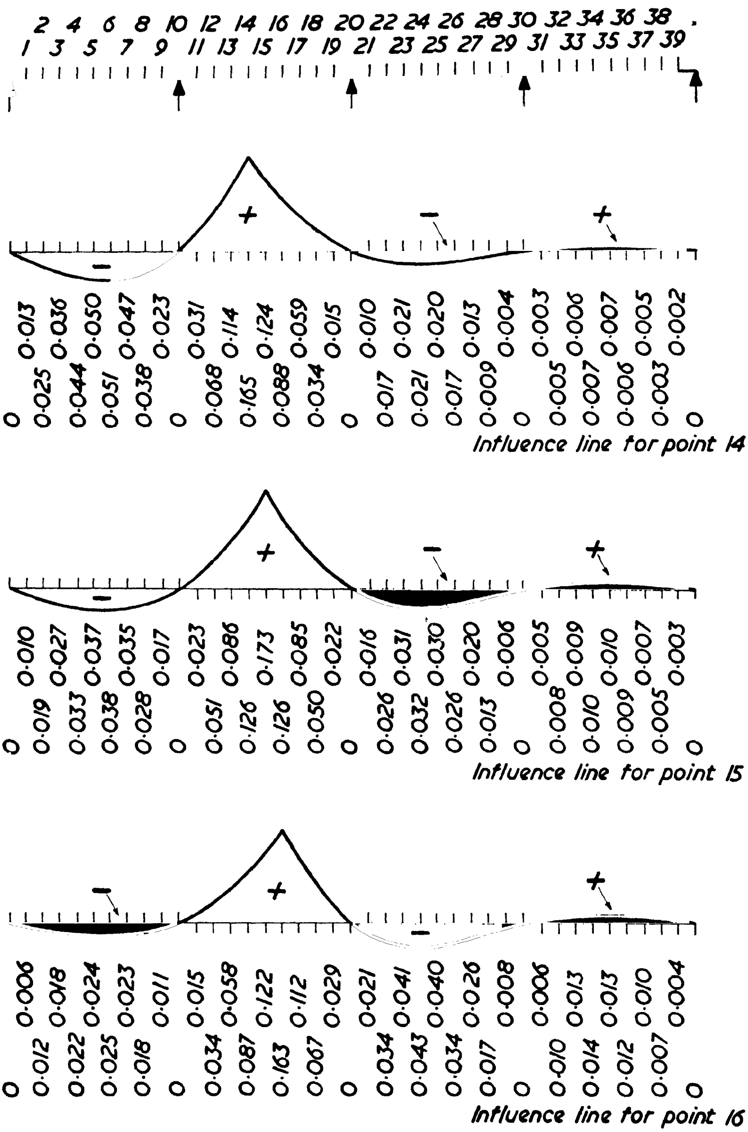


FIG. 119.—Influence lines for bending moments.
Four span beam

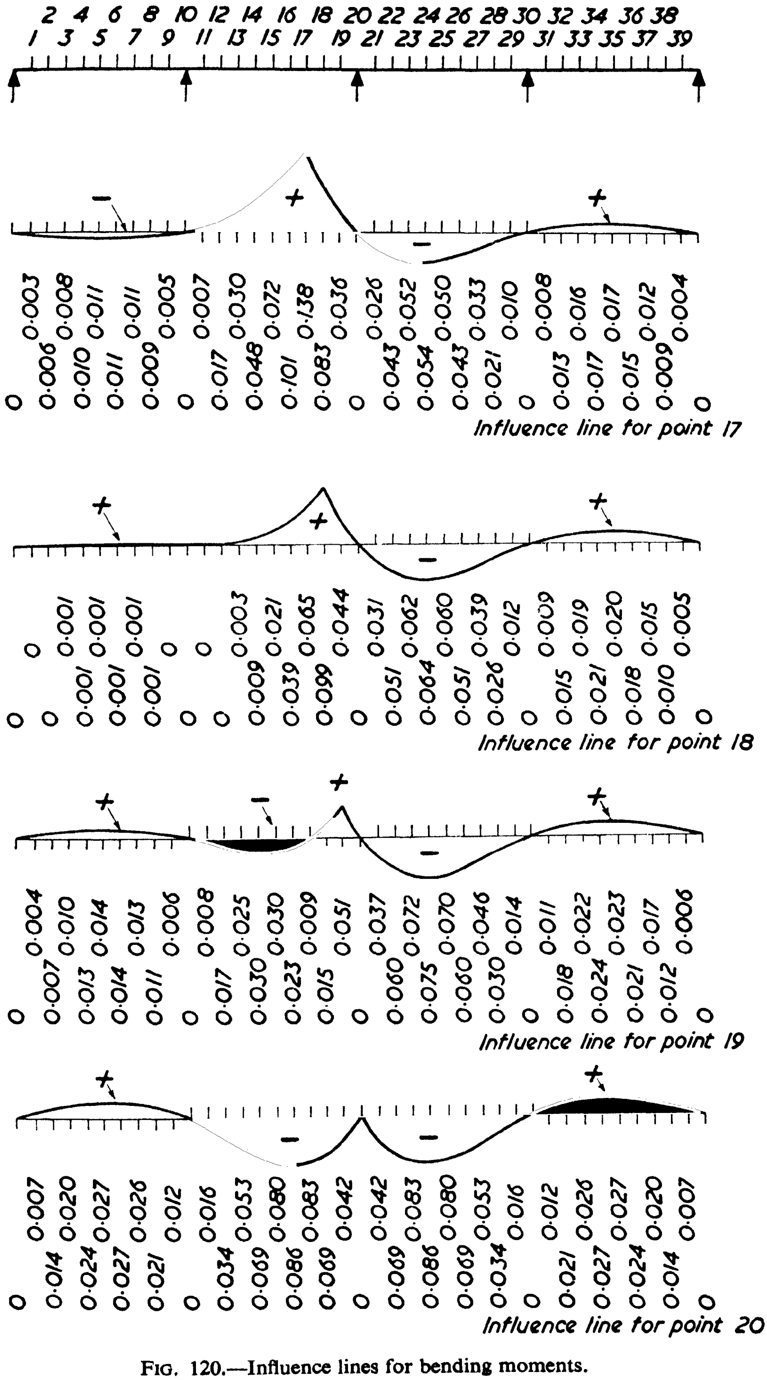


FIG. 120.—Influence lines for bending moments.
Four span beam

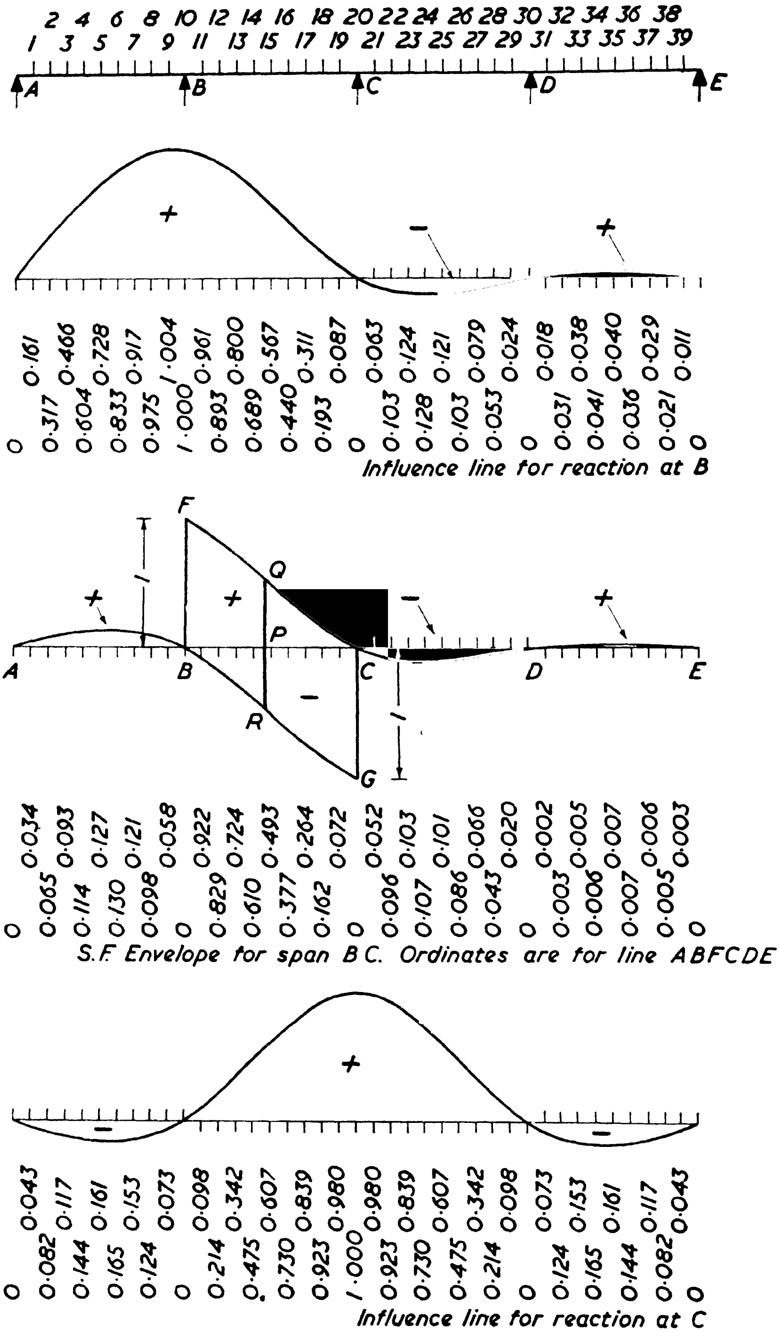


FIG. 121.—Influence lines for reactions and shear forces. Four span beam

in the third and fourth spans. This means that if the first and second spans are not loaded, then for any case of loading on the third and fourth spans there will always be a point of contraflexure at the 1/5th point of the second span.

3. Again a comparison between Figs. 114 to 116 and Figs. 118 to 120 will show that there is little difference between the moments at the corresponding points in internal spans in the cases of three- and four-span beams. It follows that there will be even less discrepancy between corresponding points in four- and five-span beams.

Accordingly the influence lines shown in Figs. 118 to 120 may be taken as the influence lines for all internal spans of continuous beams of any series of spans exceeding three in number. The top diagram of Fig. 118 may be taken as the influence line for the end support but one, while the bottom diagram of Fig. 120 may be taken as the influence line for all other internal supports of a series of spans exceeding three in number.

Shear Force

The influence lines for S.F. are best constructed by means of envelopes. If reference is made to Fig. 91 (c), it will readily be seen that all the influence lines for S.F. in a simply supported span are contained between the two enveloping sloping lines A_2Y and B_2Z, and that the influence line for any particular point, such as P_2 in the figure referred to, is constructed by drawing a vertical through P_2 to meet the two enveloping lines. Continuous beams may be treated in a similar manner.

Two-span Beam

It is required to construct the influence lines for the reactions at the supports of a two-span continuous beam. The following example will illustrate the method of construction.

Reaction at A. To construct the influence line for the reaction at A, one of the end supports of a two-span beam of equal spans. See Fig. 109.

If a unit load is placed at the point X distant a from A, then M_B, the negative moment at B, as shown in Fig. 109,

$$=\frac{a(L^2-a^2)}{4L^2}.$$

The free reaction at A would be $1\times\frac{L-a}{L}$, and the actual reaction in the given case would be

$$\frac{L-a}{L}-\frac{M_B}{L}=\frac{L-a}{L}-\frac{a(L^2-a^2)}{4L^3}.$$

When $L=1$, the reaction at $A=1-a-\frac{a-a^3}{4}$.

The values are plotted in the following table:

a	$1-a$	$\frac{a-a^3}{4}$	$1-a-\frac{a-a^3}{4}$
0	1·000	0	1·000
0·1	0·900	0·025	0·875
0·2	0·800	0·048	0·752
0·3	0·700	0·068	0·632
0·4	0·600	0·084	0·516
0·5	0·500	0·094	0·406
0·6	0·400	0·096	0·304
0·7	0·300	0·089	0·211
0·8	0·200	0·072	0·128
0·9	0·100	0·043	0·057
1·0	0	0	0

If the load is in the span BC then it is convenient to measure the distance a from C. In this case:

$$R_A = 0 - \frac{M_B}{L} = -\frac{a(L^2 - a^2)}{4L^3}$$

or

$$-\frac{a-a^3}{4} \quad \text{when } L = 1.$$

The corresponding table is shown below:

a	$-\frac{a-a^3}{4}$
0	0
0·1	−0·025
0·2	−0·048
0·3	−0·068
0·4	−0·084
0·5	−0·094
0·6	−0·096
0·7	−0·089
0·8	−0·072
0·9	−0·043
1·0	0

It will be appreciated that these figures are applicable to R_C when the span AB is loaded.

The complete influence line for the reaction at A is drawn by the line DBC in Fig. 113.

If the line AE is drawn parallel to DB and distant one unit from it, then the lines DB and AE are the boundary lines of the S.F. envelope for all points in the span AB.

If the influence line for any point, e.g. P, is required, a vertical line QPR is drawn through P to meet DB and AE in Q and R respectively. The influence line for S.F. at P is $ARPQBC$.

Reaction at B. The influence line for R_B is built up as follows:

$$R_B = 1 - R_A - R_C$$

$$= 1 - \left(1 - a - \frac{a - a^3}{4}\right) - \left(-\frac{a - a^3}{4}\right)$$

$$= 1 - \left(1 - a - \frac{a - a^3}{4}\right) + \left(\frac{a - a^3}{4}\right)$$

The corresponding table is shown below:

a	$1 - a - \frac{a - a^3}{4}$	$\frac{a - a^3}{4}$	$1 - \left(1 - a - \frac{a - a^3}{4}\right) + \frac{a - a^3}{4}$
0	1·000	0	0
0·1	0·875	0·025	0·150
0·2	0·752	0·048	0·296
0·3	0·632	0·068	0·436
0·4	0·516	0·084	0·568
0·5	0·406	0·094	0·688
0·6	0·304	0·096	0·792
0·7	0·211	0·089	0 878
0·8	0·128	0·072	0·944
0·9	0 057	0·043	0 986
1·0	0	0	1·000

The complete influence line for the reaction at B is shown in Fig. 113.

It should be noted that the reaction at B is greater than the S.F. at B in the span AB, because the reaction is composed of the sum of the shears at B in both spans.

Three-span Beam

Consider the top diagram in Fig. 117.

The line $EBCD$ shows the influence line for the reaction at A. Once again, if AF is drawn parallel to EB and distant one unit from it, then the lines EB and AF are the boundary lines of the S.F. envelope for all points in the span AB. For example, $ARPQBCD$ is the influence line for shear at the point P.

As in the case of the influence lines for B.M. for points in the end span AB, the addition of any number of spans beyond D can make negligible difference to the S.F. in the span AB, and so the envelope shown can be taken as the envelope for the end span of any series of spans exceeding two in number.

The influence line for the reaction at B and the S.F. envelope for the centre span BC are also shown in Fig. 117. In the bottom diagram, BF is parallel to EC and distant one unit from it, and the lines EC and BF are the boundary lines of the S.F. envelope for all points in the span BC. Therefore, $ABRP$-QCD is the influence line for shear at the point P.

Four or More Spans

The influence line for the reaction at B, the end but one support of a four-span beam, is shown in Fig. 121 and may be taken as that for the end support but one of any number of spans greater than three.

The S.F. envelope for an internal span of a four-span beam is also shown in Fig. 121, *FC* and *BG* being the boundary lines. Hence, *ABRPQCDE* is the influence line for shear at the point *P*. The envelope shown may be taken to be that for any internal span of a continuous beam of more than three spans.

The bottom diagram of the same figure shows the influence line for the reaction at *C*, the central support of a four-span beam. This may be taken as the influence line for any internal support of a continuous beam of more than three spans, other than the end support but one.

12. MODERN METHODS OF STRUCTURAL ANALYSIS

Introduction

In the opening sections of this book methods of calculation are given for built-in and continuous beams, the methods being based on the theorems of Mohr and Clapeyron respectively, which were developed in the nineteenth century.

The systems of analysis of statically indeterminate structures to be described in this section of the book are of more recent development but are based on the work of earlier engineers, particularly Mohr. It should be emphasised that higher mathematics is not required in the employment of these systems, and therefore they should be of general appeal. The methods are as follows:

1. Moment Distribution.
2. Column Analogy.
3. Slope Deflection.
4. Semi-graphical Integration.

The two methods described in greatest detail are Moment Distribution and Column Analogy, both of which were evolved by the American engineer Professor Hardy Cross. They are followed by Slope Deflection, which was developed somewhat earlier by the Americans Wilson and Maney, and by some notes on Semi-graphical Integration.

Where convenient or practicable, the same numerical examples have been employed to illustrate the various methods of analysis, although, however, the examples have been chosen to cover as many aspects of structural engineering as possible. By so doing, it is hoped that readers will be able to form some idea of the relative merits of the methods for a particular type of problem. Generalising, Moment Distribution and Slope Deflection are of universal application, but the work involved is lengthy where sidesway or asymmetrical frames are involved. Column Analogy may be used for all types of single-bay frames or closed rings, being particularly useful for dealing with members of varying moment of inertia. Semi-graphical Integration is most usefully employed for asymmetrical single-bay frames with hinged feet.

This section of the book is followed by formulæ and charts for the design of certain standard types of rigid frames. Readers may find it useful to consider the notes in the preamble to these tables and charts in conjunction with the following methods of analysis.

(a) Moment Distribution

Moment Distribution is a mechanical process of dealing with indeterminate structures by means of successive approximations in which the moments themselves are treated directly, the calculations involved being purely arithmetical.

The method is unique in that all joints are initially considered to be fixed against rotation. The fixed end moments are determined for each member as though it were an encastré beam and then the joints are allowed to rotate, either separately or all at once, the moments induced by the rotations being distributed among the members until the algebraic sum of the moments at each internal joint is zero.

The sign convention most commonly adopted for Moment Distribution is that all moments acting on individual members from supports or other

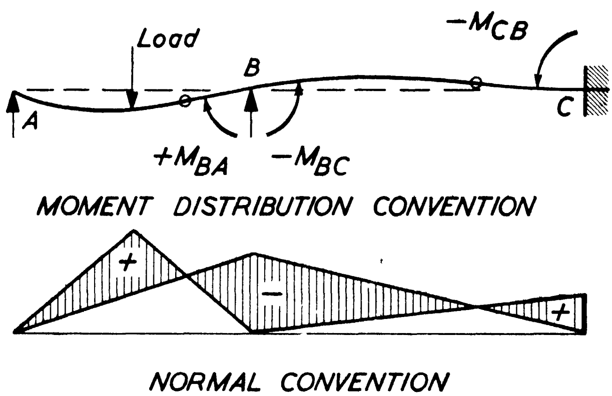


FIG. 122

members of a frame are positive if clockwise in application and negative if anti-clockwise. Before a B.M. diagram is drawn, this convention must be translated into the normal convention whereby in continuous beams, for example, sagging moments are positive and hogging moments are negative. The two conventions are compared in Fig. 122.

It will be found that the operations of Moment Distribution are more readily understood and checked if the reader considers initially how the structure deflects under load. Consequently, deflection diagrams are incorporated in many of the examples.

Although the structural principles on which Moment Distribution is based are well known, it is advisable to consider them in a definite sequence.

Fig. 123 shows a beam AB of constant cross-section, i.e. a prismatic beam, fixed in position and direction at A and fixed in position, but not in direction, at B. When the moment M_{BA} is applied at B a moment M_{AB} is induced

at A. It can be shown that

$$M_{AB}=\tfrac{1}{2}M_{BA} \quad \text{(i)}$$

$$\text{and } M_{BA}=4E\tan\theta\frac{I}{L}$$

$$=4E\theta\frac{I}{L} \quad \text{(for small values of } \theta) \quad \text{(ii)}$$

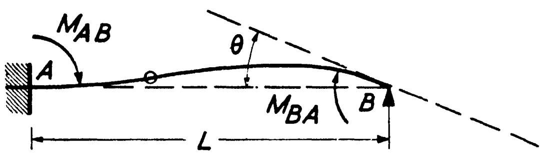


FIG. 123

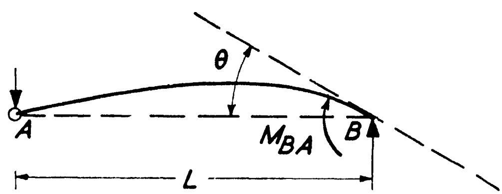


FIG. 124

Similarly, if A is fixed in position but not in direction, as in Fig. 124, then

$$M_{BA}=3E\theta\frac{I}{L} \quad \text{(for small values of } \theta). \quad \text{(iii)}$$

These equations give the three fundamental principles of Moment Distribution applicable to continuous beams on unyielding supports.

Principle I (Equation (i)). When a moment is applied at one end of a prismatic beam, that end remaining fixed in position but not in direction, the other end being fixed both in position and direction, a moment of half the amount and the same sign is induced at the second end.

Principle II (Equation (ii)). When one end of a beam remains fixed in position and direction, the moment required to produce a rotation of a given angle at the other end of the beam, which remains fixed in position, is proportional to the value I/L for the beam, provided that E is constant. The value I/L, known by the symbol K, is the stiffness factor for the particular beam in question.

Principle III (Equation (iii)). When one end of a beam is rotated through a given angle, remaining fixed in position, and the other end remains fixed in position but not in direction, the moment required at the first end is $\frac{3}{4}$ of that required if the second end were fixed both in position and direction, i.e. the equivalent stiffness factor for the beam is $\frac{3}{4}I/L=\frac{3}{4}K$.

The three foregoing principles alone are applied when the supports do not yield. However, when the joints change their positions B.M.s have to be modified accordingly. Consider Fig. 125 in which the end *A* of an encastré beam *AB*, of span L, has settled an amount *d*, the ends *A* and *B* remaining parallel in direction.

It is shown on page 27 that

$$M_{AB} = M_{BA} = \frac{6EId}{L^2} \quad \text{. . . . . (iv)}$$

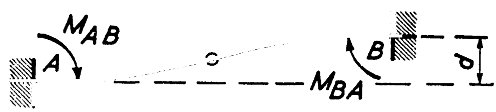


Fig. 125

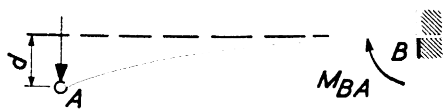


Fig. 126

Similarly, in Fig. 126, where the end *A* is hinged, i.e. not fixed in direction,

$$M_{BA} = \frac{3EId}{L^2} = \frac{6EId}{2L^2} \quad \text{. . . . . (v)}$$

From these equations, the following further principles may be derived.

Principle IV (Equation (iv)). When one end of a beam is deflected through a given distance, that end remaining parallel to its original position and the other remaining fixed in position and direction, equal moments of the same sign are induced at each end, proportional to the I/L^2 value of the beam.

Principle V (Equation (v)). When a hinged end of a beam is deflected through a given distance, the other end remaining fixed in position and direction, a moment is induced at the second end, proportional to the $I/2L^2$ value of the beam.

Having stated the principles, the moment distribution processes may be explained by considering some simple examples.

Continuous Beams

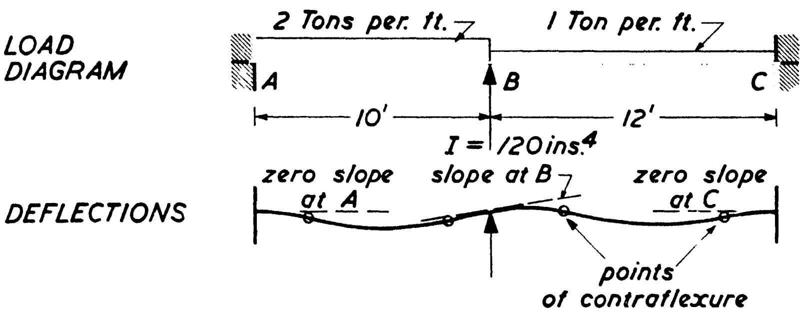
Example 1. Fig. 127 shows a continuous beam *ABC*, of constant cross-section, which is fixed in position and direction at *A* and *C* and simply supported at *B* and which carries uniformly distributed loads of 2 tons per ft. on *AB* and 1 ton per ft. on *BC*. This beam has been analysed on pp. 39 to 41.

Under these loads the beam will rotate in an anti-clockwise direction at *B* and, as it is a fundamental assumption in the theory of continuous beams that the slope does not change over a support, the beam will rotate the same amount θ on either side of *B*. However, assume that the beam does not rotate at *B*, but through some locking device remains horizontal after the

loads are applied. Then AB and BC are in effect two separate encastré beams and the moments at the end of each span are fixed-end moments (F.E.M.s) depending only on the functions of the span and the loading.

The functions of a span are (1) L, its length, and (2) I, its moment of inertia.

When I is constant throughout a span, the process is straightforward. Hence, in this example I may be ignored.



DISTRIBUTION TABLE

	A	B	B	C
Distribution Factors		0·545	0·455	
Fixed End Moments	−16·67	+16·67	−12·00	+12·00
Distribution		−2·55	−2·12	
Carry Over	−1·27			−1·06
Final Moments	−17·94	+14·12	−14·12	+10·94

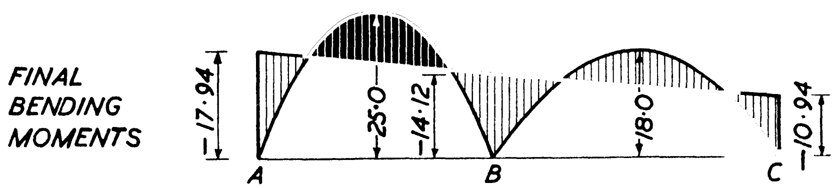


FIG. 127

Consider the F.E.M.s for the span AB. From the tables on pp. 216 to 220

F.E.M.$_{AB}$ (being anti-clockwise)

$$= -\frac{WL}{12} = \frac{-2 \times 10 \times 10}{12}$$

$$= -16{\cdot}67 \text{ tons ft.}$$

F.E.M.$_{BA}$ (being clockwise)

$$= +16{\cdot}67 \text{ tons ft.}$$

Similarly, for the span BC,

$$\text{F.E.M.}_{BC} = -\frac{WL}{12} = \frac{-1\times 12\times 12}{12}$$

$$= -12 \text{ tons ft.}$$

$$\text{F.E.M.}_{CB} = +12 \text{ tons ft.}$$

Having made these calculations the beam can be released at B and allowed to rotate in an anti-clockwise direction. Now the algebraic sum of the moments on either side of this support must be zero. However, when the beam was horizontal at B, the value of M_{AB} was $+16{\cdot}67$ tons ft. and that of M_{BC} was -12 tons ft. Therefore, to produce equilibrium at B, the total moment induced by the rotation of the beam there must be $-4{\cdot}67$ tons ft., since the moments are out of balance by $+16{\cdot}67-12=+4{\cdot}67$ tons ft.

But the span ends meeting at B rotate through the same angle θ. Consequently, the moments induced by rotation on either side of B are proportional to the stiffnesses of AB and BC (Principle II). In other words, the moment of $-4{\cdot}67$ tons ft. is distributed between AB and BC in proportion to their stiffnesses, i.e. in the proportion $K_{AB}/(K_{AB}+K_{BC})$ to the left and $K_{BC}/(K_{AB}+K_{BC})$ to the right.

These proportions are known as the distribution factors (D.F.) for the spans. Although it may sometimes be more accurate to employ fractions, these factors are usually expressed in decimals, but, in any case, the factors for a support or joint must always add up to unity.

Now $I=120$ in.^{4}, $AB=120$ in. and $BC=144$ in.

Hence $$K_{AB}=\frac{120}{120}=1$$

and $$K_{BC}=\frac{120}{144}=0{\cdot}833.$$

Therefore $$\text{D.F.}_{AB}=\frac{1}{1+0{\cdot}833}=0{\cdot}545$$

$$\text{D.F.}_{BC}=\frac{0{\cdot}833}{1+0{\cdot}833}=0{\cdot}455.$$

The operation of moment distribution is shown in the distribution table in Fig. 127, $0{\cdot}545\times -4{\cdot}67=-2{\cdot}55$ tons ft. being added to the end BA and $0{\cdot}455\times -4{\cdot}67=-2{\cdot}12$ tons ft. being added to the end BC.

From a consideration of Principle I moments are induced at the outer ends of the beam at A and C, equal to half the moments distributed between the spans at B and of the same signs.

Hence, $0{\cdot}5\times -2{\cdot}55=-1{\cdot}27$ tons ft. must be transferred to the end A and $0{\cdot}5\times -2{\cdot}12=-1{\cdot}06$ tons ft. to the end C. This process, which is known as the 'carry-over' process, is shown in Fig. 127.

The final moments in the beam are found by adding each column algebraically. When constructing a B.M. diagram it is convenient to remember that the moment to the right of a support in a distribution table bears the

same sign as the support moment in the B.M. diagram (in the normal sign convention). Therefore, the final moments at A, B and C are respectively —17·94, $-14{\cdot}12$ and $-10{\cdot}94$ tons ft.

The maximum static or 'free' B.M.s for AB and BC are obtained from the formula $+WL/8$ and equal 25 and 18 tons ft. respectively.

Example 2. The continuous beam $ABCDE$, which is shown in Fig. 128 and which has been analysed on p. 38, is simply supported at A and overhangs the other outside support D. Consequently, the beam is free to rotate at A and D, although it is restrained to a certain extent at D by the load at E, and when deriving F.E.M.s the beam is assumed to be fixed in a horizontal position at B and C only. Therefore, Principle III applies to spans AB and CD, and the stiffness factors for these spans equal $\frac{3}{4}K$. The stiffness factor for $BC=K$.

Now the moment of inertia I differs for each span, although it is constant throughout a span, as shown in Fig. 128.

Hence,
$$\tfrac{3}{4}K_{AB}=\frac{3\times 205}{4\times 168}=0{\cdot}915$$

$$K_{BC}=\frac{146}{144}=1{\cdot}014$$

$$\tfrac{3}{4}K_{CD}=\frac{3\times 122}{4\times 144}=0{\cdot}635$$

$$\text{D.F.}_{BA}=\tfrac{3}{4}K_{AB}/(\tfrac{3}{4}K_{AB}+K_{BC})=\frac{0{\cdot}915}{0{\cdot}915+1{\cdot}014}=0{\cdot}474$$

$$\text{D.F.}_{BC}=K_{BC}/(\tfrac{3}{4}K_{AB}+K_{BC})=\frac{1{\cdot}014}{0{\cdot}915+1{\cdot}014}=0{\cdot}526$$

$$\text{D.F.}_{CB}=K_{BC}/(K_{BC}+\tfrac{3}{4}K_{CD})=\frac{1{\cdot}014}{1{\cdot}014+0{\cdot}635}=0{\cdot}615$$

$$\text{D.F.}_{CD}=1-0{\cdot}615=0{\cdot}385.$$

Now AB and CD are treated as fixed at one end only.

Consider the tables of F.E.M.s (Pages 216 to 220)

$$\text{F.E.M.}_{BA}=+\frac{3PL}{16}=\frac{3\times 12\times 14}{16}=31{\cdot}50 \text{ tons ft.}$$

$$\text{F.E.M.}_{BC}=-\frac{2PL}{9}=-\frac{2\times 7\times 12}{9}=-18{\cdot}67 \text{ tons ft.}$$

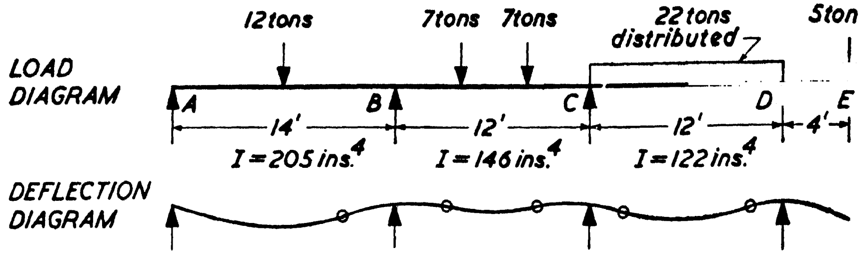
$$\text{F.E.M.}_{CB}=+18{\cdot}67 \text{ tons ft.}$$

$$\text{F.E.M.}_{CD}=-\frac{WL}{8}=-\frac{22\times 12}{8}=-33{\cdot}00 \text{ tons ft.}$$

Now under any circumstances in the remainder of the beam the moment at D can only be that due to the load on the cantilever.

The cantilever moment

$$M_{DE}=-M_{DC}=-5\times 4=-20{\cdot}00 \text{ tons ft.}$$



DISTRIBUTION TABLE

	A	B		C		D	E
D.F.		0·474	0·526	0·615	0·385		
Cantilever M						+20·00	20·00
C.O.					+ 10·00		
F.E.M.	0	+31·50	−18·67	+18·67	−33·00		
Distribution		− 6·08	− 6·75	+ 2·66	+ 1·67		
C.O.			+ 1·33	− 3·37			
Distribution		− 0·63	− 0·70	+ 2·07	+ 1·30		
C.O.			+ 1·03	− 0·35			
Distribution		− 0·49	− 0·54	+ 0·22	+ 0·13		
C.O.			+ 0·11	− 0·27			
Distribution		− 0·05	− 0·06	+ 0·17	+ 0·10		
C.O.			+ 0·09	− 0·03			
Distribution		− 0·04	− 0·05	+ 0·02	+ 0·01		
Final Moments	0	+24·21	−24·21	+19·79	− 19·79	+20·00	20·00

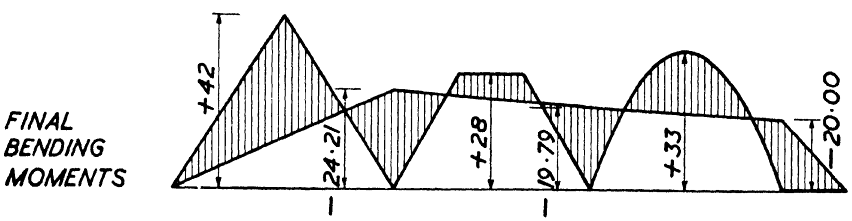


FIG. 128

Sufficient data have been accumulated to analyse the beam by moment distribution as shown in Fig. 128.

The inexperienced may prefer to deal with the cantilever first. The moments M_{DE} and M_{DC} are inserted in the appropriate columns, and half M_{DC} is carried over to the other end of the span CD. Subsequently the support D is ignored until the final moments are summated.

The preliminary operations at support C demand some explanation. When the beam is released the unbalanced moment

$$= +10 + (18{\cdot}67 - 33{\cdot}00) = -4{\cdot}33 \text{ tons ft.}$$

To balance this moment $+4{\cdot}33$ tons ft. must be distributed between the ends CB and CD.

It will be observed that the work is not completed in one cycle of distribution and carry-over as in the previous example. After one operation of distribution the moments are balanced at the supports B and C. Unfortunately, the carry-over moments from B to C and from C to B throw the support moments out of balance, and further cycles of distribution and carry-over are required until the moments distributed are so small that the process may be halted.

As the beam is free to rotate at A, there can be no moment there, neither can there be at D, other than the cantilever moment. Hence, no moments are carried over from B to A or from C to D.

It is correct to carry-over moments to an outside fixed-ended support after distribution, as in Example 1, but for all internal supports a distribution table is finished with a line of distribution. Otherwise, the carry-over moments would leave these internal supports out of balance.

It is convenient at this stage to summarise some of the points relating to continuous beams which have been brought out by the foregoing examples.

1. *Cantilever Ends.* The only moment which can occur at the supports is that due to loads on the cantilever. After carry-over to the adjacent interior support, treat the end span as being simply supported at its outer end.
2. *Simply Supported End Spans.* As the beam is free to rotate about an end support, no moment can occur there and, consequently, there is no carry over to an end support.
3. *Fixed Ends.* Moments must be carried over to the fixed end from the adjacent interior support, but not in the opposite direction as a fixed end cannot rotate.
4. *Distribution Table.* After each distribution draw a line to signify that at that stage the moments at the support are in equilibrium.
5. *Completion.* Always finish at a fixed-ended support with a carry-over. Always finish at an interior support with a line of distribution.

   Any degree of accuracy may be obtained, but the longer the cycles of moment distribution and carry-over are continued the nearer will the results be to those obtained by one of the classical methods.

   In practice, however, it is sufficiently accurate to stop the process when the moment distributed is about 2 per cent of the original F.E.M.

Example 3. Fig. 129 shows a continuous beam ABC which is freely supported at A and fixed at C. The B.M.s induced by the settlement of B can be calculated by Moment Distribution.

Let $E = 13{,}000$ tons per sq. in., and the I of the beam $= 120$ in.^{4}.

Then $$\tfrac{3}{4}K_{AB} = \frac{3 \times 120}{4 \times 120} = 0{\cdot}75$$

while $$K_{BC} = \frac{120}{180} = 0{\cdot}67$$

whence $$\text{D.F.}_{DA} = \frac{0{\cdot}75}{0{\cdot}75 + 0{\cdot}67} = 0{\cdot}53$$

and $$\text{D.F.}_{BC} = 1 - 0{\cdot}53 = 0{\cdot}47.$$

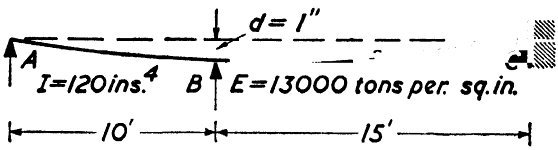
Now A is simply supported. Hence, remembering the derivation of Principle V,

$$\text{F.E.M.}_{BA} = -\frac{3EId}{L^2}$$

$$= -\frac{3 \times 13{,}000 \times 120 \times 1}{120 \times 120}$$

$$= -325 \text{ tons in.}$$

DATA DIAGRAM



DISTRIBUTION TABLE

	A	B	B	C
D.F		0·53	0·47	
F.E.M.	0	−325·0	+288·9	+288·9
Distribution		+ 19·1	+ 17·0	
C.O.				+ 8·5
Final Moments	0	−305·9	+305·9	+297·4

BENDING MOMENT DIAGRAM

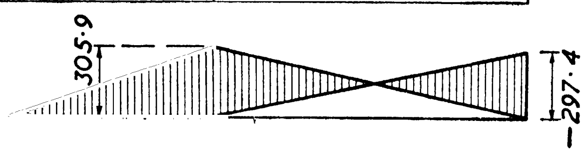


FIG. 129

The beam is fixed at C. Therefore, remembering the derivation of Principle IV,

$$\text{F.E.M.}_{BC} = \text{F.E.M.}_{CB} = +\frac{6EId}{L^2}$$

$$= \frac{6 \times 13{,}000 \times 120 \times 1}{180 \times 180}$$

$$= 288{\cdot}9 \text{ tons in.}$$

The distribution table and final moments are shown in Fig. 129.

Example 4. Fig. 130 shows a beam fixed in position and direction at A and C and in which a clockwise moment of 10 tons ft. is applied at B.

The beam may be analysed by employing two stages. In the first stage the beam is propped at B so that it cannot change its position under the action of the moment. Consequently, the beam may be treated as a continuous beam. In the second stage a load equal in size and opposite in direction to the prop is applied at B. When the moments resulting from the two stages are added algebraically, the final moments are obtained.

Moment Distribution is employed only in the first stage. The beam being of constant EI, the stiffness of BC is three times that of AB. The load on

the prop is found by considering the shears associated with the Stage I moments. Thus,

$$R_B = \frac{1{\cdot}25 + 2{\cdot}5}{15} - \frac{7{\cdot}5 + 3{\cdot}75}{5}$$

$$= -2 \text{ tons (acting downwards).}$$

The moments resulting from the application of an upward load of 2 tons at B can be calculated by formula.

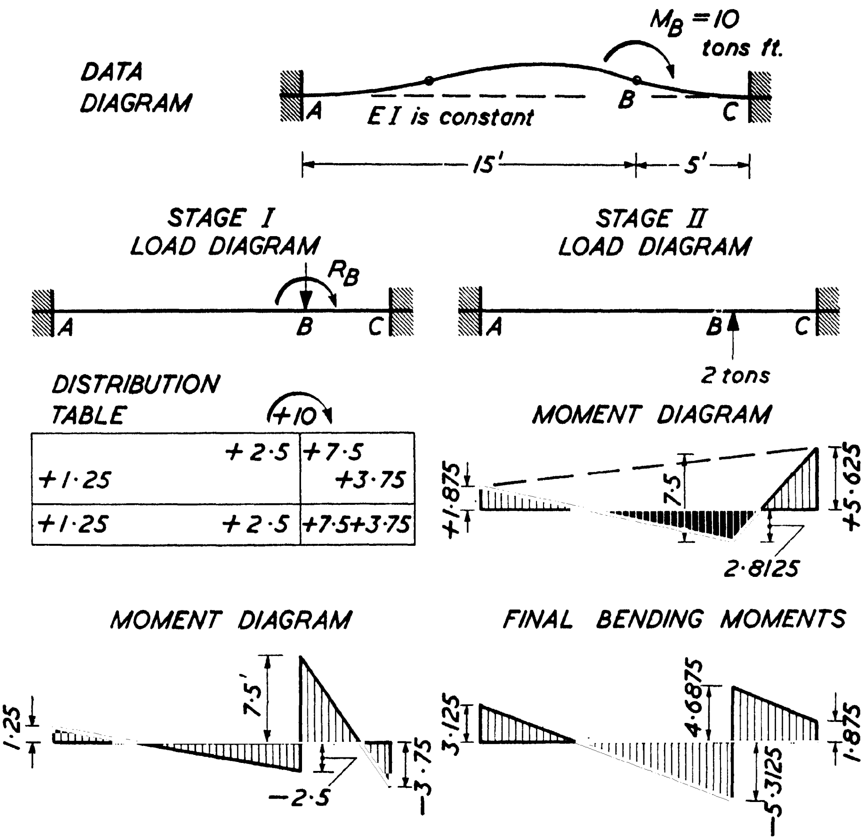


FIG. 130

The 'free' bending moment under the load

$$= -\frac{Wab}{L} = -\frac{2 \times 15 \times 5}{20} = -7.5 \text{ tons ft.}$$

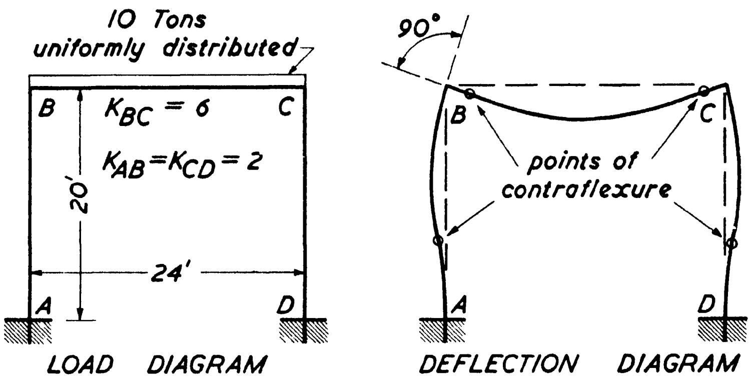
$$\text{F.E.M.}_{AB} = \frac{Wab^2}{L^2} = 1{\cdot}875 \text{ tons ft.}$$

$$\text{F.E.M.}_{CB} = \frac{Wa^2b}{L^2} = 5{\cdot}625 \text{ tons ft.}$$

The Stage I, Stage II and final B.M.s are shown in Fig. 130.

Portal Frames

Example 5. *ABCD* in Fig. 131 is a symmetrical portal frame with fixed feet and with columns of equal stiffness. When *BC* is loaded symmetrically as shown the frame does not side-sway (i.e. *B* and *C* remain symmetrical about the vertical axis, although both sink a small amount and each moves slightly towards the vertical axis). Consequently, the moments may be found in the frame by flattening it out into a continuous beam *ABCD.*



DISTRIBUTION TABLE

	A	B		C		D
D.F.		0·25	0·75	0·75	0·25	
F.E.M.			−20·00	+20·00		
Dist.		+5·00	+15·00	−15·00	−5·00	
C.O.	+2·50		−7·50	+7·50		−2·50
Dist.		+1·87	+5·63	Figures can be repeated here with opposite signs		
C.O.	+0·94		−2·81			
Dist.		+0·70	+2·11			
C.O.	+0·35		−1·05			
Dist.		+0·26	+0·79			
C.O.	+0·13		−0·39			
Dist.		+0·10	+0·29			
C.O.	+0·05					
Final Moments	+3·97	+7·93	−7·93	+7·93	−7·93	−3·97

FIG. 131

There is justification for this action because it is a fundamental assumption that the connections between the beams and columns are rigid, just as it is assumed that there is no sudden change of slope in a deformed continuous beam at its supports.

$$\text{Now} \qquad \text{F.E.M.}_{BC} = -\text{F.E.M.}_{CB} = -\frac{WL}{12} = -\frac{10 \times 24}{12}$$

$$= -20 \text{ tons ft.}$$

$$K_{AB} : K_{BC} : K_{CD} = 1 : 3 : 1.$$

Then
$$\text{D.F.}_{BA} = \text{D.F.}_{CD} = \frac{1}{1+3} = 0{\cdot}25$$

and
$$\text{D.F.}_{BC} = \text{D.F.}_{CB} = \frac{3}{1+3} = 0{\cdot}75.$$

The distribution table is shown in Fig. 131.

It should be noted that it is not essential to carry over from B to A or from C to D after each distribution. It would suffice to carry over half the final moments at B and C to A and D respectively.

It is appropriate at this stage to include a practical 'short cut' to reduce the amount of work in the distribution table. Just as it is convenient to

DISTRIBUTION TABLE

	A	B	B	C	C	D
D.F.		0·4	0·6	0·6	0·4	
F.E.M.			−20·00	+20·00		
Dist.		+8·00	+12·00	−12·00	−8·00	
C.O.	+4·00					−4·00
Final Moments	+4·00	+8·00	−8·00	+8·00	−8·00	−4·00

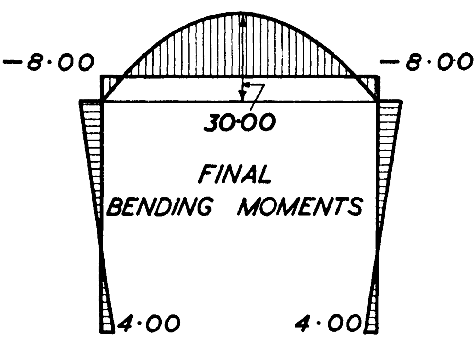


FIG. 132

consider that the equivalent stiffness of a simply supported end span is $\frac{3}{4}K$, so can one modify the stiffness factors of other members under certain conditions.

The portal frame of Fig. 131 and its load are symmetrical about the centre of the beam BC.

If, in this case, the stiffness factor for BC is taken as $K/2$, then there is no need to carry-over between B and C.

As $K_{AB} : K_{BC}/2 : K_{CD} = 1 : 1{\cdot}5 : 1$,

$$\text{D.F.}_{BA} = \text{D.F.}_{CD} = \frac{1}{1+1{\cdot}5} = 0{\cdot}4$$

and
$$\text{D.F.}_{BC} = \text{D.F.}_{CB} = 1 - 0{\cdot}4 = 0{\cdot}6.$$

Employing this method a very short distribution table results as shown in Fig. 132.

The final moments are exact.

The above procedure may be adopted for all members which are subject to *equal* end rotations in opposite directions. The principle will be used again later.

Example 6. When a portal frame is asymmetrical in shape or is asymmetrically loaded, it tends to sway to one side, and analysis by Moment Distribution has to be carried out in two stages. In the first stage the moments are derived assuming that the frame is propped against sway, while in the second moments induced by the sway are calculated.

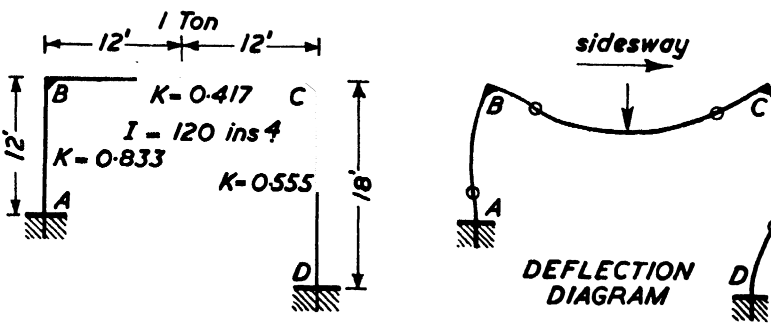


FIG. 133

Consider the frame shown in Fig. 133, which has a constant I of 120 in.^{4}.

$$K_{AB} = 120/144 = 0{\cdot}833$$

$$K_{BC} = 120/288 = 0{\cdot}417$$

$$K_{CD} = 120/216 = 0{\cdot}555.$$

Hence
$$\text{D.F.}_{BA} = \frac{0{\cdot}833}{0{\cdot}833 + 0{\cdot}417} = 0{\cdot}67$$

$$\text{D.F.}_{BC} = 0{\cdot}33$$

$$\text{D.F.}_{CB} = \frac{0{\cdot}417}{0{\cdot}417 + 0{\cdot}555} = 0{\cdot}43$$

$$\text{D.F.}_{CD} = 0{\cdot}57.$$

Now
$$\text{F.E.M.}_{BC} = -\text{F.E.M.}_{CB} = -\frac{WL}{8} = -\frac{1 \times 24}{8}$$

$$= -3{\cdot}00 \text{ tons ft.}$$

When the frame is prevented from swaying, the moments are those obtained in Fig. 134.

Now a frame sways because of unbalanced horizontal thrust.
The thrust at A

$$=\frac{+1\cdot25+2\cdot50}{12}=0\cdot31 \text{ ton}$$

while the thrust at D

$$=\frac{-2\cdot06-1\cdot03}{18}=-0\cdot17 \text{ ton.}$$

Hence, the propping force equals 0·14 ton and acts in a horizontal direction from C towards B.

DISTRIBUTION TABLE FOR STAGE I MOMENTS.

	A	B	B	C	C	D
D.F.		0·67	0·33	0·43	0·57	
F.E.M.			−3·00	+3·00		
Dist.		+2·00	+1·00	−1·29	−1·71	
C.O.	+1·00		−0·64	+0·50		−0·85
Dist.		+0·43	+0·21	−0·21	−0·29	
C.O.	+0·21		−0·11	+0·11		−0·15
Dist.		+0·07	+0·04	−0·05	−0·06	
C.O.	+0·04					−0·03
Stage I Moments	+1·25	+2·50	−2·50	+2·06	−2·06	−1·03

FIG. 134

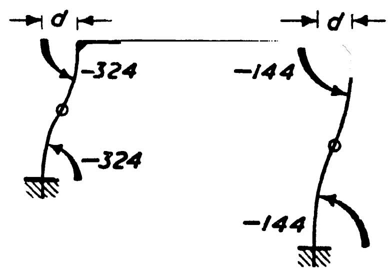


FIG. 135

The second stage of the calculations is to find what moments result when a force of 0·14 ton acts in a horizontal direction from B towards C.

Unfortunately, there is no direct method of achieving this object. Nevertheless, within the elastic range of the material, the moments produced in a frame are proportional to the applied forces. Hence, if it can be calculated that a certain B.M. produces a known lateral force, then the bending moment resulting from another lateral force in the same place may be calculated by proportion.

Let the frame sway an amount d along the line BC, the joints B and C being prevented from rotation, as shown in Fig. 135.

By Principle IV the moments induced in AB and CD are proportional to their I/L^2 values.

Hence,

$$\text{F.E.M.}_{AB} : \text{F.E.M.}_{BA} : \text{F.E.M.}_{CD} : \text{F.E.M.}_{DC} = 1/12^2 : 1/12^2 : 1/18^2 : 1/18^2$$
$$= -324 : -324 : -144 : -144.$$

Using these arbitrary moments, release the joints B and C and calculate the resulting moments in the frame, as shown in Fig. 136.

DISTRIBUTION TABLE FOR SIDESWAY

	A	B	B	C	C	D
D.F.		0·67	0·33	0·43	0·57	
F.E.M.	−324	−324			−144	−144
Dist.		+216	+108	+62	+82	
C.O.	+108		+31	+54		+41
Dist.		−20	−11	−23	−31	
C.O.	−10		−11	−5		−15
Dist.		+7	+4	+2	+3	
C.O.	+3					+1
Final Moments	−223	−121	+121	+90	−90	−117

FIG. 136

The resulting shears

$$= \frac{-223-121}{12} + \frac{-90-117}{18}$$

$$= -40{\cdot}2 \text{ tons.}$$

This force is $40{\cdot}2/0{\cdot}14 = 287$ times as great as the propping force in Stage I. Hence, the Stage II moments are 1/287 of those calculated above.

	A	B	C	D
Stage I Moments	+1·25	−2·50	−2·06	+1·03
Stage II Moments	−0·78	+0·42	−0·31	+0·41
Final Moments	+0·47	−2·08	−2·37	+1·44

FIG. 137

The Stage I and Stage II moments are shown in Figs. 137 and 138. When added algebraically they provide the final B.M.s.

Example 7. In general, rigid frames with hinged feet are a little more easy to analyse than those with fixed feet, and the analysis is particularly easy with frames symmetrical in shape and in loading. However, the frame shown in Fig. 139 is somewhat complicated as it is asymmetrical and requires analysis in two stages as in the last example.

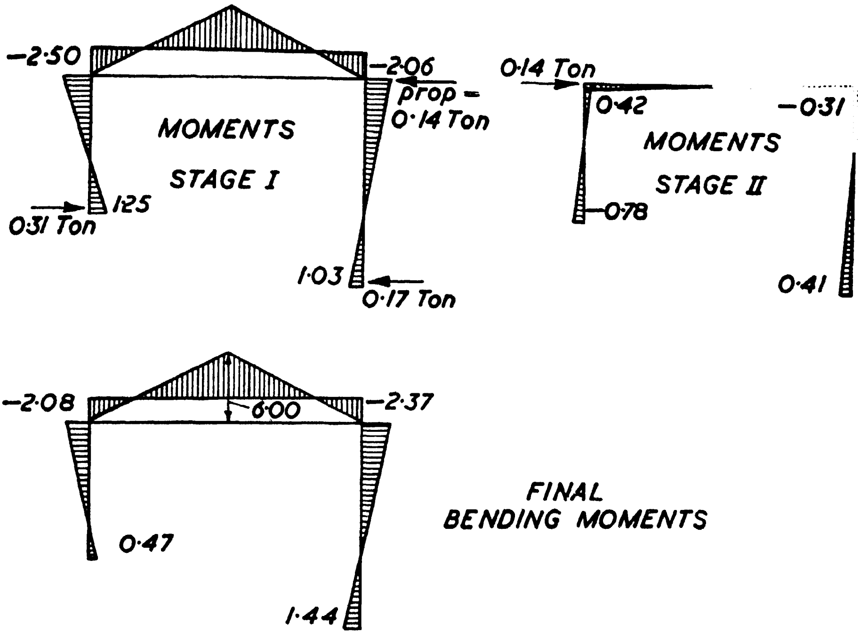


FIG. 138

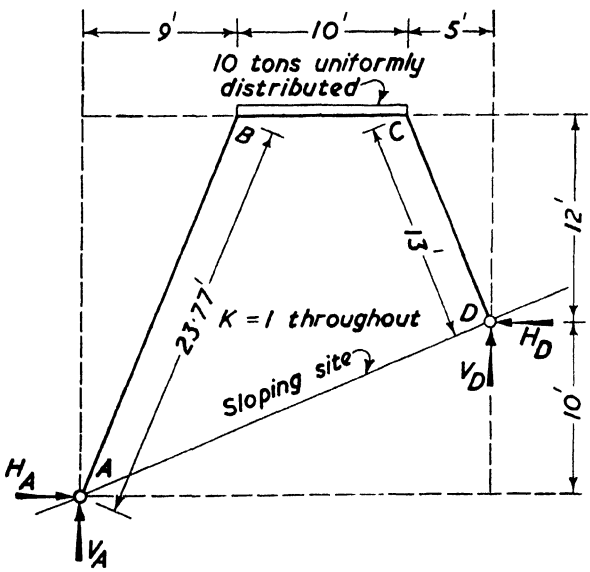


FIG. 139

The approach is new because the legs are sloping and account must be taken of the vertical reactions V_A and V_D which affect the moments at B and C.

Taking moments to the left of B,

$$9V_A + M_{BA} - 22H_A = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (i)$$

Taking moments to the left of C,

$$19V_A + M_{CB} - 22H_A \quad (10 \times 5) = 0 \quad . \quad . \quad . \quad . \quad . \quad (ii)$$

Taking moments to the left of D,

$$24V_A - 10H_A - (10 \times 10) = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (iii)$$

Subtracting (i) from (ii),

$$10V_A + M_{CB} - M_{BA} - 50 = 0 \quad . \quad . \quad . \quad . \quad . \quad (iv)$$

Multiplying (iii) by 22/10,

$$52{\cdot}8V_A - 22H_A - 220 = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (v)$$

Subtracting (i) from (v),

$$43{\cdot}8V_A - M_{BA} - 220 = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (vi)$$

Dividing (vi) by 43·8/10,

$$10V_A - 0{\cdot}228M_{BA} - 50{\cdot}23 = 0 \quad . \quad . \quad . \quad . \quad . \quad (vii)$$

Subtracting (vii) from (iv),

$$M_{CB} - 0{\cdot}772M_{BA} + 0{\cdot}23 = 0$$

or, as $-M_{BA} = +M_{BC}$, $\quad 0{\cdot}772M_{BC} + M_{CB} = -0{\cdot}23.$

This is the fundamental equation of equilibrium of the frame.

The Stage I moments are found as shown in Fig. 140.

STAGE I DISTRIBUTION TABLE

	A	B	B	C	C	D
D.F.		0·6	0·4	0·4	0·6	
F.E.M.	0		−8·33	+8·33		0
Distribution		+5·00	+3·33	−3·33	−5·00	
Stage I Moments	0	+5·00	−5·00	+5·00	−5·00	0

FIG. 140

Note that the stiffness coefficients are symmetrical about the centra member and that the load is symmetrical. Providing that the frame is propped, the K value of the central member can be halved and carry-over dispensed with between B and C for this Stage, as in Example 5.

From these moments,

$$0{\cdot}772M_{BC} + M_{CB} = (0{\cdot}772 \times -5{\cdot}00) + 5{\cdot}00 = 1{\cdot}14.$$

As this equation does not give the same result as the fundamental equation, the frame side-sways with a resulting modification of moments so that

$$0{\cdot}772M_{BC} + M_{CB} = -1{\cdot}37.$$

If it is assumed that the member BC side-sways a horizontal amount d to the left, as shown in Fig. 141, then

$$d = 22\phi_1 = 12\phi_3$$

i.e. $$\phi_3 = 1{\cdot}833\phi_1.$$

The vertical movement of B with regard to C,

$$= 10\phi_2 = 9/22d + 5/12d$$
$$= (9/22 + 5/12)22\phi_1$$
$$= 18{\cdot}16\phi_1$$

i.e. $$\phi_2 = 1{\cdot}816\phi_1.$$

From Principles IV and V,

$$\text{F.E.M.}_{BA} = \frac{3EKd}{L} = +3EK_1\phi_1$$
$$\text{F.E.M.}_{BC} = \text{F.E.M.}_{CB} = -6EK_2\phi_2$$
$$\text{F.E.M.}_{CD} = +3EK_3\phi_3$$

The K values all equal 1.

If $\phi_1 = 1$, then $\phi_2 = 1{\cdot}816$ and $\phi_3 = 1{\cdot}833$.

Hence,

$\text{F.E.M.}_{BA} : \text{F.E.M.}_{BC} : \text{F.E.M.}_{CB} : \text{F.E.M.}_{CD}$

$$= 1 : -2(1 \times 1{\cdot}816) : -2(1 \times 1{\cdot}816) : (1 \times 1{\cdot}833)$$
$$= 1 : -3{\cdot}632 : -3{\cdot}632 : 1{\cdot}833.$$

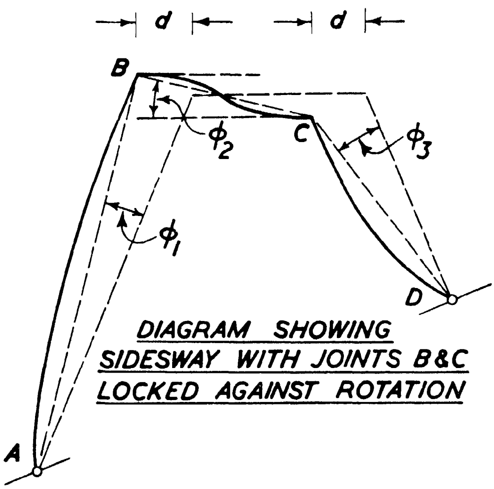


FIG. 141

Employing those moments as arbitrary F.E.M.s the Stage II distribution is carried out as shown in Fig. 142.

From these moments,

$$0{\cdot}772M_{BC} + M_{CB} = -3{\cdot}848.$$

But the Stage II moments should be such that

$$0{\cdot}772M_{BC}+M_{CB}=-1{\cdot}37.$$

Hence, the appropriate correction coefficient for the Stage II moments

$$=\frac{-1{\cdot}37}{-3{\cdot}848}=0{\cdot}356.$$

STAGE II DISTRIBUTION TABLE

	A	B		C		
D.F.		0·43	0·57	0·57	0·43	
F.E.M.	0	+1·000	−3·632	−3·632	+1·833	0
Dist.		+1·128	+1·504	+1·028	+0·771	
C.O.			+0·514	+0·752		
Dist.		−0·220	−0·294	−0·430	−0·322	
C.O.			−0·215	−0·147		
Dist.		+0·092	+0·123	+0·084	+0·063	
C.O.			+0·042	+0·062		
Dist.		−0·018	−0·024	−0·035	−0·027	
Final Moments	0	+1·982	−1·982	−2·318	+2·318	0

FIG. 142

Correcting the Stage II moments and adding them to the Stage I moments, the final moments are obtained as follows:

$$M_{BC}=0{\cdot}356\times(-1{\cdot}982)-5{\cdot}00$$
$$=-5{\cdot}71 \text{ tons ft.}$$
$$M_{CB}=0{\cdot}356\times(-2{\cdot}318)+5{\cdot}00$$
$$=+4{\cdot}17 \text{ tons ft.}$$

The final B.M. diagram is shown in Fig. 143.

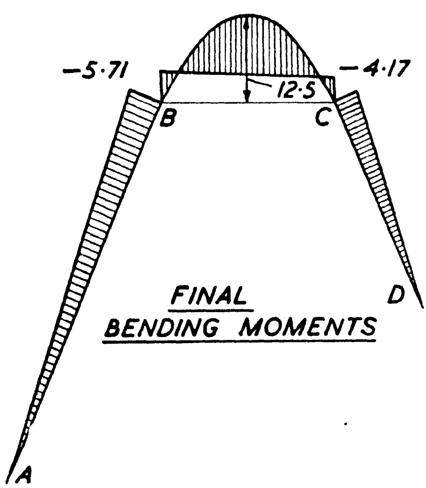


FIG. 143

Frames with Pitched Roofs

Example 8. When one joint of a three-member frame, such as that in Example 6, is displaced the displacement of other joints can be found from the geometry of the frame. When, however, four member frames, such as

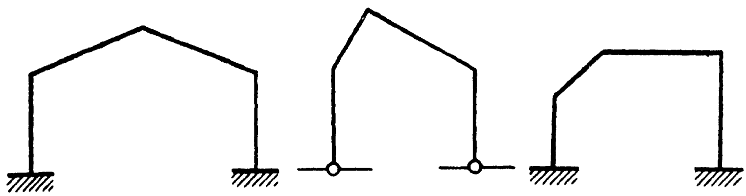


FIG. 144

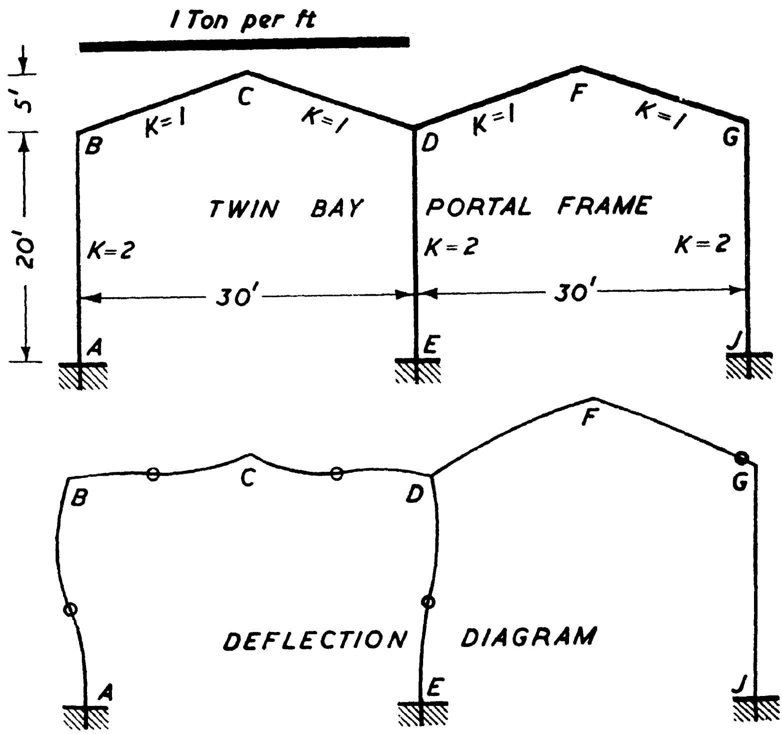


FIG. 145

those in Fig. 144, are under consideration it is necessary to know the movements of two joints before those of the remaining joints can be calculated.

Similarly when the twin portal in Fig. 145 is loaded the effect of the movement of the three joints B, D and G must be known before the frame can be analysed.

Analysis is carried out in stages as follows:

I. By assuming that the joints B, D and G are fixed in position but free to rotate.

II. By allowing each joint separately to sway under arbitrary loading while the other two remain fixed in position.

III. By proportioning the thrusts resulting from the previous stages so that no induced horizontal forces remain at B, D and G.

IV. By adjusting the moments at the various stages in proportion to the final thrusts.

Care is needed with signs. When *comparing* thrusts, those acting from right to left are considered to be negative. Similarly, downward vertical reactions are negative. As before, clockwise moments are positive.

Stage I:

$$K_{AB}=K_{DE}=K_{GJ}=2$$

$$K_{BC}=K_{CD}=K_{DF}=K_{FG}=1.$$

Hence,

$$\text{D.F.}_{BA}=\text{D.F.}_{GJ}=0{\cdot}67$$

$$\text{D.F.}_{BC}=\text{D.F.}_{GF}=0{\cdot}33$$

$$\text{D.F.}_{CB}=\text{D.F.}_{CD}=\text{D.F.}_{FD}=\text{D.F.}_{FG}=0{\cdot}5$$

$$\text{D.F.}_{DC}=\text{D.F.}_{DF}=0{\cdot}25$$

$$\text{D.F.}_{DE}=0{\cdot}50$$

$$\text{F.E.M.}_{BC}=-\text{F.E.M.}_{CB}=\text{F.E.M.}_{CD}=-\text{F.E.M.}_{DC}=-\frac{WL}{12}$$

$$=-\frac{1\times 15^2}{12}=-18{\cdot}75 \text{ tons ft.}$$

The distribution table for Stage I is shown in Fig. 146, and the appropriate B.M. diagram in Fig. 147.

$$H_A=\frac{6{\cdot}31+12{\cdot}63}{20}=\frac{18{\cdot}94}{20}=0{\cdot}947 \text{ ton}$$

$$H_E=\frac{-9{\cdot}56-4{\cdot}78}{20}=-\frac{14{\cdot}34}{20}=-0{\cdot}717 \text{ ton}$$

$$H_J=\frac{-0{\cdot}39-0{\cdot}19}{20}=-\frac{0{\cdot}58}{20}=-0{\cdot}03 \text{ ton.}$$

Taking moments to the left of D,

$$30V_A+6{\cdot}31+13{\cdot}77-20H_A-\frac{(30\times 30)}{2}=0$$

$$V_A=\frac{-6{\cdot}31 \quad 13{\cdot}77+18{\cdot}94+450}{30}$$

$$=14{\cdot}96 \text{ tons.}$$

DISTRIBUTION TABLE FOR STAGE I MOMENTS

	AB	BA	BC	CB	CD	DC	DE	ED	DF	FD	FG	GF	GJ	JG
D.F.	0	0·67	0·33	0·5	0·5	0·25	0·5	0	0·25	0·5	0·5	0·33	0·67	0
F.E.M			−18·75	+18·75	−18·75	+18·75								
Dist.		+12·50	+6·25			−4·69	−9·37		−4·69					
C.O.	+6·25			+3·12	−2·34			−4·69		−2·34				
Dist.				−0·39	−0·39					+1·17	+1·17			
C.O.			−0·19			−0·19			+0·58			+0·58		
Dist.		+0·13	+0·06			−0·10	−0·19		−0·10			−0·19	−0·39	
C.O.	+0·06			+0·03	−0·05			−0·09			−0·10			−0·19
Dist				+0·01	+0·01					+0·05	+0·05			
	+6·31	+12·63	−12·63	+21·52	−21·52	+13·77	−9·56	−4·78	−4·21	−1·12	+1·12	+0·39	−0·39	−0·19

FIG. 146

Taking moments to the left of C,

$$15V_A + 6{\cdot}31 + 21{\cdot}52 - 25H_A - \frac{(15 \times 15)}{2} - 5H_B = 0$$

$$H_B = \frac{224{\cdot}4 + 6{\cdot}31 + 21{\cdot}52 - 23{\cdot}67 - 112{\cdot}5}{5}$$

$$= 23{\cdot}21 \text{ tons.}$$

Taking moments to the right of D,

$$20H_J - 30V_J - 0{\cdot}19 - 4{\cdot}21 = 0$$

$$V_J = \frac{0{\cdot}58 - 0{\cdot}19 - 4{\cdot}21}{30}$$

$$= -0{\cdot}127 \text{ tons (i.e. } V_J \text{ acts downwards).}$$

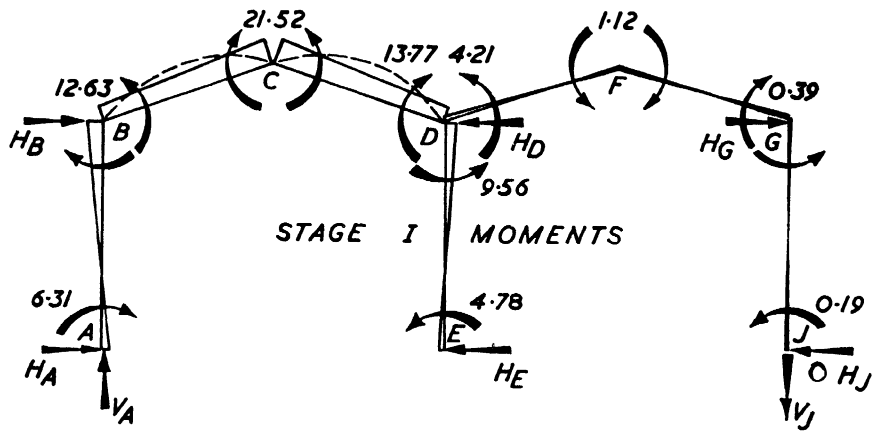


FIG. 147

Taking moments to the right of F,

$$25H_J + 15V_J - 0{\cdot}19 + 1{\cdot}12 - 5H_G = 0$$

$$H_G = \frac{0{\cdot}73 + 1{\cdot}905 - 0{\cdot}19 + 1{\cdot}12}{5}$$

$$= 0{\cdot}713 \text{ tons.}$$

Now $\Sigma H = 0$

Therefore, $-H_D = H_A + H_B + H_E + H_G + H_J$

$$= 0{\cdot}947 + 23{\cdot}21 - 0{\cdot}717 + 0{\cdot}713 - 0{\cdot}03$$

$$H_D = -24{\cdot}12 \text{ (i.e. } H \text{ acts from right to left).}$$

Summarising the Stage I induced thrusts at eaves and valley level:

$$H_B = 23{\cdot}21 \text{ tons}$$

$$H_D = -24{\cdot}12 \text{ tons}$$

$$H_G = 0{\cdot}71 \text{ ton.}$$

Stage II:

During this stage the props are removed from joints B, D and G and the moments due to sway are calculated for each joint.

Consider joint B in Fig. 148. If D and G remain fixed in position and B deflects an amount $2d$ to the left, then the ridge C will move a horizontal distance d. But C will also drop vertically. Provided d is small then C will drop an amount equal to

$$\frac{\text{span}}{2\times\text{rise}}\times d=\frac{30}{10}\times d=3d.$$

Now the angle of pitch of the roof θ is $18°\ 26'$. Hence, provided the ends of the rafters do not change their slopes, the deflection of the ends with respect to one another $=3d\cos\theta+d\sin\theta$.

Therefore, the deflection $=3{\cdot}163d$.

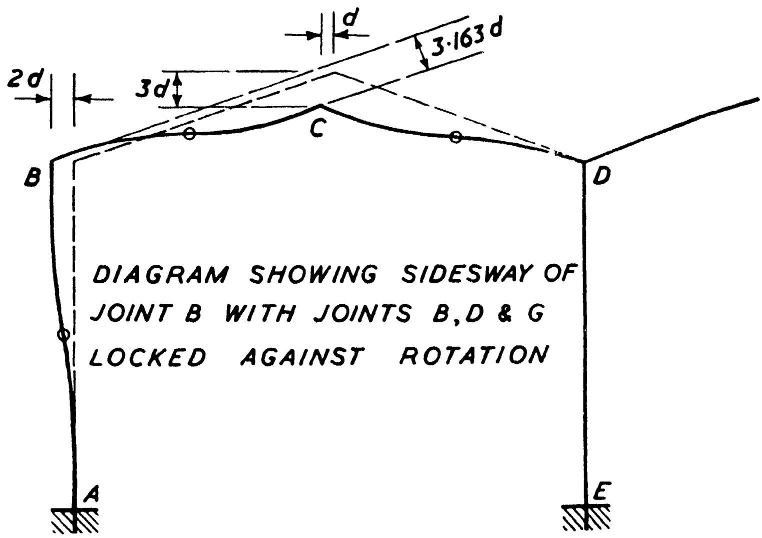


FIG. 148

Providing that the joints B and C do not rotate, the deflection diagram is as shown in Fig. 148.

The F.E.M.s are proportional to K/L and to the amount of deflection.

Hence, $$\text{F.E.M.}_{AB}=\text{F.E.M.}_{BA}:\frac{K2d}{L}=\frac{4d}{20}$$

and $$\text{F.E.M.}_{BC}=\text{F.E.M.}_{CB}=-\text{F.E.M.}_{CD}=-\text{F.E.M.}_{DC}:-\frac{K3{\cdot}163d}{L}$$

$$=-\frac{3{\cdot}163d}{15{\cdot}81}.$$

By coincidence, $$\frac{4d}{20}=\frac{3{\cdot}163d}{15{\cdot}81}$$

DISTRIBUTION TABLE WHEN JOINT B SIDESWAYS

	AB	BA	BC	CB	CD	DC	DE	ED	DF	FD	FG	GF	GJ	JG
D.F.	0	0·67	0·33	0·5	0·5	0·25	0·5	0	0·25	0·5	0·5	0·33	0·67	0
F.E.M.	+20·00	+20·00	−20·00	−20·00	+20·00	+20·00								
Dist.						−5·00	−10·00		−5·00					
C.O.					−2·50			−5·00		−2·50				
Dist.				+1·25	+1·25					+1·25	+1·25			
C.O.			+0·62			+0·62			+0·62			+0·62		
Dist.		−0·41	−0·21			−0·31	−0·62		−0·31			−0·21	−0·41	
C.O.	−0·21			−0·10	−0·15			−0·31		−0·15	−0·10			−0·21
Dist.				+0·13	+0·12					+0·12	+0·13			
	+19·79	+19·59	−19·59	−18·72	+18·72	+15·31	−10·62	−5·31	−4·69	−1·28	+1·28	+0·41	−0·41	−0·21

FIG. 149

If arbitrary moments of, say, 20 tons ft. are applied to the ends of the member *AB*, then the same amount, with appropriate signs, can be applied to the members *BC* and *CD*.

If all the joints previously fixed against rotation are allowed to rotate while *D* and *G* remain fixed in position, then the distribution table is as shown in Fig. 149.

The appropriate B.M. diagram is shown in Fig. 150.

$$H_A = \frac{19{\cdot}59 + 19{\cdot}79}{20} = \frac{39{\cdot}38}{20} = 1{\cdot}969 \text{ tons}$$

$$H_E = \frac{-10{\cdot}62 - 5{\cdot}31}{20} = -\frac{15{\cdot}93}{20} = -0{\cdot}796 \text{ ton}$$

$$H_J = \frac{-0{\cdot}41 - 0{\cdot}21}{20} = -\frac{0{\cdot}62}{20} = -0{\cdot}031 \text{ ton.}$$

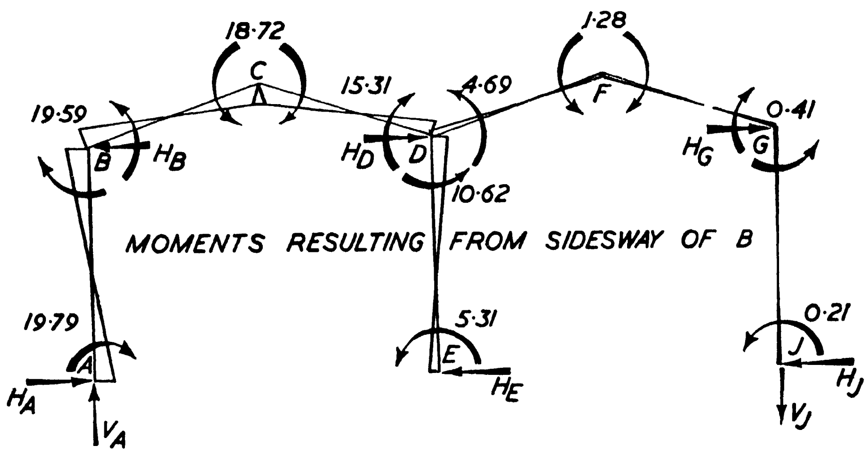


FIG. 150

Taking moments to the left of *D*,

$$30V_A + 19{\cdot}79 + 15{\cdot}31 - 20H_A = 0$$

$$V_A = \frac{39{\cdot}38 - 19{\cdot}79 - 15{\cdot}31}{30}$$

$$= 0{\cdot}142 \text{ ton.}$$

Taking moments to the left of *C*,

$$5H_B + 15V_A - 25H_A + 19{\cdot}79 - 18{\cdot}72 = 0$$

$$H_B = \frac{49{\cdot}23 + 18{\cdot}72 - 2{\cdot}14 - 19{\cdot}79}{5}$$

$$= \frac{46{\cdot}02}{5} = 9{\cdot}20 \text{ tons.}$$

H_B acts from right to left.

Taking moments to the right of D,

$$30V_J + 20H_J - 4{\cdot}69 - 0{\cdot}21 = 0$$

$$V_J = \frac{4{\cdot}69 + 0{\cdot}21 - 0{\cdot}62}{30}$$

$$= 0{\cdot}142 \text{ ton.}$$

V_J acts downwards.

Taking moments to the right of F,

$$-5H_G + 15V_J + 25H_J - 0{\cdot}21 + 1{\cdot}28 = 0$$

$$H_G = \frac{2{\cdot}13 + 0{\cdot}78 - 0{\cdot}21 + 1{\cdot}28}{5}$$

$$= 0{\cdot}80 \text{ ton.}$$

Now $\Sigma H = 0$.

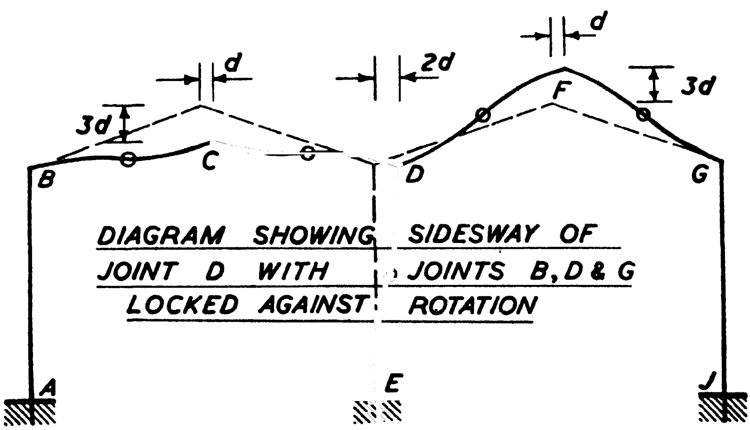


FIG. 151

Then $\quad -H_D = 1{\cdot}969 - 0{\cdot}796 - 0{\cdot}031 - 9{\cdot}20 + 0{\cdot}80$

$\quad H_D = 7{\cdot}26$ tons.

Summarising the induced thrusts at the eaves and valley due to the side-sway of B:

$$H_B = -9{\cdot}20 \text{ tons}$$

$$H_D = 7{\cdot}26 \text{ tons}$$

$$H_G = 0{\cdot}80 \text{ ton.}$$

Now consider joint D. When B and G remain fixed in position and D moves $2d$ to the right, C drops $3d$ vertically and F rises $3d$ vertically. Both C and F move to the right a horizontal amount d. Hence the deflection diagram is as shown in Fig. 151.

The types of deflection being similar to those when B side-swayed, the arbitrary F.E.M.s chosen are 20 tons ft. The distribution table is shown in Fig. 152, and the appropriate B.M. diagram is shown in Fig. 153.

DISTRIBUTION TABLE WHEN JOINT D SIDESWAYS

	AB	BA	BC	CB	CD	DC	DE	ED	DF	FD	FG	GF	GJ	JG
D.F.	0	0·67	0·33	0·5	0·5	0·25	0·5	0	0·25	0·5	0·5	0·33	0·67	0
F.E.M.			−20·00	−20·00	+20·00	+20·00	−20·00	−20·00	+20·00	+20·00	−20·00	−20·00		
Dist.		+13·33	+ 6·67			− 5·00	−10·00		−5·00			+ 6·67	+13·33	
C.O.	+6·67			+ 3·33	− 2·50			−5·00		− 2·50	+ 3·33			+6·67
Dist.				− 0·42	− 0·41					− 0·41	− 0·42			
C.O.			− 0·21			− 0·21			− 0·21			− 0·21		
Dist.		+ 0·14	+ 0·07			+ 0·10	+ 0·22		+ 0·10			+ 0·07	+ 0·14	
C.O.	+ 0·07							+ 0·11						+ 0·07
	+ 6·74	+13·47	−13·47	−17·09	+17·09	+14·89	−29·78	−24·89	+14·89	+17·09	−17·09	−13·47	+13·47	+ 6·74

FIG. 152

$$H_A=\frac{13{\cdot}47+6{\cdot}74}{20}=\frac{20{\cdot}21}{20}=1{\cdot}01 \text{ tons}$$

$$H_E=\frac{-29{\cdot}78-24{\cdot}89}{20}=-\frac{54{\cdot}67}{20}=-2{\cdot}73 \text{ tons}$$

$$H_J=\frac{13{\cdot}47+6{\cdot}74}{20}=1{\cdot}01 \text{ tons.}$$

Taking moments to the left of D,

$$30V_A+6{\cdot}74+14{\cdot}89-20H_A=0$$

$$V_A=\frac{20{\cdot}21-6{\cdot}74-14{\cdot}89}{30}$$

$$=-0{\cdot}05 \text{ ton.}$$

V_A acts downwards.

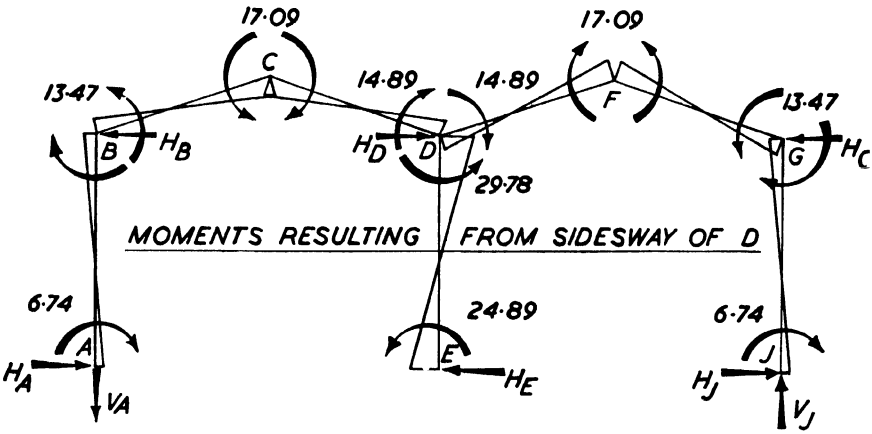


FIG. 153

Similarly,

$$V_J=0{\cdot}05 \text{ ton.}$$

V_J acts upwards.

Taking moments to the left of C,

$$5H_B+6{\cdot}74-17{\cdot}09-25H_A-15V_A=0$$

$$H_B=\frac{17{\cdot}09+25{\cdot}26+0{\cdot}75-6{\cdot}74}{5}$$

$$=-7{\cdot}27 \text{ tons.}$$

Similarly,

$$H_G=-7{\cdot}27 \text{ tons.}$$

Now $\Sigma H=0$.

Hence, $$-H_D=1{\cdot}01+1{\cdot}01-2{\cdot}73-7{\cdot}27-7{\cdot}27$$

and $$H_D=15{\cdot}25 \text{ tons.}$$

Summarising the induced thrusts at the eaves and valley due to the side-sway of D:

$$H_B = -7{\cdot}27 \text{ tons}$$

$$H_D = 15{\cdot}25 \text{ tons}$$

$$H_G = -7{\cdot}27 \text{ tons.}$$

Now consider the side-sway of joint G. When joint G moves inwards an amount $2d$, the joints B and D being fixed in position, the effect is similar to that when B moved outwards an amount $2d$ with joints D and G fixed in position. The final figures for bending moment on the right side of the frame are of the same amount but of reverse sign to those on the left side of the frame when B moved outwards. Consequently, the corresponding thrusts are of the same amount but are opposite in direction.

Therefore, the thrusts at the eaves and valley due to the side-sway of G are:

$$H_B = 0{\cdot}80 \text{ ton}$$

$$H_D = 7{\cdot}26 \text{ tons}$$

$$H_G = -9{\cdot}20 \text{ tons.}$$

Stage III:

For the equilibrium of the frame the Stage I induced thrusts at the eaves and valley must be balanced by the Stage II induced thrusts. Let x be the correction coefficient for the Stage II thrusts due to the side-sway of joint B, and y and z be the coefficients for the side-sway of joints D and G respectively.

Then at joint B: $23{\cdot}21 = 9{\cdot}20x + 7{\cdot}26y - 0{\cdot}80z$

at joint D: $24{\cdot}12 = 7{\cdot}27x + 15{\cdot}25y + 7{\cdot}27z$

at joint G: $0{\cdot}71 = -0{\cdot}80x + 7{\cdot}26y + 9{\cdot}20z.$

When these simultaneous equations are solved,

$$x = 1{\cdot}45$$

$$y = 1{\cdot}27$$

$$z = -0{\cdot}80.$$

As the value of z is negative, the joint G moves outwards when the frame is loaded, not inwards as originally supposed from a consideration of the Stage I thrusts.

Stage IV:

When the Stage II moments are multiplied by the appropriate coefficient they may be added algebraically to the Stage I Moments as shown in Fig. 154.

The final B.M. diagram is shown in Fig. 155.

It is interesting to compare these final moments with those which would result if the frame were loaded with 1 ton per ft. over the whole roof and also with those for a single frame loaded over the whole roof, as shown in

COMPUTATION OF FINAL MOMENTS

	AB	*BC*	*CD*	*DC*	*DE*
Stage I Moments	+ 6·31	−12·63	−21·52	+13·77	− 9·56
Sidesway of $B \times 1{\cdot}45$	+28·70	−28·41	+27·14	· +22·20	−15·40
Sidesway of $D \times 1{\cdot}27$	+ 8·56	−17·11	+21·70	+18·91	−37·82
Sidesway of $G \times -0{\cdot}80$	+ 0·17	− 0·33	+ 1·02	+ 3·75	+ 8·50
Final Moments	+43·74	−58·48	+28·34	+58·63	−54·28

	ED	*DF*	*FG*	*GJ*	*JG*
Stage I Moments	− 4·78	− 4·21	+ 1·12	− 0·39	− 0·19
Sidesway of $B \times 1{\cdot}45$	− 7·70	− 6·80	+ 1·86	− 0·59	− 0·30
Sidesway of $D \times 1{\cdot}27$	−36·61	+18·91	−21·70	+17·11	+ 8·56
Sidesway of $G \times -0{\cdot}80$	+ 4·25	−12·25	+14·98	−15·67	−15·83
Final Moments	−39·84	− 4·35	− 3·74	+ 0·46	− 7·76

FIG. 154

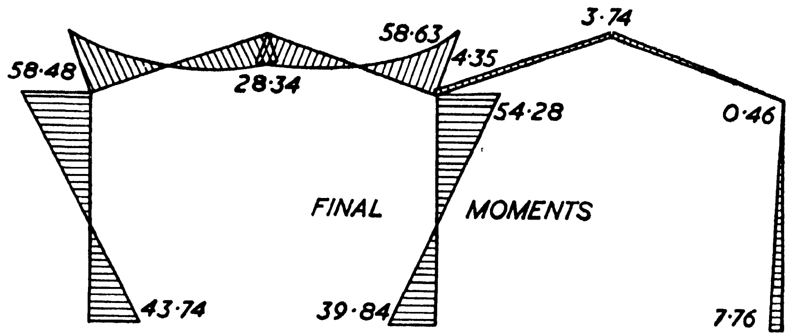


FIG. 155

Fig. 156. It will be observed that there is little variation of moment at joints analogous to *A*, *B*, *C* and *D*.

Multi-storey Frames

Example 9. Consider the symmetrical building frame in Fig. 157. When the uniformly distributed load is applied to the first floor the frame deflects in such a manner that the ends of the beams are subjected to equal rotation in opposite directions. Hence, we may reduce the stiffness of the beams to $K/2$ and dispense with carry-over between *B* and *E* and between *C* and *D*, as was done with the symmetrical rectangular frames.

As *EI* is constant,

$$K_{AB} : K_{BC} : K_{BE} : K_{CD} = 2 : 2 : 1 : 1.$$

Now $\text{F.E.M.}_{BE} = \text{F.E.M.}_{EB} = -\frac{WL}{12} = -\frac{1 \times 20^2}{12} = -33{\cdot}33$ tons ft.

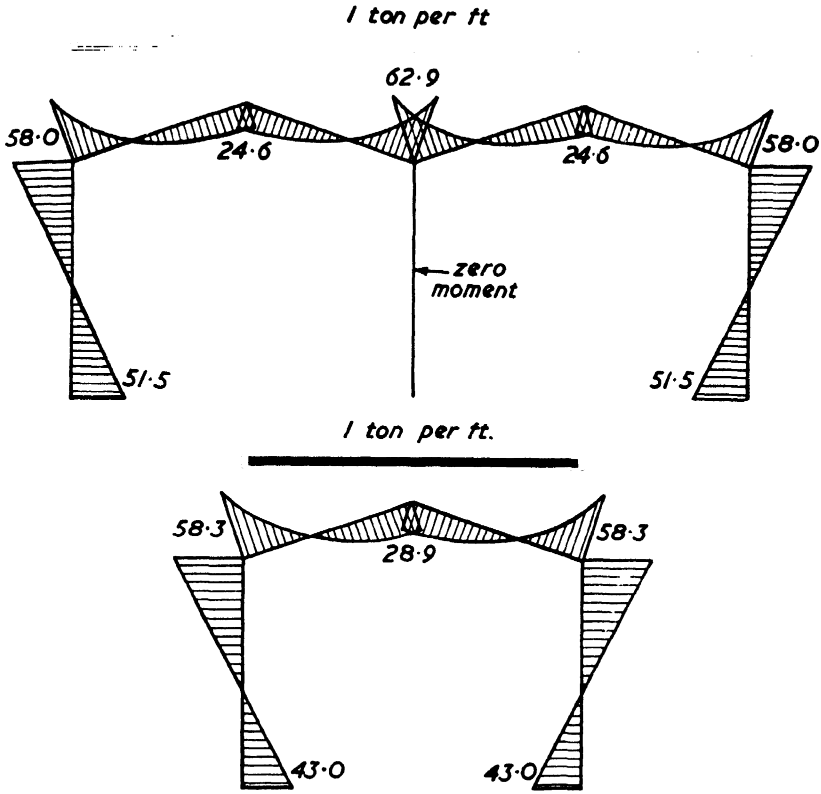


FIG. 156

There is no necessity to consider more than one-half of the frame and the distribution may be carried out as shown in Fig. 157.

The final B.M. diagram is shown in Fig. 158.

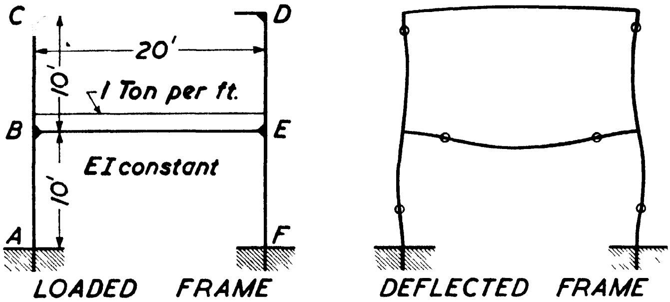
Side Loading on Symmetrical Frames

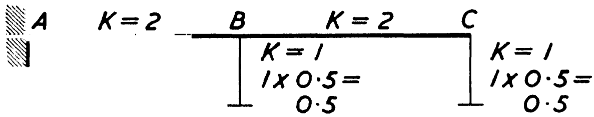
In an article entitled 'Side Sway in Symmetrical Building Frames', published in the *Structural Engineer* of April 1950, Mr. N. Naylor, B.Sc., A.M.I.Struct.E., presented a very useful modification of Moment Distribution which will be derived and illustrated by Examples 10 to 15.

Fig. 159 shows a continuous column PQR which is fixed in position and direction at P, fixed in direction but not in position at R and rigidly attached to the end Q of the beam QS, the other end of the beam being hinged and free to move in a horizontal direction.

Let the I/L values of the upper and lower columns and beams be K_U, K_L and K_B respectively.

Now let a clockwise moment M be applied at the joint Q. Then R, Q and S will move horizontally to the right, and each member meeting at Q





DISTRIBUTION TABLE

	A		B			C
Distribution Factors		4/9	1/9	4/9	4/5	1/5
F.E.M.			-33·33			
Distribution		+14·81	+ 3·71	+14·81		
C.O.	+7·40				+7·40	
Distribution					−5·92	−1·48
C.O.				− 2·96		
Distribution		+ 1·32	+ 0·32	+ 1·32		
C.O.	+0·66				+0·66	
Distribution					−0·53	−0·13
Final Moments	+8·06	+16·13	−29·30	+13·17	+ 1·61	−1·61

FIG. 157

FINAL BENDING MOMENTS

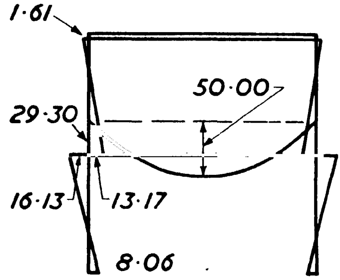


FIG. 158

will slope an amount θ at that joint. Each column will behave like a cantilever and the general effect will be as shown in Fig. 160.

It can be shown that M is distributed among the members, so that

$$M_{QP}=E\,.\,K_L\,.\,\theta$$
$$M_{QR}=E\,.\,K_U\,.\,\theta$$
$$M_{QS}=3E\,.\,K_B\,.\,\theta,$$

that is, in the proportion $K_L : K_U : 3K_B$.

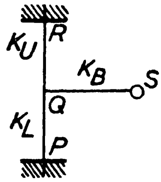


FIG. 159

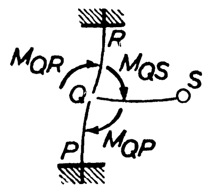


FIG. 160

As each column acts as a cantilever, the carry-over factor from Q to P or from Q to R is -1. Since the beam is hinged at S, there is no carry-over to S. Moment distribution can be applied to the numerical example shown in Fig. 161.

The joint Q is initially assumed to be locked so that the column ends remain vertical and the beam end remains horizontal. Under these conditions the side loads induce moments at each end of the column lengths, their magnitude being $Ph/2$, where P is the appropriate load and h the column height.

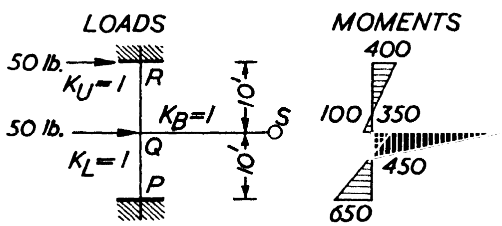


FIG. 161

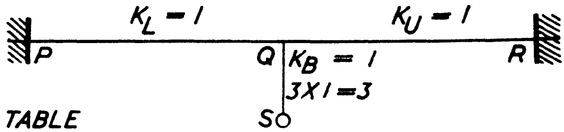
Hence, $\quad \text{F.E.M.}_{QR}=\text{F.E.M.}_{RQ}=Ph/2=\dfrac{50\times 10}{2}=250$ lb. ft.

while $\quad \text{F.E.M.}_{QP}=\text{F.E.M.}_{PQ}=(50+50)\times\dfrac{10}{2}=500$ lb. ft.

The D.F. for QR and $QP=1/(1+3+1)=0{\cdot}2$, while that for $QS=3/(1+3+1)=0{\cdot}6$.

The joint Q can now be unlocked when distribution takes place as shown in Fig. 162.

Note the value and sign of the moments carried over. The final B.M. diagram is as shown in Fig. 161.



DISTRIBUTION TABLE

	P	Q	Q	Q	R
D.F.		0·2	0·6	0·2	
F.E.M.	−500	−500		−250	−250
Distribution		+150	+450	+150	
C.O. (factor = −1)	−150				−150
Final Moments	−650	−350	+450	−100	−400

FIG. 162

The principles employed in this example may be applied to symmetrical frames of the type shown in Fig. 163.

Under the side loading shown points of contraflexure exist in the centres of the beams, and the moments induced in the right-hand half of the frame are numerically equal to those in the left-hand half.

Consequently, the moments may be calculated in a symmetrical frame subjected to side loads by considering one-half only of the frame. It should be noted that the beam length will be halved, and therefore the beam to be considered will be twice as stiff. Hence, the original K value must be multiplied by 6 when considering D.F.s for half the frame.

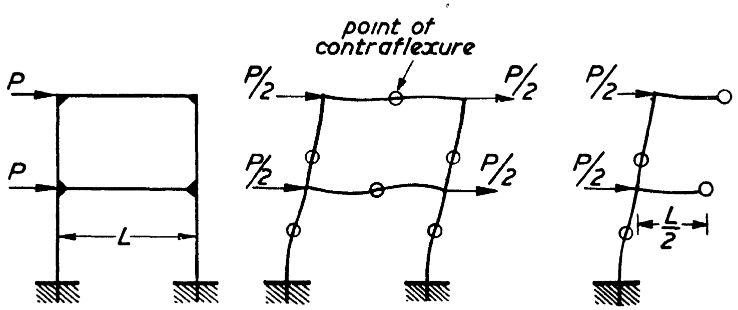


FIG. 163

Single-bay Portal Frame

Example 10. The most simple application is to the ordinary symmetrical portal frame with fixed feet, as shown in Fig. 164.

The stiffness factor for $AB = K = 1$, while that for $BC = 6 \times K = 6$.

$$\text{D.F.}_{BA} = 1/7; \quad \text{D.F.}_{BC} = 6/7$$

$$\text{F.E.M.}_{AB} = \text{F.E.M.}_{BA} = \frac{P}{2} \times \frac{h}{2} = \frac{100}{2} \times \frac{10}{2} = 250 \text{ lb. ft.}$$

The distribution table is shown in Fig. 165, and the final B.M. diagram in Fig. 164.

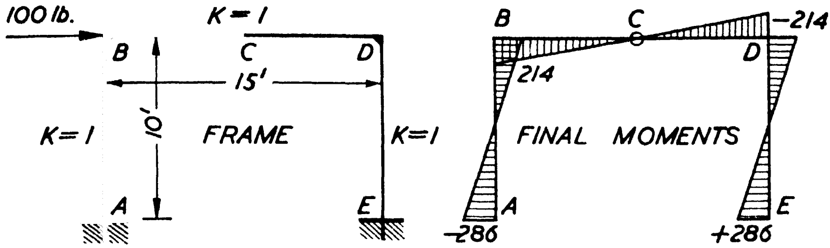
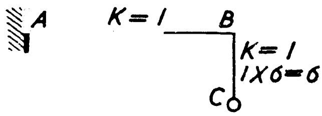


FIG. 164



DISTRIBUTION TABLE

	A	B	
D.F.		0·143	0·857
F.E.M.	−250	−250	
Distribution		+ 36	+214
C.O. (factor = −1)	− 36		
Final Moments	−286	−214	+214

FIG. 165

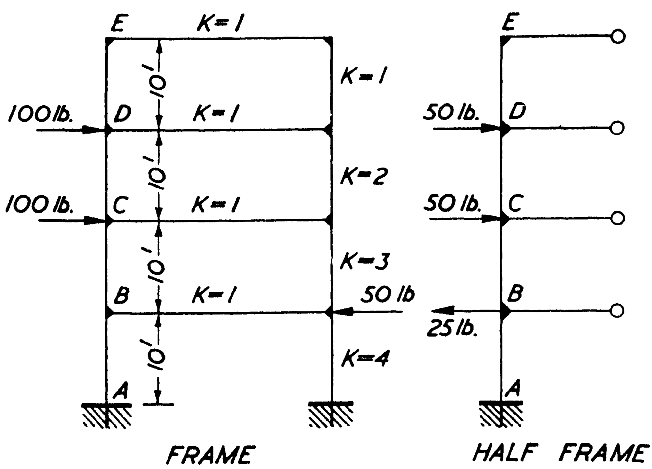
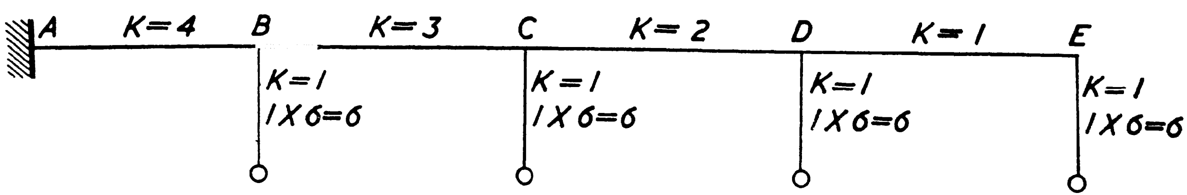


FIG. 166



	A		B			C			D			E
D.F.	0	0·31	0·46	0·23	0·27	0·55	0·18	0·22	0·67	0·11	0·14	0·86
F.E.M.	−375	−375		−500	−500		−250	−250		0	0	
Dist.		+269	+404	+202				+ 55	+168	+27		
C.O.	−269				−202		− 55				−27	
Dist.					+272	+554	+181				+ 4	+23
C.O.				−272				−181		− 4		
Dist.		+ 84	+125	+ 63				+ 41	+124	+20		
C.O.	− 84				− 63		− 41				−20	
Dist.					+ 28	+ 57	+ 19				+ 3	+17
C.O.				− 28				− 19		− 3		
Dist.		+ 8	+ 13	+ 7				+ 5	+ 15	+ 2		
C.O.	− 8											
Final Moments	−736	−14	+542	−528	−465	+611	−146	−349	+307	+42	−40	+40

FIG. 167

Multi-storey Frames

Frames with Fixed Feet

Example 11. See Fig. 166.

There are no F.E.M.s for DE but,

$$\text{F.E.M.}_{DC}=\text{F.E.M.}_{CD}=50\times\frac{10}{2}=250 \text{ lb. ft.}$$

$$\text{F.E.M.}_{CB}=\text{F.E.M.}_{BC}=(50+50)\times\frac{10}{2}=500 \text{ lb. ft.}$$

$$\text{F.E.M.}_{BA}=\text{F.E.M.}_{AB}=(50+50-25)\times\frac{10}{2}=375 \text{ lb. ft.}$$

The distribution table is shown in Fig. 167, and the final B.M. diagram in Fig. 168.

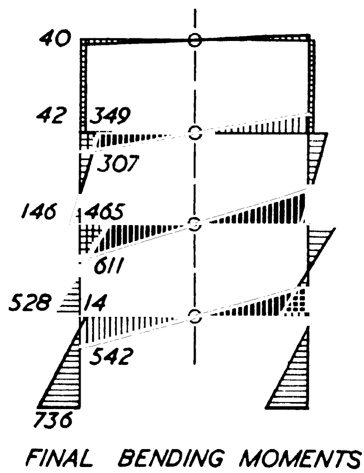


FIG. 168

Frames with Hinged Feet

Example 12. The frame shown in Fig. 169 is a symmetrical frame having hinges at the feet A and F. Hence, AB and EF behave like cantilevers loaded at A and F with a force $H/2=0{\cdot}5$ ton.

Consequently,

$$M_{BA}=M_{EF}=0{\cdot}5\times 10=5 \text{ tons ft.}$$

In the upper storey,

$$\text{F.E.M.}_{CB}=\text{F.E.M.}_{BC}=P/2\times h/2$$

$$=0{\cdot}5\times\frac{10}{2}=2{\cdot}5 \text{ tons ft.}$$

The cantilever moment M is applied to joint B and distributed between the upper column and beam, there being an appropriate carry-over to C which is absorbed in the first distribution at that joint. Then the F.E.M.s for the upper column are introduced at B and C and distribution takes place between the beams and upper column in the normal way, as shown in Fig. 170, the lower column being completely ignored. Of course, the cantilever moment and F.E.M.s may be introduced on the same line in the distribution table, but initially in calculations of this kind it may assist to separate them as shown above. The final moments are shown in Fig. 171.

Frames with Non-prismatic Columns

Example 13. In industrial buildings containing crane gantries it is usual to have columns of varying section. A typical example is shown in Fig. 172.

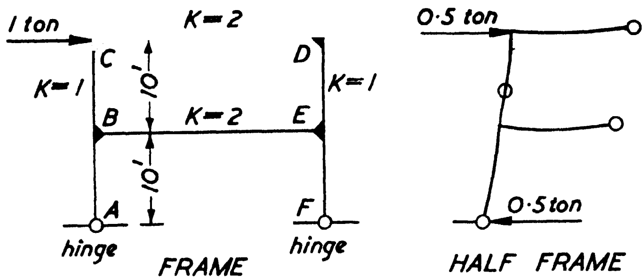
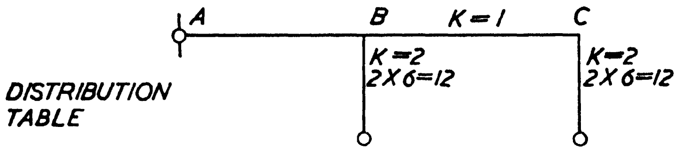


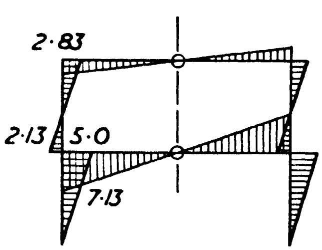
FIG. 169



	A		B		C	
D.F.		0	12/13	1/13	1/13	12/13
Cantilever M	O	−5·00				
Distribution			+4·62	+0·38		
C.O.					−0·38	
F.E.M.				−2·50	−2·50	
Distribution			+2·31	+0·19	+0·22	+2·66
C.O.				−0·22	−0·19	
Distribution			+0·20	+0·02	+0·02	+0·17
Final Moments	O	−5·00	+7·13	−2·13	−2·83	+2·83

FIG. 170

Under the side loading shown the columns undergo similar deflections and the crane beam positions B and E remain the same distance apart. Therefore, it can be assumed that B and E are joined by a beam of zero stiffness, and the frame analysed like a two-storey frame.



FINAL BENDING MOMENTS

FIG. 171

$$\text{F.E.M.}_{CB} = \text{F.E.M.}_{BC}$$

$$= -\frac{200 \times 10}{2 \times 2} = -500 \text{ lb. ft.}$$

$$\text{F.E.M.}_{BA} = \text{F.E.M.}_{AB}$$

$$= -\frac{200 \times 30}{2 \times 2} = -1{,}500 \text{ lb. ft.}$$

The distribution table is shown in Fig. 173, while the final B.M. diagram is shown in Fig. 172.

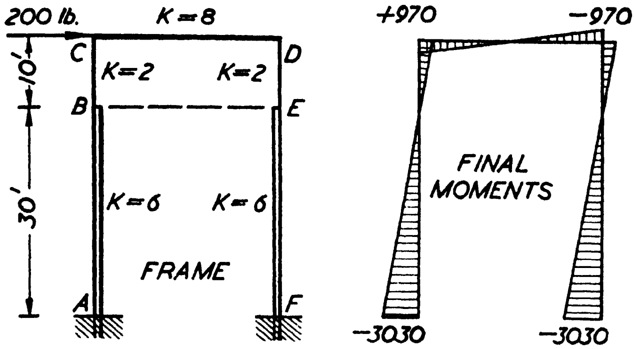
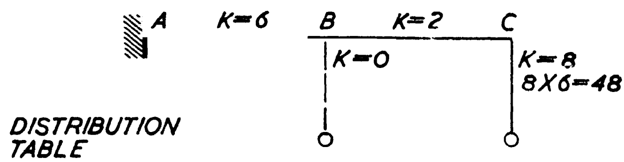


FIG. 172



	A	B			C	
D.F.		0·75	0	0·25	0·04	0·96
F.E.M.	−1500	−1500		−500	−500	
Dist.		+1500	0	+500	+ 20	+480
C.O.	−1500			− 20	−500	
Dist.		+ 15	0	+ 5	+ 20	+480
C.O.	− 15			− 20	− 5	
Dist.		+ 15		+ 5	0	+ 5
C.O.	− 15			0	− 5	
Dist.		0	0	0	0	+ 5
Final Moments	−3030	+30	0	− 30	−970	+970

FIG. 173

Example 14. The frame in Example 13 could be analysed in a similar manner if loads of the same magnitude and direction were applied at *B* and *E*. However, the problem shown in Fig. 174 is not so simple because the two columns do not distort in the same manner. Nevertheless, the problem can be solved by carrying out the analysis in two stages as shown in Figs. 174–7.

In Stage I the work follows the same lines as in the last four examples, but in Stage II the calculations are similar to those in Examples 1 to 10.

Stage I:

$$\text{F.E.M.}_{CB} = \text{F.E.M.}_{BC} = 0$$

$$\text{F.E.M.}_{BA} = \text{F.E.M.}_{AB} = 250 \times \frac{40}{2}$$

$$= 5{,}000 \text{ lb. ft.}$$

The distribution table is shown in Fig. 175.

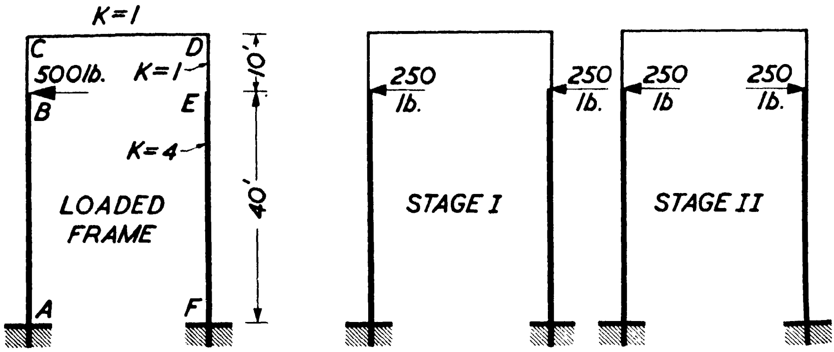
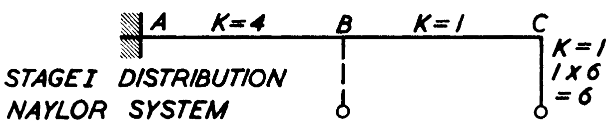


FIG. 174



	A		B			C
D.F.	0	0·8	0	0·2	0·14	0·86
F.E.M.	+5000	+5000				
Dist.		−4000	0	−1000		
C.O.	+4000				+1000	
Dist.					− 140	−860
C.O.				+ 140		
Dist.		− 112	0	− 28		
C.O.	+ 112				+ 28	
Dist.					− 4	− 24
Moments	+9112	+ 888	0	− 888	+ 884	−884

FIG. 175

Stage II:

Because of symmetry it is possible to make these calculations by considering only one-half of the frame. The K value of CD is halved and carry-over between C and D is omitted.

The F.E.M.s for AB and BC caused by a deflection at B will vary with the $I/L^2 = K/L$ value of the column. If arbitrary moments are applied at each end of the members, the propping force can be calculated. As a result the moments induced by the given load of 250 lb. can be calculated.

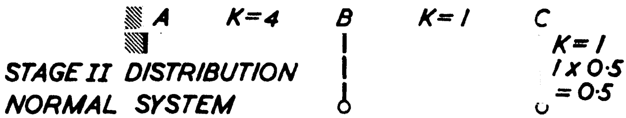
Now K/L for $AB = 4/40 = 0{\cdot}1$

and K/L for $BC = 1/10 = 0{\cdot}1$.

Assume that the arbitrary moments for $AB = 100$ lb. ft.

Then those for $BC = -100 \times \frac{0{\cdot}1}{0{\cdot}1} = -100$ lb. ft.

The distribution table is shown in Fig. 176.



	A	B			C	
D.F.	0	0·8	0	0·2	0·67	0·33
F.E.M.	+100	+100		−100	−100	
Dist.		0	0	0	+67	+33
C.O.				+33		
Dist.		−26	0	−7		
C.O.	−13				−3	
Dist.					+2	+1
Moments	+87	+74	0	−74	−34	+34

FIG. 176

The propping force required to avoid any deflection at B

$$= \frac{87+74}{40} + \frac{74+34}{10} = 14{\cdot}8.$$

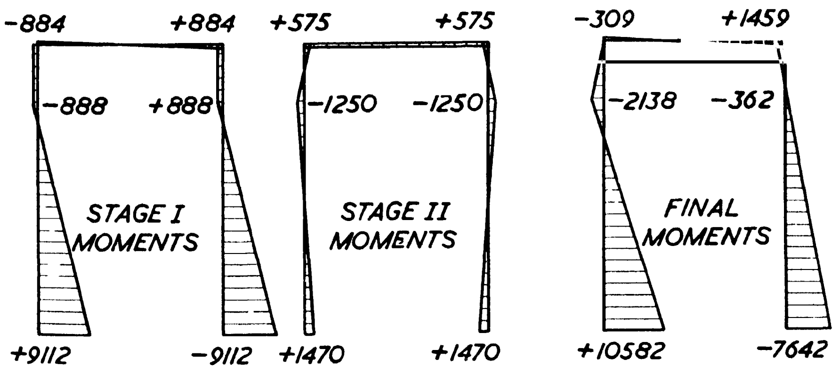


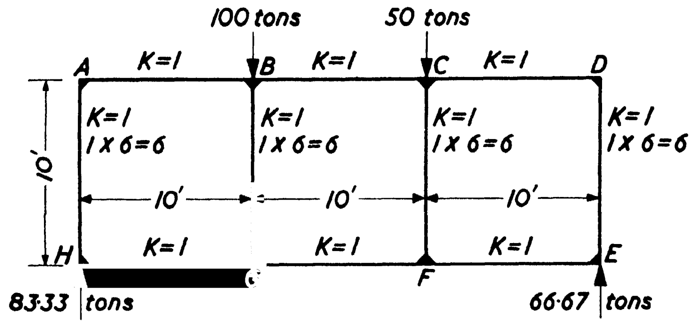
FIG. 177

But the load on the frame $= 250$ lb. Therefore, the moments resulting from the foregoing distribution table must be multiplied by $250/14{\cdot}8$ giving: $M_{AB} = +1{,}470$, $M_{BC} = -1{,}250$ and $M_{CD} = +575$ lb. ft.

The B.M. diagrams for Stages I and II are shown in Fig. 177 together with the final moments resulting from a combination of the stages.

Vierendeel Girders

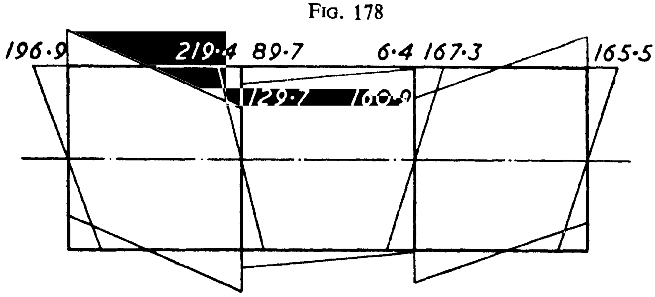
Example 15. When the top and bottom chords of any panel of Vierendeel girders are of the same section, the girders may be analysed in the same way as multi-storey frames. The F.E.M.s are derived from the panel shears.



SHEAR	83·33 tons	−16·67 tons	−66·67 tons
F.E.M's	$\frac{-83{\cdot}33 \times 10}{2 \times 2}$	$\frac{16{\cdot}67 \times 10}{2 \times 2}$	$\frac{66{\cdot}67 \times 10}{2 \times 2}$
	= −208·3	= 41·7	= 166·6

	A			B			C			D
D.F.	0·86	0·14	0·125	0·75	0·125	0·125	0·75	0·125	0·14	0·86
F.E.M.		−208·3	−208·3		+41·7	+41·7		+166·6	+166·6	
Dist.	+178·6	+29·7	+20·8	+125·0	+20·8	−26·0	−156·3	−26·0	−23·8	−142·8
C.O.		−20·8	−29·7		+26·0	−20·8		+23·8	+26·0	
Dist.	+17·8	+3·0	+0·5	+2·7	+0·5	−0·4	−2·2	−0·4	−3·7	−22·3
C.O.		−0·5	−3·0		+0·4	−0·5		+3·7	+0·4	
Dist.	+0·5	+0·0	+0·3	+2·0	+0·3	−0·4	−2·4	−0·4	0·0	−0·4
Final Moments	+196·9	−196·9	−219·4	+129·7	+89·7	−6·4	−160·9	+167·3	+165·5	−165·5

FIG. 178



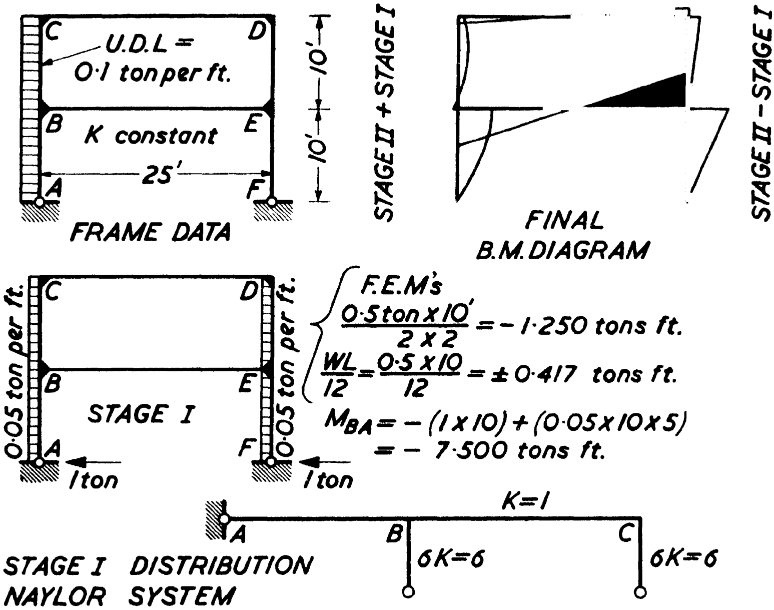
FINAL BENDING MOMENTS

FIG. 179

The girder in Fig. 178 is analysed as shown, and the final B.M. diagram is shown in Fig. 179.

Interpanel Loading

In most multi-storey buildings wind loads are transmitted to the columns at floor level, but occasionally in industrial buildings, and often with Vierendeel girders, loads are applied between the panel points.



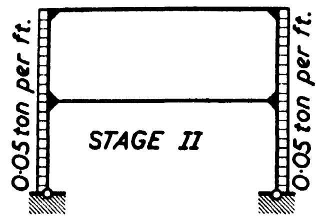
	A		B		C	
D.F.	0	0	0·86	0·14	0·14	0·86
Cantilever M	0	−7·500				
Distribution			+6·429	+1·071		
Carry Over					−1·071	
F.E.M.	0			−1·250	−1·250	
F.E.M.	0			−0·417	+0·417	
Distribution			+1·429	+0·238	+0·272	+1·632
Carry Over				−0·272	−0·238	
Distribution			+0·233	+0·039	+0·035	+0·203
Carry Over				−0·035	−0·039	
Distribution			+0·030	+0·005	+0·006	+0·033
Stage I Moments	0	−7·500	+8·121	−0·621	−1·868	+1·868

FIG. 180*a*

Although Naylor's method involves two stages of analysis, the work is simplified compared with the normal method. An example will demonstrate the method of treatment.

Example 16. The U.D.L. applied to the left column of the frame in Fig. 180 is divided in a manner similar to that adopted for Example 14, and the

distribution is carried out in two stages as shown. Note that it is only necessary to consider the left half of the frame in each case.

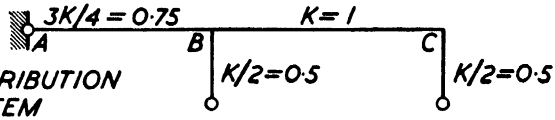


F.E.M's

$$\frac{WL}{12} = \frac{0{\cdot}5 \times 10}{12} = \pm\, 0{\cdot}417 \text{ tons ft.}$$

$$\frac{WL}{8} = \frac{0{\cdot}5 \times 10}{8} = +\, 0{\cdot}625 \text{ tons ft.}$$

STAGE II DISTRIBUTION
NORMAL SYSTEM



	A		B		C	
D.F.	0	0·33	0·22	0·44	0·67	0·33
F.E.M.	0	+0·625		−0·417	+0·417	
Distribution		−0·069	−0·046	−0·093	−0·278	−0·139
Carry Over				−0·139	−0·046	
Distribution		+0·046	+0·031	+0·062	+0·031	+0·015
Carry Over				+0·015	+0·031	
Distribution		−0·005	−0·003	−0·007	−0·021	−0·010
Stage II Moments	0	+0·597	−0·018	−0·579	+0·134	−0·134
Stage I Moments	0	−7·500	+8·121	−0·621	−1·868	+1·868
Stage II + Stage I	0	−6·903	+8·103	−1·200	−1·734	+1·734
Stage II − Stage I	0	+8·097	−8·139	+0·042	+2·002	−2·002
	F		E		D	

FIG. 180b

The final B.M. diagram is shown in detail in Fig. 180c.

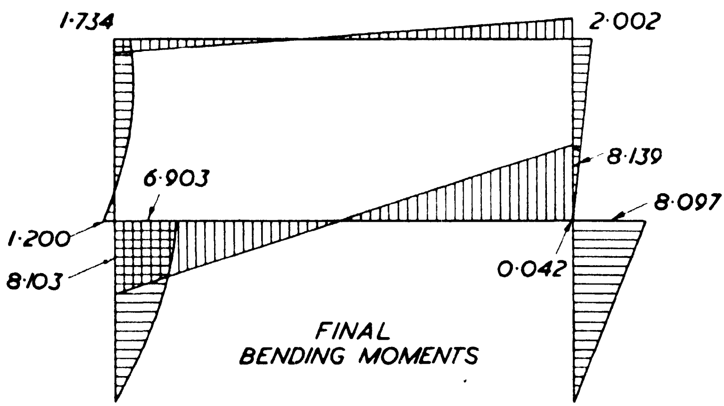


FIG. 180c

Unsymmetrical Vertical Loads

Unsymmetrical vertical loads can be divided in a similar manner, as shown in Fig. 181, the Naylor method being employed for the first stage and the normal method for the second.

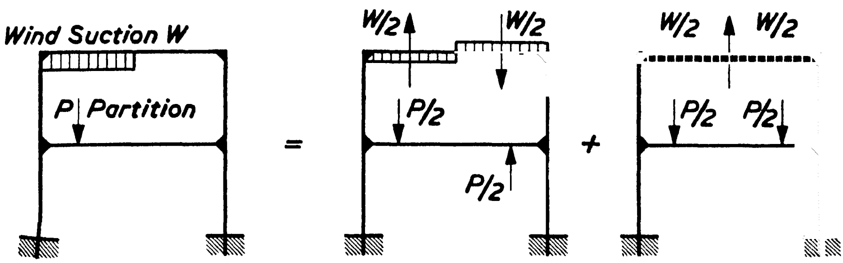


FIG. 181

As a further aid, the F.E.M.s may be divided as shown in Fig. 182.

Although the method of division of load described must not be confused with the Column Analogy immediately following this section on Moment Distribution, it is similar to the treatment of eccentric loads on columns, where the direct stresses P/A are calculated for the load centrally applied,

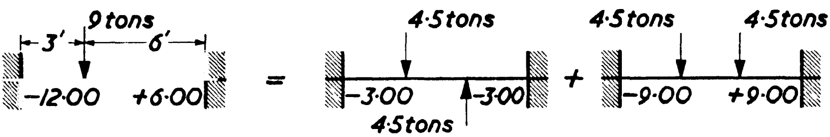


FIG. 182

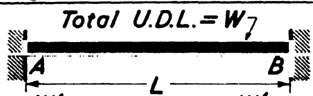
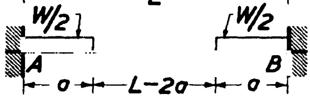
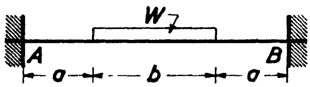
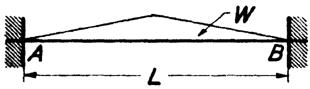
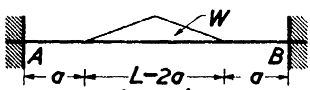
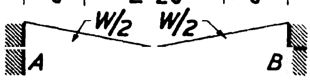
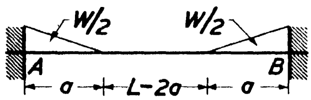
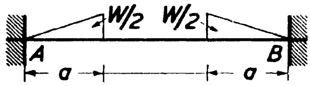
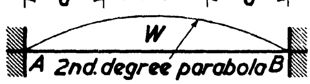
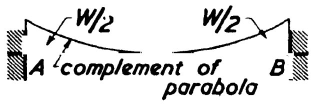
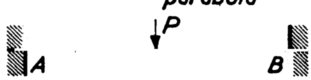
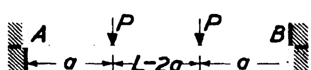
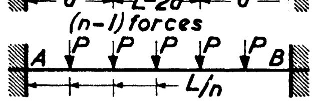
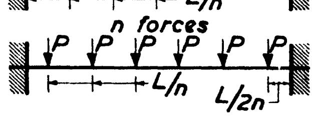
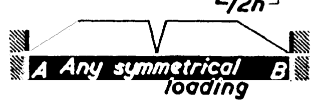
corresponding to Stage II, and the bending stresses due to the eccentricity $\pm P.e/Z$, corresponding to Stage I, are then added algebraically.

The Treatment of Non-prismatic Members

A method of applying Moment Distribution to non-prismatic members is given at the end of the section on Column Analogy.

FIXED-END MOMENTS

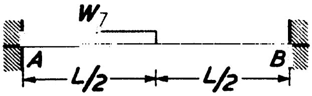
BUILT-IN BEAMS OF CONSTANT CROSS-SECTION

Symmetrical loadings	Values of $+M_A$ and $-M_B$
	$-\frac{WL}{12}$
	$-\frac{Wa}{12L}(3L-2a)$
	$-\frac{W}{24L}(3L^2-b^2)$
	$-\frac{5WL}{48}$
	$-\frac{W}{48L}(5L^2+4aL-4a^2)$
	$-\frac{WL}{16}$
	$-\frac{Wa}{12L}(2L-a)$
	$-\frac{Wa}{12L}(4L-3a)$
	$-\frac{WL}{10}$
	$-\frac{WL}{20}$
	$-\frac{PL}{8}$
	$-\frac{Pa}{L}(L-a)$
	$-\frac{PL}{12n}(n^2-1)$
	$-\frac{PL}{24n}(2n^2+1)$
	$-A_S/L$ Where A_S is the area of the 'free' bending moment diagram

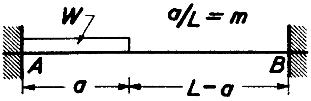
FIXED-END MOMENTS

BUILT-IN BEAMS OF CONSTANT CROSS-SECTION

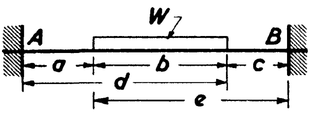
Asymmetrical loadings — *Values of M_A and M_B*



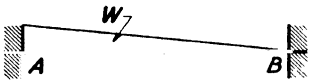
$$M_A = -\frac{11}{96}WL$$
$$M_B = +\frac{5}{96}WL$$



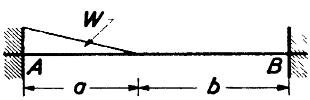
$$M_A = -\frac{WL}{12}\cdot m(3m^2-8m+6)$$
$$M_B = +\frac{WL}{12}\cdot m^2(4-3m)$$



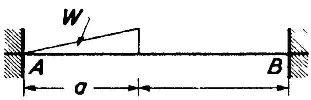
$$M_A = -\frac{W}{12L^2b}\left[e^3(4L-3e)-c^3(4L-3c)\right]$$
$$M_B = +\frac{W}{12L^2b}\left[d^3(4L-3d)-a^3(4L-3a)\right]$$



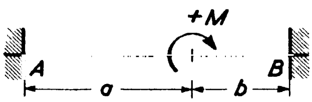
$$M_A = -\frac{WL}{10}$$
$$M_B = +\frac{WL}{15}$$



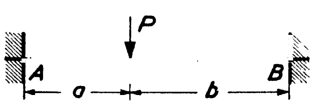
$$M_A = -\frac{Wa}{30L^2}(3a^2+10bL)$$
$$M_B = +\frac{Wa^2}{30L^2}(5L-3a)$$



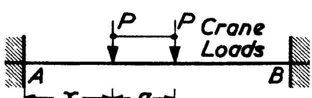
$$M_A = -\frac{Wa}{15L}(10L^2-15aL+6a^2)$$
$$M_B = +\frac{Wa^2}{10L^2}(5L-4a)$$



$$M_A = +M\frac{b}{L^2}(3a-L)$$
$$M_B = +M\frac{a}{L^2}(3b-L)$$



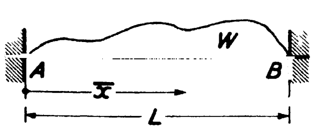
$$M_A = -\frac{Pab^2}{L^2}$$
$$M_B = +\frac{Pa^2b}{L^2}$$



When $x = \frac{4L-3a-\sqrt{4L^2-9a^2}}{6}$

$$M_A\,max = -\frac{P}{54L^2}\left[8L^3+(4L^2-9a^2)^{3/2}\right]$$

corresponding $M_B = +\frac{P}{54L^2}\left[4L^3+(2L^2+9a^2)\sqrt{4L^2-9a^2}\right]$

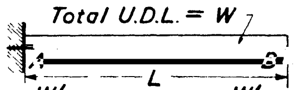
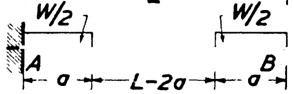
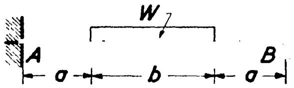
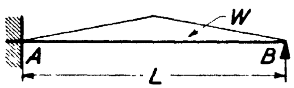
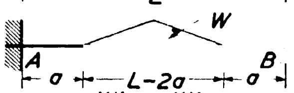
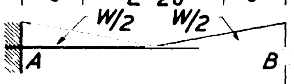
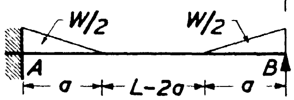
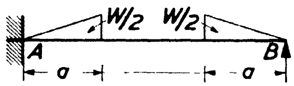
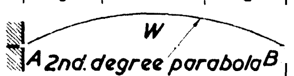
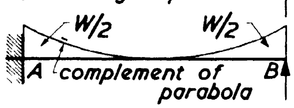
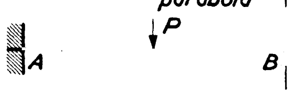
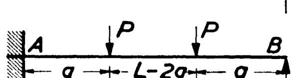
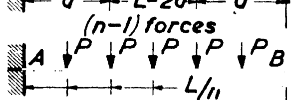
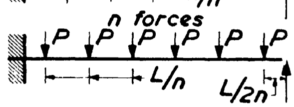
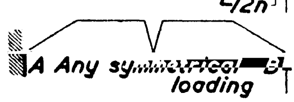


$$M_A = -\left[\frac{4A_S}{L}-\frac{6A_S\bar{x}}{L^2}\right]$$
$$M_B = +\left[\frac{6A_S\bar{x}}{L^2}-\frac{2A_S}{L}\right]$$

Where A_S is the area of the 'free' B.M. Diagram and $\bar{x}$ is the distance from A to its centroid

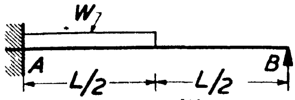
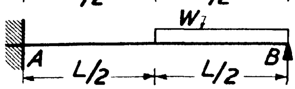
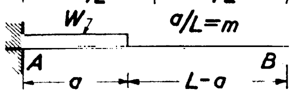
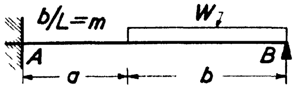
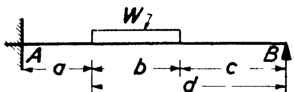
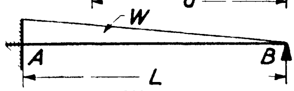
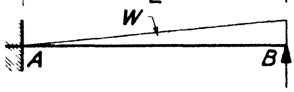
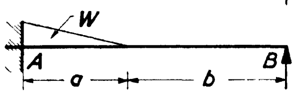
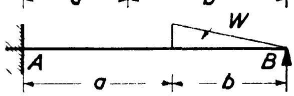
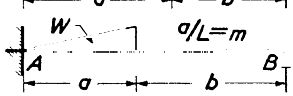
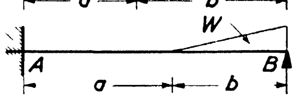
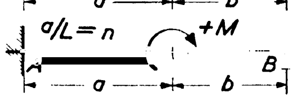
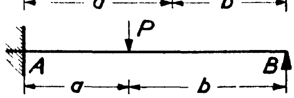
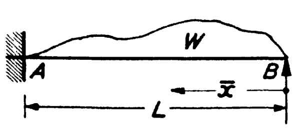
FIXED-END MOMENTS

PROPPED CANTILEVERS OF CONSTANT CROSS-SECTION

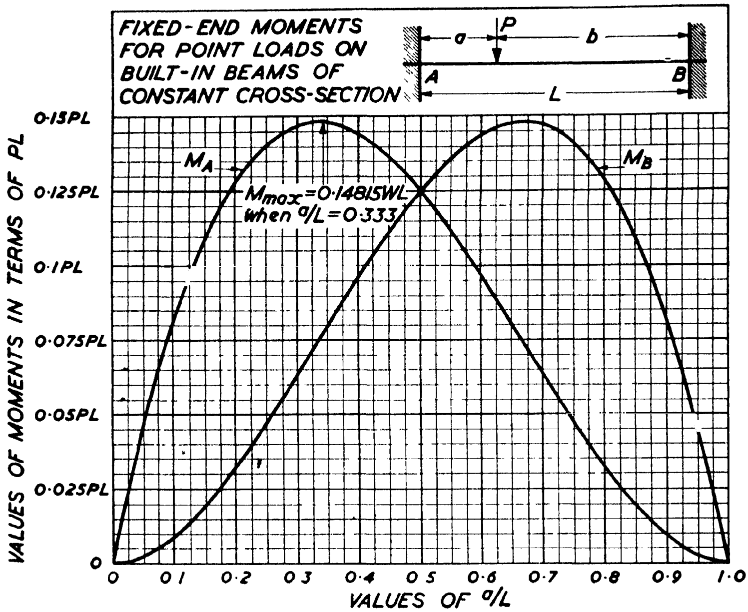
Symmetrical loadings	Values of fixing moment M_A
	$-\frac{WL}{8}$
	$-\frac{Wa}{8L}(3L-2a)$
	$-\frac{W}{16L}(3L^2-b^2)$
	$-\frac{5WL}{32}$
	$-\frac{W}{32L}(5L^2+4aL-4a^2)$
	$-\frac{3WL}{32}$
	$-\frac{Wa}{8L}(2L-a)$
	$-\frac{Wa}{8L}(4L-3a)$
	$-\frac{3WL}{20}$
	$-\frac{3WL}{40}$
	$-\frac{3PL}{16}$
	$-\frac{3Pa}{2L}(L-a)$
	$-\frac{PL}{8n}(n^2-1)$
	$-\frac{PL}{16n}(2n^2+1)$
	$-3A_S/2L$ Where A_S is the area of the 'free' bending moment diagram

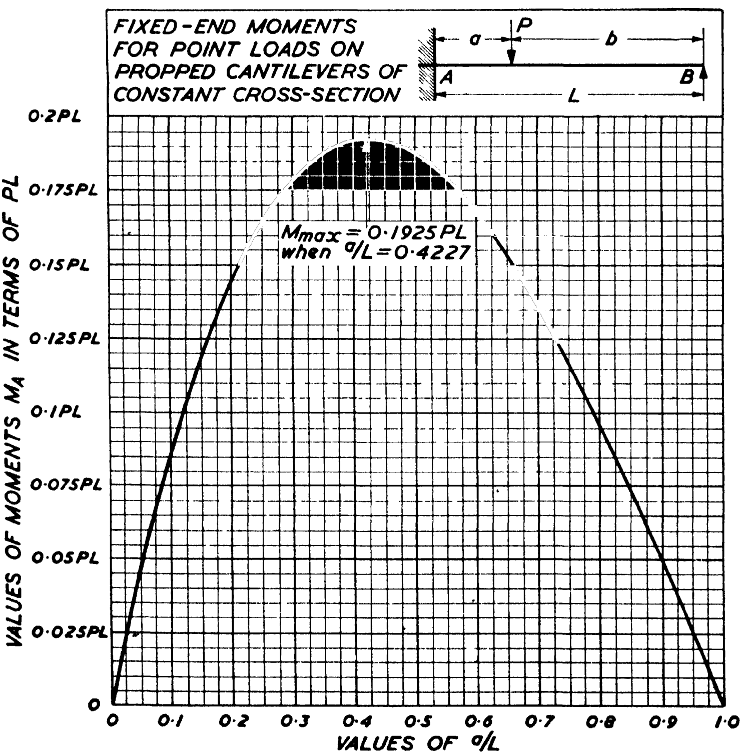
For cantilevers of opposite hand, the fixing moments M_B are of opposite sign.

FIXED-END MOMENTS

PROPPED CANTILEVERS OF CONSTANT CROSS-SECTION	
Asymmetrical loadings	Values of fixing moment M_A
	$-\frac{9}{64}WL$
	$-\frac{7}{64}WL$
	$-\frac{Wa}{8}(2-m)^2$
	$-\frac{Wb}{8}(2-m^2)$
	$-\frac{W}{8L^2b}(d^2-c^2)(2L^2-c^2-d^2)$
	$-\frac{2}{15}WL$
	$-\frac{7}{60}WL$
	$-\frac{Wa}{60L^2}(3a^2-15aL+20L^2)$
	$-\frac{Wb}{15L^2}(5L^2-3b^2)$
	$-Wa\left(\frac{m^2}{5}-\frac{3m}{4}+\frac{2}{3}\right)$
	$-\frac{Wb}{60L^2}(10L^2-3b^2)$
	$+\frac{M}{2}(2-6n+3n^2)$
	$-\frac{Pb}{2L^2}(L^2-b^2)$
	$-\frac{3A_S\bar{x}}{L}$ Where A_S is the area of the 'free' B.M. diagram, considering AB as a beam, and $\bar{x}$ is the distance from B to its centroid

For cantilevers of opposite hand, the fixing moments M_B are of opposite sign.


FIXED-END MOMENTS
FOR POINT LOADS ON
BUILT-IN BEAMS OF
CONSTANT CROSS-SECTION
a
P
b
A
B
L
M_A
M_B
M_{max}=0·14815WL
when a/L=0·333
0·15PL
0·125PL
0·1PL
0·075PL
0·05PL
0·025PL
0
VALUES OF MOMENTS IN TERMS OF PL
0 0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0
VALUES OF a/L


FIXED-END MOMENTS
FOR POINT LOADS ON
PROPPED CANTILEVERS OF
CONSTANT CROSS-SECTION
a
P
b
A
B
L
M_{max} = 0·1925 PL
when a/L = 0·4227
0·2PL
0·175PL
0·15PL
0·125PL
0·1PL
0·075PL
0·05PL
0·025PL
0
VALUES OF MOMENTS M_A IN TERMS OF PL
0 0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0
VALUES OF a/L

(b) The Column Analogy Method

Column Analogy makes use of the mathematical identity which exists between the moments produced by continuity or restraint in a beam, arch or closed frame and the fibre stresses produced when a short column is eccentrically loaded. In certain cases the method offers considerable advantages over other methods, especially when the members are of varying moment of inertia or when they are asymmetrically arranged. With this method, unlike that of Moment Distribution, no special calculations are required for side-sway.

The Analogy

Consider the curved beam AC shown in Fig. 183.

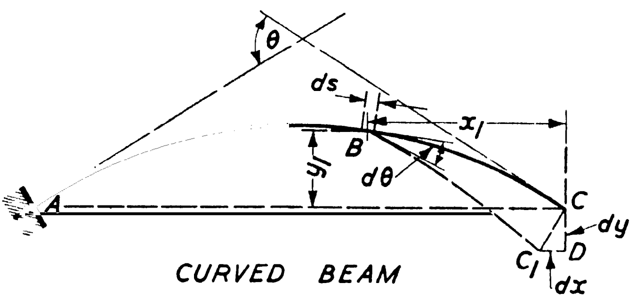


FIG. 183

Let M = the B.M. at any position B,

x_1 and y_1 = the co-ordinates of B with respect to C,

θ = the angular rotation of the beam from A to C due to bending,

$d\theta$ = the angular rotation of an element ds of the beam at B,

I = the moment of inertia at any point

and E = the modulus of elasticity, assumed to be constant.

Then $$d\theta = \frac{M \, . \, ds}{EI}$$

Consider the bending of ds only, AB being held in position and BC allowed to deflect into the position BC_1. Then C_1D and CD respectively represent the horizontal and vertical deflections dx and dy.

By similar triangles,

$$CD = x_1 \, . \, d\theta$$

and $$C_1D = y_1 \, . \, d\theta.$$

Therefore $$dy = x_1 \, . \, d\theta = \frac{Mx_1 \, . \, ds}{EI}$$

and $$dx = y_1 \, . \, d\theta = \frac{My_1 \, . \, ds}{EI}$$

Consequently, if the whole beam is considered,

the total angular rotation from A to $C = \theta = \int_C^A \frac{M \, . \, ds}{EI}$,

the total vertical deflection of C with respect to $A = y = \int_C^A \frac{Mx \, . \, ds}{EI}$,

the total horizontal deflection of C with respect to $A = x = \int_C^A \frac{My \, . \, ds}{EI}$.

These are the basic equations normally employed for any member in applications of the Elastic Theory. However, it will be remembered that in the particular cases of encastré beams and closed rings or frames, the area of the free or static moment diagram/EI is equal to the area of the restraint or indeterminate moment diagram/EI.

This may be expressed mathematically as:

$$\int \frac{M_s \, . \, ds}{EI} = \int \frac{M_i \, . \, ds}{EI},$$

where M_s = the static moment

and M_i = the indeterminate moment.

Now consider the short eccentrically loaded column shown in Fig. 184.

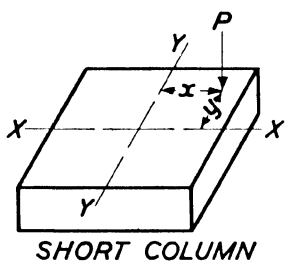


FIG. 184

Let P = the total load,

x and y = the co-ordinates with respect to the YY and XX axes respectively of the point of application of P,

f = the fibre stress,

A = the cross-sectional area of the column

and M_{XX} and M_{YY} = the moments about the XX and YY axes respectively.

Then:

$$P = \int f \, . \, dA$$
$$M_{YY} = \int fx \, . \, dA$$
$$M_{XX} = \int fy \, . \, dA.$$

It will be observed that an analogy exists between the two sets of equations. Thus,

$$\int \frac{M_s \, . \, ds}{EI} = \theta \text{ corresponds to } P,$$

$$M_i \text{ corresponds to } f.$$

It follows, therefore, that when an analogous short column is derived for an indeterminate member in which:

(a) the depth of the cross-section of the column equals the length of the member;

(b) the profile of the cross-section of the column follows the profile of the member;

(c) the breadth of the cross-section of the column at any point equals the value $1/EI$ for the corresponding section of the member; and
(d) the column cross-section is loaded with an intensity at any point equal to the static B.M. at a corresponding section of the member;

then the fibre stress at any point on the column section is numerically equal to the indeterminate moment at the corresponding section of the member.

Column Stress Formulæ

When the cross-section of a column is symmetrical about one axis, say the YY axis, the following formula is employed:

$$f=\frac{P}{A}\pm\frac{M_{XX}y}{I_{XX}}\pm\frac{M_{YY}x}{I_{YY}}$$

where x and y = the co-ordinates of the point where the stress is being calculated,

and I_{XX} and I_{YY} = the moments of inertia of the XX and YY axes respectively.

This formula, often with simplifications, is the one most commonly employed in Column Analogy, although a more complicated one, incorporating products of inertia (i.e. I_{XY}), is required for columns which are not symmetrical about either principal axis, as in Example 5.

Sign Convention

It is the usual practice to consider downward loads, compressive stresses and moments causing tension on the inside of a frame to be positive.

Static Moments

Consider the prismatic beam AB in Fig. 185 (a), which is fixed at both ends and carries a load of 300 lb. 8 ft. from A. The beam could be made statically determinate in any one of the three ways shown in Fig. 185 (b), (c) and (d) viz.:

1. by releasing both the end restraints;
2. by releasing the end restraint and removing the support at A; or
3. by releasing the end restraint and removing the support at B.

Encastré Beams

Example 1. As an example of the use of the column analogy, the fixed end moments at A and B will be determined for the beam shown in Fig. 185.

Analogous Column

Following the rules laid down, the analogous column for the beam has a cross-section with a length of 12 ft. and a width equal to $1/EI$.

Hence the cross-sectional area $A=12/EI$ units.

The column is shown in Fig. 186.

Analogous Load and Moments

The static moment diagram which was shown in Fig. 185 (b) will be used to provide the analogous load as shown in Fig. 186.

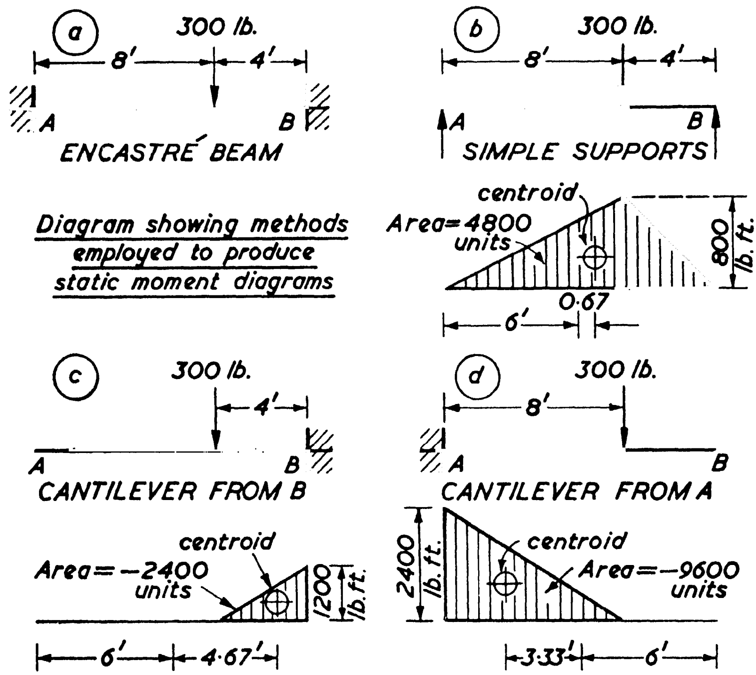


FIG. 185

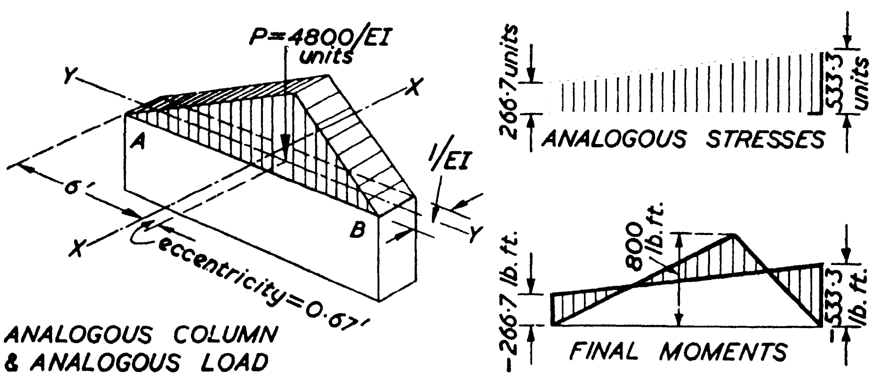


FIG. 186

The area of the diagram

$$=\frac{800\times 12}{2}=4800 \text{ units.}$$

Now the diagram is projected over the full width of the analogous column section.

Hence, the analogous load $P=4800/EI$ units.

P has an eccentricity e_x of 0·67 ft. about the XX axis but is symmetrical about the YY axis.

Hence
$$M_{XX}=P\,.\,e_x$$
$$=\frac{4800}{EI}\times 0{\cdot}67$$
$$=\frac{3200}{EI}\text{ units.}$$
$$M_{YY}=P\,.\,e_y=P\times 0$$
$$=0.$$

Analogous Moment of Inertia I_{XX}

The moment of inertia of the analogous column I_{XX} about its XX axis $=bd^3/12$.

Hence
$$I_{XX}=\frac{1}{EI}\times\frac{12^3}{12}$$
$$=\frac{144}{EI}\text{ units.}$$

Analogous Stresses

The analogous stresses in the outside edges of the column—corresponding to B and A in the beam—

$$f=\frac{P}{A}\pm\frac{M_{XX}y}{I_{XX}}$$
$$=\left(\frac{4{,}800}{EI}\times\frac{EI}{12}\right)\pm\left(\frac{3{,}200}{EI}\times\frac{EI}{144}\times 6\right)$$
$$=400\pm 133{\cdot}3$$
$$=533{\cdot}3\quad\text{or}\quad 266{\cdot}7\text{ units.}$$

The analogous stresses are numerically equal to the indeterminate moments at B and A in the original beam, the units being the same as those employed for the static moments.

Final Bending Moments

The final bending moment at any point in a structure is obtained when the indeterminate moment is deducted algebraically from the static moment, i.e.

$$M=M_s-M_i.$$

Since the static moments at A and B are zero,

$$\text{F.E.M.}_A=0-266{\cdot}7$$
$$=-266{\cdot}7\text{ lb. ft.}$$
$$\text{F.E.M.}_B=0-533{\cdot}3$$
$$=-533{\cdot}3\text{ lb. ft.}$$

The diagram for the final B.M. is also shown in Fig. 186.

It should be noted that the value $1/EI$ does not appear in the answers. Hence, the width of the analogous column for a single prismatic member can be assumed to be unity.

Example 1 (*Alternative solution*). Another solution may be found by employing the static B.M. diagram shown in Fig. 185 (c).

The analogous column has the same dimensions as before, as shown in Fig. 187.

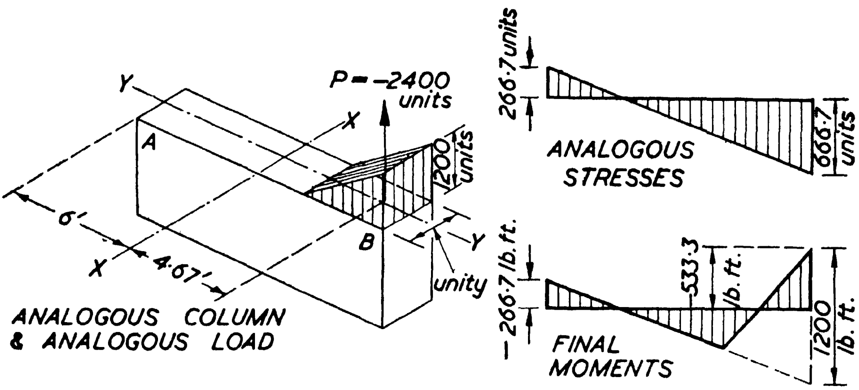


FIG. 187

Analogous Load and Moments

$$P=\frac{-1{,}200\times 4}{2}=-2400 \text{ units.}$$

It will be noted that P acts upwards.

The centroid of the analogous load has an eccentricity e_x of 4·67 ft. about the XX axis.

Hence,
$$M_{XX}=P\,.\,e_x$$
$$=-2400\times 4{\cdot}67$$
$$=-11{,}200 \text{ units.}$$
$$M_{YY}=0.$$

Analogous Stresses at B and A

$$f=\frac{P}{A}\pm\frac{M_{XX}y}{I_{XX}}$$
$$=-\frac{2400}{12}\pm\frac{-11{,}200\times 6}{144}$$
$$=-666{\cdot}7 \quad \text{or} \quad +266{\cdot}7.$$

Final Moments

Now $M=M_s-M_i$.

The value of the static moment at A is zero, while that at B is -1200 lb. ft.

Hence,
$$\text{F.E.M.}_A = 0 - 266{\cdot}7$$
$$= -266{\cdot}7 \text{ lb. ft.}$$
$$\text{F.E.M.}_B = -1200 - (-666{\cdot}7)$$
$$= -533{\cdot}3 \text{ lb. ft.}$$

The final B.M. diagram is shown in Fig. 187.

Example 2. The beam shown in Fig. 188 is built-in at A but simply supported at B.

Now the flexural rigidity EI of a hinge is zero, therefore, the area of the

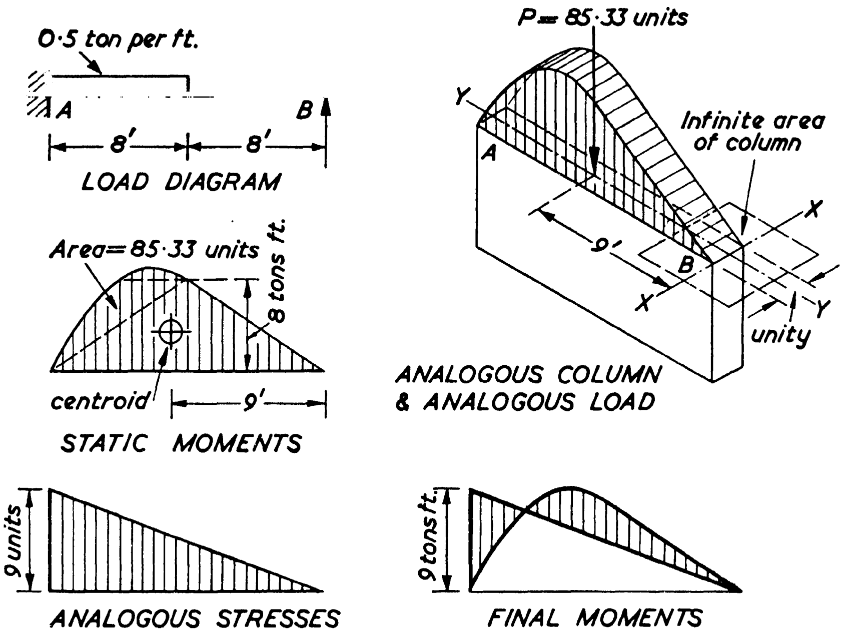


FIG. 188

analogous column $\int ds/EI$ is infinite at that point, and, in consequence, the XX axis of the analogous column passes through B.

As $P/A = P/\infty = 0$, the term P/A in the standard formula disappears.

Hence,
$$f = \frac{M_{XX}y}{I_{XX}}.$$

Analogous Load P and Moment M_{XX}

The static moment diagram consists of a triangle and a parabola, and the analogous load P

$$= \frac{8 \times 16}{2} + \left(\frac{4 \times 8}{8} \times \frac{2 \times 8}{3}\right)$$
$$= 64 + 21{\cdot}33$$
$$= 85{\cdot}33 \text{ units.}$$

Now $$M_{XX} = P \cdot e_x$$
$$= (64 \times 8) + (21{\cdot}33 \times 12)$$
$$= 768 \text{ units.}$$

Note that if the whole load of 85·33 units is considered to be concentrated at its centre of gravity, the eccentricity $e_x = 9$ ft.

Analogous Moment of Inertia I_{XX}

$$I_{XX} = \frac{bd^3}{3}$$
$$= \frac{1 \times 16^3}{3}.$$

Analogous Stress at A

$$f = \frac{M_{XX}y}{I_{XX}}$$
$$= 768 \times 16 \times \frac{3}{16^3}$$
$$= 9 \text{ units.}$$

Final Bending Moment at A

$$\text{F.E.M.}_A = M_s - M_i$$
$$= 0 - 9$$
$$= -9 \text{ tons ft.}$$

The final B.M. diagram is shown in Fig. 188.

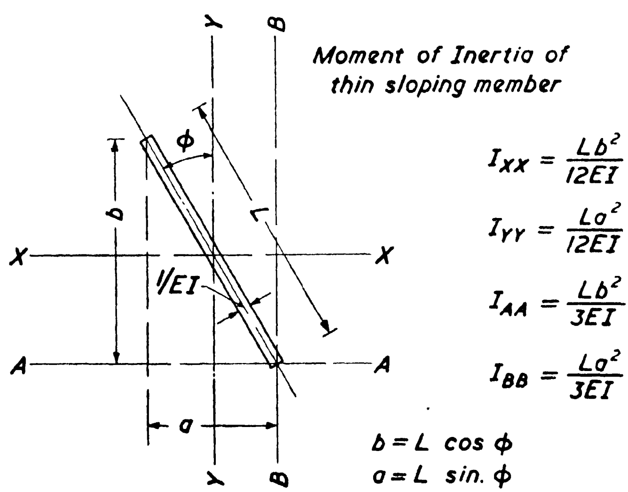


FIG. 189

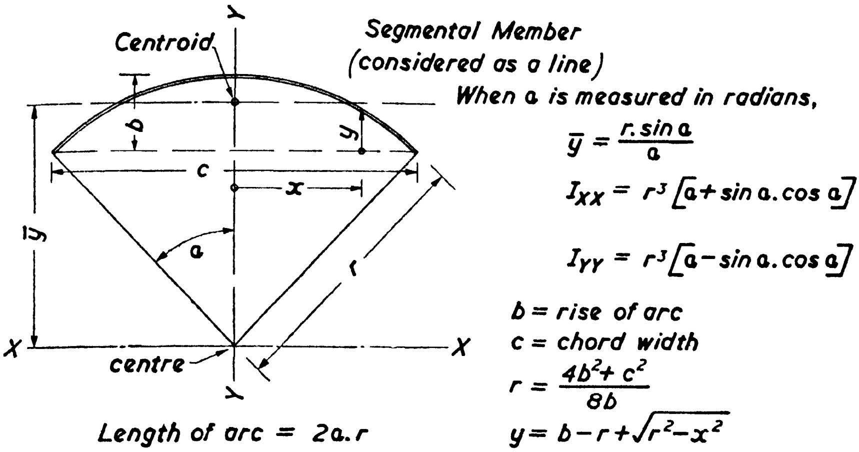


FIG. 190

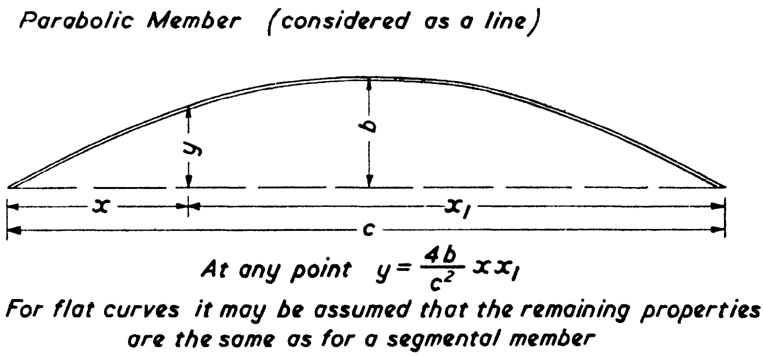


FIG. 191

Geometrical Properties of Irregular Sections

The geometrical properties of certain plane sections have been given on pages 423 to 425, but there are other sections which are required from time to time in Column Analogy problems.

The properties of sloping thin straight members, arcs of circles and parabolas are given in Figs. 189, 190 and 191.

Symmetrical Portal Frames

The analysis of a portal frame by Column Analogy is a simple operation when the frame is symmetrical about the YY axis.

Example 3. Consider the frame in Fig. 192.

Analogous Column

E is assumed to be constant throughout and will be ignored in this and subsequent operations.

The moments of inertia I_{AB}, I_{BC} and I_{CD} are in the proportions 1 : 3·6 : 1.

Now the width of the analogous column varies with the value $1/EI$ for the frame. Hence, the widths of the members AB, BC and CD in the analogous column may be taken as 3·6, 1 and 3·6 units respectively, as shown in Fig. 192.

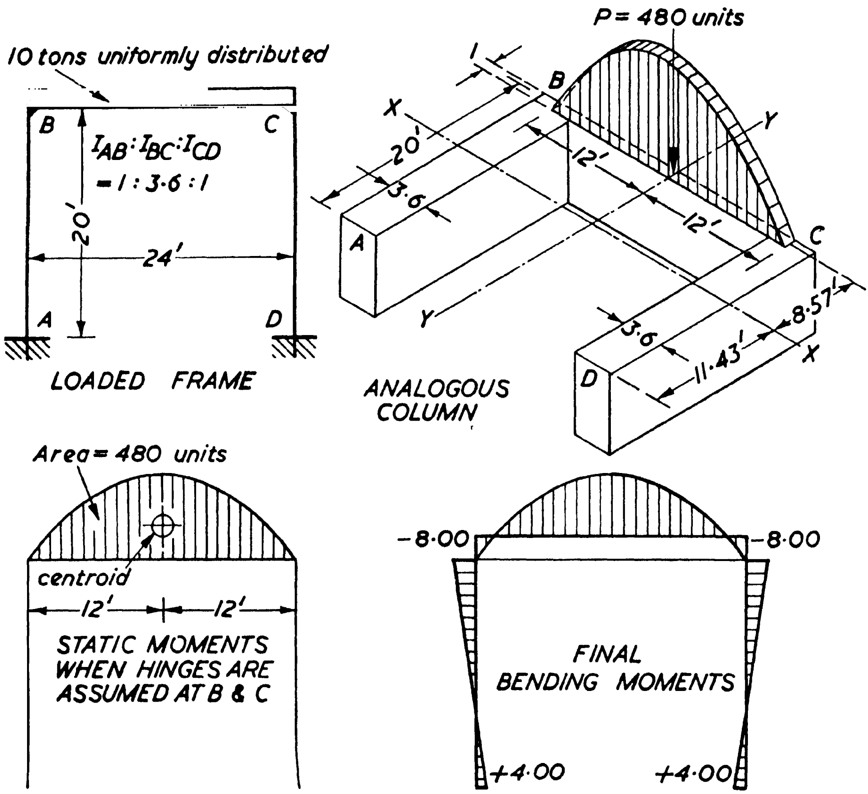


FIG. 192

The realisation that channel sections are used as columns will assist readers to visualise the analogous column for the frame.

The YY axis passes vertically through the centre of the frame.

The position of the XX axis must be calculated. Taking moments about the axis BC,

$$\bar{y} = \frac{2(20 \times 3{\cdot}6 \times 10) + (24 \times 1 \times 0)}{2(20 \times 3{\cdot}6) + (24 \times 1)}$$

$$= 8{\cdot}57 \text{ ft.}$$

Analogous Load P and Moments M_{XX} and M_{YY}

The static bending moment diagram is parabolic and has an area of $WL/8 \times 2L/3$.

Hence,
$$P = \frac{1 \times 10 \times 24 \times 2 \times 24}{8 \times 3}$$
$$= 480 \text{ units.}$$
$$M_{XX} = P \cdot e_x$$
$$= 480 \times 8{\cdot}57$$
$$= 4114 \text{ units.}$$
$$M_{YY} = 0.$$

Analogous Moment of Inertia I_{XX}

The moment of inertia of each member about the XX axis of the analogous column may be found from the standard formula

$$I_{XX} = I_{xx} + Ay^2,$$

where I_{xx} = the moment of inertia of any member about an axis parallel to XX and through its own centroid,

A = the area of the member,

and y = the eccentricity of the member about the XX axis.

In calculating the properties of the analogous column, the moments of inertia of the individual members about their longitudinal axes are neglected. Hence,

$$I_{XX} = 2\left(\frac{3{\cdot}6 \times 20^3}{12} + 3{\cdot}6 \times 20 \times 1{\cdot}43^2\right) + (1 \times 24 \times 8{\cdot}57^2)$$
$$= 6857.$$

Analogous Stresses

These may be found from the formula

$$f = \frac{P}{A} \pm \frac{M_{XX}y}{I_{XX}}.$$

At A and D,
$$f = \frac{480}{168} - \frac{4114 \times 11{\cdot}43}{6857}$$
$$= 2{\cdot}86 - 6{\cdot}86$$
$$= -4 \text{ units.}$$

At B and C,
$$f = \frac{480}{168} + \frac{4114 \times 8{\cdot}57}{6857}$$
$$= 2{\cdot}86 + 5{\cdot}14$$
$$= 8 \text{ units.}$$

Final Bending Moments

$$M = M_s - M_i.$$

Hence,
$$M_A = M_D = 0 - (-4)$$
$$= 4 \text{ tons ft.}$$
$$M_B = M_C = 0 - 8$$
$$= -8 \text{ tons ft.}$$

The final B.M. diagram is shown in Fig. 192.

Example 4. Fig. 193 shows a two-hinged frame with a segmental girder carrying loads at the quarter points of the span.

This example will be analysed fully, but it is one in which many approximations could be made which would save a considerable amount of work and which would involve very little inaccuracy. For instance, each of the curved sections *BC, CD, DE* and *EF* could be assumed to be straight. Also, the moment of inertia of the arc about axis *xx* is so small that it could be ignored.

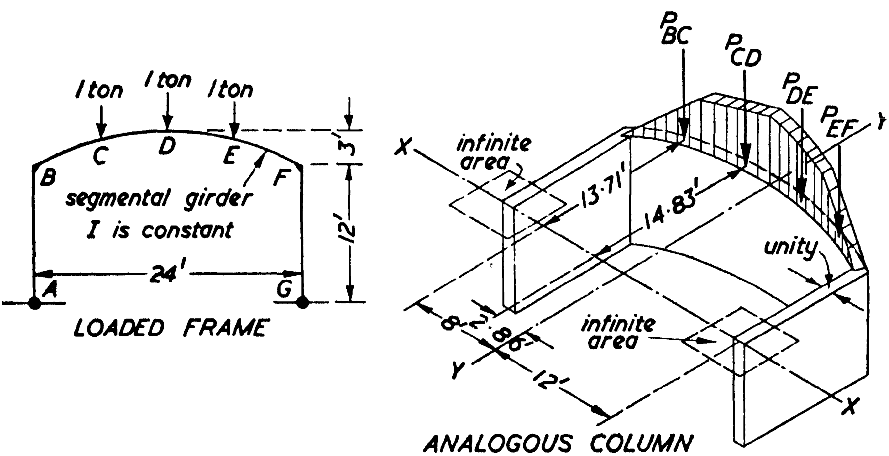


FIG. 193

Analogous Column

The frame being of constant section the analogous column has a constant thickness of unity as shown in Fig. 193.

The feet of the frame are hinged. Consequently, as with the hinge in Example 2, it is assumed that infinite areas of column are centred on these points. Hence, the *XX* axis passes through *A* and *G* and the *YY* axis passes vertically midway between *A* and *G*.

Some attention must be given to the geometrical properties of the arc *BF*. Consider Fig. 194.

The chord width $c = 24$ ft.; the rise $b = 3$ ft. Hence,

$$\text{the radius } r = \frac{4b^2 + c^2}{8b}.$$
$$= 25{\cdot}5 \text{ ft.}$$

Let the angle subtended by the arc BD be α (alpha).

Then, $$\sin\alpha = \frac{c}{2r} = \frac{24}{51} = 0{\cdot}4706$$

whence $$\alpha = 0{\cdot}49 \text{ radian.}$$

The length of the arc BF

$$= 2\alpha \times r$$
$$= 2 \times 0{\cdot}49 \times 25{\cdot}5$$
$$= 24{\cdot}99 \text{ ft.}$$

The height of the centroid of the arc above the centre

$$\bar{y} = \frac{r\,.\,\sin\alpha}{\alpha} = \frac{25{\cdot}5 \times 0{\cdot}4706}{0{\cdot}49}$$
$$= 24{\cdot}49 \text{ ft.}$$

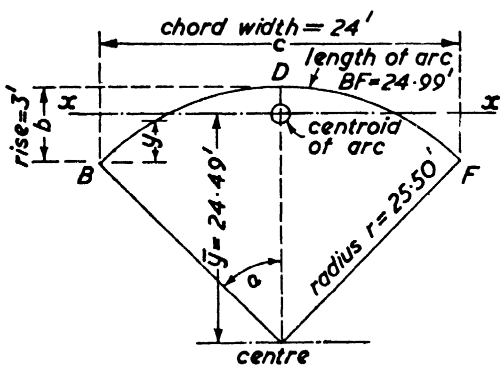


FIG. 194

Therefore, the centroid is 1·99 ft. above the springing points B and F.

The moment of inertia of the arc about a horizontal axis through the centre is derived from the formula

$$I = r^3(\alpha + \sin\alpha\,.\,\cos\alpha)$$
$$= 25{\cdot}5^3(0{\cdot}49 + 0{\cdot}4706 \times 0{\cdot}8823)$$
$$= 15{,}000.$$

Now the moment of inertia about the centroid of the arc

$$I_{xx} = 15{,}000 - (24{\cdot}99 \times 24{\cdot}49^2)$$
$$= 10 \text{ units.}$$

Analogous Load P and Moment M_{XX}

The frame may be made statically determinate by assuming hinges at B and F.

The static moment diagram shown in Fig. 195 may be divided into four parts, viz. two triangles and two trapezia.

Consider half the arc say, BD. Using the YY axis as origin the horizontal distance x to the centroid of the M_s diagram for BC is 8 ft., while that for CD

$$=\frac{(9\times6\times3)+(1\cdot5\times6\times2)}{(9\times6)+(1\cdot5\times6)}$$

$$=2\cdot86 \text{ ft.}$$

The corresponding eccentricities above the chord BF may be found from the formula

$$y=b-r+\sqrt{(r^2-x^2)} \text{ (See Fig. 194),}$$

the values being 1·71 and 2·83 ft. respectively. Similarly the point C is 2·28 ft. above the chord BF. Using the latter figure, the chords BC and CD may be found to be 6·41 ft. and 6·05 ft. in length respectively. Now, in this case, the arcs BC and CD are very nearly proportional to their chords. The

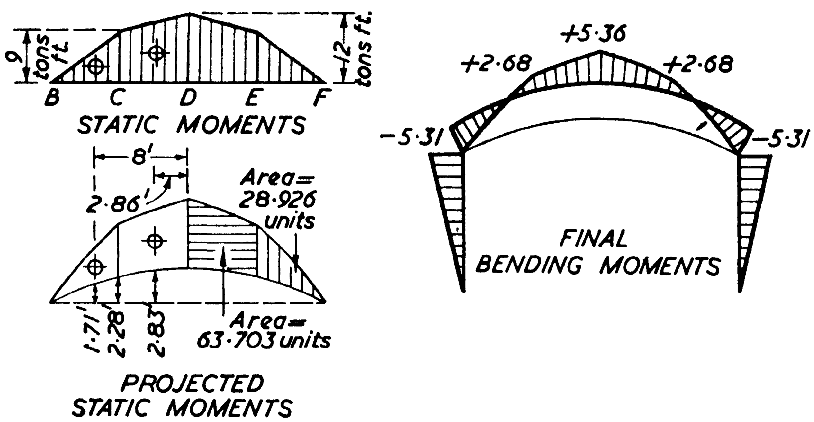


FIG. 195

arc BD is 12·495 ft. long. Hence, the arc $BC=6\cdot428$ ft. and the arc $CD=6\cdot067$ ft.

Now the static moment diagram must be projected on to these arcs to give the loads as represented in Fig. 193.

Therefore, for BC or EF,

$$P_{BC}=P_{EF}=\frac{9\times6\cdot428}{2}=28\cdot926 \text{ units.}$$

and for CD or DE,

$$P_{CD}=P_{DE}=10\cdot5\times6\cdot067=63\cdot703 \text{ units.}$$

Now $$M_{XX}=P\,.\,e_x$$

$$=2(28\cdot926\times13\cdot71)+2(63\cdot703\times14\cdot83)$$

$$=2681 \text{ units.}$$

Analogous Moment of Inertia I_{XX}

Members AB and FG:

$$I_{XX}=2\left(\frac{bd^3}{3}\right)$$
$$=1152 \text{ units.}$$

Arc BF:

$$I_{XX}=I_{xx}+Ay^2$$
$$=10+(24{\cdot}99\times 13{\cdot}99^2)$$
$$=4904.$$
$$\text{Total } I_{XX}=6056 \text{ units.}$$

Analogous Stresses

Since there are infinite areas situated at A and G in the analogous column the standard formula reduces to

$$f=\frac{M_{XX}y}{I_{XX}}.$$

Hence,

At B or F,
$$f=\frac{2681\times 12}{6056}$$
$$=5{\cdot}31 \text{ units.}$$

At C or E,
$$f=\frac{2681\times 14{\cdot}28}{6056}$$
$$=6{\cdot}32 \text{ units.}$$

At D,
$$f=\frac{2681\times 15}{6056}$$
$$=6{\cdot}64 \text{ units.}$$

Final Bending Moments

$$M=M_s-M_i.$$

Hence,
$$M_B=M_F=0-5{\cdot}31$$
$$=-5{\cdot}31 \text{ tons ft.}$$
$$M_C=M_E=9-6{\cdot}32$$
$$=+2{\cdot}68 \text{ tons ft.}$$
$$M_D=12-6{\cdot}64$$
$$=+5{\cdot}36 \text{ tons ft.}$$

Asymmetrical Portal Frames

Example 5. The formulæ used for stress and flexure in the previous examples are those applicable to structures having at least one axis of symmetry. For a frame such as that shown in Fig. 196 the formula is more complicated, viz.:

$$f=\frac{P}{A}\pm\frac{M_{YY}I_{XX}-M_{XX}I_{XY}}{I_{XX}I_{YY}-(I_{XY})^2}\cdot x\pm\frac{M_{XX}I_{YY}-M_{YY}I_{XY}}{I_{XX}I_{YY}-(I_{XY})^2}\cdot y$$

The frame is made statically determinate by assuming that there are hinges at T and U. Then the static moment diagram forming the analogous load is triangular and the area

$$=\frac{WL}{4}\times\frac{L}{2}=\frac{1\times24\times24}{4\times2}=72 \text{ units.}$$

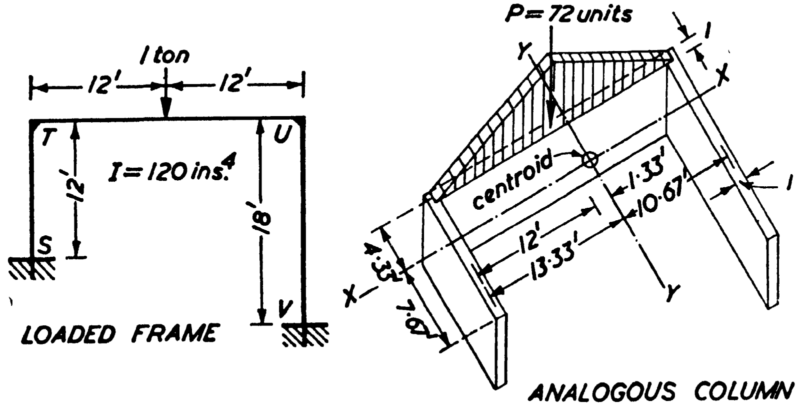


FIG. 196

Analogous Column and Moments of Inertia I_{XX}, I_{YY} and I_{XY}

The moment of inertia of the original frame is constant throughout, and consequently the analogous column is assumed to have unit thickness.

The properties of the analogous column are more easily calculated if a table as shown in Fig. 197 is prepared.

Member	*Length L in ft.*	*Value I/EI*	*A (=L/EI)*	*Horizontal distance x from S*	*Ax*	*Vertical distance y from S*	*Ay*	*Eccentricity e_y from centroid*	*Eccentricity e_x from centroid*	I_{XX} $(=Ae_x^2+I_{xx})$	I_{YY} $(=Ae_y+I_{yy})$	I_{XY} $(=Ae_xe)$
ST	12	1	12	0	0	6	72	− 13·33	− 1·67	177·33	2133·33	+ 266·67
TU	24	1	24	12	288	12	288	− 1·33	+ 4·33	451·67	1194·67	− 138·67
UV	18	1	18	24	432	3	54	+ 10·67	− 4·67	878·00	2048·00	− 896·00
Σ			54		720		414			1507·00	5376·00	− 768·00

FIG. 197

S is used as the origin to find the position of the centroid. From the data in the first eight columns in the table,

$$\bar{x}=\frac{\Sigma Ax}{\Sigma A}=\frac{720}{54}=13{\cdot}33 \text{ ft.}$$

$$\bar{y}=\frac{\Sigma Ay}{\Sigma A}=\frac{414}{54}=7{\cdot}67 \text{ ft.}$$

Having found the position of the centroid, the remaining columns can be filled in.

Analogous Load P and Moments M_{XX} and M_{YY}

Now the analogous load $P = 72 \times 1 = 72$ units.

Hence,
$$M_{XX} = P \,.\, e_x = 72 \times 4{\cdot}33 = 312$$
$$M_{YY} = P \,.\, e_y = 72 \times -1{\cdot}33 = -96.$$

Sufficient information is available to apply the basic formula to find the analogous stresses at the four joints of the frame.

Joint S:

$$= \frac{P}{A} - \frac{M_{YY}I_{XX} - M_{XX}I_{XY}}{I_{XX}I_{YY} - (I_{XY})^2} \,.\, x - \frac{M_{XX}I_{YY} - M_{YY}I_{XY}}{I_{XX}I_{YY} - (I_{XY})^2} \,.\, y$$

$$= \frac{72}{54} - \frac{(-96)(1507) - (312)(-768)}{(1507)(5376) - (-768)^2} 13{\cdot}33 - \frac{(312)(5376) - (-96)(-768)}{(1507)(5376) - (-768)^2} \,.\, 7{\cdot}67$$

$$= 1{\cdot}33 - 0{\cdot}013(13{\cdot}33) - 0{\cdot}214(7{\cdot}67)$$

$$= -0{\cdot}48.$$

Joint T:

$$f = 1{\cdot}33 - 0{\cdot}013(13{\cdot}33) + 0{\cdot}214(4{\cdot}33)$$
$$= +2{\cdot}087.$$

Joint U:

$$f = 1{\cdot}33 + 0{\cdot}013(10{\cdot}67) + 0{\cdot}214(4{\cdot}33)$$
$$= +2{\cdot}396.$$

Joint V:

$$f = 1{\cdot}33 + 0{\cdot}013(10{\cdot}67) - 0{\cdot}214(13{\cdot}67)$$
$$= -1{\cdot}451.$$

Final Bending Moments

Now
$$M = M_s - M_i.$$

At each joint $M_s = 0$.

Hence,
$$M_{ST} = +0{\cdot}48 \text{ tons ft.}$$
$$M_{TS} = -2{\cdot}087 \text{ tons ft.}$$
$$M_{UV} = -2{\cdot}396 \text{ tons ft.}$$
$$M_{VU} = +1{\cdot}451 \text{ tons ft.}$$

The final B.M. diagram is shown in Fig. 198.

Example 6. In the last example the frame had fixed feet and being asymmetrical about both principal axes, the formula employed was rather cumbersome. However, the frame shown in Fig. 199, having hinged feet, embraces a simplification of formula.

Since the analogous column is assumed to have infinite section at A and D, the basic formula reduces to

$$f = \frac{M_{XX}y}{I_{XX}}$$

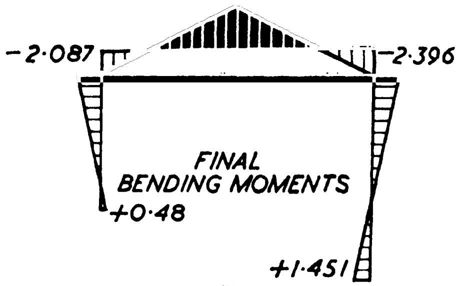


FIG. 198

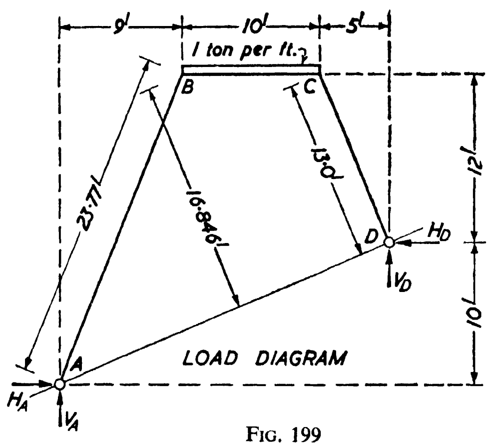


FIG. 199

Analogous Column

The analogous column is shown in Fig. 200.

K is constant throughout the frame. Hence,

$$I_{AB} : I_{BC} : I_{CD} = 23{\cdot}77 : 10 : 13.$$

Therefore, the thickness of the column section $1/EI$ representing AB, BC and CD may be taken as 1/23·77, 1/10 and 1/13 respectively.

Analogous Moment M_{XX}

The frame may be made statically determinate by releasing the horizontal thrust at a foot, say D. The analogous load P, which may be divided into five geometrical forms, is obtained by projecting the static B.M. diagram on to the analogous column, as shown in Fig. 200. (See also Fig. 228, p. 268.)

The analogous moment M_{XX}

$$= \left[\frac{37{\cdot}5 \times 23{\cdot}77}{2} \times \frac{1}{23{\cdot}77}\right] 11{\cdot}231 + \left[29{\cdot}17 \times 10 \times \frac{1}{10}\right] 14{\cdot}923$$

$$+ \left[\frac{8{\cdot}33 \times 10}{2} \times \frac{1}{10}\right] 15{\cdot}564 + \left[\frac{2 \times 12{\cdot}5 \times 10}{3} \times \frac{1}{10}\right] 14{\cdot}923$$

$$+ \left[\frac{29{\cdot}17 \times 13}{2} \times \frac{1}{13}\right] 8{\cdot}67$$

$$= 961{\cdot}49 \text{ units.}$$

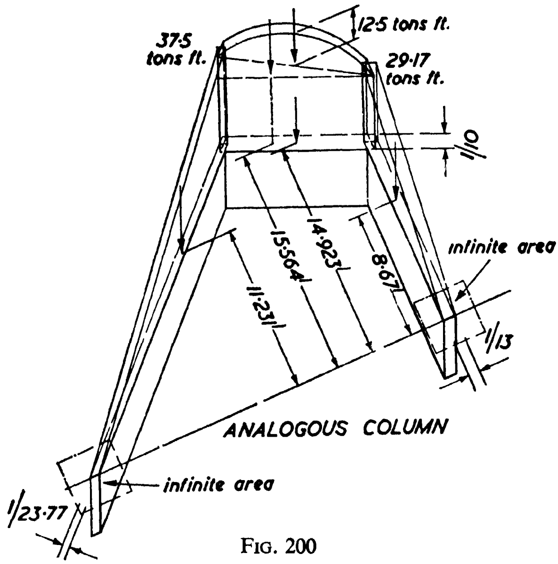


FIG. 200

Analogous Moment of Inertia

CD is perpendicular to *AD*, but for *AB* and *BC*, the reader should refer to the formula for sloping members given in Fig. 189.

AB: $$I_{xx}=\frac{La^2}{3EI}=\frac{23\cdot77\times16\cdot846^2}{3\times23\cdot77}=94\cdot60$$

BC: $$I_{xx}=Ay^2+\frac{La^2}{12EI}$$

$$=\left(\frac{1}{10}\times10\times14\cdot923^2\right)$$

$$+\left(\frac{10\times3\cdot846^2}{12\times10}\right)=223\cdot93$$

CD: $$I_{xx}=\frac{bd^3}{3}=\frac{1}{13}\times\frac{13^3}{3}=56\cdot33$$

$$\text{Total } I_{XX} \text{ for the frame}=374\cdot86$$

Analogous Stresses

At *B*: $$f=\frac{M_{XX}y}{I_{XX}}=\frac{961\cdot49\times16\cdot846}{374\cdot86}$$

$$=43\cdot21.$$

At *C*: $$f=\frac{M_{XX}y}{I_{XX}}=\frac{961\cdot49\times13\cdot00}{374\cdot86}$$

$$=33\cdot34.$$

Final Bending Moments

$$M = M_s - M_i.$$

Hence,

$$M_{BA} = M_{BC} = 37{\cdot}5 - 43{\cdot}21$$
$$= -5{\cdot}71 \text{ tons ft.}$$
$$M_{CB} = M_{CD} = 29{\cdot}17 - 33{\cdot}34$$
$$= -4{\cdot}17 \text{ tons ft.}$$

The final B.M. diagram is shown in Fig. 201.

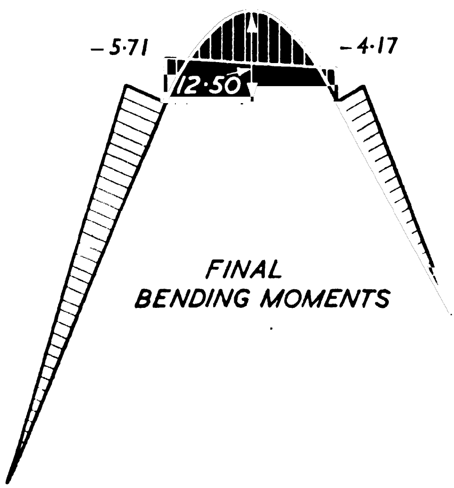


Fig. 201

Closed Frames

Example 7. Although the joints of the rectangular frame shown in Fig. 202 are rigid, the frame is simply supported at S and V, so that under load it may take up the deflected form shown.

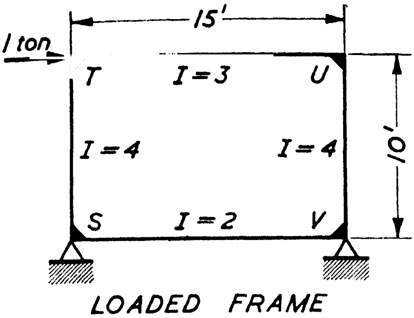
Analogous Column

Now
$$I_{ST} = I_{UV} = 4$$
$$I_{SV} = 2$$
$$I_{TU} = 3.$$

Hence,
$$1/I_{ST} : 1/I_{UV} : 1/I_{SV} : 1/I_{TU} = 1 : 1 : 2 : 4/3.$$

These thicknesses of column section are employed for the analogous column shown in Fig. 203.

The analogous column is symmetrical about the YY axis, but the position of the XX axis must be calculated.



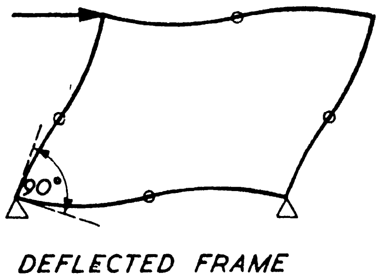


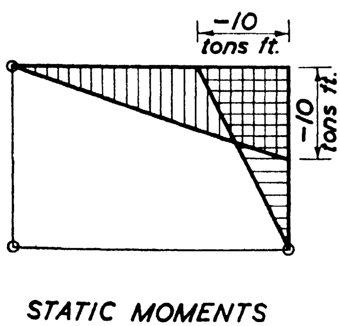
FIG. 202

Using S as origin,

$$\bar{y} = \frac{\Sigma Ay}{\Sigma A}$$

$$= \frac{2(1 \times 10 \times 10/2) + (4/3 \times 15 \times 10) + (2 \times 15 \times 0)}{2(1 \times 10) + (4/3 \times 15) + (2 \times 15)}$$

$$= 4{\cdot}286 \text{ ft.}$$



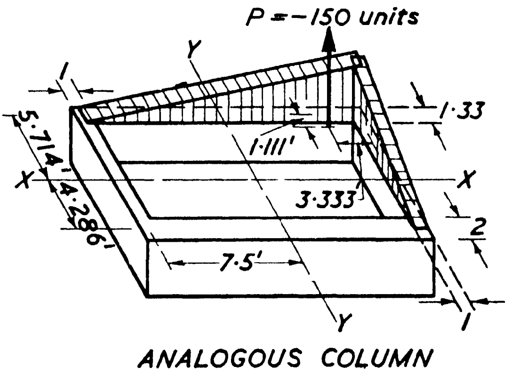


FIG. 203

Analogous Load P and Moments M_{XX} and M_{YY}

The frame may be made statically determinate by inserting hinges at S, T and V, when the appropriate static moments are those shown in Fig. 203.

The centroid of the analogous load P lies inside the frame.

Using U as origin,

$$\bar{x}=\frac{\Sigma Ax}{\Sigma A}$$

$$=\frac{\left(\frac{4}{3}\times\frac{15\times 10}{2}\times\frac{15}{3}\right)+\left(1\times\frac{10\times 10}{2}\times 0\right)}{\left(\frac{4}{3}\times\frac{15\times 10}{2}\right)+\left(1\times\frac{10\times 10}{2}\right)}$$

$$=3{\cdot}333 \text{ ft.}$$

$$\bar{y}=\frac{\Sigma Ay}{\Sigma A}$$

$$=\frac{\left(\frac{4}{3}\times\frac{15\times 10}{2}\times 0\right)+\left(1\times\frac{10\times 10}{2}\times\frac{10}{3}\right)}{\left(\frac{4}{3}\times\frac{15\times 10}{2}\right)+\left(1\times\frac{10\times 10}{2}\right)}$$

$$=1{\cdot}111 \text{ ft.}$$

Now

$$P=-\left(\frac{4}{3}\times\frac{15\times 10}{2}\right)-\left(1\times\frac{10\times 10}{2}\right)$$

$$=-150 \text{ units.}$$

Hence,

$$M_{XX}=P\,.\,e_x$$

$$=-150(10-1{\cdot}111-4{\cdot}286)$$

$$=-690{\cdot}4 \text{ units.}$$

$$M_{YY}=P\,.\,e_y$$

$$=-150(7{\cdot}5-3{\cdot}33)$$

$$=-625 \text{ units.}$$

Analogous Moments of Inertia I_{XX} and I_{YY}

$$I_{XX}=2\left(\frac{1\times 10^3}{12}+1\times 10\times 0{\cdot}714^2\right)+(2\times 15\times 4{\cdot}286^2)+(1{\cdot}33\times 15\times 5{\cdot}714^2)$$

$$=1380{\cdot}9 \text{ units.}$$

$$I_{YY}=2(10\times 1\times 7{\cdot}5^2)+(1{\cdot}33+2)(15^3/12)$$

$$=2062{\cdot}5 \text{ units.}$$

Analogous Stresses

The analogous stresses are obtained by employing the standard formula for symmetrical frames, viz.

$$f=\frac{P}{A}\pm\frac{M_{XX}y}{I_{XX}}\pm\frac{M_{YY}x}{I_{YY}}.$$

Joint S:

$$f=-\frac{150}{70}+\frac{690{\cdot}4\times 4{\cdot}286}{1380{\cdot}9}+\frac{625\times 7{\cdot}5}{2062{\cdot}5}$$

$$=-2{\cdot}143+2{\cdot}143+2{\cdot}273$$

$$=2{\cdot}273.$$

Joint T:

$$f = -2{\cdot}143 - \frac{690{\cdot}4 \times 5{\cdot}714}{1380{\cdot}9} + 2{\cdot}273$$
$$= -2{\cdot}143 - 2{\cdot}857 + 2{\cdot}273$$
$$= -2{\cdot}727.$$

Joint U:

$$f = -2{\cdot}143 - 2{\cdot}857 - 2{\cdot}273$$
$$= -7{\cdot}273.$$

Joint V:

$$f = -2{\cdot}143 + 2{\cdot}143 - 2{\cdot}273$$
$$= -2{\cdot}273.$$

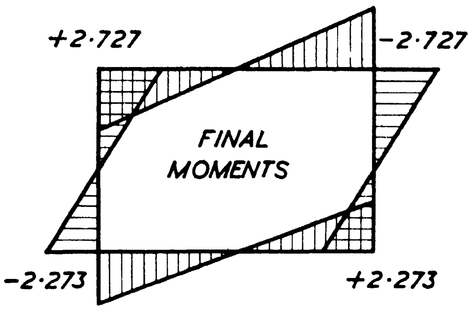


FIG. 204

Final Bending Moments

$$M = M_s - M_i.$$

At U, $M = -10$ tons ft.

At the other joints M_s is zero.

Hence,

$$M_S = 0 - 2{\cdot}273 = -2{\cdot}273 \text{ tons ft.}$$
$$M_T = 0 - (-2{\cdot}727) = +2{\cdot}727 \text{ tons ft.}$$
$$M_U = -10 - (-7{\cdot}273) = -2{\cdot}727 \text{ tons ft.}$$
$$M_V = 0 - (-2{\cdot}273) = +2{\cdot}273 \text{ tons ft.}$$

The final B.M. diagram is shown in Fig. 204.

Example 8. The rigid frame shown in Fig. 205, together with its analogous column, is typical of those erected for industrial buildings, there being a monitor roof and columns designed to accommodate gantry cranes. The frame will be analysed for the uniform side loading of 0·1 ton per ft. and for the gantry moment of 10 tons ft., both of which tend to make the frame sway to the right.

The frame may be rendered statically determinate in several ways, hinges being introduced to make the frame act like a mechanism. If hinges are assumed at F, G and J, then the member AF acts as a cantilever fixed in posi-

tion and direction at A, but under no other restraint. The resultant static moments shown in Fig. 206 consist of the moments due to the side loading

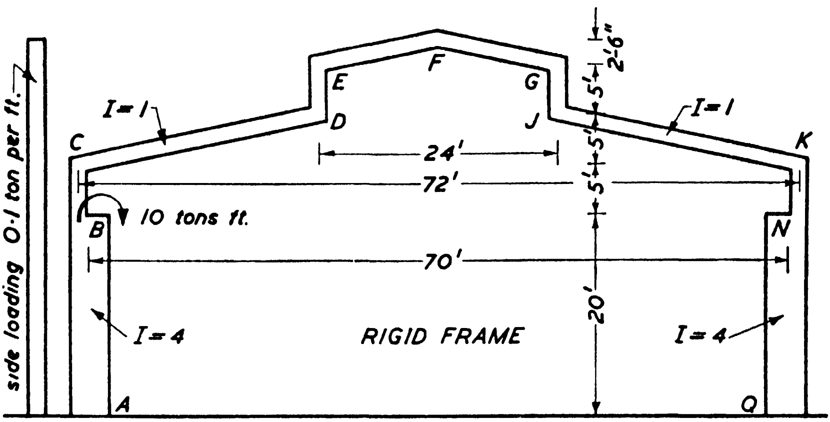


FIG. 205

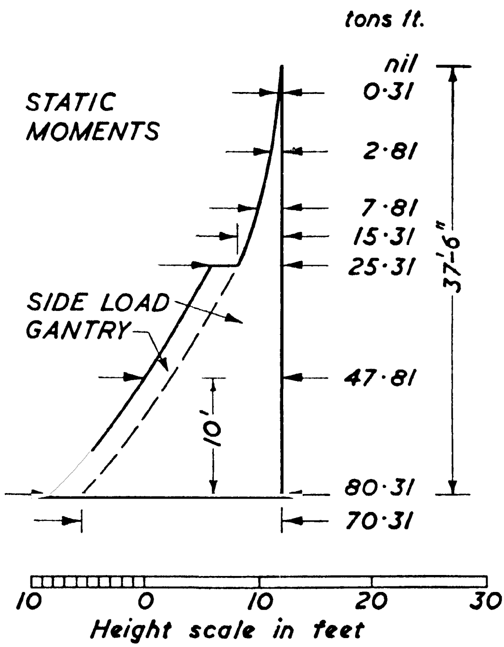


FIG. 206

which vary parabolically from zero at F to a maximum of 70·31 tons ft. at A, together with a uniform moment of 10 tons ft. between B and A. These moments provide the analogous load P.

The properties of the analogous column are reasonably easy to calculate, being shown in Figs. 207 and 209.

As the frame is symmetrical about the *YY* axis, calculations are made only for one-half of the frame and then doubled to embrace the whole frame. Similarly the base *AQ* is used initially as horizontal axis for simplicity of calculation. Later all calculations are corrected to the *XX* axis.

Although unnecessary in the previous examples, a strict sign convention is essential in this example.

Analogous Load

The principal problem in the analysis of the frame is the computation of the value and composition of the analogous load. The static moment diagram has to be projected on to the left-hand side of the frame, and a number of calculations made for each of the five different members between *A* and *F*.

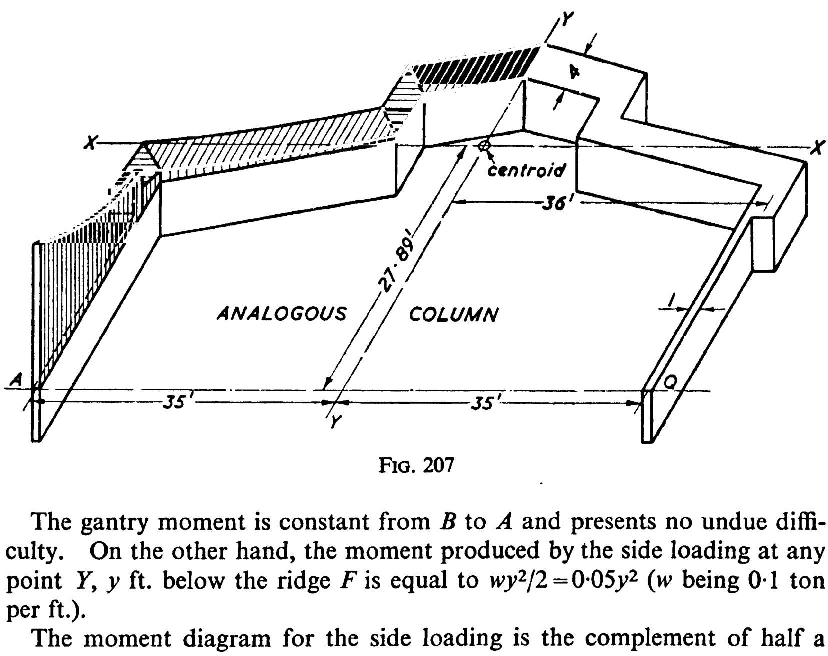


FIG. 207

The gantry moment is constant from *B* to *A* and presents no undue difficulty. On the other hand, the moment produced by the side loading at any point *Y*, *y* ft. below the ridge *F* is equal to $wy^2/2 = 0{\cdot}05y^2$ (*w* being 0·1 ton per ft.).

The moment diagram for the side loading is the complement of half a second-degree parabola. Consequently, the area between the ridge and *Y* is $wy^2/2 \times y/3 = 0{\cdot}0167y^3$. Also, the centroid of the diagram occurs at a distance of $0{\cdot}75y$ below the ridge *F*.

Using the same reasoning and similar terms, the distance from *F* to the centroid of an area between points *Y* and *Z* may be found from the formula

$$\bar{z} = \frac{(0{\cdot}0167z^3 \times 0{\cdot}75z) - (0{\cdot}0167y^3 \times 0{\cdot}75y)}{0{\cdot}0167(z^3 - y^3)}.$$

Employing the foregoing formulæ and including the gantry moment a table may be prepared as shown in Fig. 208 giving the areas of the sections

of the M_s diagram, together with the distances of their centroids from the ridge and from the base AQ.

Section	*Area of static moment diagram*	*Value of* $\bar{z}$	$37\cdot5-\bar{z}$
F to E	0·26	1·88	35·62
E to D	(7·03 − 0·26) = 6·77	5·75	31·75
D to C	(32·55 − 7·03) = 25·52	10·41	27·09
C to B	(89·32 − 32·55) = 56·77	15·27	22·23
B to A	(878·91 + 200·00 − 89·32) = 989·59	29·34	8·16

FIG. 208

As an example, the value of $\bar{z}$ for the section between B and A

$$=\frac{(878\cdot91\times0\cdot75\times37\cdot5)+(200\times27\cdot5)-(89\cdot32\times0\cdot75\times17\cdot5)}{(878\cdot91+200-89\cdot32)}$$

$$=29\cdot34.$$

It will be appreciated that these calculations have been necessary because AF is not one single member of constant I.

Having found the areas of the sections and the positions of their centroids, the members FE and DC require further attention as they are not vertical. Consider DC, for example.

The vertical height is 5 ft., but the length is 24·51 ft. Hence the value of the static moment diagram projected on to DC is $(32\cdot55-7\cdot03)\times24\cdot51/5$ $=125\cdot10$ units. The centroid occurs at a distance 10·41 ft. vertically below F or 2·91 ft. below D. Hence, the horizontal distance from the YY axis $=-(12+2\cdot91/5\times24)=-25\cdot97$ ft.

The data required for the analogous loads and moments are shown in Fig. 209.

Analogous Stresses

The analogous stresses are computed from the formula

$$f=\frac{P}{A}\pm y\,.\,\frac{M_{XX}}{I_{XX}}\pm x\,.\,\frac{M_{YY}}{I_{YY}}$$

At A:

$$f=-\frac{437\cdot31}{103\cdot52}-\left(27\cdot89\times\frac{5266}{5830}\right)-\left(35\times\frac{14{,}044}{58{,}414}\right)$$

$$=-4\cdot22-(27\cdot89\times0\cdot9033)-(35\times0\cdot2404)$$

$$=-37\cdot83.$$

At B:

$$f=-4\cdot22-(7\cdot89\times0\cdot9033)-(35\times0\cdot2404)$$

$$=-19\cdot77.$$

Immediately above B:

$$f=-4\cdot22-(7\cdot89\times0\cdot9033)-(36\times0\cdot2404)$$

$$=-20\cdot01.$$

Member	Length L in feet	Value $1/EI$	$A\ (=L/EI)$	Horizontal distance x from YY axis	Vertical distance y from base AQ	Ax^2	Ay	Ay^2	I_{xx}	I_{AQ} $(=Ay^2+I_{xx})$	I_{yy}	I_{YY} $(=Ax^2+I_{yy})$	Analogous Load P $(=1/EI\times M_s)$	Eccentricity e_x	$M_{AQ}(=P\,.\,e_x)$	Eccentricity e_y	$M_{YY}(=P\,.\,e_y)$
AB	20	0·25	5·0	−35	10·0	6125	50	500	167	667	0	6125	−247·40	8·16	−2019	−35	+8659
BC	5	1·0	5·0	−36	22·5	6480	113	2531	10	2541	0	6480	−56·77	22·23	−1262	−36	+2044
CD	24·51	1·0	24·51	−24	27·5	14,118	674	18,535	51	18,586	1176	15,294	−125·10	27·09	−3389	−25·97	+3249
DE	5	1·0	5·0	−12	32·5	720	163	5281	10	5291	0	720	−6·77	31·75	−215	−12	+81
EF	12·25	1·0	12·25	−6	36·25	441	444	16,097	6	16,103	147	588	−1·27	35·62	−45	−9	+11
Total for half the frame			51·76	—	—	—	1444	—	—	43,188	—	29,207	−437·31	—	−6930	—	+14,044
Total for the whole frame			103·52	—	—	—	2888	—	—	86,376	—	58,414	−(−437·31 × 27·89) =		+12,196		
Correcting to *XX* axis					27·89	$\left(=\frac{2888}{103\cdot52}\right)$			I_{XX} =	5830	$(=86{,}376-103\cdot52\times27\cdot89^2)$			M_{XX} =	+5266		

FIG. 209

At C:

$$f = -4{\cdot}22 - (2{\cdot}89 \times 0{\cdot}9033) - (36 \times 0{\cdot}2404)$$
$$= -15{\cdot}50.$$

At D:

$$f = -4{\cdot}22 + (2{\cdot}11 \times 0{\cdot}9033) - (12 \times 0{\cdot}2404)$$
$$= -5{\cdot}19.$$

At E:

$$f = -4{\cdot}22 + (7{\cdot}11 \times 0{\cdot}9033) - (12 \times 0{\cdot}2404)$$
$$= -0{\cdot}67.$$

At F:

$$f = -4{\cdot}22 + 9{\cdot}61(0{\cdot}9033) + 0$$
$$= +4{\cdot}46.$$

Similarly:

at G, $f = +5{\cdot}09$

J, $f = +0{\cdot}57$

K, $f = +1{\cdot}80$.

Immediately above N, $f = -2{\cdot}71$

at N, $f = -2{\cdot}94$

at Q, $f = -21{\cdot}00$.

The final moments are found by deducting the indeterminate moments from the static moments as shown in Fig. 210.

Joint	M_s	M_i	*Final M* $(=M_s - M_i)$
A	−80·31	−37·83	−42·48
B	−25·31	−19·77	− 5·54
Immediately above *B*	−15·31	−20·01	+ 4·70
C	− 7·81	−15·50	+ 7·69
D	− 2·81	− 5·19	+ 2·38
E	− 0·31	− 0·67	+ 0·36
F	0	+ 4·46	− 4·46
G	0	+ 5·09	− 5·09
J	0	+ 0·57	− 0·57
K	0	+ 1·80	− 1·80
Immediately above *N*	0	− 2·71	+ 2·71
N	0	− 2·94	+ 2·94
Q	0	−21·00	+21·00

FIG. 210

The final B.M. diagram is shown in Fig. 211.

The Treatment of Non-prismatic Members

The moment of inertia of a steel member usually changes in two ways: (a) When plates are added to the flanges of a prismatic member, such as an R.S.J., where the change is abrupt; and (b) by a uniform change in the depth of the web of a member, the sizes of the flanges remaining constant, where there is a parabolic variation of I. Both of these types of variation in I may

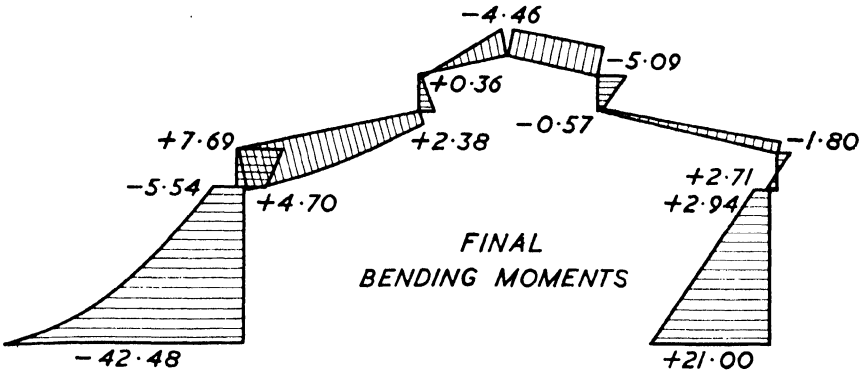


FIG. 211

be treated without undue difficulty and, indeed, the first occurred in the columns of the building just examined in Example 8. When there is any non-linear variation in I, for example as shown in Fig. 212 where one flange of a member is straight and the other curved, the member can be treated by dividing it into a number of short sections over which the value of I can be assumed to be constant and the analogous column plotted accordingly.

It may happen that the variation in I and the value of the static moment may be related by an algebraic expression when the calculations will lead to an exact answer, but usually the method shown in Fig. 212 will give reasonably reliable results.

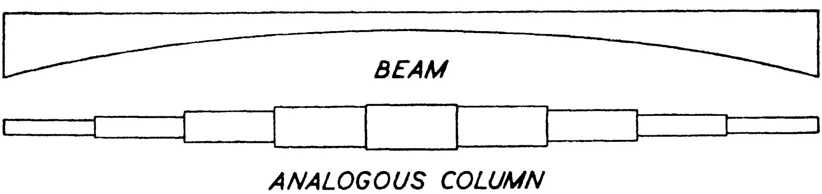


FIG. 212

Example 9. It is proposed to calculate the fixing moments at A and D for the built-in compound beam shown in Fig. 213 which carries a U.D.L. of 40 tons.

The analogous load is the M/EI diagram for the girder shown in the Figure. Although the various widths of the analogous column have been shown in terms of $1/EI$, the values for EI can be ignored, as in Example 1, and the width for the middle half assumed to be unity.

Considering AD as being simply supported, the reaction at either end is 20 tons. Therefore the equation for the static B.M. at any point distant x ft. from A (or D)

$$=20x-\frac{2x^2}{2}=20x-x^2.$$

The analogous load may be divided into three parts corresponding to AB, BC and CD.

Then the value of the load on AB,

$$P_{AB} = \frac{2}{3}\int_0^5 (20x - x^2)\, dx$$

$$= \frac{2}{3}\left[\frac{20x^2}{2} - \frac{x^3}{3}\right]_0^5 = 138{\cdot}89 \text{ units.}$$

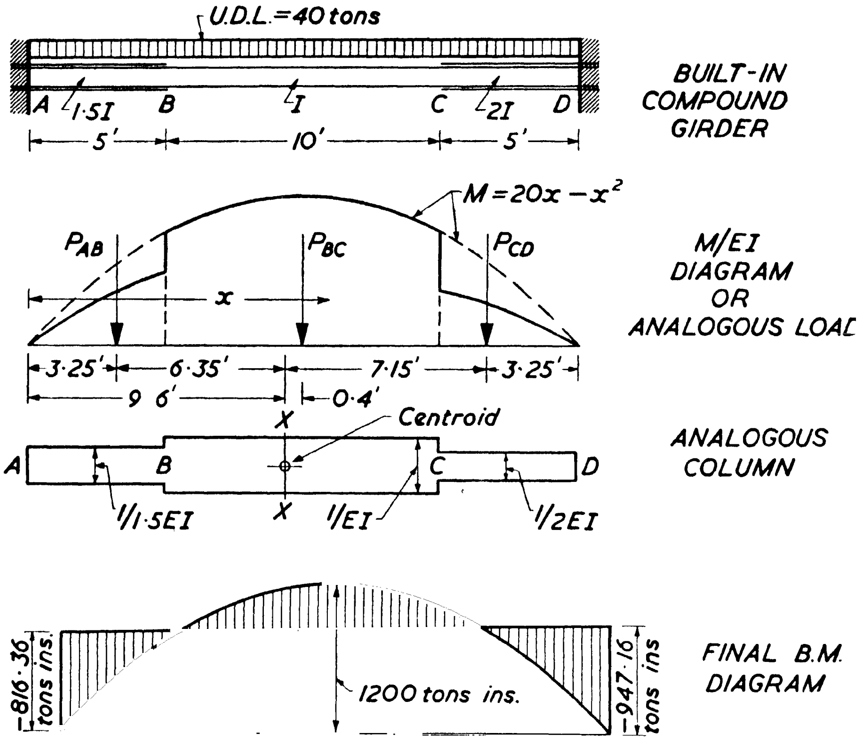


FIG. 213

The moment of the load about A

$$= \frac{2}{3}\int_0^5 (20x - x^2)x\, .\, dx$$

$$= \frac{2}{3}\int_0^5 (20x^2 - x^3)\, dx = \frac{2}{3}\left[\frac{20x^3}{3} - \frac{x^4}{4}\right]_0^5$$

$$= 451{\cdot}39 \text{ units.}$$

Then the distance from A to the C.G. of this load,

$$\bar{x} = \frac{\Sigma Ax}{\Sigma A} = \frac{451{\cdot}39}{138{\cdot}89} = 3{\cdot}25 \text{ ft.}$$

The value of the load P_{CD}

$$=P_{AB}\times\frac{\frac{1}{2}}{\frac{2}{3}}=138{\cdot}89\times\tfrac{3}{4}=104{\cdot}17 \text{ units.}$$

The distance from D to its C.G. is also 3·25 ft.
The value of the load P_{BC}

$$=\int_5^{15}(20x-x^2)\,dx=916{\cdot}67 \text{ units.}$$

As the load is symmetrical about AD, its C.G. lies in the middle of the column.

The total load $=138{\cdot}89+104{\cdot}17+916{\cdot}67=1159{\cdot}73$ units.

Now the centroid of the column section must be found. Taking A as origin,

$$\bar{x}=\frac{\Sigma Ax}{\Sigma A}=\frac{(\frac{2}{3}\times5\times2{\cdot}5)+(1\times10\times10)+(\frac{1}{2}\times5\times17{\cdot}5)}{(\frac{2}{3}\times5)+(1\times10)+(\frac{1}{2}\times5)}$$

$$=\frac{152{\cdot}083}{15{\cdot}83}=9{\cdot}6 \text{ ft.}$$

Then the moment of the loads about the centroid,

$$M_{XX}=-(138{\cdot}89\times6{\cdot}35)+(916{\cdot}67\times0{\cdot}4)+(104{\cdot}17\times7{\cdot}15)$$

$$=+229{\cdot}53 \text{ units.}$$

The value of I_{XX} for the column section

$$=\left[\frac{\frac{2}{3}\times5^3}{12}\right]+\left[\tfrac{2}{3}\times5\times7{\cdot}10^2\right]+\left[\frac{1\times10^3}{12}\right]+(10\times0{\cdot}4^2)+\left[\frac{\frac{1}{2}\times5^3}{12}\right]+(\tfrac{1}{2}\times5\times7{\cdot}9^2)$$

$=421{\cdot}14$ units.

With this information, the analogous stresses may be calculated.

$$f=\frac{P}{A}\pm\frac{M_{XX}\,.\,y}{I_{XX}}.$$

Considering point A,

$$f=\frac{1159{\cdot}73}{15{\cdot}83}-\frac{229{\cdot}53\times9{\cdot}6}{421{\cdot}14}$$

$$=73{\cdot}26-5{\cdot}23$$

$$=68{\cdot}03 \text{ units.}$$

Considering point D,

$$f=\frac{P}{A}+\frac{M_{XX}\,.\,y}{I_{XX}}$$

$$=\frac{1159{\cdot}73}{15{\cdot}83}+\frac{229{\cdot}53\times10{\cdot}4}{421{\cdot}14}$$

$$=73{\cdot}26+5{\cdot}67$$

$$=78{\cdot}93 \text{ units.}$$

At A and D, $M_s = 0$. Therefore,

$$M_A = -68{\cdot}03 \text{ tons ft.}$$
$$= -816{\cdot}36 \text{ tons ins.}$$
$$M_D = -78{\cdot}93 \text{ tons ft.}$$
$$= -947{\cdot}16 \text{ tons ins.}$$

Stiffness and Carry-over Factors

The stiffness and carry-over factors for beams of prismatic section were derived in the Principles of Moment Distribution. Considering Fig. 214 (a) the stiffness factor at A is defined as the moment to be applied in order to rotate the tangent at A by one radian. The carry-over factor is the ratio of the F.E.M. at B to the moment applied at A.

The stiffness factors for any beam, of prismatic or varying section, may be found by Column Analogy. Considering Fig. 214 (b), the stiffness factor for A can be calculated by applying an analogous load of 1 radian $= \theta_A$ to the analogous column for AB

The area A of the analogous column $= L/EI$.

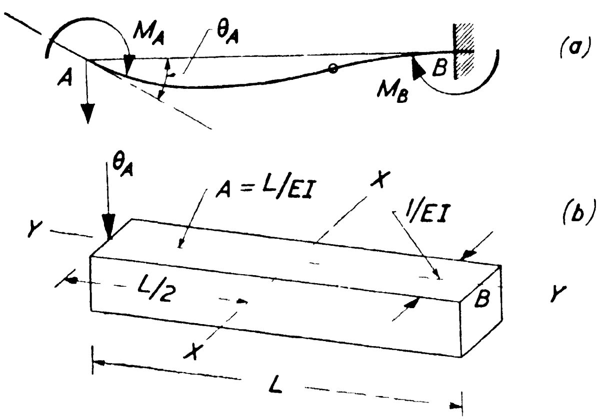


FIG 214

The moment of the load about the axis XX

$$= \theta_A \times \frac{L}{2} = \frac{\theta_A L}{2}.$$

The value of I_{XX} for the column

$$= \frac{bd^3}{12} = \frac{1}{EI} \times \frac{L^3}{12} = \frac{L^3}{12EI}.$$

Now
$$f = M_A = \frac{P}{A} + \frac{M_{XX} \cdot y}{I_{XX}}$$
$$= \frac{\theta_A}{L/EI} + \left[\frac{\theta_A L}{2} \times \frac{L}{2} \times \frac{12EI}{L^3}\right]$$
$$= \frac{4EI\theta_A}{L}.$$

which is the stiffness factor for A. (See derivation of Principles I and II in Moment Distribution.)

Similarly,

$$M_B = \frac{P}{A} - \frac{M_{XX} \cdot y}{I_{XX}} = \frac{-2EI\theta_A}{L}.$$

The carry-over factor

$$\frac{M_B}{M_A} = \frac{-2EI\theta_A}{L} \times \frac{L}{4EI\theta_A} = -\tfrac{1}{2}.$$

Using the Moment Distribution sign convention, the C.O. factor would be $+\tfrac{1}{2}$. (See Principle I.)

If B is simply supported, there is an infinite area in the analogous column, with its centroid at B, and P/A has zero value. (Cf. Example 2.)

Then the moment of the load about the XX axis,

$$M_{XX} = \theta_A \times L = \theta_A L \text{ units},$$

$$I_{XX} = \frac{bd^3}{3} = \frac{1 \times L^3}{EI \times 3} = \frac{L^3}{3EI} \text{ units}$$

and

$$f = \frac{M_{XX} \cdot y}{I_{XX}} = \frac{\theta_A L \times L \times 3EI}{L^3} = \frac{3EI\theta_A}{L}.$$

Thus demonstrating Principle III in Moment Distribution.

The same reasoning may be applied to the beam in Example 9.

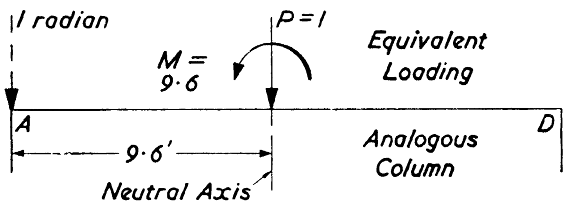


FIG. 215

Example 10. Calculate the stiffness and C.O. factors for the beam in Fig. 213.

Consider the end A first. Apply an analogous load at A of 1 radian as shown in Fig. 215 and note that calculations involving the width of the analogous column must be in terms of $1/EI$.

Then the area A of the column $= \dfrac{15{\cdot}83}{EI}$

the value of I_{XX} for the column $= \dfrac{421{\cdot}14}{EI}$

while the load $P = 1$ unit

and the moment $M_{XX} = 1 \times 9{\cdot}6 = 9{\cdot}6$ units.

Multiplying numerator and denominator by L,

$$f = M_A = \left[\frac{P}{A} + \frac{M_{XX} \,.\, y}{I_{XX}}\right]\frac{L}{L}$$

$$= \left[\frac{1 \times EI}{15{\cdot}83} + \frac{9{\cdot}6 \times 9{\cdot}6 \times EI}{421{\cdot}14}\right]\frac{20}{L}$$

$$= \frac{5{\cdot}640EI}{L}.$$

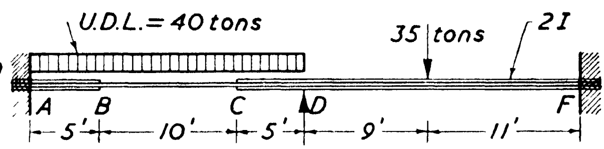
Similarly,

$$f = M_D = \left[\frac{P}{A} - \frac{M_{XX} \,.\, y}{I_{XX}}\right]\frac{20}{L}$$

$$= \left[\frac{1 \times EI}{15{\cdot}83} - \frac{9{\cdot}6 \times 10{\cdot}4 \times EI}{421{\cdot}14}\right]\frac{20}{L}$$

$$= -\frac{3{\cdot}478EI}{L}.$$

Moment Distribution for Non-Prismatic Continuous Beam



LOAD DIAGRAM

	A		D	F
Carry-Over Factors	→ 0	0·544 ←	→ 0·5	0 ←
Distribution Factors	0	0·444	0·556	0
Fixed End Moments	−816·36	+947·16	−1143·45	+935·55
Distribution		+87·15	+109·14	
Carry-Over	+47·41			+54·57
Final Moments	−768·95	+1034·31	−1034·31	+990·12

FINAL B.M. DIAGRAM

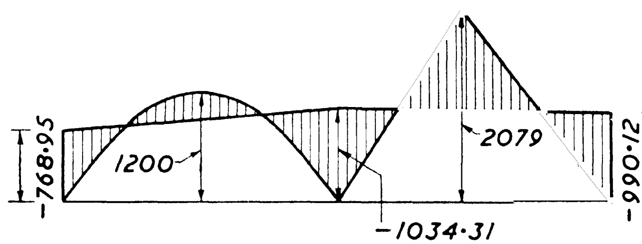


FIG. 216

Hence, the stiffness factor for $A = 5{\cdot}640EI/L$.

While the C.O. factor from A to $D = 3{\cdot}478/5{\cdot}640 = 0{\cdot}616$.

In a similar manner, by applying an analogous load of 1 radian at D, it can be shown that the stiffness factor for D is $6{\cdot}399EI/L$ and the C.O. factor from D to A is 0·544.

Having obtained the various properties of the beam it is possible to use them in a problem in Moment Distribution.

Example 11. Calculate the moments at A, D and F in the continuous beam shown in Fig. 216, where AD has the same cross-section as in Figs. 213 and 215 and where DF has a constant moment of inertia of $2I$.

The C.O. factor from D to A is 0·544, as stated in the previous Example, while that from D to F is 0·5, since DF is a prismatic beam.

The stiffness factor for the end D of the span DA is $6{\cdot}399EI/L$.

As DF is a prismatic beam and F is built-in, the appropriate factor for D is $4E(2I)/L=8EI/L$.

Hence, the distribution factor for DA

$$=\frac{6{\cdot}399}{6{\cdot}399+8{\cdot}00}=0{\cdot}444$$

and the distribution factor for DF

$$=1-0{\cdot}444=0{\cdot}556.$$

The distribution table is shown in Fig. 216.

(c) The Slope Deflection Method of Analysis

This method was made widely known by Maney and Wilson of Minnesota University in 1915, although it was based on the work of Mohr.

Joint rotations and deflections are treated as the unknown quantities and, once these have been evaluated, the moments follow automatically by substituting the values in standard equations.

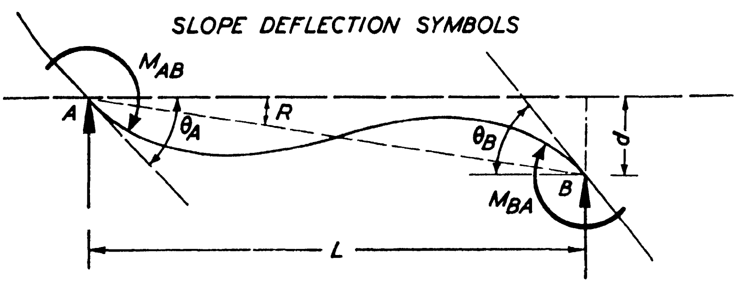


FIG. 217

Suppose that the member AB in Fig. 217 is one unloaded span of a continuous beam and that the member is of constant moment of inertia. Then, for the conditions shown,

$$M_{AB}=2EK(2\theta_A+\theta_B-3R)$$

$$M_{BA}=2EK(2\theta_B+\theta_A-3R),$$

where E and K have the normal significance,

θ_A and θ_B are the angles the joints make with the horizontal

and R is the angle of rotation of B with respect to A when B sinks an amount d (i.e. $R=d/L$).

With regard to sign convention,

θ is positive when the tangent to the beam rotates in a clockwise direction,

R is positive when the beam rotates in a clockwise direction

and M is positive when the moment acts in a clockwise direction on the beam.

Therefore, the various values of M, θ and R in Fig. 217 are all positive.

Suppose that the span AB carries a load acting downwards in the normal fashion. Then,

$$M_{AB} = 2EK(2\theta_A + \theta_B - 3R) - \text{F.E.M.}_{AB}$$

$$M_{BA} = 2EK(2\theta_B + \theta_A - 3R) + \text{F.E.M.}_{BA}.$$

F.E.M._{AB} and F.E.M._{BA} are the fixed-end moments which would exist if AB were a fixed-end beam. The values and signs used are precisely the same as in the Moment Distribution Method and the tables given on pages 216 to 220 are equally of use for the Slope Deflection Method.

When the end A of a beam AB is hinged, the formula for the moment at the other end is modified as follows:

$$M_{BA} = EK(3\theta_B - 3R) \quad \text{(unloaded condition)}$$

or $$M_{BA} = EK(3\theta_B - 3R) + \text{F.E.M.}_{BA} \quad \text{(loaded condition)}$$

An analogy for this modification exists in Moment Distribution where the stiffness factor for a beam hinged at one end is reduced to $\frac{3}{4}K$. It should be noted that the value of the F.E.M. is that applicable to beams hinged at one end and fixed in direction and position at the other.

The standard formulæ will be applied to some of the examples which appeared in the section on Moment Distribution. The reader should examine especially the signs which are given to the rotation R.

As in most other methods of analysis the value of the modulus of elasticity E can be ignored in nearly every example.

When calculating the values of F.E.M.s for loads acting downwards in the normal fashion the appropriate signs can be ignored because the fundamental formulæ automatically provide the correct signs.

The final B.M. diagram is prepared by considering all hogging moments as negative and all sagging moments as positive.

When the method of Slope Deflection is used to find the moments in a continuous beam, the slope of the beam over each internal support is calculated. The values of the slopes may be useful in calculating the deflections in interior spans, but great care is needed with signs. In the slope-deflection calculations a positive value for the slope means that the beam has rotated in a clockwise direction. In the purely mathematical sense a positive slope is one 'going upwards to the right', i.e. dy/dx is positive, as shown in Fig. 20. Furthermore, it is essential that the units employed throughout the Slope Deflection calculations should be the same as those for E and I. Otherwise, the values of the slopes will not be related to the units employed in the deflection calculations.

Continuous Beams

Example 1. Consider the continuous beam in Fig. 218. Under the action of the loads, the joint B rotates in an anti-clockwise direction. When this rotation θ_B is calculated the whole beam can be analysed.

Let the suffixes 1 and 2 be applied to AB and BC respectively.

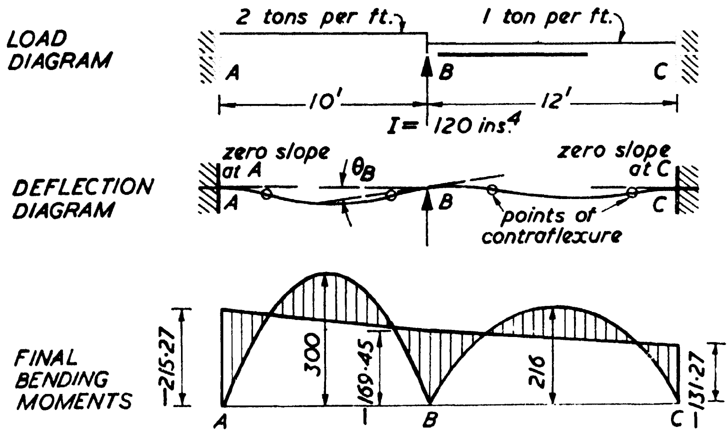


FIG. 218

A and C are fixed in direction as well as position. Hence,

$$\theta_A = 0 = \theta_C.$$

A, B and C are on the same level. Therefore,

$$R_1 = 0 = R_2.$$

$$\text{F.E.M.}_{AB} = \text{F.E.M.}_{BA} = \frac{WL}{12} = 200 \text{ tons in.}$$

$$\text{F.E.M.}_{BC} = \text{F.E.M.}_{CB} = \frac{WL}{12} = 144 \text{ tons in.}$$

Now $$M_{BA} + M_{BC} = 0.$$

Hence, $2EK_1(2\theta_B + \theta_A - 3R_1) + 200 + 2EK_2(2\theta_B + \theta_C - 3R_2) - 144 = 0.$

But $$\theta_A = 0 = \theta_C,$$

$$R_1 = 0 = R_2,$$

$$K_1 = \frac{I}{L} = \frac{120}{120}$$

and $$K_2 = \frac{I}{L} = \frac{120}{144}.$$

Therefore,

$$\left(2E \times \frac{120}{120} \times 2\theta_B\right) + \left(2E \times \frac{120}{144} \times 2\theta_B\right) = 144 - 200$$

$$E\theta_B = -7{\cdot}6364.$$

Using the basic formulæ,

$$M_{AB} = 2EK_1(\theta_B) - 200$$
$$= (-2 \times 1 \times 7{\cdot}6364) - 200 = -215{\cdot}27 \text{ tons in.}$$
$$M_{BA} = -M_{BC}$$
$$= 2EK_1(2\theta_B) + 200$$
$$= (-2 \times 1 \times 2 \times 7{\cdot}6364) + 200 = 169{\cdot}45 \text{ tons in.}$$
$$M_{CB} = 2EK_2(\theta_B) + 144$$
$$= (-2 \times 0{\cdot}833 \times 7{\cdot}6364) + 144 = 131{\cdot}27 \text{ tons in.}$$

Example 2. The continuous beam *ABCDE* in Fig. 219 involves the treatment of simply supported end spans and a cantilever.

Let the suffixes 1, 2 and 3 apply to *AB*, *BC* and *CD* respectively.

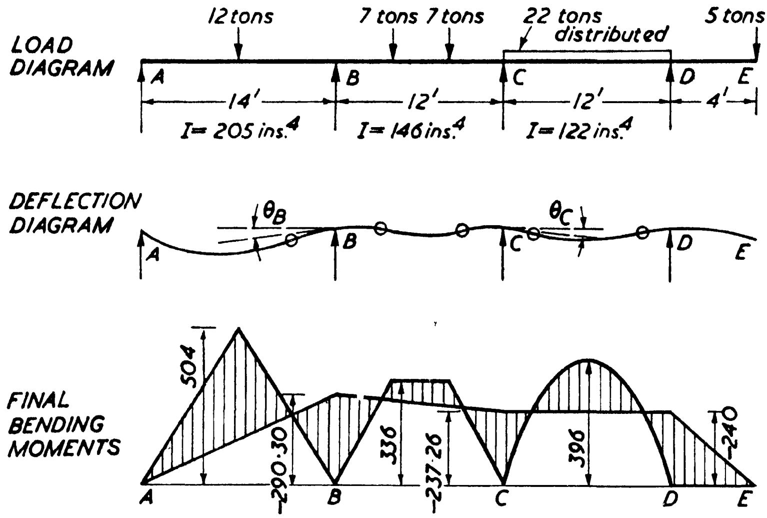


FIG. 219

Now the supports *A*, *B*, *C* and *D* are on the same level. Hence,

$$R_1 = 0 = R_2 = R_3.$$

Also,
$$M_{BA} + M_{BC} = 0$$
$$M_{CB} + M_{CD} = 0$$
$$M_{DC} = 240 \text{ tons in.}$$

The effect at *C* of the cantilever *DE* is that F.E.M.$_{CD}$ is reduced in value by half the amount of M_{DC}, i.e. by 120 tons in. (cf. Principle I in Moment Distribution).

$$EK_1(3\theta_B - 3R_1) + \text{F.E.M.}_{BA} + 2EK_2(2\theta_B + \theta_C - 3R_2) - \text{F.E.M.}_{BC} = 0$$
$$2EK_2(2\theta_C + \theta_B - 3R_2) + \text{F.E.M.}_{CB} + EK_3(3\theta_C - 3R_3) - \text{F.E.M.}_{CD} + M_{DC}/2 = 0.$$

Hence,
$$E \times 1{\cdot}22(3\theta_B) + 378 + 2E \times 1{\cdot}014(2\theta_B + \theta_C) - 224 = 0$$
$$7{\cdot}716E\theta_B + 2{\cdot}028E\theta_C + 154 = 0 \quad . \; . \; . \; . \; . \quad \text{(i)}$$

$$2E \times 1{\cdot}014(2\theta_C + \theta_B) + 224 + E \times 0{\cdot}847(3\theta_C) - 396 + 120 = 0$$

$$2{\cdot}028E\theta_B + 6{\cdot}597E\theta_C - 52 = 0.$$

Or $$7{\cdot}716E\theta_B + 25{\cdot}10E\theta_C - 197{\cdot}8 = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(ii)}$$

Subtracting (i) from (ii),

$$23{\cdot}072E\theta_C = 351{\cdot}8$$

$$E\theta_C = 15{\cdot}25.$$

Substituting in equation (i),

$$E\theta_B = -23{\cdot}96.$$

Employing these values of $E\theta_B$ and $E\theta_C$,

$$\begin{aligned} M_{BA} &= -M_{BC} \\ &= EK_1(3\theta_B) + 378 \\ &= (-1{\cdot}22 \times 3 \times 23{\cdot}96) + 378 \\ &= 290{\cdot}30 \text{ tons in.} \end{aligned}$$

$$\begin{aligned} M_{CB} &= -M_{CD} \\ &= 2EK_2(2\theta_C + \theta_B) + 224 \\ &= 2 \times 1{\cdot}014(30{\cdot}5 - 23{\cdot}96) + 224 \\ &= 237{\cdot}26 \text{ tons in.} \end{aligned}$$

Example 3. Fig. 220 shows a continuous beam ABC, freely supported at A and fixed at C. The B.M. induced by the settlement of 1 in. at B will be calculated.

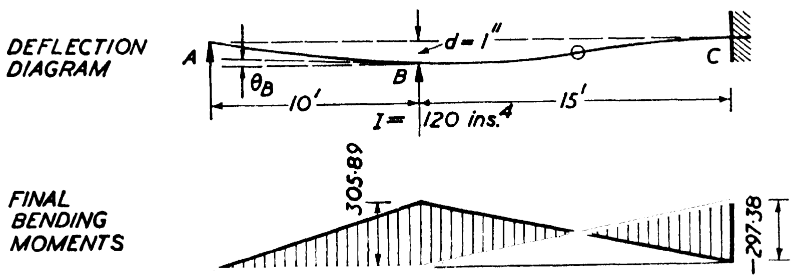


FIG. 220

Assume that $E = 13{,}000$ tons per sq. in.

Let the suffixes 1 and 2 be applied to spans AB and BC respectively.

A being simply supported, $M_{AB} = 0$.

C being fixed in direction, $\theta_C = 0$.

B sinks 1 in. $= d$.

$$R_1 = \frac{d}{L_1} = \frac{1}{120}$$

$$R_2 = -\frac{d}{L_2} = -\frac{1}{180}.$$

Now $$M_{BA} + M_{BC} = 0.$$

Hence, $$EK_1(3\theta_B - 3R_1) + 2EK_2(2\theta_B + \theta_C - 3R_2) = 0$$

$$13{,}000 \times \frac{120}{120}\left(3\theta_B - \frac{3}{120}\right) + \left(2 \times 13{,}000 \times \frac{120}{180}\right)\left(2\theta_B + \frac{3}{180}\right) = 0$$

$$5{\cdot}67\theta_B = \frac{1}{360}$$

$$\theta_B = 0{\cdot}00049.$$

Using this value of θ_B,

$$M_{BA} = EK_1(3\theta_B - 3R_1)$$

$$= 13{,}000 \times \frac{120}{120}\left(0{\cdot}00147 - \frac{3}{120}\right)$$

$$= -305{\cdot}89 \text{ tons in.}$$

Similarly,

$$M_{CB} = 2EK_2(\theta_B - 3R_2)$$

$$= 2 \times 13{,}000 \times \frac{120}{180}\left(0{\cdot}00049 + \frac{3}{180}\right)$$

$$= 297{\cdot}38 \text{ tons in.}$$

Example 4. Fig. 221 shows a beam ABC which is fixed in position and direction at A and C and in which a clockwise moment of 10 tons ft. is

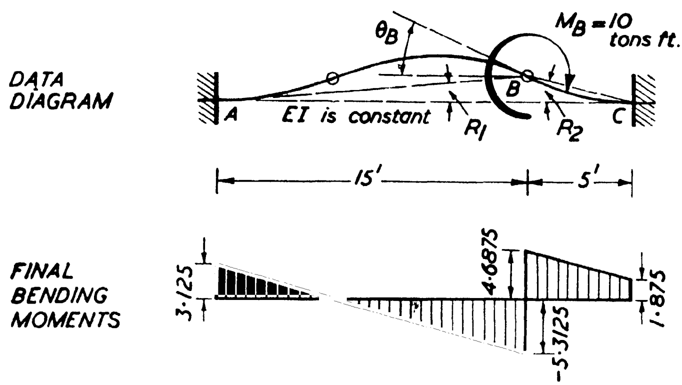


FIG. 221

applied at B. The beam is analysed by finding the rotation and displacement of B.

Now $$\theta_A = \theta_C = 0.$$

Let the suffixes 1 and 2 be applied to AB and BC respectively.

By virtue of the fact that θ_B is positive it is reasonable to assume that B moves upwards and, consequently, that R_1 is negative. If these assumptions are correct the numerical value calculated for R_1 will be positive. Alternatively it could be assumed that R_1 was positive and the sign of the numerical value would provide the correct sign for R_1.

EI being constant, $K_2 = 3K_1$,

also $R_2 = -3R_1$.

Now $M_{BA} + M_{BC} = 120$ tons in. . . . . (i)

The moment at B induces vertical shear forces at A and C and for equilibrium these forces must be equal and opposite.

Let the shear forces be S.

Then
$$M_{AB} + M_{BA} + 15S = 0$$
$$M_{BC} + M_{CB} + 5S = 0,$$
or
$$(M_{AB} + M_{BA}) - 3(M_{BC} + M_{CB}) = 0 \quad \text{. . . . . . (ii)}$$

Equation (i) can be rewritten:
$$2EK_1(2\theta_B + 3R_1) + 2E(3K_1)(2\theta_B - 3 \times 3R_1) = 120$$
$$16EK_1\theta_B - 48EK_1R_1 = 120 \quad \text{. . . . . . . (iii)}$$

Similarly, equation (ii) becomes:
$$2EK_1(\theta_B + 3R_1) + 2EK_1(2\theta_B + 3R_1) - 3\{2E(3K_1)(2\theta_B - 3 \times 3R_1) + 2E(3K_1)(\theta_B - 3 \times 3R_1)\} = 0$$
$$-48EK_1\theta_B + 336EK_1R_1 = 0 \quad \text{. . . . . . . (iv)}$$

Multiplying equation (iii) by 3,
$$48EK_1\theta_B - 144EK_1R_1 = 360 \quad \text{. . . . . (v)}$$

Adding equations (iv) and (v),
$$192EK_1R_1 = 360$$
$$EK_1R_1 = 15/8.$$

Substituting in equation (iii),
$$16EK_1\theta_B - \frac{48 \times 15}{8} = 120$$
$$EK_1\theta_B = 105/8.$$

Using these values of $EK_1\theta_B$ and EK_1R_1 in the fundamental equations,
$$M_{AB} = 2EK_1(\theta_B + 3R_1)$$
$$= \frac{210 + 90}{8}$$
$$= 37{\cdot}5 \text{ tons in.} = 3{\cdot}125 \text{ tons ft.}$$
$$M_{BA} = 2EK_1(2\theta_B + 3R_1)$$
$$= \frac{420 + 90}{8}$$
$$= 63{\cdot}75 \text{ tons in.} = 5{\cdot}3125 \text{ tons ft.}$$
$$M_{BC} = 2E(3K_1)(2\theta_B - 3 \times 3R_1)$$
$$= \frac{1260 - 810}{8}$$
$$= 56{\cdot}25 \text{ tons in.} = 4{\cdot}6875 \text{ tons ft.}$$

(Note $M_{BA} + M_{BC} = 10$ tons ft.)

$$M_{CB} = 2E(3K_1)(\theta_B - 3 \times 3R_1)$$
$$= \frac{630 - 810}{8}$$
$$= -22{\cdot}5 \text{ tons in.} = 1{\cdot}875 \text{ tons ft.}$$

The final B.M.s are shown in Fig. 221.

It will be appreciated that this problem is similar to that for a column provided with a bracket load.

Symmetrical Portal Frames

Example 5. Rigid frames which are symmetrical in shape and symmetrically loaded are easy to analyse by the Slope Deflection Method. The portal frame in Fig. 222 will be analysed as an example.

$$\text{F.E.M.}_{BC} = \text{F.E.M.}_{CB} = \frac{10 \times 24 \times 12}{12}$$
$$= 240 \text{ tons in.}$$

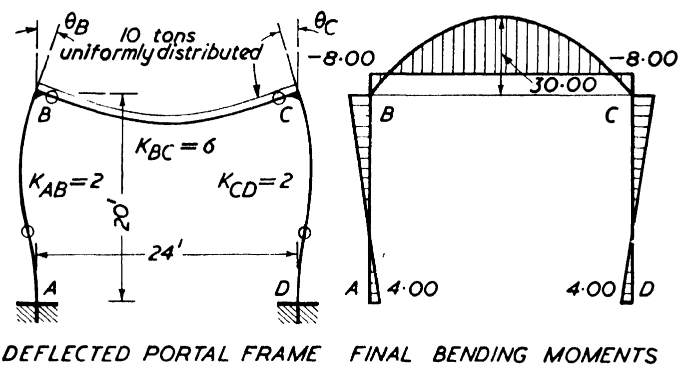


FIG. 222

Let the suffixes 1, 2 and 3 be applied to the members AB, BC and CD respectively.

Then $$K_1 : K_2 : K_3 = 1 : 3 : 1.$$

Being symmetrical the frame does not sway under load. In addition, it is assumed that the lengths of the members do not change.

Hence, $$R_1 = 0 = R_2 = R_3$$
$$\theta_B = -\theta_C.$$

As A and D are fixed in direction as well as position,

$$\theta_A = 0 = \theta_D.$$

Now $$M_{BA} + M_{BC} = 0,$$

i.e. $$2EK_1(2\theta_B) + 2EK_2(2\theta_B + \theta_C) - \text{F.E.M.}_{BC} = 0$$
$$2EK_1(2\theta_B) + 2E(3K_1)(\theta_B) - 240 = 0$$
$$10EK_1\theta_B = 240$$
$$EK_1\theta_B = 24.$$

Using the fundamental formula,

$$M_{AB}=2EK_1(\theta_B)$$
$$=48 \text{ tons in.} = 4 \text{ tons ft.}$$
$$M_{BA}=2EK_1(2\theta_B)$$
$$=96 \text{ tons in.} = 8 \text{ tons ft.}$$

Similarly,

$$M_{CD}= -8 \text{ tons ft.}$$
$$M_{DC}= -4 \text{ tons ft.}$$

Asymmetrical Portal Frames

Example 6. As the frame shown in Fig. 223 is not symmetrical it is less easy to analyse than the previous example.

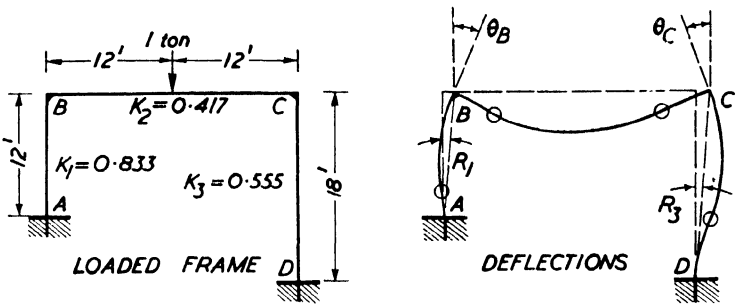


FIG. 223

Let the suffixes 1, 2 and 3 be applied to the members *AB*, *BC* and *CD* respectively.

From the data there is no slope at A and D, i.e.

$$\theta_A=0=\theta_D.$$

B and C are at the same height and under load it is assumed that they retain their positions relative to one another, i.e.

$$R_2=0.$$

$$\text{F.E.M.}_{BC}=\text{F.E.M.}_{CB}=36 \text{ tons in.}$$

The conditions of equilibrium require that

$$\Sigma M_B=0, \quad \Sigma M_C=0 \quad \text{and} \quad \Sigma H=0.$$

Hence,

$$M_{BA}+M_{BC}=0$$
$$M_{CB}+M_{CD}=0$$
$$\frac{M_{AB}+M_{BA}}{L_1}+\frac{M_{CD}+M_{DC}}{L_3}=0.$$

The side-sway of B equals that of C. Hence,

$$R_1L_1=R_3L_3.$$

Using the basic formulæ,

$$2EK_1(2\theta_B - 3R_1) + 2EK_2(2\theta_B + \theta_C) - 36 = 0$$

$$2EK_2(2\theta_C + \theta_B) + 36 + 2EK_3(2\theta_C - 3R_3) = 0$$

$$\frac{2EK_1(\theta_B - 3R_1) + 2EK_1(2\theta_B - 3R_1)}{L_1} + \frac{2EK_3(2\theta_C - 3R_3) + 2EK_3(\theta_C - 3R_3)}{L_3} = 0.$$

Now $K_1 = 0{\cdot}833, \quad K_2 = 0{\cdot}417, \quad K_3 = 0{\cdot}555$

and $R_1 = 1{\cdot}5R_3$

Hence,

$$2 \times 0{\cdot}833E(2\theta_B - 4{\cdot}5R_3) + 2 \times 0{\cdot}417E(2\theta_B + \theta_C) - 36 = 0$$

$$2 \times 0{\cdot}417E(2\theta_C + \theta_B) + 2 \times 0{\cdot}555E(2\theta_C - 3R_3) + 36 = 0$$

$$\frac{2 \times 0{\cdot}833E}{12}(3\theta_B - 9R_3) + \frac{2 \times 0{\cdot}555E}{18}(3\theta_C - 6R_3) = 0.$$

That is:

$$5E\theta_B + 0{\cdot}833E\theta_C - 7{\cdot}5ER_3 - 36 = 0 \quad \text{. . . . (i)}$$

$$0{\cdot}833E\theta_B + 3{\cdot}888E\theta_C - 3{\cdot}33ER_3 + 36 = 0 \quad \text{. . . . (ii)}$$

$$0{\cdot}417E\theta_B + 0{\cdot}185E\theta_C - 1{\cdot}62ER_3 = 0 \quad \text{. . . . (iii)}$$

Multiplying (ii) by 2·25,

$$1{\cdot}875E\theta_B + 8{\cdot}75E\theta_C - 7{\cdot}5R_3 + 81 = 0 \quad \text{. . . . (iv)}$$

Subtracting (iv) from (i),

$$3{\cdot}125E\theta_B - 7{\cdot}917E\theta_C - 117 = 0 \quad \text{. . . . . . (v)}$$

Multiplying (iii) by 4·63,

$$1{\cdot}93E\theta_B + 0{\cdot}858E\theta_C - 7{\cdot}5ER_3 = 0 \quad \text{. . . . (vi)}$$

Subtracting (vi) from (i),

$$3{\cdot}07E\theta_B - 0{\cdot}025E\theta_C - 36 = 0 \quad \text{. . . . . . (vii)}$$

Multiplying (vii) by 1·018,

$$3{\cdot}125E\theta_B - 0{\cdot}025E\theta_C - 36{\cdot}65 = 0 \quad \text{. . . . . . (viii)}$$

Subtracting (viii) from (v),

$$-7{\cdot}892E\theta_C - 80{\cdot}35 = 0$$

Whence, $E\theta_C = -10{\cdot}181$

$E\theta_B = 11{\cdot}643$

$ER_3 = 1{\cdot}831$

and $ER_1 = 2{\cdot}746.$

Using these values in the basic formulæ:

$$M_{AB} = 2EK_1(\theta_B - 3R_1)$$
$$= 2 \times 0{\cdot}833(11{\cdot}643 - 8{\cdot}238)$$
$$= 5{\cdot}675 \text{ tons in.} = 0{\cdot}47 \text{ tons ft.}$$

$$M_{BA} = 2EK_1(2\theta_B - 3R_1)$$
$$= 2 \times 0{\cdot}833(23{\cdot}286 - 8{\cdot}238)$$
$$= 25{\cdot}08 \text{ tons in.} = 2{\cdot}09 \text{ tons ft.}$$

$$M_{CD} = 2EK_3(2\theta_C - 3R_3)$$
$$= 2 \times 0{\cdot}555(-20{\cdot}362 - 5{\cdot}493)$$
$$= -28{\cdot}715 \text{ tons in.} = -2{\cdot}39 \text{ tons ft.}$$
$$M_{DC} = 2EK_3(\theta_C - 3R_3)$$
$$= 2 \times 0{\cdot}555(-10{\cdot}181 - 5{\cdot}493)$$
$$= -17{\cdot}414 \text{ tons in.} = -1{\cdot}45 \text{ tons ft.}$$

The final B.M. diagram is shown in Fig. 224.

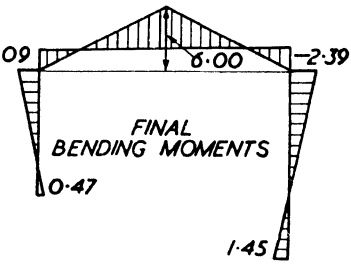


FIG. 224

Example 7. The frame shown in Fig. 225 is analysed in a similar manner to that of the last example, but the analysis is complicated by the side-sway of the joints B and C. As the legs slope and are of different length, the rela-

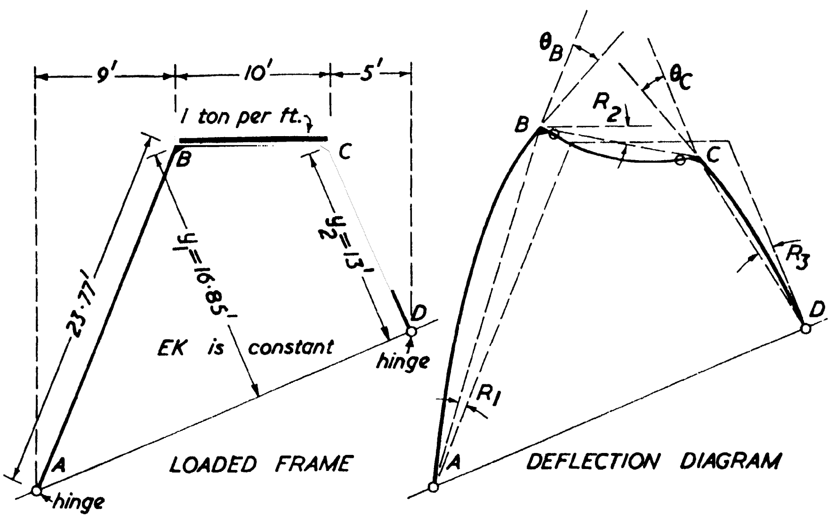


FIG. 225

tive values of R for the three members must be obtained by geometry or other means. This was done for the Stage II moments for the same frame in the section on Moment Distribution and will not be repeated here.

Let the suffixes 1, 2 and 3 apply to members AB, BC and CD respectively.

Now EK is constant throughout the frame.

Also, $$R_2 = -1{\cdot}816R_1$$

and $$R_3 = 1{\cdot}833R_1.$$

The frame may be analysed knowing that:

$$M_{BA} + M_{BC} = 0$$

$$M_{CB} + M_{CD} = 0$$

$$0{\cdot}772M_{BA} + M_{CD} = 0{\cdot}23 \text{ tons ft. (see p.186, line 20)}$$

$$= 2{\cdot}76 \text{ tons ins.}$$

Hence,

$$EK(3\theta_B + 3R_1) + 2EK(2\theta_B + \theta_C - 3 \times 1{\cdot}816R_1) - \text{F.E.M.}_{BC} = 0$$

$$7EK\theta_B + 2EK\theta_C - 7{\cdot}896EKR_1 - 100 = 0 \quad . \quad . \quad . \quad . \quad \text{(i)}$$

$$2EK(2\theta_C + \theta_B - 3 \times 1{\cdot}816R_1) + \text{F.E.M.}_{CB} + EK(3\theta_C + 3 \times 1{\cdot}833R_1) = 0$$

$$2EK\theta_B + 7EK\theta_C - 5{\cdot}396EKR_1 + 100 = 0$$

or $$7EK\theta_B + 24{\cdot}5EK\theta_C - 18{\cdot}886EKR_1 + 350 = 0 \quad . \quad . \quad . \quad \text{(ii)}$$

Subtracting equation (i) from (ii),

$$22{\cdot}5EK\theta_C - 10{\cdot}99EKR_1 + 450 = 0 \quad . \quad . \quad . \quad . \quad . \quad \text{(iii)}$$

Also $$0{\cdot}772\ EK\ (3\theta_B + 3R_1) + EK\ (3\theta_C + 5{\cdot}5R_1) = 2{\cdot}76$$

Hence,

$$2{\cdot}316EK\theta_B + 3EK\theta_C + 7{\cdot}816EKR_1 = 2{\cdot}76$$

or $$7{\cdot}0EK\theta_B + 9{\cdot}067EK\theta_C + 23{\cdot}623EKR_1 = 8{\cdot}342 \quad . \quad . \quad . \quad \text{(iv)}$$

Subtracting equation (i) from (iv),

$$7{\cdot}067EK\theta_C + 31{\cdot}519EKR_1 + 91{\cdot}658 = 0$$

or $$22{\cdot}5EK\theta_C + 100{\cdot}350EKR_1 + 291{\cdot}82 = 0 \quad . \quad . \quad . \quad . \quad \text{(v)}$$

Subtracting equation (iii) from (v),

$$111{\cdot}340EKR_1 = 158{\cdot}18$$

$$EKR_1 = 1{\cdot}42.$$

Substituting in equation (v),

$$22{\cdot}5EK\theta_C = -291{\cdot}82 - (100{\cdot}35 \times 1{\cdot}42)$$

$$EK\theta_C = -19{\cdot}30.$$

Substituting in equation (i),

$$7EK\theta_B = (2 \times 19{\cdot}30) + (7{\cdot}896 \times 1{\cdot}42) + 100$$

$$EK\theta_B = 21{\cdot}40.$$

Employing the fundamental formulæ,

$$\begin{aligned} M_{BA} &= EK(3\theta_B + 3R_1) \\ &= 3(21{\cdot}40 + 1{\cdot}42) \\ &= 68{\cdot}46 \text{ tons in.} \\ &= 5{\cdot}71 \text{ tons ft.} \\ M_{CD} &= EK(3\theta_C + 5{\cdot}5R_1) \\ &= 3(-19{\cdot}30) + 5{\cdot}5(1{\cdot}42) \\ &= -50{\cdot}09 \text{ tons in.} \\ &= -4{\cdot}17 \text{ tons ft.} \end{aligned}$$

The final B.M. diagram is shown in Fig. 226.

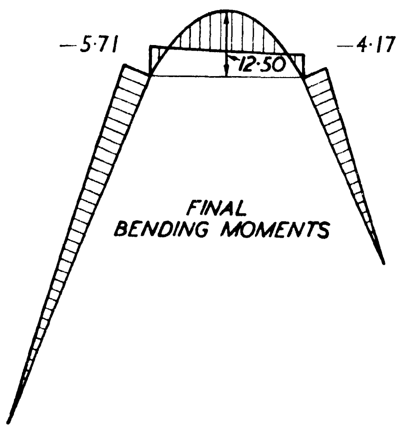


FIG. 226

(d) The Method of Semi-Graphical Integration

Semi-Graphical Integration has been described by Mr. R. G. Robertson, M.A., M.I.C.E., A.M.I.Struct.E., in the Maclachlan Lecture published in the *Structural Engineer* for November 1949. It provides an alternative solution to the classical methods of finding the horizontal thrust in portal frames.

The method consists of a combination of graphics and simple arithmetic, and its great merit is its extreme simplicity. In essence it is a graphical method of applying the principles of the arch theory which can be summarised in the expression:

$$H = \frac{\int \frac{My \, . \, ds}{I}}{\int \frac{y^2 ds}{I}} \quad \text{or} \quad \int \frac{My \, . \, ds}{I} = H \int \frac{y^2 ds}{I} \quad \text{when } E \text{ is constant.}$$

The method has particular advantage in the cases of portal frames with unequal or sloping legs and with hinged feet as shown in Fig. 227.

It should be noted that, although, in the strict interpretation of the arch theory, y should be measured vertically, it is often convenient to measure y

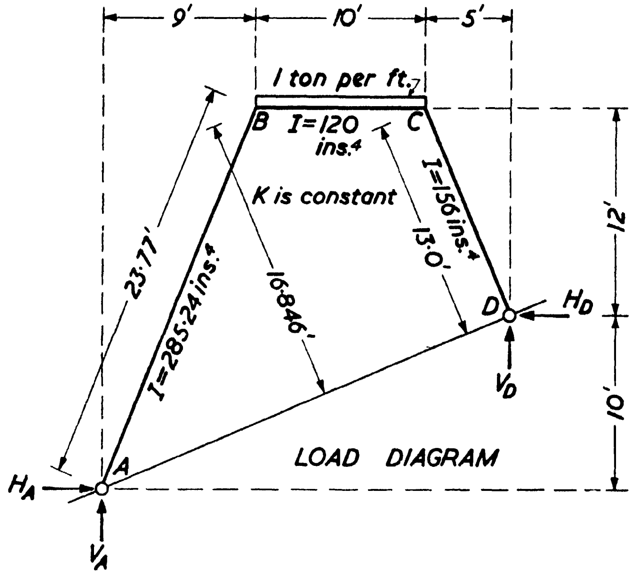


FIG. 227

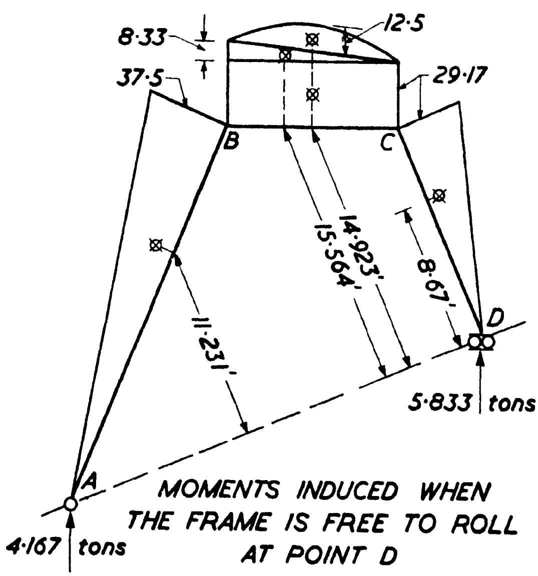


FIG. 228

at right angles to the line joining the feet of the portal, and, provided that all measurements are made to this line, the solution is not affected.

The areas of the moment diagrams are considered to be concentrated along the appropriate parts of the frame, and the centroids of these areas are first projected at right angles to those parts and then from the frame at right angles to the line joining the feet, as in Figs. 228 and 229.

The first step is to consider the point D free to roll, in which case the frame AD acts as a beam and there are no horizontal reactions at the feet, so that $V_A = 4{\cdot}167$ tons and $V_D = 5{\cdot}833$ tons.

$$M_B = 4{\cdot}167 \times 9 = 37{\cdot}5 \text{ tons ft.}$$

$$M_C = 5{\cdot}833 \times 5 = 29{\cdot}17 \text{ tons ft.}$$

The areas of the moment diagram are shown in Fig. 228, from which it will be seen that the diagram on BC consists of a rectangle, a triangle and a parabola. It is always desirable to break up the diagram into simple forms.

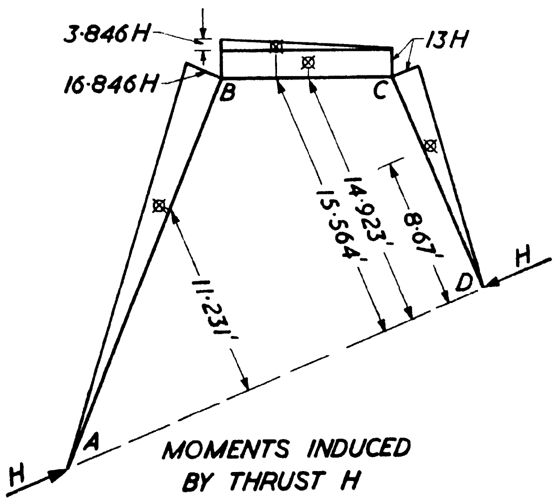


FIG. 229

Using tons ft. units, the area moments are then summated with respect to the line AD:

AB, $\displaystyle\int_B^A \frac{My \, . \, ds}{I} = \int_B^A Mds \, . \, y \, . \, \frac{1}{I}$ (where M is the static moment at any point)

$$= 37{\cdot}5 \times \frac{23{\cdot}77}{2} \times 11{\cdot}231 \times \frac{12^4}{285{\cdot}24} = 363{,}880$$

BC, $$29{\cdot}17 \times 10 \times 14{\cdot}923 \times \frac{12^4}{120} = 752{,}210$$

$$8{\cdot}33 \times \frac{10}{2} \times 15{\cdot}564 \times \frac{12^4}{120} = 112{,}020$$

$$\frac{12{\cdot}5 \times 10 \times 2}{3} \times 14{\cdot}923 \times \frac{12^4}{120} = 214{,}890$$

CD, $$29{\cdot}17 \times \frac{13}{2} \times 8{\cdot}67 \times \frac{12^4}{156} = 218{,}510$$

$$\underline{1{,}661{,}510}$$

The second step is to bring back the point D to its original position by applying two equal and opposite thrusts H at A and D acting along the line AD, as shown in Fig. 229.

Again summating the area moments with respect to the line AD:

AB, $H\int_B^A \frac{y^2 ds}{I} = \int_B^A Mds \,.\, y \,.\, \frac{1}{I}$ (where $M = Hy$ is the moment induced at any point by the thrust).

$$= 16{\cdot}846H \times \frac{23{\cdot}77}{2} \times 11{\cdot}231 \times \frac{12^4}{285{\cdot}24} = 163{,}470H$$

BC,
$$13H \times 10 \times 14{\cdot}923 \times \frac{12^4}{120} = 335{,}230H$$

$$3{\cdot}846H \times \frac{10}{2} \times 15{\cdot}564 \times \frac{12^4}{120} = 51{,}720H$$

CD,
$$13{\cdot}0H \times \frac{13}{2} \times 8{\cdot}67 \times \frac{12^4}{156} = 97{,}380H$$

$$\underline{647{,}800H}$$

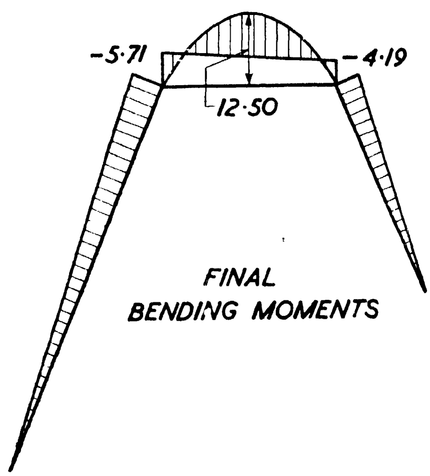


FIG. 230

For there to be no resultant moment,

$$647{,}800H = 1{,}661{,}510,$$

whence, $H = 2{\cdot}565$ tons

of which the vertical component $= 0{\cdot}987$ ton and the horizontal component $= 2{\cdot}368$ tons.

Hence, the total vertical reaction V_A

$$= 4{\cdot}167 + 0{\cdot}987 = 5{\cdot}154 \text{ tons}$$

and the total vertical reaction V_D

$$= 5{\cdot}833 - 0{\cdot}987 = 4{\cdot}846 \text{ tons.}$$

Whence $M_B = 5{\cdot}154 \times 9 - 2{\cdot}368 \times 22 = -5{\cdot}71$ tons ft.

and $M_C = 4{\cdot}846 \times 5 - 2{\cdot}368 \times 12 = -4{\cdot}19$ tons ft.

The final B.M. diagram is shown in Fig. 230.

13. SLOPES AND DEFLECTIONS IN RIGID FRAMES

When a rigid frame has been analysed by employing the Slope Deflection Method, it is possible to find the displacement and angle of rotation of every joint in the structure directly from the analysis computations, provided, of course, that the actual, rather than comparative, units have been used throughout.

It is normal to assume that there is no longitudinal strain in any member and that deflections of any point can be measured at right angles to the original position of the member concerned. This being so, rotations and deflections can be computed by using the Area-Moment Method described on pages 48 *et seq.* (Figs. 22, 23 and 24 in particular).

Example 1. It is not always necessary to carry out any detailed calculations because, in cases where there is symmetry, formulæ can frequently be used. Suppose that it is desired to find the rotation of the joints B and D

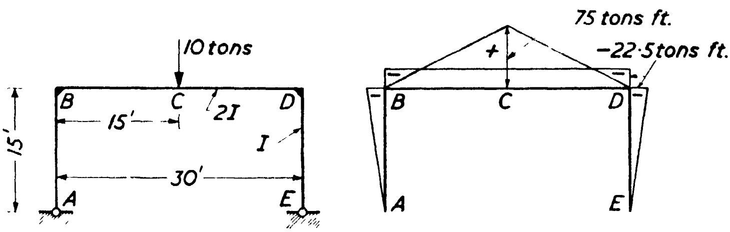


FIG. 231

in Fig. 231 and to find the maximum deflections in the legs and cross-beam. This information can be obtained from the final B.M. diagram and a knowledge of certain formulæ dispersed through various parts of this book.

Neglecting any shortening of the members due to strain or curvature, then the points B and D can be assumed not to change their position under the symmetrical loading shown. Hence, AB or DE can be considered as simply supported beams, at the ends of which, at B and D respectively, are applied moments M equal to 22·5 tons ft. Considering Fig. 23 and the associated calculations,

$$\theta = \frac{ML}{3EI}.$$

Therefore,

$$\theta_B = -\theta_D = \frac{22{\cdot}5 \times 15}{3EI}$$

$$= \frac{112{\cdot}5}{EI} \text{ radian (ft. units)}$$

where clockwise moments and rotations are considered positive.

On page 50, in the section dealing with the deflection of beams, it was shown that the deflection

$$d = -\frac{\text{Secondary B.M.}}{EI}$$

Consider the B.M. for AB shown in Fig. 231 and the properties of a similarly shaped triangular load, as shown on page 18.

Then the maximum secondary B.M. occurs at a point $(0{\cdot}5774 \times 15) = 8{\cdot}661$ ft. from A as shown in Fig. 232, and the deflection

$$d = 0{\cdot}128\,WL/EI \quad \text{(where } W = \text{the area of the B.M. diagram for } AB\text{)}$$

$$= \frac{0{\cdot}128 \times 22{\cdot}5 \times 15 \times 15}{2 \times EI}$$

$$= \frac{324}{EI} \text{ (ft. units)}$$

i.e. the deflection is outwards from the frame.

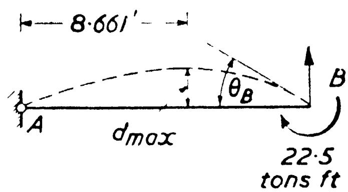


FIG. 232

Considering the cross-beam BD, the rotations of B and D are already known and it remains to find the maximum deflection, which occurs under the load.

The B.M. diagram consists of a rectangle of height $-22{\cdot}5$ tons ft. and an isosceles triangle of height $+75$ tons ft. and base BD. Consulting the appropriate properties for U.D.L.s and triangular loads on pages 17 and 18,

$$d = -\frac{(-W_1L/8) + (+W_2L/6)}{E(2I)}$$

$$= -\frac{(-22{\cdot}5 \times 30) \times 30/8 + (75 \times 30/2) \times 30/6}{E(2I)}$$

$$= -\frac{1547}{EI} \text{ (ft. units)},$$

i.e. the deflection is inwards.

It will be noted that the sign convention usual in mathematics has been adopted in this particular example, but any system may be adopted provided it is used throughout a specific example.

Example 2. The frame in Fig. 233 is subjected to a horizontal point load applied at B, which results in the B.M. and thrust diagram shown. Calculate the rotations and deflections of B and D.

It will be observed that a point of contraflexure exists at C, the mid-point of BD, as M_B and M_D are equal and opposite. Consequently, C is in line with B and D and all three points can be assumed to have retained their relative height, each moving horizontally by the same amount.

Then the portion BC may be considered to be a beam simply supported at B and C with a moment of 75 tons ft. applied at B.

$$\theta_B = \theta_D = \frac{M_BL}{3EI} = \frac{75 \times 15}{3EI} = \frac{375}{EI} \text{ radian (ft. units).}$$

Since the slope of the cantilever at A is unknown it is convenient to consider the displacement of A relative to B. Hence, consider B fixed in position and A free to move to the left. Then the relative displacement is in two parts, one due to the rotation of B and the other due to the thrust at A when BA acts as a cantilever, as shown in Fig. 234.

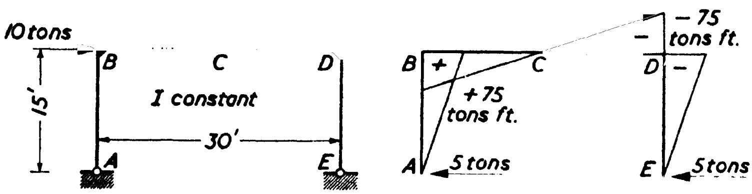


FIG. 233

Therefore the horizontal deflection of A due to the rotation of B,

$$d_\theta = AB \times \theta_B = \frac{15 \times 375}{EI}$$

$$= \frac{5625}{EI} \text{ (ft. units).}$$

The horizontal deflection due to the thrust at A,

$$d_H = \frac{PL^3}{3EI} = \frac{5 \times 15^3}{3EI} = \frac{5625}{EI} \text{ (ft. units).}$$

Therefore the total horizontal deflection of A relative to B

$$= \frac{5625 + 5625}{EI} = \frac{11{,}250}{EI} \text{ (ft. units).}$$

To determine the deflection in inches it is necessary to multiply these figures by $12^3 = 1728$.

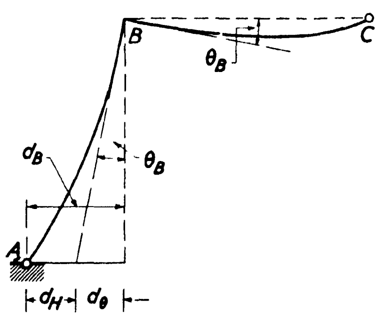


FIG. 234

Suppose the framework is composed of 18-in. × 7-in. × 75-lb. joist sections, for which $I_{xx} = 1151{\cdot}18$ ins.^{4}.

Then the deflection of B or D,

$$d = \frac{11{,}250}{EI} = \frac{11{,}250 \times 12^3}{13{,}000 \times 1151{\cdot}18}$$

$$= 1{\cdot}3 \text{ in.}$$

Example 3. The frame shown in Fig. 235 is of uniform section throughout, and it is required to find the deformed shape of the frame under the action of the crane loads only. The crane bracket loads produce moments Pc of 10 tons ft. and 4 tons ft. at Q and R respectively. Then the B.M. diagram may be constructed by consulting pages 374 and 375.

In terms of Pc, the following moments may be read off the charts:

Joint	*B.M. due to equal loads at Q and R*	*B.M. due to a single load at Q*
A	+0·319	−0·032
B	+0·218	+0·417
C	−0·148	−0·074
D	+0·218	−0·199
E	+0·319	+0·352

After multiplication by the appropriate coefficients of Pc (4 and 6 respectively for the equal and single loads) and adding algebraically the values for the two types of load, the moments shown in the Figure are obtained.

The values at Q are, respectively, above the bracket,

$$\left(3{\cdot}374-\frac{3{\cdot}374-1{\cdot}084}{5}\right)+\frac{10}{5}=+4{\cdot}916$$

and below the bracket

$$\left(3{\cdot}374-\frac{3{\cdot}374-1{\cdot}084}{5}\right)-\frac{10\times 4}{5}=-5{\cdot}084.$$

Values at R are obtained in a similar manner.

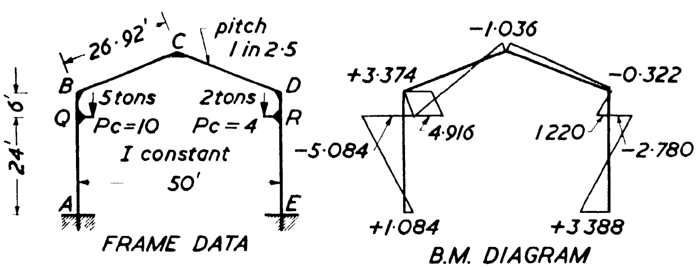


FIG. 235

From the B.M. diagram it is possible to calculate both the slope and the displacement of any point in the frame. The ends A and E are fixed in position and direction, i.e. $\theta_A=\theta_E=0$, and hence form useful points from which to commence operations.

Now
$$\theta=\frac{\Sigma A}{EI}$$

where A = area of B.M. diagram, and

$$d=\frac{\Sigma Ay}{EI}$$

where Ay = area moment of B.M. diagram.

The work can be done by commencing at A and working through to E, but it is more convenient to work from A to C and then from E to C; the results should be identical and this provides a ready check upon the accuracy of the calculations.

Commencing from A, see Fig. 236, and assuming deflections into the frame to be positive,

$$d_Q = \frac{(+1{\cdot}084 \times 24^2 \times \frac{1}{2} \times \frac{2}{3}) - (5{\cdot}084 \times 24^2 \times \frac{1}{2} \times \frac{1}{3})}{EI}$$

$$= \frac{279{\cdot}94}{EI} \text{ (ft. units).}$$

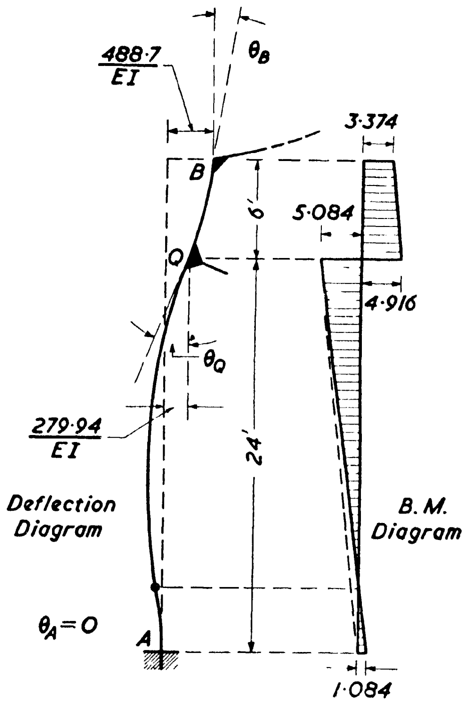


FIG. 236

The slope at Q,

$$\theta_Q = \frac{(5{\cdot}084 \times 24/2) - (1{\cdot}084 \times 24/2)}{EI}$$

$$= \frac{48}{EI} \text{ radian (clockwise rotation positive).}$$

The deflection of B,

$$d_B = +\frac{279{\cdot}94}{EI} + \frac{48 \times 6}{EI} - \frac{(3{\cdot}374 \times 6 \times 3) + (4{\cdot}916 - 3{\cdot}374) \times 6^2 \times \frac{1}{2} \times \frac{2}{3}}{EI}$$

$$= \frac{488{\cdot}70}{EI} \text{ (ft. units).}$$

The slope at B,

$$\theta_B = \text{slope at } Q + \text{rotation between } Q \text{ and } B.$$

$$= \frac{48}{EI} - \frac{(4{\cdot}916 + 3{\cdot}374) \times 6}{2 \times EI}$$

$$= \frac{23{\cdot}13}{EI} \text{ radian.}$$

Now calculate the deflection of C with respect to B, measuring normal to the slope of the rafter BC, as shown in Fig. 237.

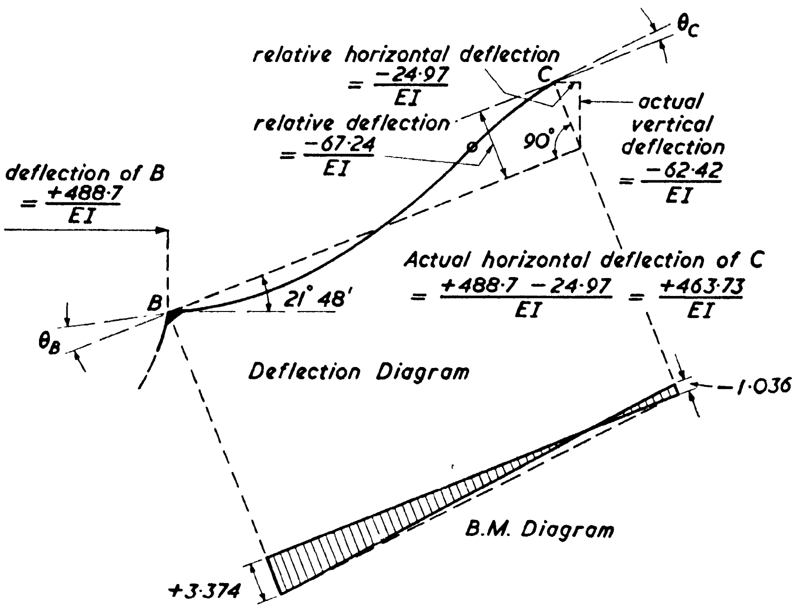


FIG. 237

Then the relative deflection of C with respect to B = the deflection due to the rotation of B + the deflection in BC,

$$= \frac{23{\cdot}13 \times 26{\cdot}92}{EI} - \frac{3{\cdot}374 \times 26{\cdot}92^2 \times \frac{1}{2} \times \frac{2}{3}}{EI} + \frac{1{\cdot}036 \times 26{\cdot}92^2 \times \frac{1}{2} \times \frac{1}{3}}{EI}$$

$$= -\frac{67{\cdot}24}{EI} \text{ (ft. units).}$$

Now the rafter has an angle of slope of 21° 48′.

$$\text{Sin } 21° \ 48' = 0{\cdot}3714.$$

$$\text{Cos } 21° \ 48' = 0{\cdot}9285.$$

Then the relative horizontal deflection of C

$$= -\frac{67{\cdot}24 \times 0{\cdot}3714}{EI} = -\frac{24{\cdot}97}{EI} \text{ (ft. units)}$$

while the actual vertical deflection of C

$$= -\frac{67\cdot24 \times 0\cdot9285}{EI} = -\frac{62\cdot42}{EI} \text{ (ft. units)}$$

Hence, the horizontal displacement of C = displacement at B + displacement of C relative to B,

$$= \frac{488\cdot70 - 24\cdot97}{EI} = \frac{463\cdot73}{EI} \text{ (ft. units)}$$

and the vertical deflection

$$= -\frac{62\cdot42}{EI} \text{ (ft. units).}$$

Therefore, the ridge C has moved upwards and to the right.

The slope at the ridge = the rotation at B + the rotation between B and C.

$$\theta_c = \frac{23\cdot13}{EI} - \frac{(3\cdot374 - 1\cdot036) \times 0\cdot5 \times 26\cdot92}{EI}$$

$$= -\frac{8\cdot33}{EI} \text{ radian.}$$

Therefore, the ridge C has rotated in an anti-clockwise direction.

Now consider the right half of the frame.

Commencing from E, the deflection at R,

$$d_R = \frac{(-3\cdot388 \times 24^2 \times \frac{1}{2} \times \frac{2}{3}) + (2\cdot780 \times 24^2 \times \frac{1}{2} \times \frac{1}{3})}{EI}$$

$$= -\frac{383\cdot61}{EI} \text{ (ft. units).}$$

The slope at R,

$$\theta_R = \frac{(3\cdot388 - 2\cdot780) \times \frac{1}{2} \times 24}{EI}$$

$$= \frac{7\cdot30}{EI} \text{ radian.}$$

The deflection of D,

$$d_D = -\frac{383\cdot61}{EI} - \frac{7\cdot30 \times 6}{EI} - \frac{1\cdot220 \times 6^2 \times \frac{1}{2} \times \frac{2}{3}}{EI} + \frac{0\cdot322 \times 6^2 \times \frac{1}{2} \times \frac{1}{3}}{EI}$$

$$= -\frac{440\cdot12}{EI} \text{ (ft. units).}$$

The slope at D,

$$\theta_D = \frac{7\cdot30 + (1\cdot220 - 0\cdot322) \times \frac{1}{2} \times 6}{EI}$$

$$= \frac{9\cdot99}{EI} \text{ radian.}$$

Considering the deflection of C with respect to D and normal to DC,

$$\text{relative } d_C = -\frac{9\cdot99 \times 26\cdot92}{EI} + \frac{0\cdot322 \times 26\cdot92^2 \times \frac{1}{2}}{EI} + \frac{(1\cdot036 - 0\cdot322) \times 26\cdot92^2 \times \frac{1}{2} \times \frac{1}{3}}{EI}$$

$$= -\frac{66\cdot02}{EI} \text{ (ft. units)}$$

which compares reasonably well with the figure of $-67{\cdot}24/EI$ for the relative deflection of C with regard to B.

Then the relative horizontal deflection of C

$$= -\frac{66{\cdot}02 \times 0{\cdot}3714}{EI} = -\frac{24{\cdot}52}{EI}$$

while the actual vertical deflection of C

$$= -\frac{66{\cdot}02 \times 0{\cdot}9285}{EI} = -\frac{61{\cdot}30}{EI}$$

Therefore the actual horizontal deflection of C, calculated along the right half of the frame,

$$= \frac{-440{\cdot}12 - 24{\cdot}52}{EI} = -\frac{464{\cdot}6}{EI}$$

compared with $463{\cdot}73/EI$ when calculated along the left half of the frame.

The calculated vertical deflection is $-61{\cdot}30/EI$ compared with $-62{\cdot}42/EI$ when calculated along the left half of the frame.

The slope at C,

$$\theta_C = \frac{9{\cdot}99}{EI} - \frac{(1{\cdot}036 + 0{\cdot}322) \times \frac{1}{2} \times 26{\cdot}92}{EI}$$

$$= -\frac{8{\cdot}29}{EI} \text{ radian,}$$

which compares favourably with the value of $-8{\cdot}33$ obtained from the previous calculations.

The slight differences in values for deflections and rotations can be due to slight inaccuracies both in the evaluation of the original B.M.s and from the calculations given here. It is likely that the greatest inaccuracy here could be traced to the values of the B.M.s.

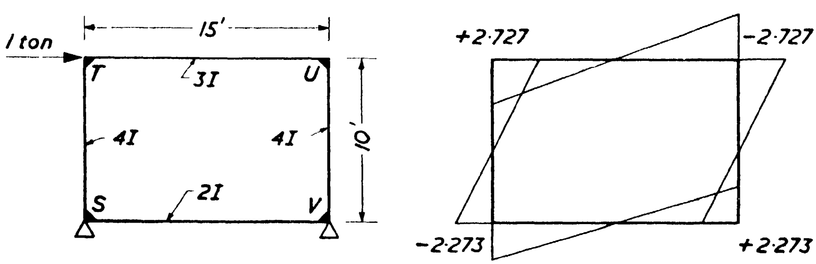


FIG. 238

Example 4. Fig. 238 shows the frame which was analysed in Example 8 of Column Analogy. The slopes and deflections of the joints will be calculated.

Both TU and SV are members at the ends of which have been applied moments of the same value and the same kind of rotation, i.e. all the rotations are clockwise. Then each member is like the member BD in Fig. 233 and the slopes at each end could be calculated as in Example 2. The easier way is to use the formula, applicable in such a case,

$$\theta = \frac{ML}{6EI}.$$

Therefore,

$$\theta_T = \theta_U = \frac{2{\cdot}727 \times 15}{6E(3I)}$$

$$= \frac{2{\cdot}273}{EI} \text{ radian,}$$

while,

$$\theta_S = \theta_V = \frac{2{\cdot}273 \times 15}{6E(2I)}$$

$$= \frac{2{\cdot}841}{EI} \text{ radian.}$$

Now S and V are fixed in position, so the deflection of both T and U can be found by working upwards either from S or V.

Commencing from S, the deflection of T = deflection due to θ_S plus the deflection due to the moments in ST. Therefore,

$$d_T = \frac{2{\cdot}841 \times 10}{EI} + \frac{2{\cdot}273 \times 10^2 \times \frac{1}{2} \times \frac{2}{3}}{E(4I)} - \frac{2{\cdot}727 \times 10^2 \times \frac{1}{2} \times \frac{1}{3}}{E(4I)}$$

$$= +\frac{35{\cdot}99}{EI} \text{ (ft units).}$$

The deflection of U is the same amount.

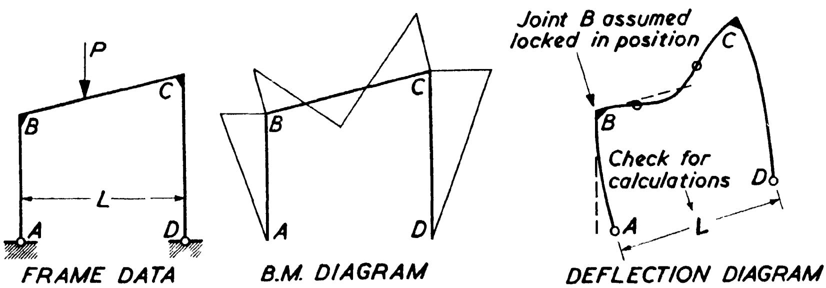


FIG. 239

Frames where no Symmetry Exists

Cases will arise where none of the expedients already stated can be employed. In this event, the method of procedure is to assume that one of the joints is locked in position and direction and to allow the frame to rotate and deflect about it. The method is outlined in Fig. 239.

The foregoing Area-Moment principles apply, but the work is a little more tedious. The deflection and rotation of A, with respect to B, due to the horizontal thrust at A are calculated and the relative position of A is plotted. Then the deflection and rotation of C relative to B are calculated and the position and slope of C are plotted. From C the relative deflection of D is due to the rotation of C and to the deflection of CD resulting from the horizontal thrust at D. All these operations have been demonstrated in the preceding Examples. When the calculations are complete, the distance between the points A and D should be L.

14. FORMULÆ FOR RIGID FRAMES

The formulæ given on pages 287 to 345 are based on Professor Kleinlogel's Rahmenformeln and Mehrstielige Rahmen, published by Wilhelm Ernst & Sohn of Berlin, to whom grateful acknowledgment is made. The formulæ are applicable to frames which are symmetrical about a central vertical axis, with the single exception of the triangular frame, and in which each member is of constant moment of inertia.

Formulæ are given for the following types of frame:

Single-storey Frame

Frame I. Hingeless rectangular portal frame.
II. Two-hinged rectangular portal frame.
III. Hingeless gable frame with vertical legs.
IV. Two-hinged gable frame with vertical legs.
V. Hingeless frame with skew corners.
VI. Two-hinged frame with skew corners.
VII. Two-hinged triangular frame.

Multi-storey Frames

VIII. Two-storey Frame.
IX. Three-storey Frame.

Multi-bay Frame

X. Twin Gable Frame with hinged feet.

The loadings are so arranged that dead, snow and wind loads may be reproduced on all the frames. For example, wind suction acting normal to the sloping rafters of a building may be divided into horizontal and vertical components, for which appropriate formulæ are given, although all the signs must be reversed because the loadings shown in the tables act inwards, not outwards as in the case of suction. Crane loads, including surge, are also shown in a number of the single-storey frames.

It should be noted that, with few exceptions, the loads between node or panel points are uniformly distributed over the *whole* member. It is appreciated that it is normal practice to impose loads on frames through purlins, side rails or beams. By using the coefficients in Fig. 240, however, allowance can be made for many other symmetrically placed loads on the cross-beams of Frames I, II, VIII and IX above, where the difference in effect is sufficient to warrant the corrections being made. The indeterminate B.M.s in the whole frame are calculated as though the loads were uniformly distributed over the beam being considered, and then all are adjusted by multiplying by the appropriate coefficient in Fig. 240. It may be of interest to state why these

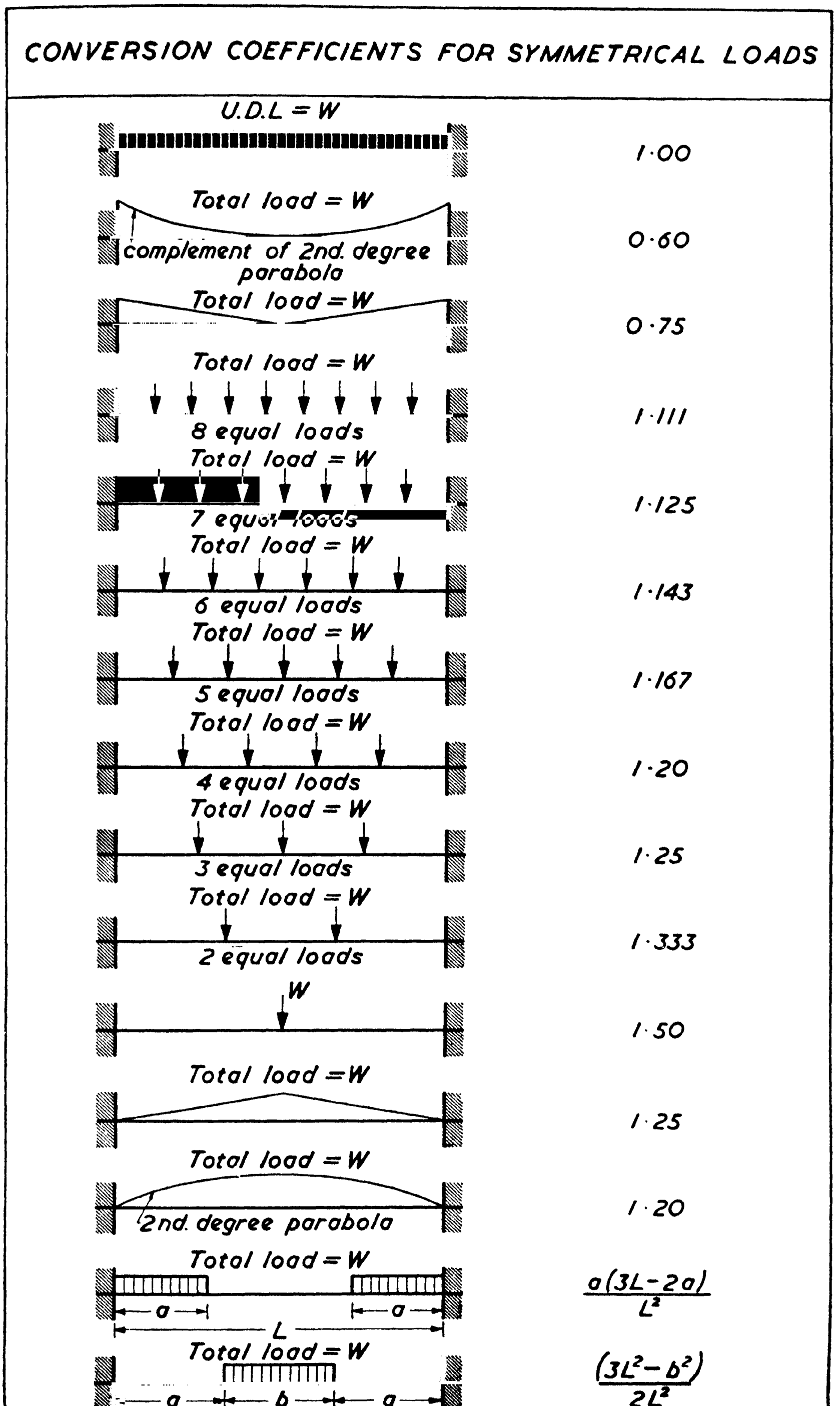

CONVERSION COEFFICIENTS FOR SYMMETRICAL LOADS
U.D.L = W
1·00
Total load = W
complement of 2nd. degree parabola
0·60
Total load = W
0·75
Total load = W
8 equal loads
1·111
Total load = W
7 equal loads
1·125
Total load = W
6 equal loads
1·143
Total load = W
5 equal loads
1·167
Total load = W
4 equal loads
1·20
Total load = W
3 equal loads
1·25
Total load = W
2 equal loads
1·333
W
1·50
Total load = W
1·25
Total load = W
2nd. degree parabola
1·20
Total load = W
a
a
L
a(3L−2a)/L²
Total load = W
a
b
a
(3L²−b²)/2L²

FIG. 240

adjustments can be made. In any statically indeterminate structure the indeterminate moments vary directly with the value of the following quantity:

$$\frac{\text{Area of the free B.M. diagram}}{EI}.$$

Where the loaded member is of constant cross-section, EI may be ignored.

Consider, as an example, the case of an encastré beam of constant cross-section and of length L carrying a U.D.L. of W. Then the area of the free B.M. diagram

$$=\frac{WL}{8}\times\frac{2L}{3}=\frac{WL^2}{12}.$$

If, however, W were a central point load, the area of the free B.M. diagram would be

$$\frac{WL}{4}\times\frac{L}{2}=\frac{WL^2}{8}.$$

The F.E.M.s due to the two types of loadings are $WL/12$ and $WL/8$ respectively, thus demonstrating that the indeterminate moments vary with the area of the free B.M. diagram and proving that the indeterminate moments are in the proportion of 1 : 1·5.

No rules can be laid down for the effect on the reactions of a change in the mode of application of the load, although sometimes they will vary with the indeterminate moments. Consider a simple rectangular portal with hinged feet. If a U.D.L. placed over the whole of the beam is replaced by a central point load of the same magnitude, then the knee moments will increase by 50 per cent with a corresponding increase in the horizontal thrusts H, while the vertical reactions V will remain the same.

Although the foregoing remarks relating to the indeterminate moments resulting from symmetrical loads apply to all rectangular portals, the rule applies for asymmetrical loads imposed upon the cross-beam of a rectangular portal frame with hinged feet. If a vertical U.D.L. on the cross-beam is replaced by any vertical load of the same magnitude, then the indeterminate moments vary with the areas of the respective free B.M. diagrams.

No doubt readers who use the tables frequently will learn short cuts, but it is not inappropriate to mention some. For example, if a U.D.L. of W over the whole of a single-bay symmetrical frame is replaced by a U.D.L. *of the same magnitude of* W over either the left-hand or right-hand half of the frame, the horizontal thrust at the feet is unaltered. If the frame has a pitched roof then the ridge moments will also be unaltered.

It will be noted that the formulæ for the load P on a single crane bracket are related to those for loads P on both crane brackets. Consider Fig. 241.

Then M_A $(=M_E)$ and M_B $(=M_D)$ in Fig. 241 (b) are equal to (M_A+M_E) and (M_B+M_D) respectively in Fig. 241 (a), while M_C in Fig. 241 (b) is double the value of M_C in Fig. 241 (a).

When frames have hinged feet, the moments resulting from surge loads P can be written down without calculation, although the frames are nominally statically indeterminate. The moments at both the loads and at the knees

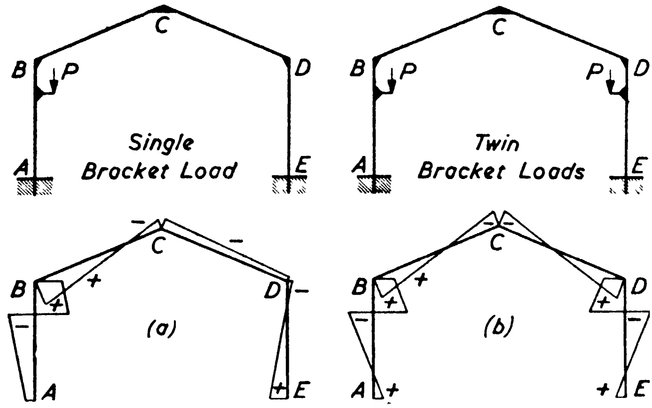


FIG. 241

are equal to Pa, where a is the height of the point of application of the loads above the feet of the frame.

The charts on pages 346 to 398 have been prepared to assist in the design of rectangular frames or frames with roof pitches of 1 in 5 or 1 in 2·5. Results for intermediate pitches may be interpolated with reasonable accuracy.

The charts on pages 399 to 412 are for two-bay portal frames with a roof pitch of 1 in 2·5 only.

Arrangement of Formulæ

Each set of formulæ is treated as a separate chapter. The data required for each frame, together with the constants to be used in the various formulæ, are given on the first page of each chapter. This general information is followed by the detailed formulæ for the various loading conditions, each of which is illustrated by two diagrams placed side by side, the left-hand diagram giving a loading condition and the right-hand one giving the appropriate B.M. and reaction diagram. It should be noted, however, that some B.M.s change their signs as the frames change their proportions. This will be appreciated by examining the charts on pages 346 to 412.

For simple frames, i.e. for single-bay, single-storey frames, the formulæ for reactions immediately follow the formulæ for B.M.s for each load. For multi-storey or multi-bay frames the formulæ for B.M.s are given first in a group and are followed by formulæ for reactions, shears and thrusts, also in a group.

Considering the simple frames only, the kind of formula depends on the degree of indeterminacy and the shape of the frame. Auxiliary Coefficients X are introduced whenever the direct expressions become complicated or for other reasons of expediency.

No hard and fast rules can be laid down for the nomenclature and it must be noted that each set of symbols and constants applies only to the particular frame under consideration, although, of course, an attempt has been made to produce similarity in the types of symbols.

The formulæ for multi-storey or multi-bay frames may seem less complicated than for simple frames, but they are based on numerous constants and composite coefficients which must be accurately computed.

Sign Conventions

All computations must be carried out algebraically, hence every quantity must be given its correct sign. The results will then be automatically correct in sign and magnitude.

The direction of the load or applied moment shown in the left-hand diagram for each load condition is considered to be positive. If the direction of the load or moment is reversed, the signs of all the results obtained from the formulæ as printed must be reversed.

For simple frames, the moments causing tension on the inside faces of the frame are considered to be positive. Upward vertical reactions and inward horizontal reactions are also positive.

For multi-storey or multi-bay frames the same general rules apply to moments and vertical reactions. However, in the case of a two-span portal frame, for example, a problem arises with the central column. It is assumed in the formulæ that the central column belongs to the left-hand bay, so that if the column bends inwards towards this bay, the moment is positive. Similarly, in a multi-storey frame, a cross-beam is associated with the storey below it, tension on the lower face providing a positive B.M.

Horizontal reactions at the feet of multi-storey or multi-bay frames have been given signs in the diagrams but the appropriate rules are given on the first page dealing with reactions in each chapter. In general it may be said that these thrusts bear the same sign as the moment which they create in the joint at the top of the column upon which they act. It should be noted that this system is opposite to that which operates for simple frames.

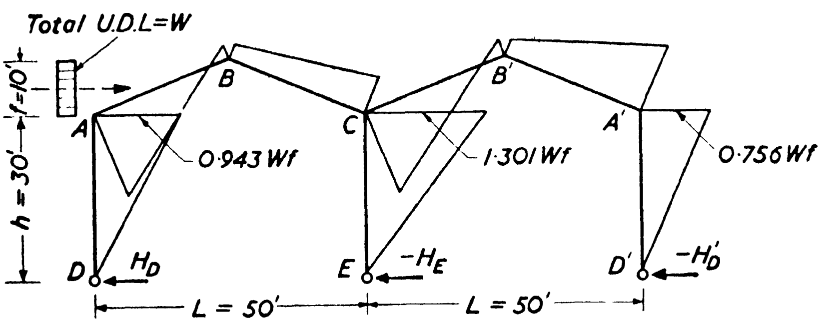


FIG. 242

Checking Calculations for Indeterminate Frames

It is possible to check calculations for indeterminate frames either by using some other method of calculation or, more simply, from a consideration of the three fundamental statical equations:

$$\Sigma H=0;\quad \Sigma V=0;\quad \Sigma M=0.$$

Consider, for example, the loaded frame shown in Fig. 242 together with its B.M. diagram, the moments being in terms of Wf.

Now, if the B.M.s at A, C and A' are correct, the algebraic sum of the horizontal thrusts at D, E and D' should be equal and opposite in direction to W. From a consideration of the B.M.s all the thrusts act from right to left and their sum is equal to

$$\frac{M_A + M_{CII} + M'_A}{h} = \frac{(0{\cdot}943 + 1{\cdot}301 + 0{\cdot}756)Wf}{h}$$

$$= \frac{3Wf}{h} = \frac{3W \times 10}{30} = W.$$

So that the moments being checked are in order.

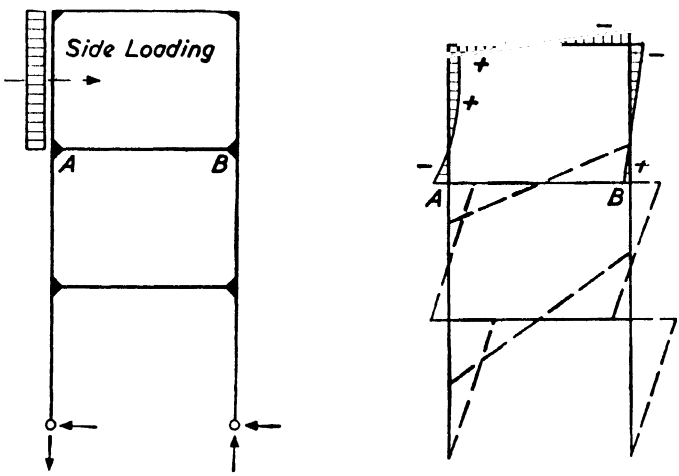


FIG. 243

In addition, it is possible to check any frame or any portion of a frame, such as that shown in Fig. 243, from a consideration of the following two rules:

1. The sum of the areas of the M/EI diagram above any line such as AB is zero if A and B are fully fixed.
2. The sum of the moments, with respect to the base AB, of the areas of the M/EI diagram above the line AB is zero if A and B are partially restrained (as shown in the Figure) or are hinged.

For example, check the diagram shown in Fig. 244 where EI is constant. Then the area of the M/EI diagram, considering the legs first, is

$$= 2\left[\frac{(+0{\cdot}0736 - 0{\cdot}0826) \times 20}{2 \times EI}\right] + 2\left[\frac{(-0{\cdot}0826 + 0{\cdot}0893) \times 26{\cdot}92}{2 \times EI}\right]$$

$$= \frac{-0{\cdot}180 + 0{\cdot}180}{EI} = 0.$$

Thus demonstrating that the calculations are correct.

It is of interest to note that the underlying principles for the last method of checking given above are those used in the application of the Column Analogy method of analysis.

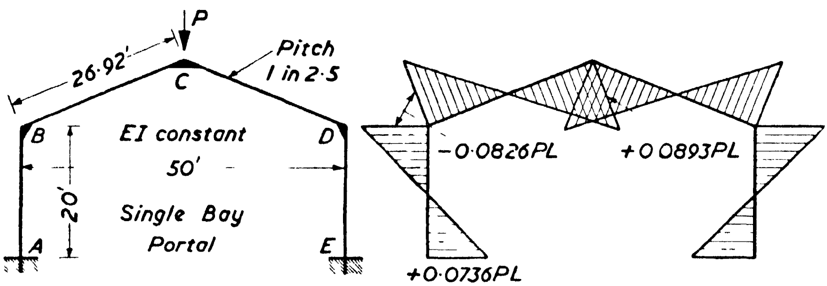
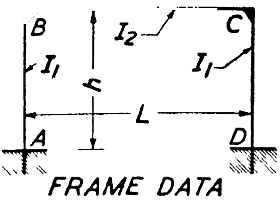


FIG. 244

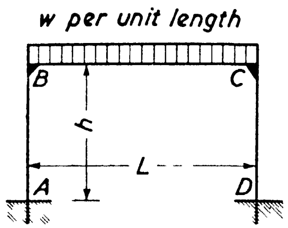
Frame I

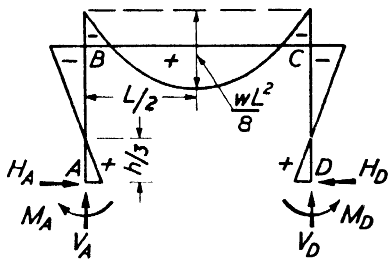


Coefficients:

$$k = \frac{I_2}{I_1} \cdot \frac{h}{L}$$

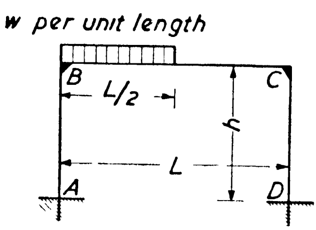
$$N_1 = k + 2 \qquad N_2 = 6k + 1$$

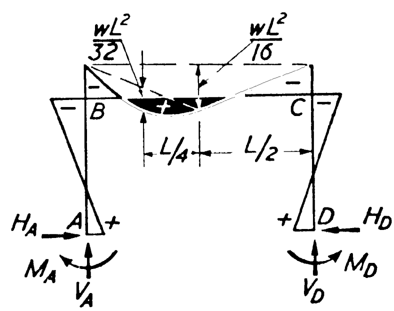




$$M_A = M_D = \frac{wL^2}{12N_1} \qquad M_B = M_C = -\frac{wL^2}{6N_1} = -2M_A$$

$$M_{\max} = \frac{wL^2}{8} + M_B \qquad V_A = V_D = \frac{wL}{2} \qquad H_A = H_D = \frac{3M_A}{h}$$



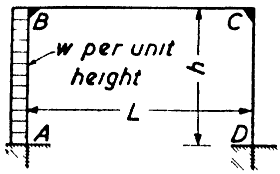


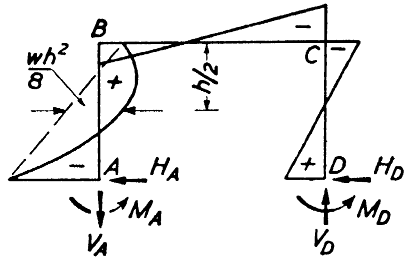
$$M_A = \frac{wL^2}{8}\left[\frac{1}{3N_1} - \frac{1}{8N_2}\right] \qquad M_B = -\frac{wL^2}{8}\left[\frac{2}{3N_1} + \frac{1}{8N_2}\right]$$

$$M_D = \frac{wL^2}{8}\left[\frac{1}{3N_1} + \frac{1}{8N_2}\right] \qquad M_C = -\frac{wL^2}{8}\left[\frac{2}{3N_1} - \frac{1}{8N_2}\right]$$

$$V_D = \frac{wL}{8}\left[1 - \frac{1}{4N_2}\right] \qquad V_A = \frac{wL}{2} - V_D \qquad H_A = H_D = \frac{wL^2}{8hN_1}$$

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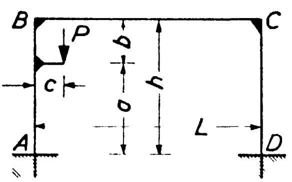


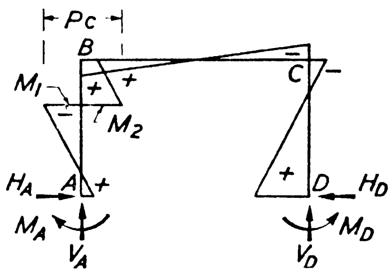


$$M_A = \frac{wh^2}{4}\left[-\frac{k+3}{6N_1} - \frac{4k+1}{N_2}\right] \qquad M_B = \frac{wh^2}{4}\left[-\frac{k}{6N_1} + \frac{2k}{N_2}\right]$$

$$M_D = \frac{wh^2}{4}\left[-\frac{k+3}{6N_1} + \frac{4k+1}{N_2}\right] \qquad M_C = \frac{wh^2}{4}\left[-\frac{k}{6N_1} - \frac{2k}{N_2}\right]$$

$$H_D = \frac{wh(2k+3)}{8N_1} \qquad H_A = -(wh - H_D) \qquad V_A = -V_D = -\frac{wh^2k}{LN_2}$$





$$\text{Constants: } a_1 = \frac{a}{h} \qquad b_1 = \frac{b}{h}$$

$$X_1 = \frac{Pc}{2N_1}[1 + 2b_1k - 3b_1^2(k+1)] \qquad X_2 = \frac{Pcka_1(3a_1 - 2)}{2N_1}$$

$$X_3 = \frac{3Pcka_1}{N_2}$$

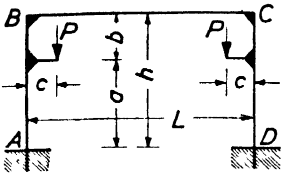
$$M_A = +X_1 - \left(\frac{Pc}{2} - X_3\right) \qquad M_B = +X_2 + X_3$$

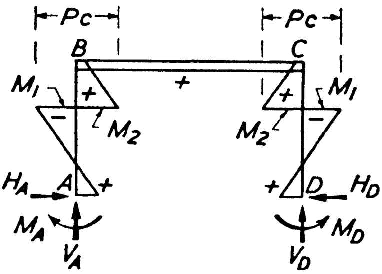
$$M_D = +X_1 + \left(\frac{Pc}{2} - X_3\right) \qquad M_C = +X_2 - X_3$$

$$H_A = H_D = \frac{Pc}{2h} + \frac{X_1 - X_2}{h} \qquad V_D = \frac{2X_3}{L} \qquad V_A = P - V_D$$

$$M_1 = M_A - H_A a \qquad M_2 = M_B + H_D b$$

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Constants: $a_1 = \frac{a}{h} \qquad b_1 = \frac{b}{h}$

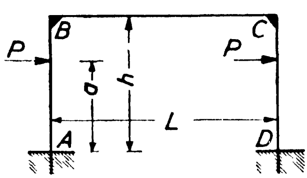
$$X_1 = \frac{Pc}{2N_1}[1 + 2b_1 k - 3b_1^2(k+1)] \qquad X_2 = \frac{Pcka_1(3a_1 - 2)}{2N_1}$$

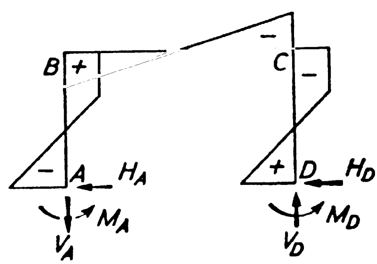
$$M_A = M_D = \frac{Pc}{N_1}[1 + 2b_1 k - 3b_1^2(k+1)] = 2X_1$$

$$M_B = M_C = \frac{Pcka_1(3a_1 - 2)}{N_1} = 2X_2$$

$$V_A = V_D = P \qquad H_A = H_D = \frac{Pc + M_A - M_B}{h}$$

$$M_1 = M_A - H_A a \qquad M_2 = M_B + H_D b$$





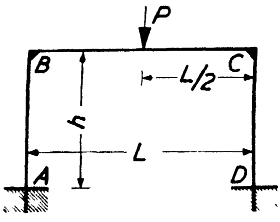
Constants: $a_1 = \frac{a}{h} \qquad X_1 = \frac{3Paa_1k}{N_2}$

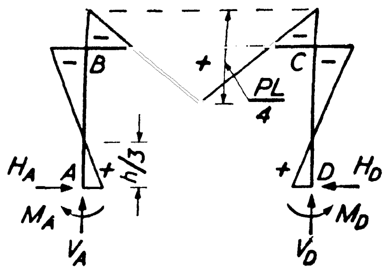
$$M_A = -Pa + X_1 \qquad M_B = X_1$$

$$M_D = +Pa - X_1 \qquad M_C = -X_1$$

$$V_A = -V_D = -\frac{2X_1}{L} \qquad H_A = -H_D = -P$$

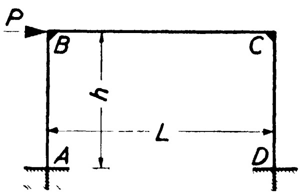
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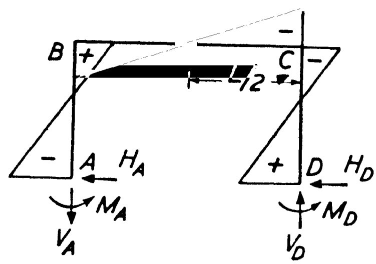




$$M_A = M_D = +\frac{PL}{8N_1} \qquad M_B = M_C = -2M_A$$

$$V_A = V_D = \frac{P}{2} \qquad H_A = H_D = \frac{3M_A}{h}$$

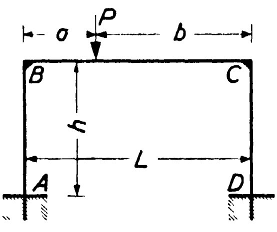


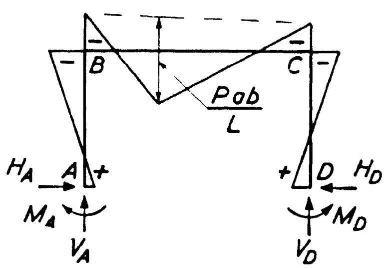


$$M_A = -\frac{Ph}{2} \cdot \frac{3k+1}{N_2} \qquad M_B = +\frac{Ph}{2} \cdot \frac{3k}{N_2}$$

$$M_D = +\frac{Ph}{2} \cdot \frac{3k+1}{N_2} \qquad M_C = -\frac{Ph}{2} \cdot \frac{3k}{N_2}$$

$$H_A = -H_D = -\frac{P}{2} \qquad V_A = -V_D = -\frac{2M_B}{L}$$





Constants: $a_1 = a/L \qquad b_1 = b/L$

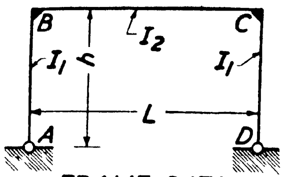
$$M_A = +\frac{Pab}{L}\left[\frac{1}{2N_1} - \frac{b_1 - a_1}{2N_2}\right] \qquad M = -\frac{Pab}{L}\left[\frac{1}{N_1} + \frac{b_1 - a_1}{2N_2}\right]$$

$$M_D = +\frac{Pab}{L}\left[\frac{1}{2N_1} + \frac{b_1 - a_1}{2N_2}\right] \qquad M_C = -\frac{Pab}{L}\left[\frac{1}{N_1} - \frac{b_1 - a_1}{2N_2}\right]$$

$$V_A = Pb_1\left[1 + \frac{a_1(b_1 - a_1)}{N_2}\right] \qquad V_D = P - V_A. \qquad H_A = H_D = \frac{3Pab}{2LhN_1}$$

Extract: '*Kleinlogel, Rahmenformeln*' 11. *Auflage Berlin—Verlag von Wilhelm Ernst & Sohn.*

Frame II

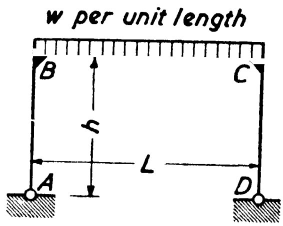


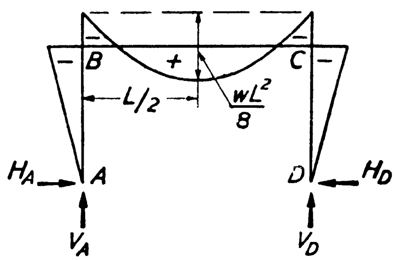
FRAME DATA

Coefficients:

$$k = \frac{I_2}{I_1} \cdot \frac{h}{L}$$

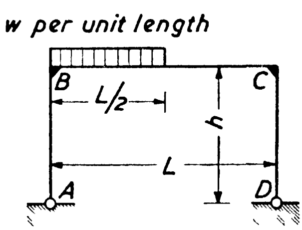
$$N = 2k + 3$$

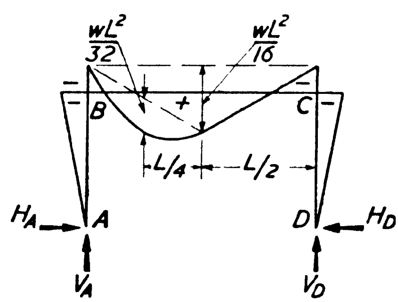




$$M_B = M_C = -\frac{wL^2}{4N} \qquad M_{\text{max}} = \frac{wL^2}{8} + M_B$$

$$V_A = V_D = \frac{wL}{2} \qquad H_A = H_D = -\frac{M_B}{h}$$

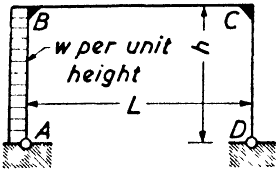


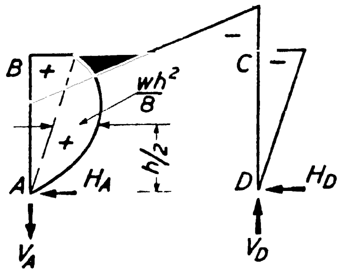


$$M_B = M_C = -\frac{wL^2}{8N}$$

$$V_A = \frac{3wL}{8} \qquad V_D = \frac{wL}{8} \qquad H_A = H_D = -\frac{M_B}{h}$$

Extract: 'Kleinlogel, Rahmenformeln' 11. Auflage Berlin—Verlag von Wilhelm Ernst & Sohn.

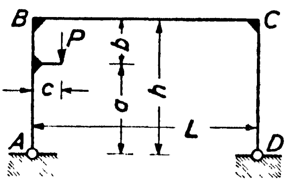


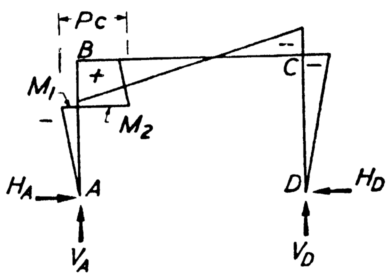


$$M_B = \frac{wh^2}{4}\left[-\frac{k}{2N}+1\right] \qquad H_D = -\frac{M_C}{h}$$

$$M_C = \frac{wh^2}{4}\left[-\frac{k}{2N}-1\right] \qquad H_A = -(wh - H_D)$$

$$V_A = -V_D = -\frac{wh^2}{2L}$$





$$\text{Constant: } a_1 = \frac{a}{h}$$

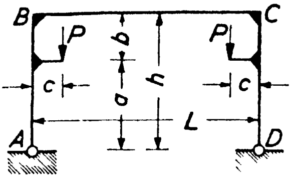
$$M_B = \frac{Pc}{2}\left[\frac{(3a_1-1)k}{N}+1\right] \qquad H_A = H_D = -\frac{M_C}{h}$$

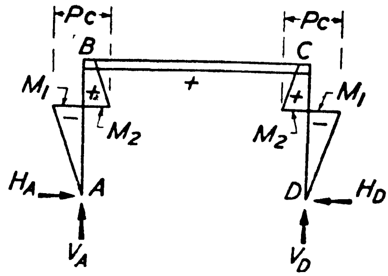
$$M_C = \frac{Pc}{2}\left[\frac{(3a_1^2-1)k}{N}-1\right]$$

$$V_D = \frac{Pc}{L} \qquad V_A = P - V_D$$

$$M_1 = -H_A a \qquad M_2 = Pc - H_A a$$

Extract: 'Kleinlogel, Rahmenformeln' 11. *Auflage Berlin—Verlag von Wilhelm Ernst & Sohn.*



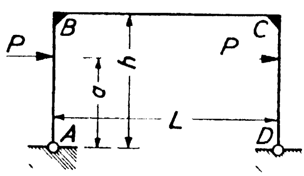


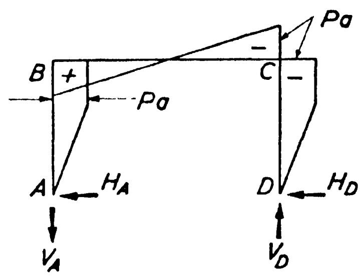
$$\text{Constant: } a_1 = \frac{a}{h}$$

$$M_B = M_C = \frac{Pc(3a_1 - 1)k}{N}$$

$$H_A = H_D = \frac{Pc - M_B}{h} \qquad V_A = V_D = P$$

$$M_1 = -H_A a \qquad M_2 = Pc - H_A a$$



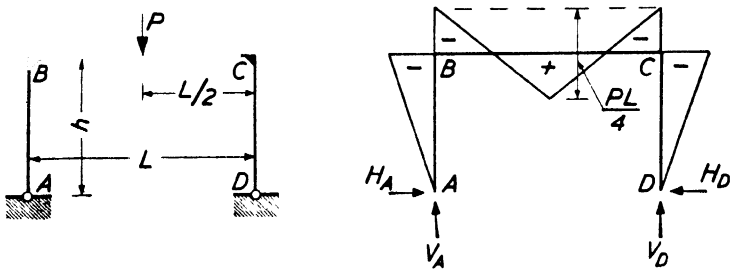


$$M_B = -M_C = Pa \qquad H_A = H_D = P$$

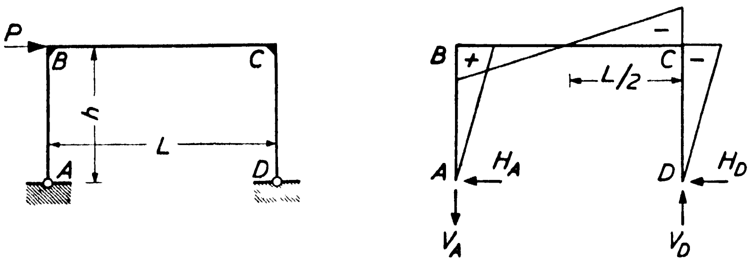
$$V_A = -V_D = -\frac{2Pa}{L}$$

$$\text{Moment at loads} = \pm Pa$$

Extract: 'Kleinlogel, Rahmenformeln' 11. *Auflage Berlin—Verlag von Wilhelm Ernst & Sohn.*

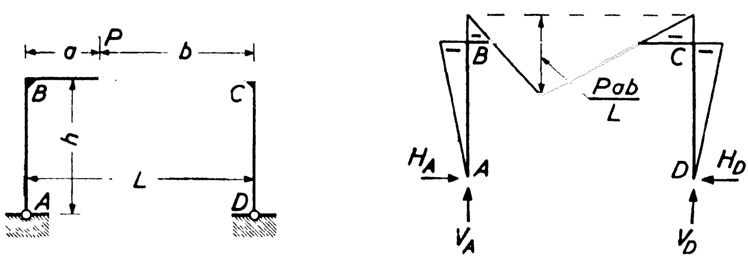


$$M_B = M_C = -\frac{3PL}{8N} \qquad V_A = V_D = \frac{P}{2} \qquad H_A = H_D = -\frac{M_B}{h}$$



$$M_B = -M_C = +\frac{Ph}{2}$$

$$V_A = -V_D = -\frac{Ph}{L} \qquad H_A = -H_D = -\frac{P}{2}$$

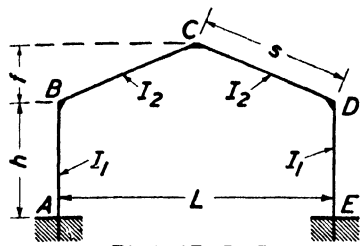


$$M_B = M_C = -\frac{Pab}{L} \cdot \frac{3}{2N}$$

$$V_A = \frac{Pb}{L} \qquad V_D = \frac{Pa}{L} \qquad H_A = H_D = -\frac{M_B}{h}$$

Extract: '*Kleinlogel, Rahmenformeln*' 11. *Auflage Berlin—Verlag von Wilhelm Ernst & Sohn.*

Frame III



FRAME DATA

Coefficients:

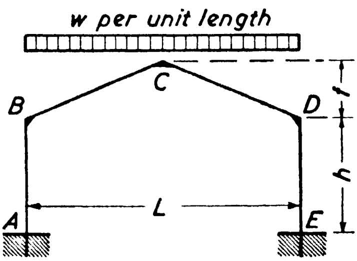
$$k = \frac{I_2}{I_1} \cdot \frac{h}{s} \qquad \phi = \frac{f}{h}$$

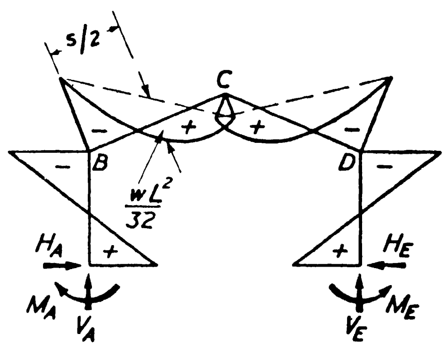
$$m = 1 + \phi$$

$$B = 3k + 2 \qquad C = 1 + 2m$$

$$K_1 = 2(k + 1 + m + m^2) \qquad K_2 = 2(k + \phi^2)$$

$$R = \phi C - k \qquad N_1 = K_1 K_2 - R^2 \qquad N_2 = 3k + B$$





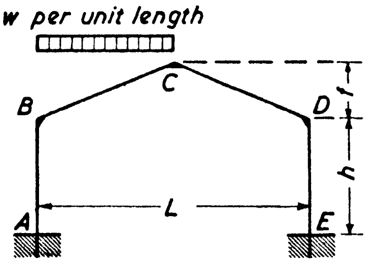
$$M_A = M_E = \frac{wL^2}{16} \cdot \frac{k(8 + 15\phi) + \phi(6 - \phi)}{N_1}$$

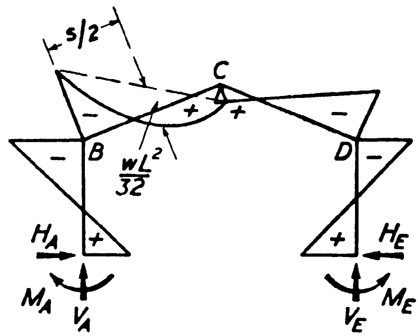
$$M_B = M_D = -\frac{wL^2}{16} \cdot \frac{k(16 + 15\phi) + \phi^2}{N_1}$$

$$M_C = \frac{wL^2}{8} - \phi M_A + m M_B$$

$$V_A = V_E = \frac{wL}{2} \qquad H_A = H_E = \frac{M_A - M_B}{h}$$

Extract: 'Kleinlogel, Rahmenformeln' 11. *Auflage Berlin—Verlag von Wilhelm Ernst & Sohn.*





$$\text{Constants: } {}^{*}X_1 = \frac{wL^2}{32} \cdot \frac{k(8 + 15\phi) + \phi(6 - \phi)}{N_1}$$

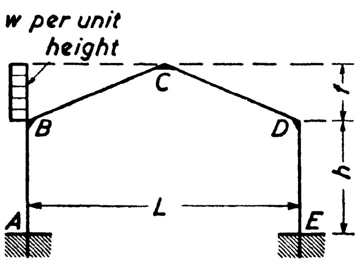
$$^{*}X_2 = \frac{wL^2}{32} \cdot \frac{k(16 + 15\phi) + \phi^2}{N_1} \qquad X_3 = \frac{wL^2}{32N_2}$$

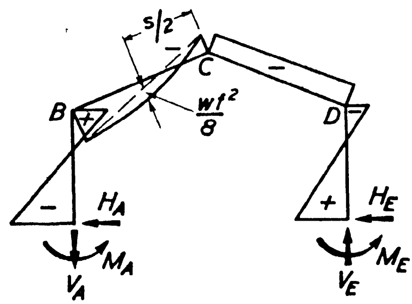
$$M_A = +X_1 - X_3 \qquad M_B = -X_2 - X_3 \qquad M_E = +X_1 + X_3 \qquad M_D = -X_2 + X_3$$

$$^{*}M_C = \frac{wL^2}{16} - \phi X_1 - mX_2$$

$$V_E = \frac{wL}{8} - \frac{2X_3}{L} \qquad V_A = \frac{wL}{2} - V_E \qquad H_A = H_E = \frac{X_1 + X_2}{h}$$

* Note that X_1, $-X_2$ and M_C are respectively half the values of M_A $(=M_E)$, M_B $(=M_D)$ and M_C from the previous set of formulæ where the whole span was loaded.





$$\text{Constants: } X_1 = \frac{wf^2}{8} \cdot \frac{k(9\phi + 4) + \phi(6 + \phi)}{N_1}$$

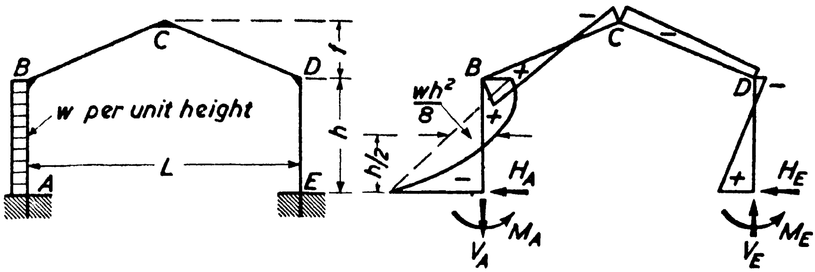
$$X_2 = \frac{wf^2}{8} \cdot \frac{k(8 + 9\phi) - \phi^2}{N_1} \qquad X_3 = \frac{wfh}{8} \cdot \frac{4B + \phi}{N_2}$$

$$M_A = -X_1 - X_3 \qquad M_B = +X_2 + \left(\frac{wfh}{2} - X_3\right)$$

$$M_E = -X_1 + X_3 \qquad M_D = +X_2 - \left(\frac{wfh}{2} - X_3\right)$$

$$M_C = -\frac{wf^2}{4} + \phi X_1 + mX_2$$

$$V_A = -V_E = -\frac{wfh(2 + \phi)}{2L} + \frac{2X_3}{L} \qquad H_E = \frac{wf}{2} - \frac{X_1 + X_2}{h} \qquad H_A = -(wf - H_E)$$



$$\text{Constants: } X_1 = \frac{wh^2}{8} \cdot \frac{k(k+6) + k\phi(15+16\phi) + 6\phi^2}{N_1}$$

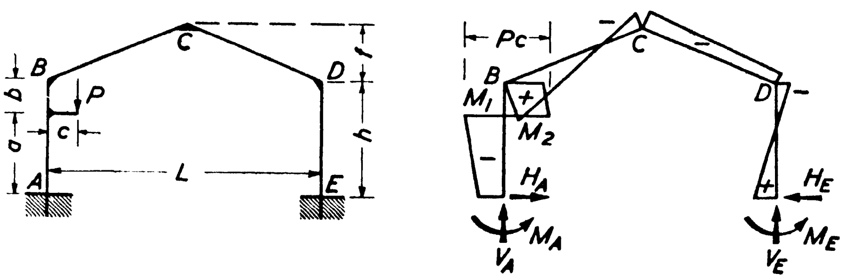
$$X_2 = \frac{wh^2 k(9\phi + 8\phi^2 - k)}{8N_1} \qquad X_3 = \frac{wh^2(2k+1)}{2N_2}$$

$$M_A = -X_1 - X_3 \qquad M_B = +X_2 + \left(\frac{wh^2}{4} - X_3\right)$$

$$M_E = -X_1 + X_3 \qquad M_D = +X_2 - \left(\frac{wh^2}{4} - X_3\right)$$

$$M_C = -\frac{whf}{4} + \phi X_1 + mX_2$$

$$V_A = -V_E = -\frac{wh^2}{2L} + \frac{2X_3}{L} \qquad H_E = \frac{wh}{4} - \frac{X_1 + X_2}{h} \qquad H_A = -(wh - H_E)$$



$$\text{Constants: } a_1 = \frac{a}{h} \qquad b_1 = \frac{b}{h}$$

$$Y_1 = Pc[2\phi^2 - (1 - 3b_1^2)k] \qquad Y_2 = Pc[\phi C + (3a_1^2 - 1)k]$$

$$X_1 = \frac{Y_1K_1 - Y_2R}{2N_1} \qquad X_2 = \frac{Y_2K_2 - Y_1R}{2N_1} \qquad X_3 = \frac{Pc}{2} \cdot \frac{B - 3(a_1 - b_1)k}{N_2}$$

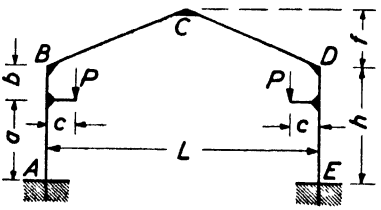
$$M_A = -X_1 - X_3 \qquad M_B = +X_2 + \left(\frac{Pc}{2} - X_3\right)$$

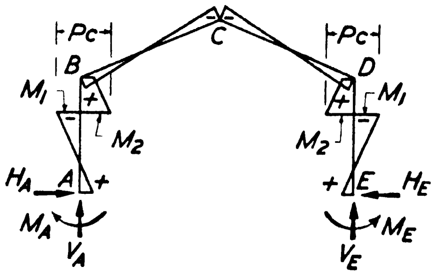
$$M_E = -X_1 + X_3 \qquad M_D = +X_2 - \left(\frac{Pc}{2} - X_3\right) \qquad M_C = -\frac{\phi Pc}{2} + \phi X_1 + mX_2$$

$$M_1 = M_A - H_A a \qquad M_2 = M_B + H_E b$$

$$V_E = \frac{Pc - 2X_3}{L} \qquad V_A = P - V_E \qquad H_A = H_E = \frac{Pc}{2h} \quad \frac{X_1 + X_2}{h}$$

Extract: 'Kleinlogel, Rahmenformeln' 11. *Auflage Berlin—Verlag von Wilhelm Ernst & Sohn.*





Constants: $a_1 = \frac{a}{h} \qquad b_1 = \frac{b}{h}$

$$Y_1 = Pc[2\phi^2 - (1 - 3b_1^2)k]$$

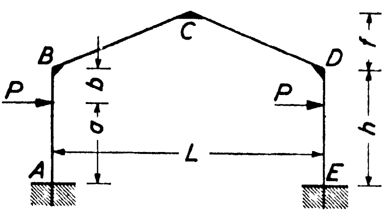
$$Y_2 = Pc[\phi C + (3a_1^2 - 1)k]$$

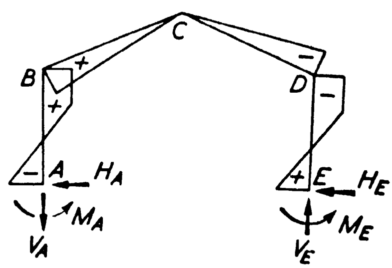
$$M_A = M_E = \frac{Y_2 R - Y_1 K_1}{N_1} \qquad M_B = M_D = \frac{Y_2 K_2 - Y_1 R}{N_1}$$

$$M_C = -\phi(Pc + M_A) + mM_B$$

$$V_A = V_D = P \qquad H_A = H_E = \frac{Pc + M_A - M_B}{h}$$

$$M_1 = M_A - H_A a \qquad M_2 = M_B + H_E b$$



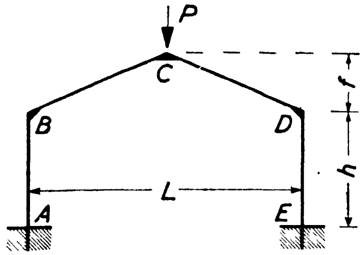


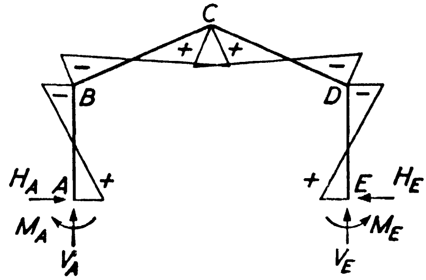
Constant: $X_1 = \frac{Pa(B + 3b_1 k)}{N_2}$

$$M_A = -M_E = -X_1 \qquad M_B = -M_D = Pa - X_1 \qquad M_C = 0$$

$$V_A = -V_E = -2\left[\frac{Pa - X_1}{L}\right] \qquad H_A = -H_E = -P$$

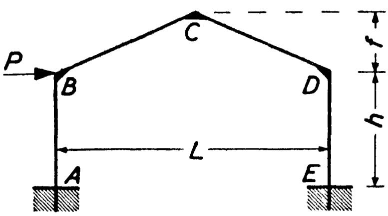
Extract: 'Kleinlogel, Rahmenformeln' 11. *Auflage Berlin—Verlag von Wilhelm Ernst & Sohn.*

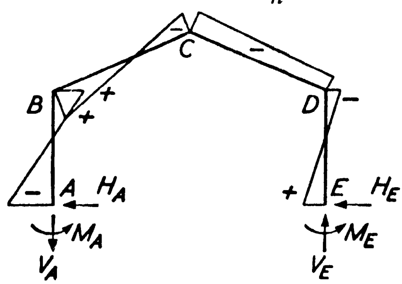




$$M_A = M_E = \frac{3PL(k + 2k\phi + \phi)}{4N_1} \qquad M_B = M_D = -\frac{3PLkm}{2N_1}$$

$$M_C = \frac{PL}{4} - \phi M_A + mM_B \qquad V_A = V_E = P/2 \qquad H_A = H_E = \frac{M_A - M_B}{h}$$





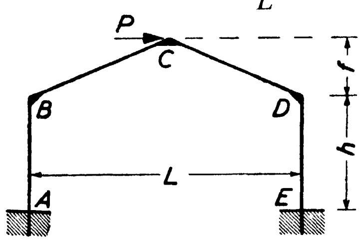
$$\text{Constants: } X_1 = \frac{3Pf(k + 2\phi k + \phi)}{2N_1} \qquad X_2 = \frac{3Pfmk}{N_1} \qquad X_3 = \frac{PhB}{2N_2}$$

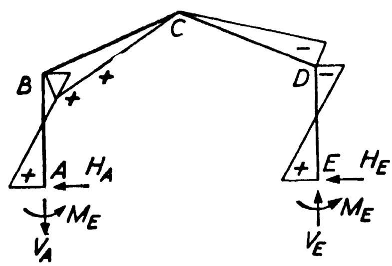
$$M_A = -X_1 - X_3 \qquad M_B = +X_2 + \left(\frac{Ph}{2} - X_3\right)$$

$$M_E = -X_1 + X_3 \qquad M_D = +X_2 - \left(\frac{Ph}{2} - X_3\right)$$

$$M_C = -\frac{Pf}{2} + \phi X_1 + mX_2$$

$$V_A = -V_E = -\frac{Ph - 2X_3}{L} \qquad H_E = \frac{P}{2} - \frac{X_1 + X_2}{h} \qquad H_A = -(P - H_E)$$



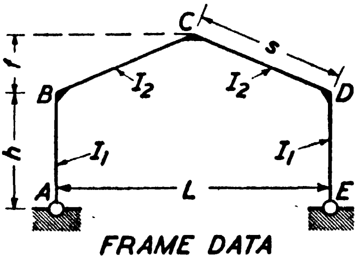


$$M_A = -M \quad = -\frac{PhB}{2N_2} \qquad M_B = -M_D = +\frac{3Phk}{2N_2} \qquad M_C = 0$$

$$V_A = -V_E = -\frac{P(h + f) + 2M_A}{L} \qquad H_A = -H_E = -\frac{P}{2}$$

Extract: 'Kleinlogel, Rahmenformeln' 11. *Auflage Berlin—Verlag von Wilhelm Ernst & Sohn.*

Frame IV



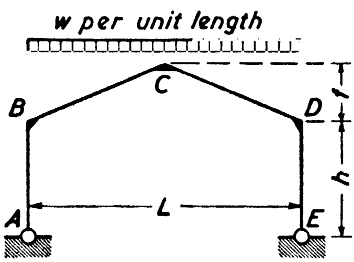
Coefficients:

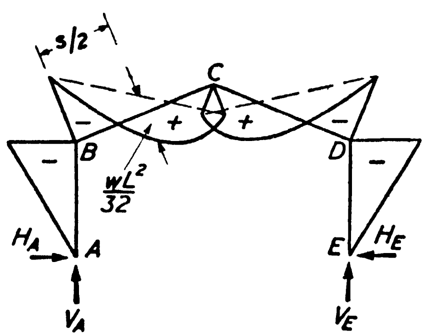
$$k=\frac{I_2}{I_1}\cdot\frac{h}{s}$$

$$\phi=\frac{f}{h}$$

$$m=1+\phi$$

$$B=2(k+1)+m \qquad C=1+2m \qquad N=B+mC$$

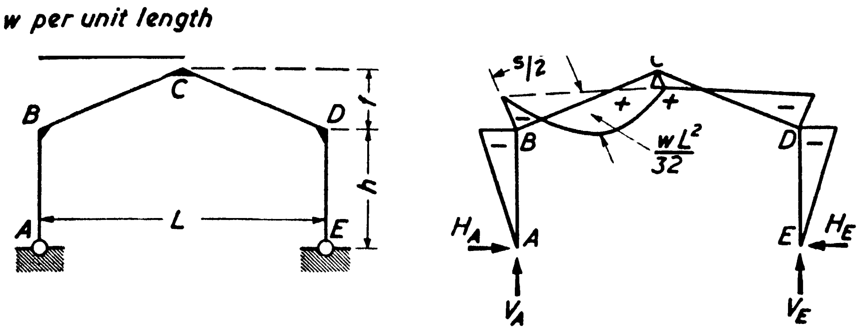




$$M_B=M_D=-\frac{wL^2(3+5m)}{16N} \qquad M_C=\frac{wL^2}{8}+mM_B$$

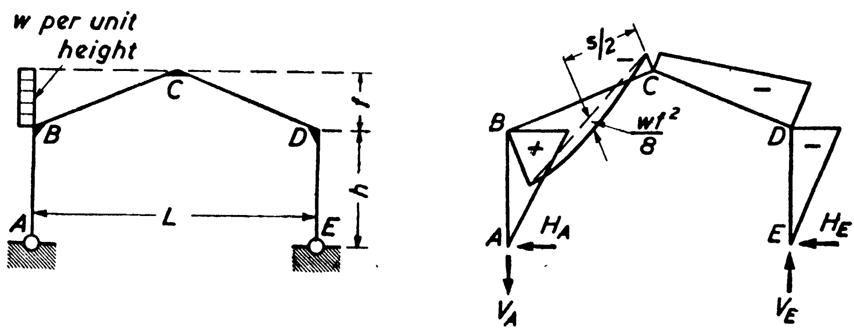
$$H_A=H_E=-\frac{M_B}{h} \qquad V_A=V_E=\frac{wL}{2}$$

Extract: '*Kleinlogel, Rahmenformeln*' 11. *Auflage Berlin—Verlag von Wilhelm Ernst & Sohn.*



$$M_B = M_D = -\frac{wL^2(3+5m)}{32N} \qquad M_C = \frac{wL^2}{16} + mM_B$$

$$H_A = H_E = -\frac{M_B}{h} \qquad V_A = \frac{3wL}{8} \qquad V_E = \frac{wL}{8}$$



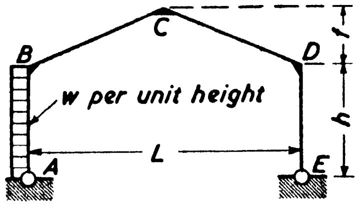
$$\text{Constant: } X = \frac{wf^2(C+m)}{8N}$$

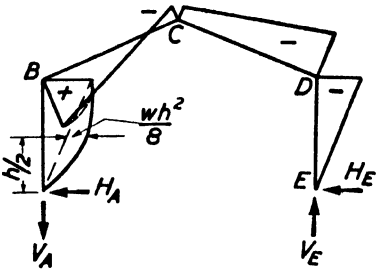
$$M_B = +X + \frac{wfh}{2} \qquad M_C = -\frac{wf^2}{4} + mX$$

$$M_D = +X - \frac{wfh}{2} \qquad V_A = -V_E = -\frac{wfh(1+m)}{2L}$$

$$H_A = -\frac{X}{h} - \frac{wf}{2} \qquad H_E = -\frac{X}{h} + \frac{wf}{2}$$

Extract: 'Kleinlogel, Rahmenformeln' 11. *Auflage Berlin—Verlag von Wilhelm Ernst & Sohn.*

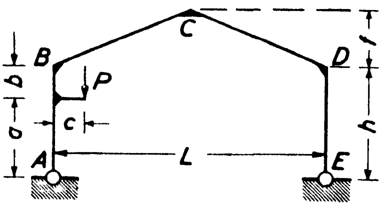


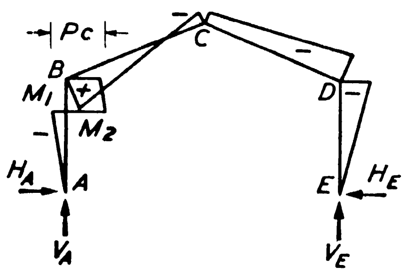


$$M_D = -\frac{wh^2}{8} \cdot \frac{2(B+C)+k}{N} \qquad M_B = \frac{wh^2}{2} + M_D$$

$$M_C = \frac{wh^2}{4} + mM_D$$

$$V_A = -V_E = -\frac{wh^2}{2L} \qquad H_E = -\frac{M_D}{h} \qquad H_A = -(wh - H_E)$$





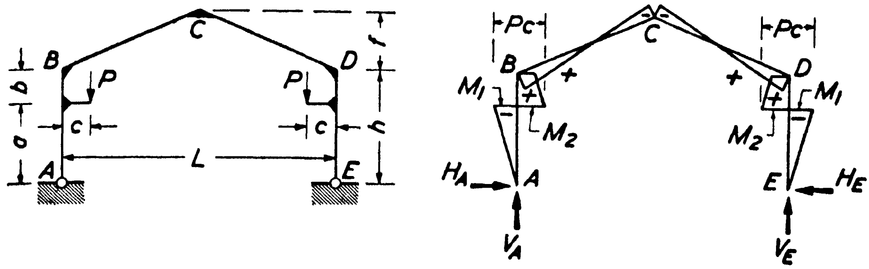
$$\text{Constants: } a_1 = \frac{a}{h} \qquad X = \frac{Pc}{2} \cdot \frac{B + C - k(3a_1^2 - 1)}{N}$$

$$M_B = Pc - X \qquad M_D = -X \qquad M_C = \frac{Pc}{2} - mX$$

$$M_1 = -a_1 X \qquad M_2 = Pc - a_1 X$$

$$V_E = \frac{Pc}{L} \qquad V_A = P - V_E \qquad H_A = H_E = \frac{X}{h}$$

Extract: 'Kleinlogel, Rahmenformeln' 11. *Auflage Berlin—Verlag von Wilhelm Ernst & Sohn.*

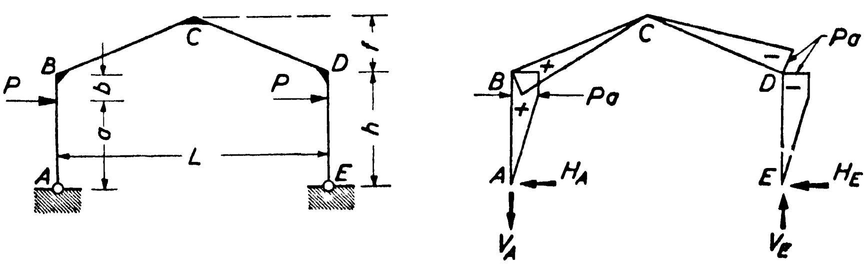


$$\text{Constant: } a_1 = \frac{a}{h}$$

$$M_B = M_D = Pc \cdot \frac{\phi C + k(3a_1^2 - 1)}{N} \qquad M_C = -\phi Pc + mM_B$$

$$H_A = H_E = \frac{Pc - M_B}{h} \qquad V_A = V_E = P$$

$$M_1 = -a_1(Pc - M_B) \qquad M_2 = (1 - a_1)Pc + a_1 M_B$$

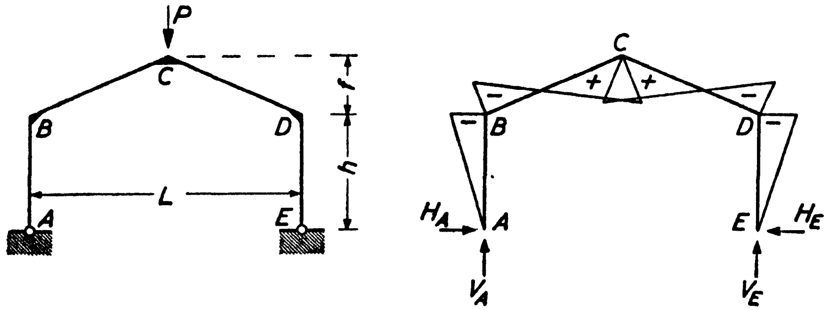


$$M_B = -M_D = Pa \qquad M_C = 0$$

$$H_A = -H_E = -P \qquad V_A = -V_E = -\frac{2Pa}{L}$$

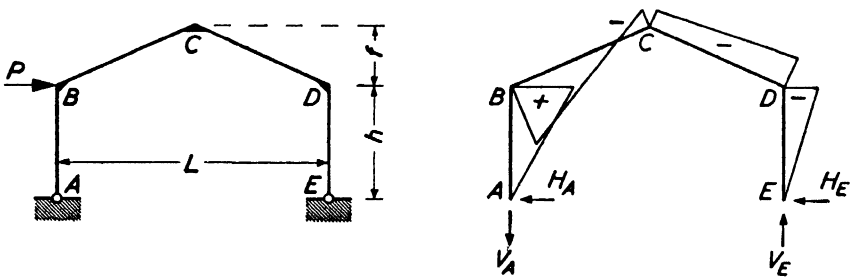
$$\text{Moment at loads} = \pm Pa$$

Extract: 'Kleinlogel, Rahmenformeln' 11. Auflage Berlin—Verlag von Wilhelm Ernst & Sohn.



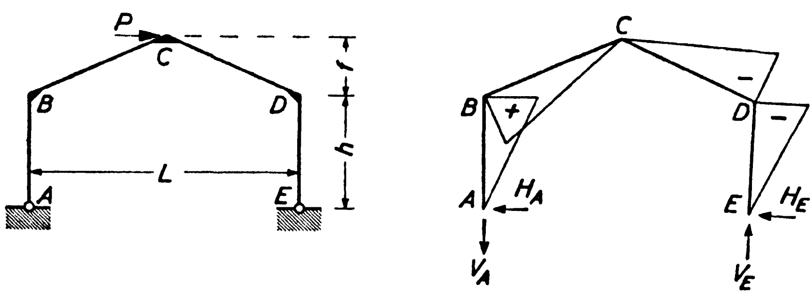
$$M_B = M_D = -\frac{PL}{4}\cdot\frac{C}{N} \qquad M_C = +\frac{PL}{4}\cdot\frac{B}{N}$$

$$V_A = V_E = \frac{P}{2} \qquad H_A = H_E = -\frac{M_B}{h}$$



$$M_D = -\frac{Ph(B+C)}{2N} \qquad M_B = Ph + M_D \qquad M_C = \frac{Ph}{2} + mM_D$$

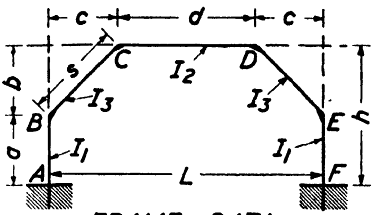
$$V_A = -V_E = -\frac{Ph}{L} \qquad H_E = -\frac{M_D}{h} \qquad H_A = -(P - H_E)$$



$$M_B = -M_D = +\frac{Ph}{2} \qquad M_C = 0 \qquad V_A = -V_E = -\frac{Phm}{L} \qquad H_A = -H_E = -\frac{P}{2}$$

Extract: 'Kleinlogel, Rahmenformeln' 11. *Auflage Berlin—Verlag von Wilhelm Ernst & Sohn.*

Frame V



FRAME DATA

Coefficients:

$$k_1 = \frac{I_3}{I_1} \cdot \frac{a}{s} \qquad k_2 = \frac{I_3}{I_2} \cdot \frac{d}{s}$$

$$c_1 = \frac{c}{L} \qquad d_1 = \frac{d}{L}$$

$$\phi = \frac{b}{a} \qquad m = \frac{h}{a} = 1 + \phi$$

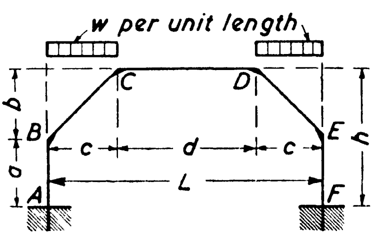
$$(2c_1 + d_1 = 1)$$

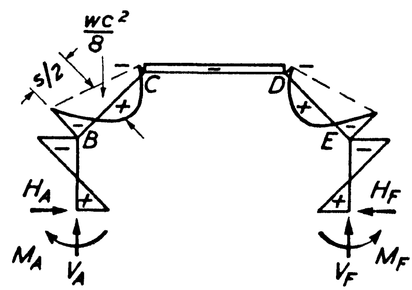
$$C_1 = \phi(2 + 3k_2) \qquad K_1 = 2(k_1 + 1) + m(1 + C_2)$$

$$C_2 = 1 + m(2 + 3k_2) \qquad K_2 = 2k_1 + \phi C_1$$

$$R = \phi C_2 - k_1 \qquad N_1 = K_1 K_2 - R^2$$

$$B = 3k_1 + 2 + d_1 \qquad C_3 = 1 + d_1(2 + k_2) \qquad N_2 = 3k_1 + B + d_1 C_3$$





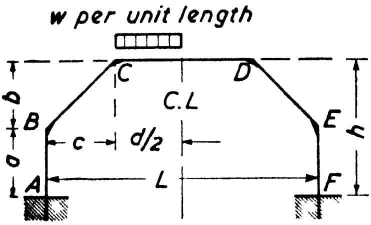
$$\text{Constants: } Y_1 = \frac{wc^2}{4}(2C_1 + \phi) \qquad Y_2 = \frac{wc^2}{4}(2C_2 + 1 + m)$$

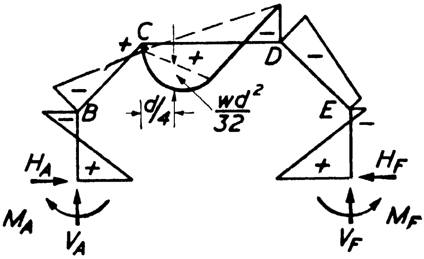
$$M_A = M_F = \frac{Y_1 K_1 - Y_2 R}{N_1} \qquad M_B = M_E = -\frac{Y_2 K_2 - Y_1 R}{N_1}$$

$$M_C = M_D = \frac{wc^2}{2} - \phi M_A + m M_B$$

$$V_A = V_F = wc \qquad H_A = H_F = \frac{M_A - M_B}{a}$$

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$$\text{Constants: } Y_3 = \frac{wd^2}{32}(8c_1C_3 + d_1k_2)$$

$$Y_1 = \frac{wd}{4}(2cC_1 + d\phi k_2) \qquad Y_2 = \frac{wd}{4}(2cC_2 + dmk_2)$$

$$X_1 = \frac{Y_1K_1 - Y_2R}{2N_1} \qquad X_2 = \frac{Y_2K_2 - Y_1R}{2N_1} \qquad X_3 = \frac{Y_3}{2N_2}$$

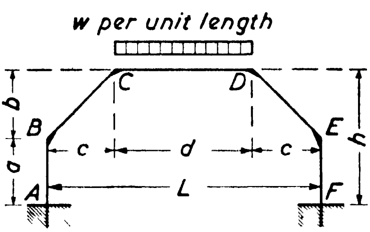
$$M_A = +X_1 - X_3 \qquad M_B = -X_2 - X_3 \qquad H_A = H_F = \frac{X_1 + X_2}{a}$$

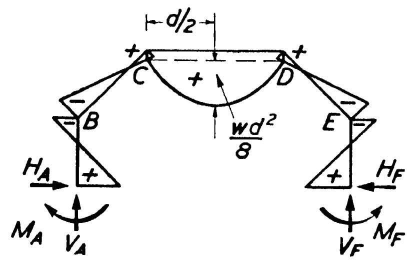
$$M_F = +X_1 + X_3 \qquad M_E = -X_2 + X_3$$

$$M_C = \frac{wdc}{4} - \phi X_1 - mX_2 + \left(\frac{c_1wd^2}{8} - d_1X_3\right)$$

$$M_D = \frac{wdc}{4} - \phi X_1 - mX_2 - \left(\frac{c_1wd^2}{8} - d_1X_3\right)$$

$$V_A = \frac{wd(4c + 3d) + 16X_3}{8L} \qquad V_I = \frac{wd(4c + d) - 16X_3}{8L}$$





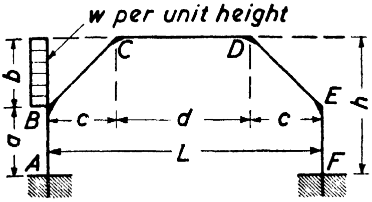
$$\text{Constants: } Y_1 = wd\left(cC_1 + \frac{d\phi k_2}{2}\right) \qquad Y_2 = wd\left(cC_2 + \frac{dmk_2}{2}\right)$$

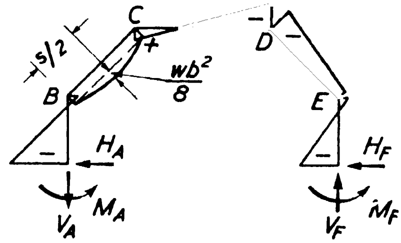
$$X_1 = \frac{Y_1K_1 - Y_2R}{2N_1} \qquad X_2 = \frac{Y_2K_2 - Y_1R}{2N_1}$$

$$M_A = M_F = X_1 \qquad M_B = M_E = -X_2 \qquad M_C = M_D = \frac{wdc}{2} - \phi X_1 - mX_2$$

$$V_A = V_F = \frac{wd}{2} \qquad H_A = H_F = \frac{X_1 + X_2}{a}$$

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$$\text{Constants: } Y_1 = \frac{wb^2}{4}(2C_1 - \phi)$$

$$Y_2 = \frac{wb^2}{4}(2C_2 - 1 - m) \qquad Y_3 = wab(B + d_1C_3) + \frac{wb^2}{4}(2d_1C_3 + 1 + d_1)$$

$$X_1 = \frac{Y_1K_1 - Y_2R}{2N_1} \qquad X_2 = \frac{Y_2K_2 - Y_1R}{2N_1} \qquad X_3 = \frac{Y_3}{2N_2}$$

$$M_A = -X_1 - X_3 \qquad M_B = +X_2 + \frac{wab}{2} - X_3$$

$$M_F = -X_1 + X_3 \qquad M_E = +X_2 - \frac{wab}{2} + X_3$$

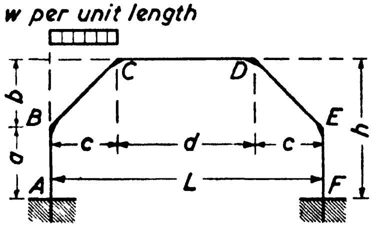
$$M_C = -\frac{wb^2}{4} + \phi X_1 + mX_2 + \frac{d_1}{2}\left(wab + \frac{wb^2}{2} - 2X_3\right)$$

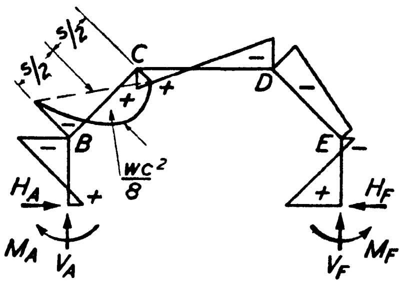
$$M_D = -\frac{wb^2}{4} + \phi X_1 + mX_2 - \frac{d_1}{2}\left(wab + \frac{wb^2}{2} - 2X_3\right)$$

$$V_A = -V_F = \frac{2wab + wb^2 - 4X_3}{2L}$$

$$H_F = \frac{wb}{2} - \frac{X_1 + X_2}{a} \qquad H_A = -(wb - H_F)$$

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Constants: $Y_1 = \frac{wc^2}{4}(2C_1 + \phi)$ $\quad Y_2 = \frac{wc^2}{4}(2C_2 + 1 + m)$ $\quad Y_3 = \frac{wc^2}{4}(2d_1C_3 + 1 + d_1)$

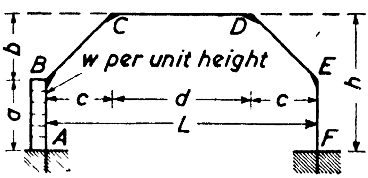
$$X_1 = \frac{Y_1K_1 - Y_2R}{2N_1} \qquad X_2 = \frac{Y_2K_2 - Y_1R}{2N_1} \qquad X_3 = \frac{Y_3}{2N_2}$$

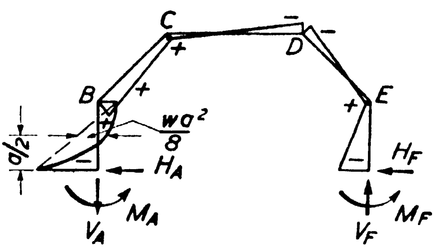
$$M_A = +X_1 - X_3 \qquad M_B = -X_2 - X_3 \qquad M_F = +X_1 + X_3 \qquad M_E = -X_2 + X_3$$

$$M_C = \frac{wc^2}{4} - \phi X_1 - mX_2 + \frac{d_1}{2}\left(\frac{wc^2}{2} - 2X_3\right)$$

$$M_D = \frac{wc^2}{4} - \phi X_1 - mX_2 - \frac{d_1}{2}\left(\frac{wc^2}{2} - 2X_3\right)$$

$$V_F = \frac{wc^2 - 4X_3}{2L} \qquad V_A = wc - V_F \qquad H_A = H_F = \frac{X_1 + X_2}{a}$$





Constants: $Y_1 = \frac{wa^2}{4}(2\phi C_1 + k_1)$ $\quad Y_2 = \frac{wa^2}{4}(2\phi C_2 - k_1)$ $\quad Y_3 = \frac{wa^2}{2}(B + d_1C_3 + k_1)$

The formulæ for X_1, X_2 and X_3 are the same as above.

$$M_A = -X_1 - X_3 \qquad M_B = +X_2 + \left(\frac{wa^2}{4} - X_3\right)$$

$$M_F = -X_1 + X_3 \qquad M_E = +X_2 - \left(\frac{wa^2}{4} - X_3\right)$$

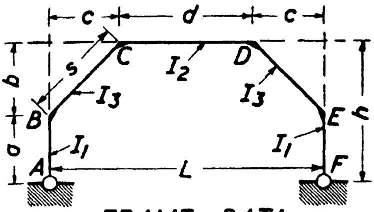
$$M_C = -\frac{wa^2\phi}{4} + \phi X_1 + mX_2 + d_1\left(\frac{wa^2}{4} - X_3\right)$$

$$M_D = -\frac{wa^2\phi}{4} + \phi X_1 + mX_2 - d_1\left(\frac{wa^2}{4} - X_3\right)$$

$$V_A = -V_F = -\frac{2}{L}\left(\frac{wa^2}{4} - X_3\right) \qquad H_F = \frac{wa}{4} - \frac{X_1 + X_2}{a} \qquad H_A = -(wa - H_F)$$

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Frame VI



FRAME DATA

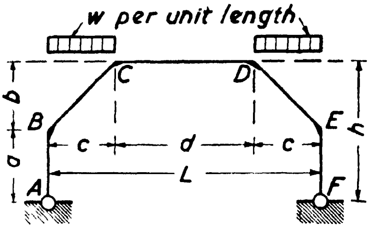
Coefficients:

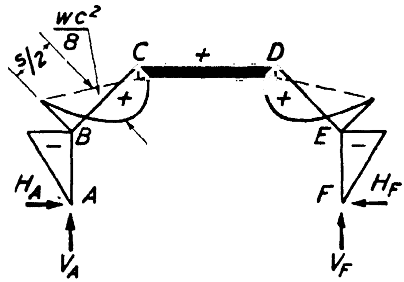
$$k_1=\frac{I_3}{I_1}\cdot\frac{a}{s} \qquad k_2=\frac{I_3}{I_2}\cdot\frac{d}{s}$$

$$a_1=\frac{a}{h} \qquad c_1=\frac{c}{L}$$

$$B=2a_1(k_1+1)+1 \qquad C=a_1+2+3k_2$$

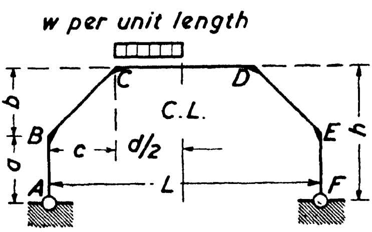
$$N=a_1B+C$$

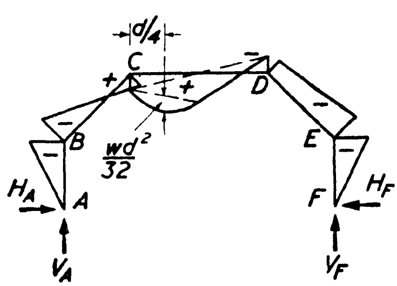




Constant: $X=\frac{wc^2}{4}\cdot\frac{3a_1+5+6k_2}{N}$

$$M_B=M_E=-a_1X \qquad M_C=M_D=\frac{wc^2}{2}-X \qquad V_A=V_F=wc \qquad H_A=H_F=\frac{X}{h}$$



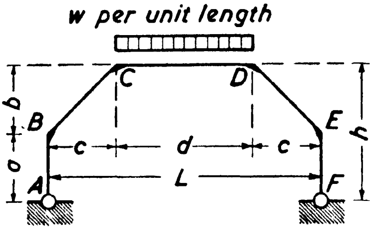


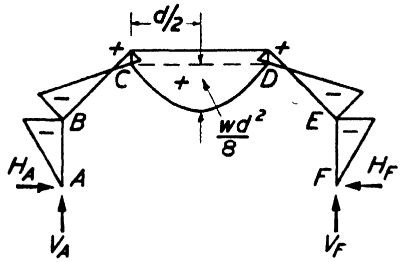
Constant: $X=\frac{wd(2cC+dk_2)}{8N}$

$$M_C=\frac{c_1wd}{8}(3d+4c)-X \qquad M_D=\frac{c_1wd}{8}(4c+d)-X \qquad M_B=M_E=-a_1X$$

$$V_A=\frac{wd(3d+4c)}{8L} \qquad V_F=\frac{wd(4c+d)}{8L} \qquad H_A=H_F=\frac{X}{h}$$

Extract: 'Kleinlogel, Rahmenformeln' 11. *Auflage Berlin—Verlag von Wilhelm Ernst & Sohn.*

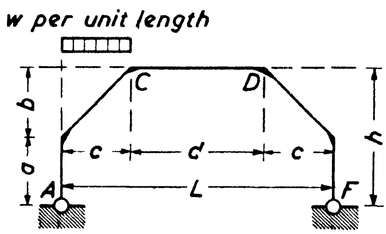


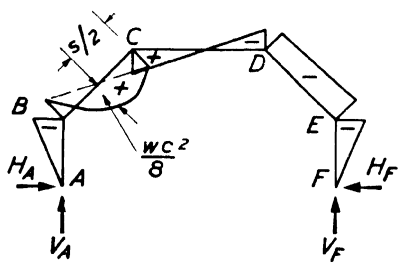


$$\text{Constant: } X = \frac{wd}{4} \cdot \frac{2cC + dk_2}{N}$$

$$M_B = M_E = -a_1 X \qquad M_C = M_D = \frac{wdc}{2} - X$$

$$V_A = V_F = \frac{wd}{2} \qquad H_A = H_F = \frac{X}{h}$$



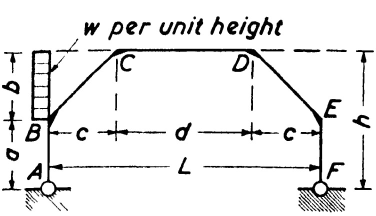


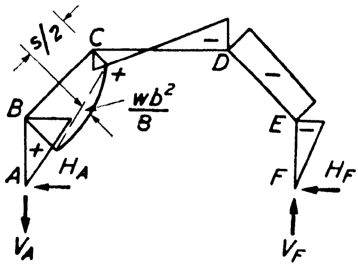
$$\text{Constant: } X = \frac{wc^2(2C + a_1 + 1)}{8N}$$

$$M_B = M_E = -a_1 X \qquad M_C = (1 - c_1)\frac{wc^2}{2} - X$$

$$V_F = \frac{wc^2}{2L} \qquad V_A = wc - V_F \qquad H_A = H_F = \frac{X}{h}$$

Extract: 'Kleinlogel, Rahmenformeln' 11. Auflage Berlin—Verlag von Wilhelm Ernst & Sohn.



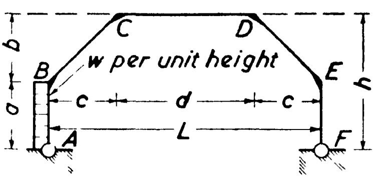


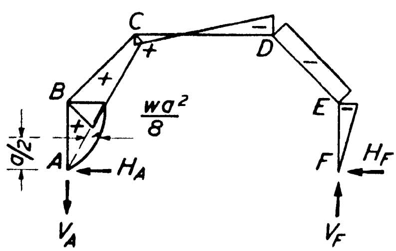
$$\text{Constant: } X=\frac{wb}{8}\cdot\frac{4a(B+C)+b(2C+a_1+1)}{N}$$

$$M_B=wba-a_1X \qquad M_E=-a_1X$$

$$M_C=V_F(L-c)-X \qquad M_D=-X+V_F\,.\,c$$

$$V_A=-V_F=-\frac{wb(a+h)}{2L} \qquad H_F=\frac{X}{h} \qquad H_A=-(wb-H_F)$$





$$\text{Constant: } X=\frac{wa^2}{8}\cdot\frac{2(B+C)+a_1k_1}{N}$$

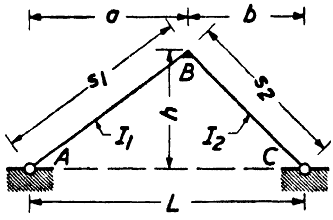
$$M_B=\frac{wa^2}{2}-a_1X \qquad M_E=-a_1X$$

$$M_C=V_F(L-c)-X \qquad M_D=-X+V_F\,.\,c$$

$$V_A=-V_F=-\frac{wa^2}{2L} \qquad H_F=\frac{X}{h} \qquad H_A=-(wa-H_F)$$

Extract: 'Kleinlogel, Rahmenformeln' 11. *Auflage Berlin—Verlag von Wilhelm Ernst & Sohn.*

Frame VII



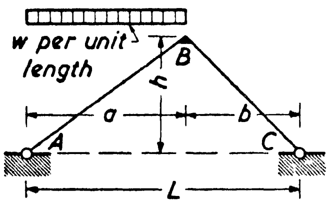
FRAME DATA

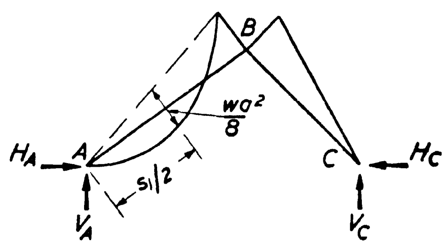
Coefficients:

$$k = \frac{I_1}{I_2} \cdot \frac{s_2}{s_1} \qquad N = 1 + k$$

$$a_1 = \frac{a}{L} \qquad b_1 = \frac{b}{L}$$

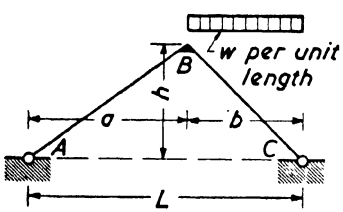
$$(a_1 + b_1 = 1)$$

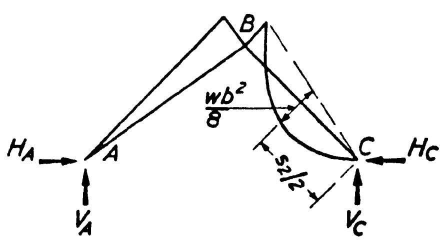




$$M_B = -\frac{wa^2}{8N} \qquad V_C = \frac{wa^2}{2L} \qquad V_A = wa - V_C$$

$$H_A = H_C = \frac{wa^2 b_1}{2h} - \frac{M_B}{h}$$

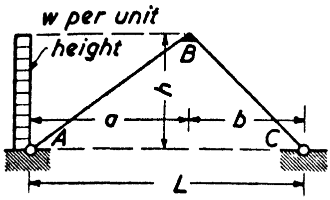


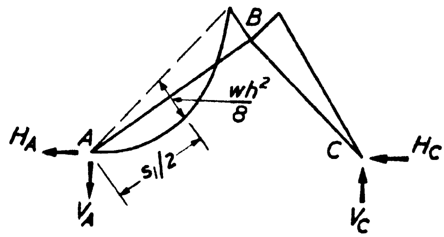


$$M_B = -\frac{wb^2 k}{8N} \qquad V_A = \frac{wb^2}{2L} \qquad V_C = wb - V_A$$

$$H_A = H_C = \frac{wb^2 a_1}{2h} - \frac{M_B}{h}.$$

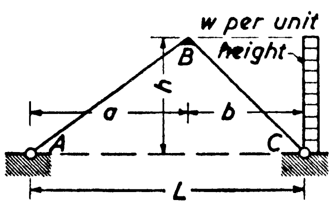
Extract: 'Kleinlogel, Rahmenformeln' 11. *Auflage Berlin—Verlag von Wilhelm Ernst & Sohn.*

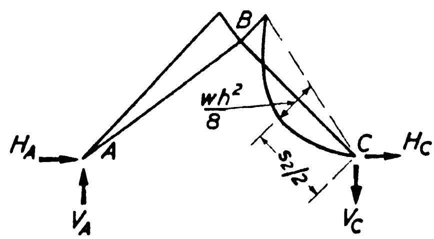




$$M_B = -\frac{wh^2}{8N} \qquad V_A = -V_C = -\frac{wh^2}{2L}$$

$$H_C = \frac{whb_1}{2} - \frac{M_B}{h} \qquad H_A = -(wh - H_C)$$



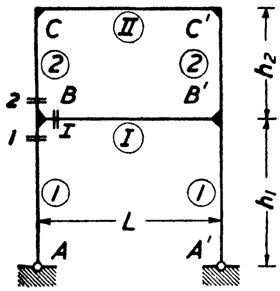


$$M_B = -\frac{wh^2k}{8N} \qquad V_A = -V_C = \frac{wh^2}{2L}$$

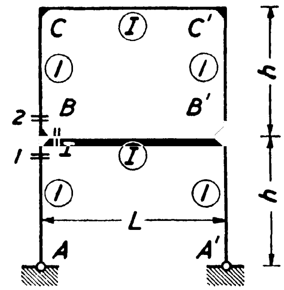
$$H_A = \frac{wha_1}{2} - \frac{M_B}{h} \qquad H_C = -(wh - H_A)$$

Extract: 'Kleinlogel, Rahmenformeln' 11. *Auflage Berlin—Verlag von Wilhelm Ernst & Sohn.*

Frame VIII



FRAME DATA



SPECIAL CASE

Constants: I_c = any convenient value chosen for I.

$$L_1 = \frac{I_c}{I_I} \qquad L_2 = \frac{I_c}{I_{II}} \qquad x_1 = \frac{I_c h_1}{I_1 L} \qquad x_2 = \frac{I_c h_2}{I_2 L} \qquad n = \frac{h_1}{h_2} \qquad m = \frac{h_2}{h_1}$$

$$K_1 = 2x_1 + 3L_1 \qquad K_2 = 2x_2 + 3L_2 \qquad N_1 = 3s_2(x_2 + 2L_2) + 2r_1 x_1$$

$$s_2 = \frac{x_2}{K_2} \qquad r_1 = \frac{3L_1}{K_1} \qquad t_1 = \frac{2x_1}{K_1} \qquad k_1 = \frac{1}{2K_1} \qquad k_2 = \frac{1}{2K_2}$$

$$B = L_1 + 3x_2 \qquad C = 3x_2 + L_2 \qquad F = 6x_2 + L_2 \qquad N_2 = B + C$$

Special case*:

$$I_c = I_I = I_{II} \qquad I_1 = I_2 \qquad h_1 = h_2 = h$$

$$L_1 = L_2 = 1 \qquad x_1 = x_2 = x \qquad n = 1 \qquad m = 1$$

Influence Coefficients:

$$n_{22} = \frac{1}{2N_1} \qquad n_{21} = r_1 n_{22} \qquad n_{23} = s_2 n_{22}$$

$$n_{11} = k_1 + r_1 n_{21} \qquad n_{31} = s_2 n_{21}$$

$$n_{12} = r_1 n_{22} \qquad n_{32} = s_2 n_{22} \qquad m_1 = \frac{1}{2N_2}$$

$$n_{13} = r_1 n_{23} \qquad n_{33} = k_2 + s_2 n_{23}$$

$$u = L_1 m_1 = \frac{L_1}{2N_2} \qquad v_2 = \frac{C}{2N_2} \qquad v_3 = \tfrac{1}{2} - v_2 = \frac{B}{2N_2}$$

* All those constants referring to the special case which can be simplified to the general constants are enclosed in light ruled lines on the following pages.

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Composite Influence Coefficients:

$$r_{11} = +n_{11} - n_{21} = k_1 - t_1 n_{21} = n_{1I} \qquad s_{21} = +n_{21} - n_{31}$$

$$r_{12} = -n_{12} + n_{22} = +t_1 n_{22} = n_{2I} \qquad s_{22} = +n_{22} - n_{32}$$

$$r_{13} = -n_{13} + n_{23} = +t_1 n_{23} = n_{3I} \qquad s_{23} = -n_{23} + n_{33}$$

$$r_{1I} = +n_{1I} + n_{2I} = +r_{11} + r_{12} \qquad s_{2I} = +n_{2I} - n_{3I} = -s_{21} + s_{22}$$

$$u_I = \tfrac{1}{2} - u = \frac{F}{2N_2} \qquad L_{22} = -u' + \frac{v_2}{2}$$

$$u' = nu \qquad L_{23} = +u' + \frac{v_3}{2}$$

$$u'_I = nu_I \qquad L_{2I} = +u'_I + \frac{v_2}{2} = \frac{n}{2} + L_{22}$$

Special case:

$$a_1 = 1 + x \qquad a_2 = 2 + x \qquad a_3 = 3 + x \qquad a_4 = 4 + x$$

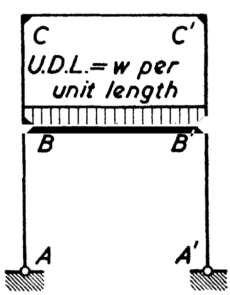
$$K = 2x + 3 \qquad d = 1 + 3x \qquad e = 1 + 6x$$

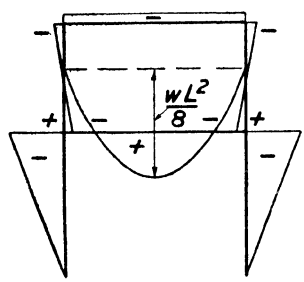
$$n_{11} = \frac{a_1 a_3}{2Kxa_4} \qquad n_{12} = n_{21} = \frac{1}{2xa_4} \qquad r_{11} = \frac{a_2}{2Ka_4} = n_{1I}$$

$$n_{22} = \frac{K}{6xa_4} \qquad n_{13} = n_{31} = \frac{1}{2Ka_4} \qquad r_{12} = \frac{1}{3a_4} = n_{2I}$$

$$n_{33} = \frac{2a_3}{3Ka_4} \qquad n_{23} = n_{32} = \frac{1}{6a_4} \qquad r_{13} = \frac{x}{3Ka_4} = n_{3I}$$

$$m_1 = \frac{1}{4d} = u = u' \qquad v_2 = \tfrac{1}{4} = v_3 \qquad u_I = \frac{e}{4d} = u'_I$$





$$M_{B1} = M'_{B1} = -S_I r_{11}$$

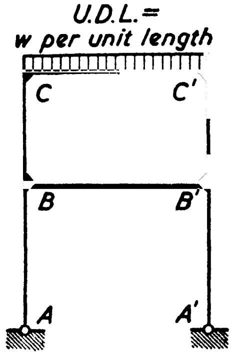
$$M_{B2} = M'_{B2} = +S_I r_{12}$$

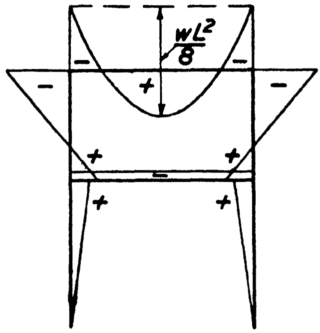
$$M_C = M'_C = -S_I r_{13}$$

$$M_{BI} = M'_{BI} = -S_I r_{1I}$$

where $S_I = \frac{wL^2}{2} \times L_1$

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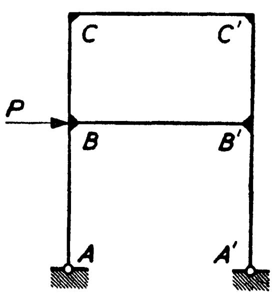
$M_{B1} = M'_{B1} = +S_{II}n_{31}$

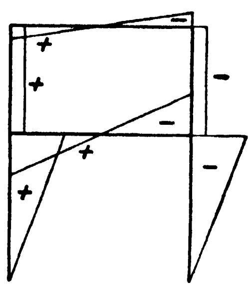
$M_{B2} = M'_{B2} = +S_{II}n_{32}$

$M_C = M'_C = -S_{II}n_{33}$

$M_{BI} = M'_{BI} = -S_{II}n_{3I}$

where $S_{II} = \frac{wL^2}{2} \times L_2$



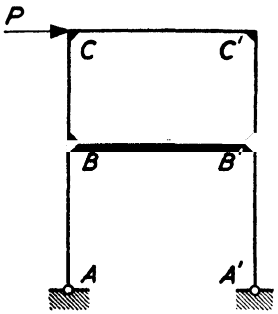


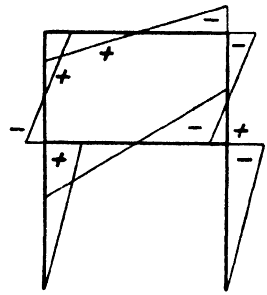
$M_{B1} = +M/2 \qquad M_{B2} = M_C = +Mu \qquad M_{BI} = +Mu_I$

$M'_{B1} = -M/2 \qquad M'_{B2} = M'_C = -Mu \qquad M'_{BI} = -Mu_I$

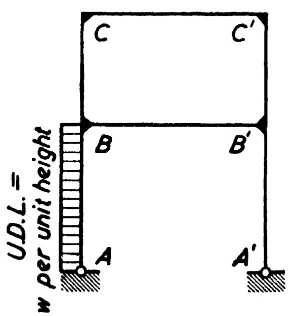
where: $M = Ph_1$

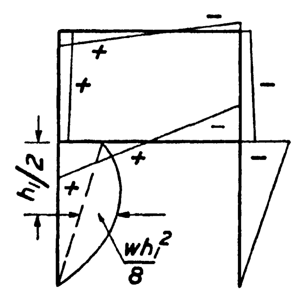
Extract: 'Kleinlogel, Mehrstielige Rahmen', Band I + II. Berlin—Verlag von Wilhelm Ernst & Sohn.





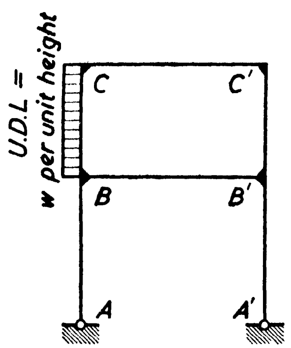
$$M_{B1} = +M/2 \qquad M_{BI} = +Mp_I$$
$$M'_{B1} = -M/2 \qquad M'_{BI} = -Mp_I$$
$$M_{B2} = -Mp_2$$
$$M'_{B2} = +Mp_2 \qquad \text{where:} \quad M = Ph_1$$
$$M_C = +Mp_3 \qquad p_2 = -u + mv_2 \qquad p_3 = +u + mv_3$$
$$M'_C = -Mp_3 \qquad p_I = +u_I + mv_2 = \tfrac{1}{2} + p_2$$

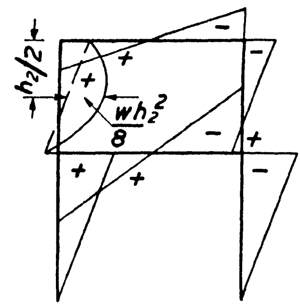




$$M_{B1} = R_1(-n_{11}x_1 + 1) \qquad M'_{B1} = R_1(-n_{11}x_1 - 1)$$
$$M_{B2} = R_1(-n_{12}x_1 + 2u) \qquad M'_{B2} = R_1(-n_{12}x_1 - 2u)$$
$$M_C = R_1(+n_{13}x_1 + 2u) \qquad M'_C = R_1(+n_{13}x_1 - 2u)$$
$$M_{BI} = R_1(-n_{1I}x_1 + 2u_I) \qquad M'_{BI} = R_1(-n_{1I}x_1 - 2u_I)$$
$$\text{where} \quad R_1 = \frac{w(h_1)^2}{4}$$

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$$M_{B1} = R_2(-x_2 s_{21} + 2n) \qquad M'_{B1} = R_2(-x_2 s_{21} - 2n)$$

$$M_{B2} = R_2[x_2(-s_{22} - 2m_1) - 4L_{22}] \qquad M'_{B2} = R_2[x_2(-s_{22} + 2m_1) + 4L_{22}]$$

$$M_C = R_2[x_2(-s_{23} - 2m_1) + 4L_{23}] \qquad M'_C = R_2[x_2(-s_{23} + 2m_1) - 4L_{23}]$$

$$M_{BI} = R_2[x_2(+s_{2I} + 2m_1) + 4L_{2I}] \qquad M'_{BI} = R_2[x_2(+s_{2I} - 2m_1) - 4L_{2I}]$$

where $R_2 = \dfrac{w(h_2)^2}{4}$

Formulæ for Support Reactions, Shear and Axial Forces, together with relationships between moments at the joints.

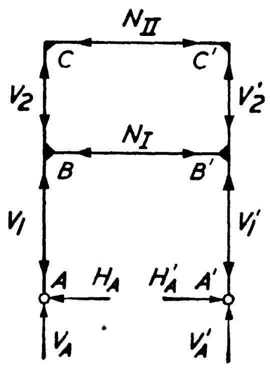
Nomenclature:

V = vertical reaction or axial force in columns.

N = axial force in beams.

T = shear force at ends of beams or columns.

H = horizontal reactions in columns.



(a) For all loads:

$$V_I = \frac{-M_{BI} + M'_{BI}}{L} \qquad V_{II} = \frac{-M_C + M'_C}{L}$$

$$H_1 = \frac{+M_{B1}}{h_1} \qquad H_2 = \frac{-M_{B2} + M_C}{h_2}$$

$$H'_1 = \frac{+M'_{B1}}{h_1} \qquad H'_2 = \frac{-M'_{B2} + M'_C}{h_2}$$

When columns take no exterior load:

$$T_1 = -T'_1 = H_1 = H'_1 = H_A = H'_A$$

$$T_2 = -T'_2 = H_2 = H'_2$$

$$N_I = -H_1 + H_2 \qquad N_{II} = -H_2$$

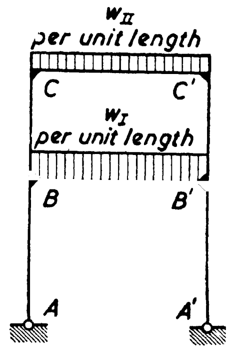
Extract 'Kleinlogel, Mehrstielige Rahmen', Band I + II. Berlin—Verlag von Wilhelm Ernst & Sohn.

Moment relationships:

$$M_{B1} - M_{B2} - M_{BI} = 0$$

$$M'_{B1} - M'_{B2} - M'_{BI} = 0$$

(b) For vertical U.D.L.s on first floor or roof:

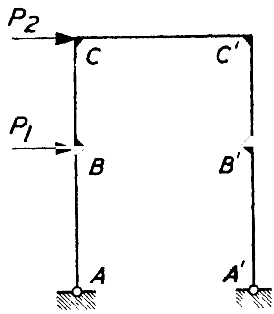


$$T_I = T'_I = w_I L/2$$

$$T_{II} = T'_{II} = w_{II} L/2 = V_2 = V'_2$$

$$V_1 = V_2 + T_I = V_A \qquad V'_1 = V'_2 + T'_I = V'_A$$

(c) For horizontal point loads applied at floor or roof level:



$$T_I = \frac{-2M_{BI}}{L} \qquad T_{II} = \frac{-2M_{CII}}{L}$$

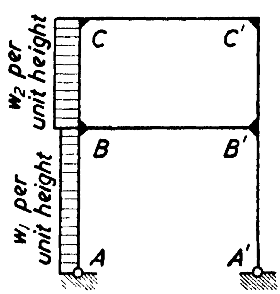
$$T_1 = T'_1 = H_1 = -H'_1 = \frac{P_B + P_C}{2} = H_A = -H'_A$$

$$T_2 = T'_2 = H_2 = -H'_2 = \frac{P_C}{2}$$

$$V_2 = +T_{II} = -V'_2 \qquad V_1 = V_2 + T_I = -V'_1$$

$$N_I = \frac{P_B}{2} \qquad N_{II} = \frac{P_C}{2}$$

(d) For horizontal U.D.L.s applied to any of the storeys:



$$T_A = +H_1 + \frac{w_1 h_1}{2} \qquad T_{B1} = +H_1 - \frac{w_1 h_1}{2}$$

$$T_{B2} = +H_2 + \frac{w_2 h_2}{2} \qquad T_C = +H_2 - \frac{w_2 h_2}{2}$$

$$H_A = +T_A \qquad T'_1 = -H'_1$$

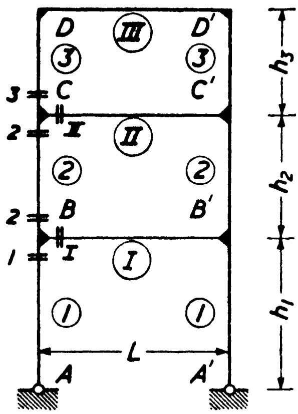
$$H'_A = +H'_1 \qquad T'_2 = -H'_2$$

$$T_I = +V_I \qquad V_2 = +V_{II} = -V'_2 \qquad N_I = +H'_2 - H'_1 = -T_{B1} + T_{B2}$$

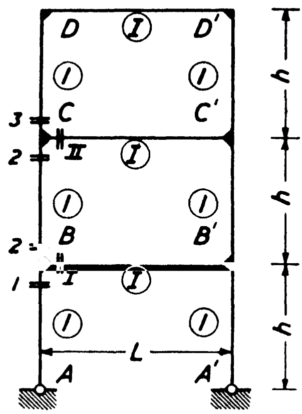
$$T_{II} = +V_{II} \qquad V_1 = V_2 + V_I = -V'_1 \qquad N_{II} = -H'_2 = -T_C$$

Extract: 'Kleinlogel, Mehrstielige Rahmen', Band I + II. Berlin—Verlag von Wilhelm Ernst & Sohn.

Frame IX



FRAME DATA



SPECIAL CASE

Constants: I_c = any convenient value chosen for I.

$$L_1=\frac{I_c}{I_I}\qquad L_2=\frac{I_c}{I_{II}}\qquad L_3=\frac{I_c}{I_{III}}\qquad x_1=\frac{I_c h_1}{I_1 L}\qquad x_2=\frac{I_c h_2}{I_2 L}\qquad x_3=\frac{I_c h_3}{I_3 L}$$

$$K_1=2x_1+3L_1\qquad s_2=\frac{x_2}{K_2}\qquad r_1=\frac{3L_1}{K_1}\qquad t_1=\frac{2x_1}{K_1}\qquad k_2=\frac{1}{2K_2}$$

$$K_2=2x_2+3L_2$$

$$K_3=2x_3+3L_3\qquad s_3=\frac{x_3}{K_3}\qquad r_2=\frac{3L_2}{K_2}\qquad k_1=\frac{1}{2K_1}\qquad k_3=\frac{1}{2K_3}$$

$$n_2=\frac{h_1}{h_2}\qquad n_3=\frac{h_1}{h_3}\qquad e=\frac{h_2}{h_3}\qquad m_2=\frac{h_2}{h_1}\qquad m_3=\frac{h_3}{h_1}$$

$$B_2=L_1+3x_2\qquad C_2=3x_2+L_2\qquad G_2=L_1+6x_2+L_2=B_2+C_2$$

$$C_3=L_2+3x_3\qquad D_3=3x_3+L_3\qquad G_3=L_2+6x_3+L_3=C_3+D_3$$

$$F=L_1+6x_2\qquad F_2=L_2+6x_2\qquad F_3=L_3+6x_3\qquad N_2=G_2G_3-L_2^2$$

Special case*:

$$I_I=I_{II}=I_{III}=I_c\qquad I_1=I_2=I_3$$

$$h_1=h_2=h_3=h\qquad n_2=n_3=1\qquad e=1\qquad m_2=m_3=1$$

$$L_1=L_2=L_3=1\qquad x_1=x_2=x_3=x$$

* All those constants referring to the special case which can be simplified to the general constants are enclosed in light ruled lines on the following pages.

Extract: 'Kleinlogel, Mehrstielige Rahmen', Band I + II. Berlin—Verlag von Wilhelm Ernst & Sohn.

Influence Coefficients:

$$Z_{22}=3s_3(x_3+2L_3)+2r_2x_2 \qquad Z_{44}=3s_2(x_2+2L_2)+2r_1x_1$$

$$Z_{24}=r_2x_2=3s_2L_2 \qquad N_1=Z_{22}Z_{44}-(Z_{24})^2$$

$$n_{22}=\frac{Z_{22}}{2N_1} \qquad n_{44}=\frac{Z_{44}}{2N_1} \qquad n_{24}=\frac{Z_{24}}{2N_1}=n_{42}$$

$$n_{21}=r_1n_{22}=n_{12} \qquad n_{41}=r_1n_{42}=n_{14}$$

$$n_{23}=s_2n_{22}+r_2n_{24}=n_{32} \qquad n_{43}=s_2n_{42}+r_2n_{44}=n_{34}$$

$$n_{25}=s_3n_{24}=n_{52} \qquad n_{45}=s_3n_{44}=n_{54}$$

$$n_{11}=k_1+r_1n_{21} \qquad n_{31}=s_2n_{21}+r_2n_{41} \qquad n_{51}=s_3n_{41}$$

$$n_{12}=r_1n_{22} \qquad n_{32}=s_2n_{22}+r_2n_{42} \qquad n_{52}=s_3n_{42}$$

$$n_{13}=r_1n_{23} \qquad n_{33}=k_2+s_2n_{23}+r_2n_{43} \qquad n_{53}=s_3n_{43}$$

$$n_{14}=r_1n_{24} \qquad n_{34}=s_2n_{24}+r_2n_{44} \qquad n_{54}=s_3n_{44}$$

$$n_{15}=r_1n_{25} \qquad n_{35}=s_2n_{25}+r_2n_{45} \qquad n_{55}=k_3+s_3n_{45}$$

$$m_{22}=\frac{G_3}{2N_2} \qquad u_2=\frac{L_1G_3}{2N_2} \qquad v_{22}=\frac{C_2G_3-(L_2)^2}{2N_2} \qquad y_2=\frac{L_2D_3}{2N_2}$$

$$m_{23}=m_{32}=\frac{L_2}{2N_2} \qquad u_3=\frac{L_1L_2}{2N_2} \qquad v_{23}=\frac{B_2G_3}{2N_2} \qquad y_{34}=\frac{D_3G_2}{2N_2}$$

$$m_{33}=\frac{G_2}{2N_2} \qquad v_3=\frac{L_2B_2}{2N_2} \qquad y_{35}=\frac{C_3G_2-(L_2)^2}{2N_2}$$

Special case:

$a_1=1+x$	$b_1=1+2x$	$c_6=6+6x+x^2$	$A=6xa_5a_3b_3$
$a_2=2+x$	$b_3=3+2x$	$c_7=7+6x+x^2$	$B=6xa_5a_3$
$a_3=3+x$	$b_5=5+2x$	$d=1+3x$	$C=6xa_5b_3$
$a_4=4+x$	$b_7=7+2x$	$j=1+6x$	$D=6xa_5$
$a_5=5+x$	$b_9=9+2x$	$z=1+12x+18x^2$	$N=3b_1$

$$n_{11}=\frac{3a_2c_6}{A} \qquad n_{12}=n_{21}=\frac{3a_4}{B} \qquad n_{13}=n_{31}=\frac{3a_1}{C}$$

$$n_{22}=\frac{a_4b_3}{B} \qquad n_{23}=n_{32}=\frac{a_1}{D} \qquad n_{24}=n_{42}=\frac{b_3}{B}$$

$$n_{33}=\frac{4a_1a_3}{C} \qquad n_{34}=n_{43}=\frac{4}{D} \qquad n_{35}=n_{53}=\frac{4x}{C}$$

$$n_{44}=\frac{a_4b_3}{B} \qquad n_{45}=n_{54}=\frac{xa_4}{B} \qquad n_{14}=n_{41}=\frac{3}{B}$$

$$n_{55}=\frac{xb_5b_9}{A} \qquad n_{15}=n_{51}=\frac{3x}{A} \qquad n_{25}=n_{52}=\frac{x}{B}$$

$$m_{22}=m_{33}=m=\frac{d}{N} \qquad m_{23}=m_{32}=r=\frac{1}{2N} \qquad u_2=m \quad u_3=r$$

$$v_{22}=y_{35}=\frac{z}{2N} \qquad v_{23}=y_{34}=\frac{d^2}{N} \qquad v_3=y_2=\frac{m}{2}$$

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Composite Influence Coefficients:

$$r_{11} = +n_{11} - n_{21} = k_1 - t_1 n_{21} = n_{1I}$$
$$r_{12} = -n_{12} + n_{22} = \quad +t_1 n_{22} = n_{2I}$$
$$r_{13} = -n_{13} + n_{23} = \quad +t_1 n_{23} = n_{3I}$$
$$r_{14} = -n_{14} + n_{24} = \quad +t_1 n_{24} = n_{4I}$$
$$r_{15} = -n_{15} + n_{25} = \quad +t_1 n_{25} = n_{5I}$$
$$r_{1I} = +n_{1I} + n_{2I} = r_{11} + r_{12}$$
$$r_{1II} = -n_{1II} + n_{2II} = r_{13} - r_{14} = r_{2I}$$

$$r_{21} = +n_{31} - n_{41} = n_{1II}$$
$$r_{22} = +n_{32} - n_{42} = n_{2II}$$
$$r_{23} = +n_{33} - n_{43} = n_{3II}$$
$$r_{24} = -n_{34} + n_{44} = n_{4II}$$
$$r_{25} = -n_{35} + n_{45} = n_{5II}$$

$$r_{2I} = +n_{3I} - n_{4I} = -r_{21} + r_{22} = r_{1II}$$
$$r_{2II} = +n_{3II} + n_{4II} = +r_{23} + r_{24}$$

$$s_{21} = +n_{21} - n_{31} \qquad s_{31} = +n_{41} - n_{51}$$
$$s_{22} = +n_{22} - n_{32} \qquad s_{32} = +n_{42} - n_{52}$$
$$s_{23} = -n_{23} + n_{33} \qquad s_{33} = +n_{43} - n_{53}$$
$$s_{24} = -n_{24} + n_{34} \qquad s_{34} = +n_{44} - n_{54}$$
$$s_{25} = -n_{25} + n_{35} \qquad s_{35} = -n_{45} + n_{55}$$

$$s_{2I} = +n_{2I} - n_{3I} = -s_{21} + s_{22}$$
$$s_{2II} = -n_{2II} + n_{3II} = +s_{23} - s_{24}$$
$$s_{3I} = +n_{4I} - n_{5I} = -s_{31} + s_{32}$$
$$s_{3II} = +n_{4II} - n_{5II} = -s_{33} + s_{34}$$

$$m_{2II} = +m_{22} - m_{32} = \frac{F_3}{2N_2} \qquad u_{II} = +u_2 - u_3 = \frac{L_1 F_3}{2N_2}$$
$$m_{3II} = -m_{23} + m_{33} = \frac{F}{2N_2} \qquad v_{II} = +v_{23} - v_3 = \frac{B_2 F_3}{2N_2}$$
$$u_I = \tfrac{1}{2} - u_2 \qquad y_{II} = -y_2 + y_{34} = \frac{D_3 F}{2N_2}$$

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$$v'_{22} = -n_3u_2 + ev_{22}$$

$$u'_2 = n_2u_2 \qquad v'_{23} = +n_3u_2 + ev_{23}$$

$$u'_3 = n_2u_3 \qquad v'_3 = +n_3u_3 + ev_3$$

$$u'_I = n_2u_I = \frac{n_2}{2} - u'_2 \qquad v'_I = +n_3u_I + ev_{22} = \frac{n_3}{2} + v'_{22}$$

$$u'_{II} = n_2u_{II} = u'_2 - u'_3 \qquad v'_{II} = +n_3u_{II} + ev_{II} = v'_{23} - v'_3$$

$$L_{22} = -u'_2 + \frac{v_{22}}{2} \qquad L_{32} = +v'_{22} + \frac{y_2}{2}$$

$$L_{23} = +u'_2 + \frac{v_{23}}{2} \qquad L_{33} = +v'_{23} - \frac{y_2}{2}$$

$$L_{24} = +u'_3 + \frac{v_3}{2} \qquad L_{34} = -v'_3 + \frac{y_{34}}{2}$$

$$L_{25} = L_{24} \qquad L_{35} = +v'_3 + \frac{y_{35}}{2}$$

$$L_{2I} = +u'_I + \frac{v_{22}}{2} = \frac{n_2}{2} + L_{22} \qquad L_{3I} = +v'_I + \frac{y_2}{2} = \frac{n_3}{2} + L_{32}$$

$$L_{2II} = +u'_{II} + \frac{v_{II}}{2} = L_{23} - L_{24} \qquad L_{3II} = +v'_{II} + \frac{y_{II}}{2} = L_{33} + L_{34}$$

Special case:

$$r_{11} = \frac{3xc_7}{A} \qquad r_{21} = \frac{3xa_2}{A} \qquad n_2 = n_3 = e = 1$$

$$r_{12} = \frac{2xa_4}{B} \qquad r_{22} = \frac{xa_2}{B}$$

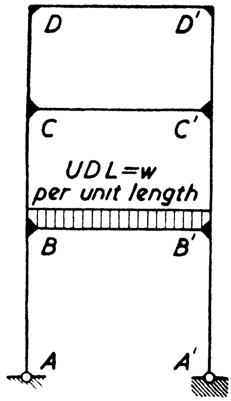
$$r_{13} = \frac{2xa_1}{C} \qquad r_{23} = \frac{4xa_2}{C}$$

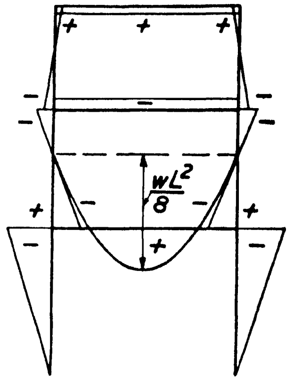
$$r_{14} = \frac{2x}{B} \qquad r_{24} = \frac{xb_7}{B} \qquad u_I = \tfrac{1}{2} - m \qquad v_{II} = y_{II} = \frac{d}{6b_1}$$

$$r_{15} = \frac{2x^2}{A} \qquad r_{25} = \frac{x^2b_7}{A} \qquad u'_I = u_I \qquad u_{II} = u'_{II}$$

$$u'_2 = m \qquad u'_3 = r$$

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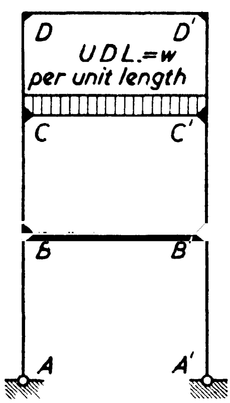
$M_{B1} = M'_{B1} = -S_I r_{11}$

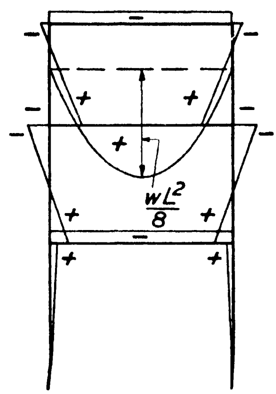
$M_{B2} = M'_{B2} = +S_I r_{12}$ $\qquad M_{BI} = M'_{BI} = -S_I r_{1I}$

$M_{C2} = M'_{C2} = -S_I r_{13}$

$M_{C3} = M'_{C3} = -S_I r_{14}$ $\qquad M_{CII} = M'_{CII} = -S_I r_{1II}$

$M_D = M'_D = +S_I r_{15}$ $\qquad$ where $S_I = \frac{wL^2}{2} \times L_1$





$M_{B1} = M'_{B1} = +S_{II} r_{21}$ $\qquad M_{BI} = M'_{BI} = -S_{II} r_{2I}$

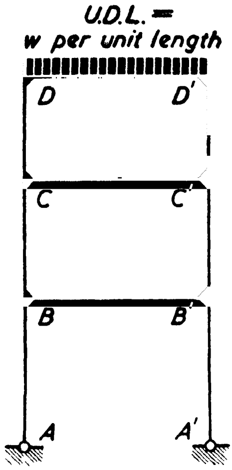
$M_{B2} = M'_{B2} = +S_{II} r_{22}$

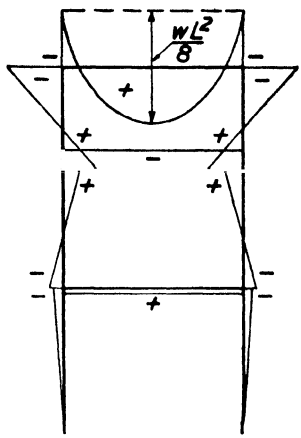
$M_{C2} = M'_{C2} = -S_{II} r_{23}$

$M_{C3} = M'_{C3} = +S_{II} r_{24}$ $\qquad M_{CII} = M'_{CII} = -S_{II} r_{2II}$

$M_D = M'_D = -S_{II} r_{25}$ $\qquad$ where $S_{II} = \frac{wL^2}{2} \times L_2$

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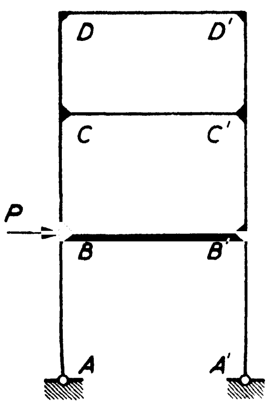
$$M_{B1}=M'_{B1}=-S_{III}n_{51} \qquad M_{BI}=M'_{BI}=+S_{III}n_{5I}$$

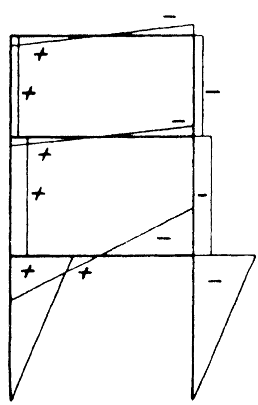
$$M_{B2}=M'_{B2}=-S_{III}n_{52}$$

$$M_{C2}=M'_{C2}=+S_{III}n_{53}$$

$$M_{C3}=M'_{C3}=+S_{III}n_{54} \qquad M_{CII}=M'_{CII}=-S_{III}n_{5II}$$

$$M_D=M'_D=-S_{III}n_{55} \qquad \text{where} \quad S_{III}=\frac{wL^2}{2}\times L_3$$





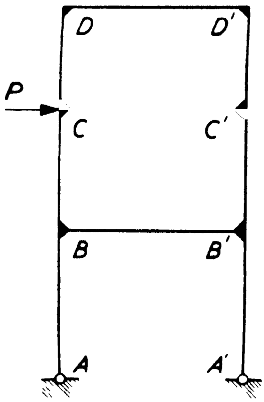
$$M_{B1}=+M/2 \qquad M_{B2}=M_{C2}=+Mu_2 \qquad M_{C3}=M_D=+Mu_3$$

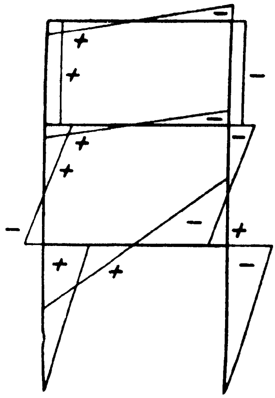
$$M'_{B1}=-M/2 \qquad M'_{B2}=M'_{C2}=-Mu_2 \qquad M'_{C3}=M'_D=\ Mu_3$$

$$M_{BI}=+Mu_I \qquad M_{CII}=+Mu_{II}$$

$$M'_{BI}=-Mu_I \qquad M'_{CII}=-Mu_{II} \qquad \text{where} \quad M=Ph_1$$

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$$M_{B1} = +M/2$$

$$M'_{B1} = -M/2 \qquad M_{BI} = +Mp_I$$

$$M_{B2} = -Mp_{22} \qquad M'_{BI} = -Mp_I$$

$$M'_{B2} = +Mp_{22}$$

$$M_{C2} = +Mp_{23}$$

$$M'_{C2} = -Mp_{23} \qquad M'_{II} = -Mp_{II}$$

$$M_{C3} = M_D = +Mp_3 \qquad M_{CII} = +Mp_{II}$$

$$M'_{C3} = M'_D = -Mp_3$$

where

$$p_{22} = -u_2 + m_2 v_{22} = m_3 v'_{22}$$

$$p_{23} = +u_2 + m_2 v_{23} = m_3 v'_{23} \qquad M = Ph_1$$

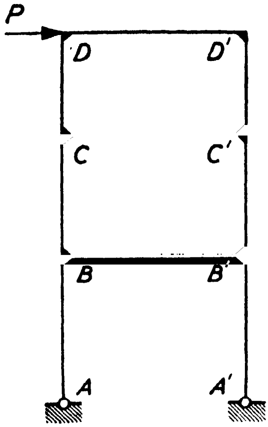
$$p_3 = +u_3 + m_2 v_3 = m_3 v'_3$$

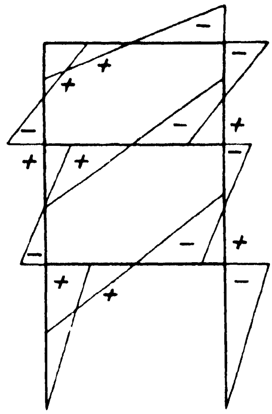
$$p_I = +u_I + m_2 v_{22} = m_3 v'_I = \tfrac{1}{2} + p_{22}$$

$$p_{II} = +u_{II} + m_2 v_{II} = m_3 v'_{II} = p_{23} - p_3$$

Special case: $m_2 = m_3 = 1$

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$$M_{B1} = +M/2$$
$$M'_{B1} = -M/2 \qquad M_{BI} = +Mq_I$$
$$M_{B2} = -Mq_2 \qquad M'_{BI} = -Mq_I$$
$$M'_{B2} = +Mq_2$$
$$M_{C2} = +Mq_3$$
$$M'_{C2} = -Mq_3 \qquad M_{CII} = +Mq_{II}$$
$$M_{C3} = -Mq_4 \qquad M'_{CII} = -Mq_{II}$$
$$M'_{C3} = +Mq_4$$
$$M_D = +Mq_5$$
$$M'_D = -Mq_5$$

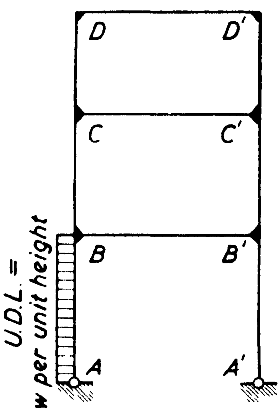
where

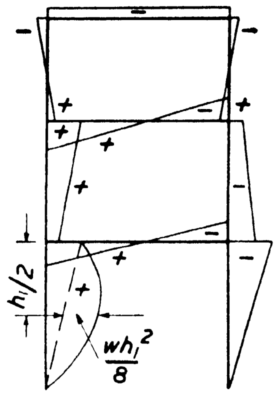
$$q_2 = m_3(+v'_{22} + y_2) = +p_{22} + m_3 y_2$$
$$q_3 = m_3(+v'_{23} - y_2) = +p_{23} - m_3 y_2$$
$$q_4 = m_3(-v'_3 + y_{34}) = -p_3 + m_3 y_{34}$$
$$q_5 = m_3(+v'_3 + y_{35}) = +p_3 + m_3 y_{35}$$
$$q_I = m_3(+v'_I + y_2) = \tfrac{1}{2} + q_2$$
$$q_{II} = m_2(+v'_{II} + y_{II}) = q_3 + q_4$$
$$M = Ph_1$$

Special case: $m_3 = 1$

The values of p are as on the previous page.

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$$M_{B1} = R_1(-x_1 n_{11} + 1) \qquad M'_{B1} = R_1(-x_1 n_{11} - 1)$$

$$M_{B2} = R_1(-x_1 n_{12} + 2u_2) \qquad M'_{B2} = R_1(-x_1 n_{12} - 2u_2)$$

$$M_{C2} = R_1(+x_1 n_{13} + 2u_2) \qquad M'_{C2} = R_1(+x_1 n_{13} - 2u_2)$$

$$M_{C3} = R_1(+x_1 n_{14} + 2u_3) \qquad M'_{C3} = R_1(+x_1 n_{14} - 2u_3)$$

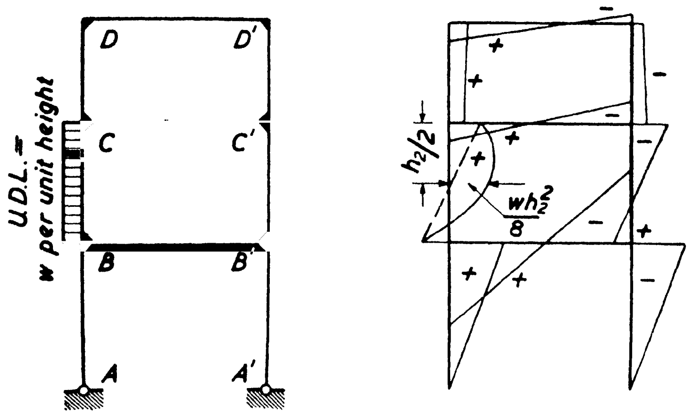
$$M_D = R_1(-x_1 n_{15} + 2u_3) \qquad M'_D = R_1(-x_1 n_{15} - 2u_3)$$

$$M_{BI} = R_1(-x_1 n_{1I} + 2u_I) \qquad M'_{BI} = R_1(-x_1 n_{1I} - 2u_I)$$

$$M_{CII} = R_1(+x_1 n_{1II} + 2u_{II}) \qquad M'_{CII} = R_1(+x_1 n_{1II} - 2u_{II})$$

where $R_1 = \dfrac{w(h_1)^2}{4}$

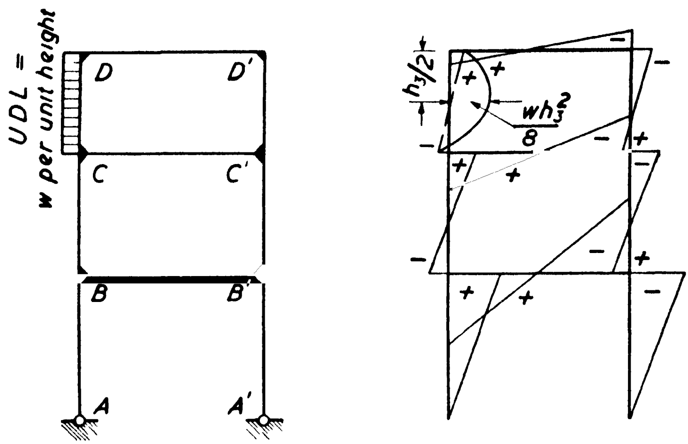
Extract: 'Kleinlogel, Mehrstielige Rahmen', Band I + II. Berlin—Verlag von Wilhelm Ernst & Sohn.



$$M_{B1} = R_2[-x_2 s_{21} + 2n_2]$$
$$M'_{B1} = R_2[-x_2 s_{21} - 2n_2]$$
$$M_{B2} = R_2[-x_2(s_{22} + 2m_{22}) - 4L_{22}]$$
$$M'_{B2} = R_2[-x_2(s_{22} - 2m_{22}) + 4L_{22}]$$
$$M_{C2} = R_2[-x_2(s_{23} + 2m_{22}) + 4L_{23}]$$
$$M'_{C2} = R_2[-x_2(s_{23} - 2m_{22}) - 4L_{23}]$$
$$M_{C3} = R_2[-x_2(s_{24} + 2m_{23}) + 4L_{24}]$$
$$M'_{C3} = R_2[-x_2(s_{24} - 2m_{23}) - 4L_{24}]$$
$$M_D = R_2[+x_2(s_{25} - 2m_{23}) + 4L_{25}]$$
$$M'_D = R_2[+x_2(s_{25} + 2m_{23}) - 4L_{25}]$$
$$M_{BI} = R_2[+x_2(s_{2I} + 2m_{22}) + 4L_{2I}]$$
$$M'_{BI} = R_2[+x_2(s_{2I} - 2m_{22}) - 4L_{2I}]$$
$$M_{CII} = R_2[-x_2(s_{2II} + 2m_{2II}) + 4L_{2II}]$$
$$M'_{CII} = R_2[-x_2(s_{2II} - 2m_{2II}) - 4L_{2II}]$$

where $R_2 = \dfrac{w(h_2)^2}{4}$

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$$M_{B1} = R_3[+x_3 s_{31} + 2n_3]$$

$$M'_{B1} = R_3[+x_3 s_{31} - 2n_3]$$

$$M_{B2} = R_3[+x_3(s_{32} - 2m_{32}) - 4L_{32}]$$

$$M'_{B2} = R_3[+x_3(s_{32} + 2m_{32}) + 4L_{32}]$$

$$M_{C2} = R_3[-x_3(s_{33} + 2m_{32}) + 4L_{33}]$$

$$M'_{C2} = R_3[-x_3(s_{33} - 2m_{32}) - 4L_{33}]$$

$$M_{C3} = R_3[-x_3(s_{34} + 2m_{33}) - 4L_{34}]$$

$$M'_{C3} = R_3[-x_3(s_{34} - 2m_{33}) + 4L_{34}]$$

$$M_D = R_3[-x_3(s_{35} + 2m_{33}) + 4L_{35}]$$

$$M'_D = R_3[-x_3(s_{35} - 2m_{33}) - 4L_{35}]$$

$$M_{BI} = R_3[-x_3(s_{3I} - 2m_{32}) + 4L_{3I}]$$

$$M'_{BI} = R_3[-x_3(s_{3I} + 2m_{32}) - 4L_{3I}]$$

$$M_{CII} = R_3[+x_3(s_{3II} + 2m_{3II}) + 4L_{3II}]$$

$$M'_{CII} = R_3[+x_3(s_{3II} - 2m_{3II}) - 4L_{3II}]$$

$$\text{where} \quad R_3 = \frac{w(h_3)^2}{4}$$

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Formulæ for Support Reactions, Shear and Axial Forces, together with relationships between moments at the joints.

Nomenclature:

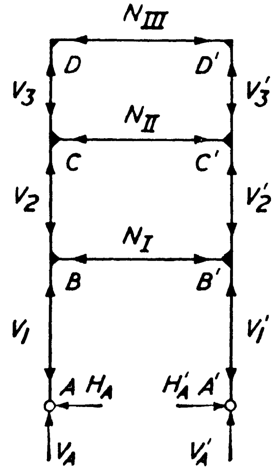
V = vertical reaction or axial force in columns.

N = axial force in beams.

T = shear force at ends of beams or columns.

H = horizontal reactions in columns.

(a) For all loads:



$$V_I = \frac{-M_{BI} + M'_{BI}}{L} \qquad V_{II} = \frac{-M_{CII} + M'_{CII}}{L}$$

$$V_{III} = \frac{-M_D + M'_D}{L}$$

$$H_1 = \frac{+M_{B1}}{h_1} \qquad H'_1 = \frac{+M'_{B1}}{h_1}$$

$$H_2 = \frac{-M_{B2} + M_{C2}}{h_2} \qquad H'_2 = \frac{-M'_{B2} + M'_{C2}}{h_2}$$

$$H_3 = \frac{-M_{C3} + M_D}{h_3} \qquad H'_3 = \frac{-M'_{C3} + M'_D}{h_3}$$

When columns take no exterior load:

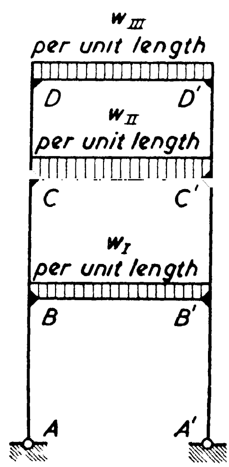
$$T_1 = -T'_1 = H_1 = H'_1 = H_A = H'_A$$

$$T_2 = -T'_2 = H_2 = H'_2$$

$$T_3 = -T'_3 = H_3 = H'_3$$

$$N_I = -H_1 + H_2 \qquad N_{II} = -H_2 + H_3 \qquad N_{III} = -H_3$$

Moment relationships:



$$M_{B1} - M_{B2} - M_{BI} = 0 \qquad M'_{B1} - M'_{B2} - M'_{BI} = 0$$

$$M_{C2} - M_{C3} - M_{CII} = 0$$

$$M'_{C2} - M'_{C3} - M'_{CII} = 0$$

(b) For vertical U.D.L.s on the roof or floors:

$$T_I = T'_I = w_I L/2$$

$$T_{II} = T'_{II} = w_{II} L/2$$

$$T_{III} = T'_{III} = w_{III} L/2 = V_3 = V'_3$$

$$V_2 = V_3 + T_{II}$$

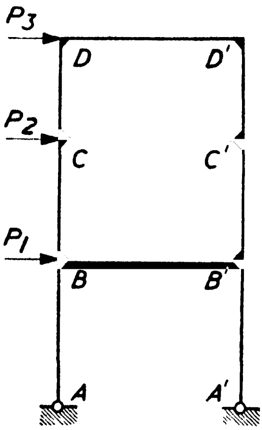
$$V'_2 = V'_3 + T'_{II}$$

$$V_1 = V_2 + T_I = V_A$$

$$V'_1 = V'_2 + T'_I = V'_A$$

Extract: 'Kleinlogel, Mehrstielige Rahmen', Band I + II. Berlin—Verlag von Wilhelm Ernst & Sohn.

(c) For horizontal point loads applied at floor or roof level:



$$T_I = \frac{-2M_{BI}}{L} \qquad T_{II} = \frac{-2M_{CII}}{L}$$

$$T_{III} = \frac{-2M_D}{L}$$

$$T_1 = T_1' = H_1 = -H_1' = \frac{P_B + P_C + P_D}{2}$$

$$= H_A = -H_A'$$

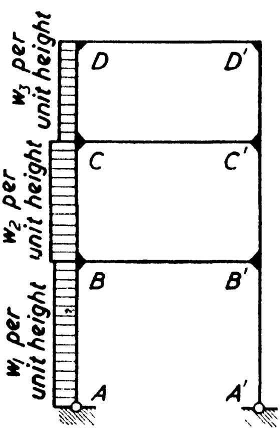
$$T_2 = T_2' = H_2 = -H_2' = \frac{P_C + P_D}{2}$$

$$T_3 = T_3' = H_3 = -H_3' = \frac{P_D}{2}$$

$$N_I = \frac{P_B}{2} \qquad N_{II} = \frac{P_C}{2} \qquad N_{III} = \frac{P_D}{2}$$

$$V_3 = +T_{III} = -V_3' \qquad V_2 = V_3 + T_{II} = -V_2' \qquad V_1 = V_2 + T_I = -V_1'$$

(d) For horizontal U.D.L.s applied to any of the storeys:



$$T_A = +H_1 + \frac{w_1 h_1}{2} \qquad T_{B1} = +H_1 - \frac{w_1 h_1}{2}$$

$$T_{B2} = +H_2 + \frac{w_2 h_2}{2} \qquad T_{C2} = +H_2 - \frac{w_2 h_2}{2}$$

$$T_{C3} = +H_3 + \frac{w_3 h_3}{2} \qquad T_D = +H_3 - \frac{w_3 h_3}{2}$$

$$H_A = T_A \qquad H_A' = H_1'$$

$$T_1' = -H_1' \qquad T_I = +V_I$$

$$T_2' = -H_2' \qquad T_{II} = +V_{II}$$

$$T_3' = -H_3' \qquad T_{III} = +V_{III}$$

$$V_3 = \quad V_{III} = -V_3'$$

$$V_2 = V_3 + V_{II} = -V_2'$$

$$V_1 = V_2 + V_I = -V_1' = V_A = -V_A'$$

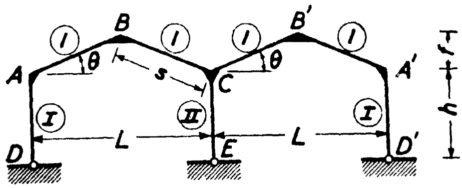
$$N_I = +H_2' - H_1' = -T_{B1} + T_{B2}$$

$$N_{II} = +H_3' - H_2' = -T_{C2} + T_{C3}$$

$$N_{III} = -H_3' = -T_D$$

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Frame X



FRAME DATA

Constants:

$$\phi=\frac{f}{h}\qquad x_1=\frac{I_1h}{I_Is}\qquad x_{II}=\frac{I_1h}{I_{II}s}\qquad \cos\theta=\frac{L}{2s}\qquad \sin\theta=\frac{f}{s}$$

$$N_1=8x_I+12(1+\phi)+7\phi^2\qquad N_2=2x_I+12(1+\phi)+4\phi^2+4x_{II}$$

Influence Coefficients:

$$n_{11}=\frac{2}{N_1}\qquad n_{12}=n_{21}=\frac{2+3\phi}{2N_1}\qquad n_{22}=\frac{x_I+2+3\phi+2\phi^2}{N_1}$$

$$L_1=\frac{2+5\phi}{4N_1}\qquad c_1=\frac{6+7\phi}{4N_1}\qquad y_{11}=2\phi c_1$$

$$L_2=\frac{2x_I+2-\phi-2\phi^2}{4N_1}\qquad c_2=\frac{6x_I+6+\phi}{4N_1}\qquad y_{12}=2\phi c_2$$

$$n=\frac{1}{2N_2}\qquad L'=\frac{1+\phi}{2N_2}\qquad c=\frac{3+2\phi}{2N_2}\qquad r=\frac{1}{N_2}$$

$$y_{13}=\frac{6+9\phi+4\phi^2+4x_{II}}{2N_2}\qquad y_{23}=\frac{6+3\phi+4x_{II}}{2N_2}$$

$$y_{14}=\frac{2x_I+6+3\phi}{2N_2}\qquad y_{24}=\frac{2x_I+6+9\phi+4\phi^2}{2N_2}$$

$$(y_{13}+y_{14}=0{\cdot}5)\qquad (y_{23}+y_{24}=0{\cdot}5)$$

Composite Influence Coefficients:

$$s_1=+n_{11}+L_1\qquad s_1'=-L_1+n_{21}\qquad r_1=-n_{11}+n_{21}$$

$$s_2=+n_{12}-L_2\qquad s_2'=+L_2+n_{22}\qquad r_2=-n_{12}+n_{22}$$

$$s'=L'+n\qquad r=2n$$

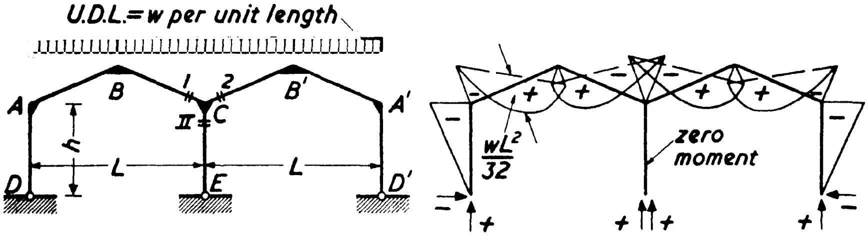
$$m_1=\frac{+s_1-s_1'}{2}=L_1-\frac{r_1}{2}\qquad m_2=\frac{-s_2+s_2'}{2}=L_2+\frac{r_2}{2}$$

$$y_3=y_{23}+c\phi\qquad y_{13}-y_{23}-2c\phi=0$$

$$y_4=y_{24}-c\phi\qquad y_{14}-y_{24}+2c\phi=0$$

Note.—The four rafters of equal length AB, BC, CB' and $B'A'$ with the member number 1 are allocated another series of member numbers according to the figure on page 341 as a distinguishing mark for the static values referring to these rafters.

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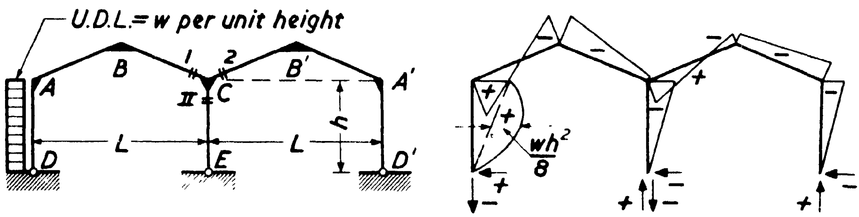


$$M_A = M'_A = -\frac{wL^2}{4}(m_1 + 2c_1)$$

$$M_{C1} = M_{C2} = -\frac{wL^2}{4}(m_2 + 2c_2)$$

$$M_{CII} = 0$$

$$M_B = M'_B = +\frac{wL^2}{8} + \frac{M_A(1+2\phi) + M_{C1}}{2}$$



$$M_A = \frac{wh^2}{4} \cdot x_I(-n_{11} - n) + \frac{wh^2}{2}(+y_{11} + y_{13})$$

$$M'_A = \frac{wh^2}{4} \cdot x_I(-n_{11} + n) + \frac{wh^2}{2}(+y_{11} - y_{13})$$

$$M_{C1} = \frac{wh^2}{4} \cdot x_I(+n_{12} - n) + \frac{wh^2}{2}(+y_{12} - y_{14})$$

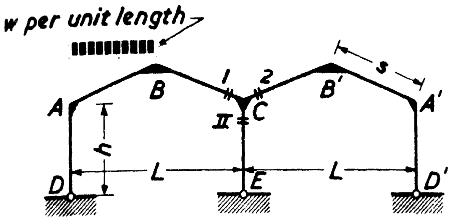
$$M_{C2} = \frac{wh^2}{4} \cdot x_I(+n_{12} + n) + \frac{wh^2}{2}(+y_{12} + y_{14})$$

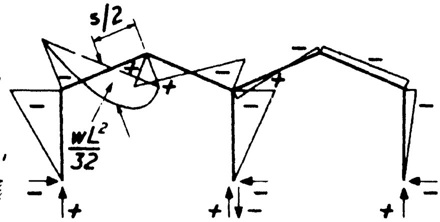
$$M_{CII} = -2\left[\frac{wh^2}{4} \cdot x_I n + \frac{wh^2}{2} \cdot y_{14}\right]$$

$$M_B = -\frac{wh^2\phi}{2} + \frac{M_A(1+2\phi) + M_{C1}}{2}$$

$$M'_B = \frac{M_{C2} + M'_A(1+2\phi)}{2}$$

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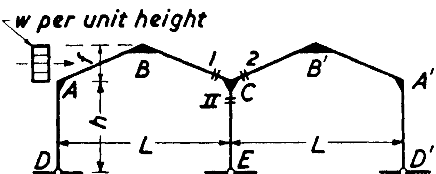


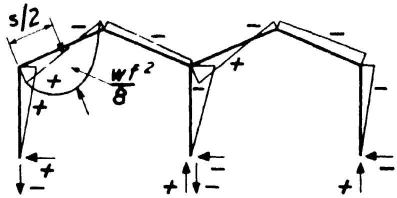
$$M_A = \frac{wL^2}{16}(-s_1 - s' - 2c_1 - 2c) \qquad M'_A = \frac{wL^2}{16}(-s_1 + s' - 2c_1 + 2c)$$

$$M_{C1} = \frac{wL^2}{16}(+s_2 - s' - 2c_2 - 2c) \qquad M_{C2} = \frac{wL^2}{16}(+s_2 + s' - 2c_2 + 2c)$$

$$M_{CII} = -\frac{wL^2}{8}(s' + 2c)$$

$$M_B = \frac{wL^2}{16} + \frac{M_A(1+2\phi) + M_{C1}}{2} \qquad M'_B = \frac{M_{C2} + M'_A(1+2\phi)}{2}$$





Constant: $W_1 = wfh$

$$M_A = \frac{wf^2}{4}(-s_1 - s') + \frac{3wf^2}{2}(+c_1 + c) + W_1 y_{23}$$

$$M'_A = \frac{wf^2}{4}(-s_1 + s') + \frac{3wf^2}{2}(+c_1 - c) - W_1 y_{23}$$

$$M_{C1} = \frac{wf^2}{4}(+s_2 - s') + \frac{3wf^2}{2}(+c_2 + c) - W_1 y_{24}$$

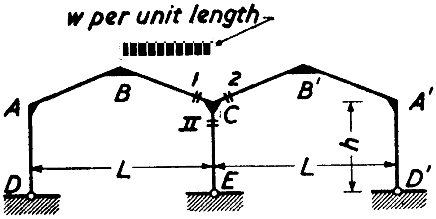
$$M_{C2} = \frac{wf^2}{4}(+s_2 + s') + \frac{3wf^2}{2}(+c_2 - c) + W_1 y_{24}$$

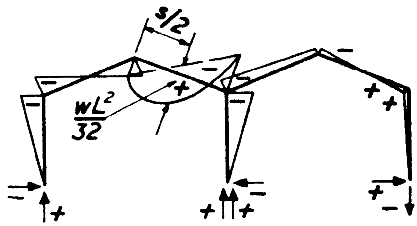
$$M_{CII} = -\frac{wf^2 s'}{2} + 3wf^2 c - 2W_1 y_{24}$$

$$M_B = -\frac{3wf^2}{4} + \frac{M_A(1+2\phi) + M_{C1}}{2}$$

$$M'_B = \frac{M_{C2} + M'_A(1+2\phi)}{2}$$

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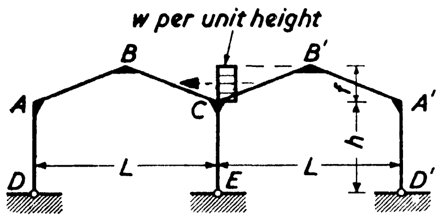


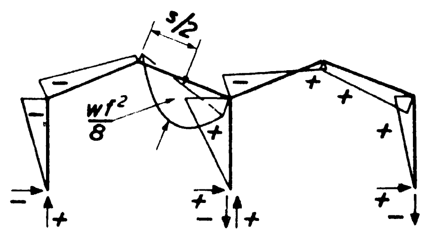
$$M_A = \frac{wL^2}{16}(+s_1' - s' - 2c_1 - 2c) \qquad M_A' = \frac{wL^2}{16}(+s_1' + s' - 2c_1 + 2c)$$

$$M_{C1} = \frac{wL^2}{16}(-s_2' - s' - 2c_2 - 2c) \qquad M_{C2} = \frac{wL^2}{16}(-s_2' + s' - 2c_2 + 2c)$$

$$M_{CII} = -\frac{wL^2}{8}(s' + 2c)$$

$$M_B = \frac{wL^2}{16} + \frac{M_A(1 + 2\phi) + M_{C1}}{2} \qquad M_B' = \frac{M_{C2} + M_A'(1 + 2\phi)}{2}$$





Constant: $W_1 = wfh$

$$M_A = \frac{wf^2}{4}(+s_1' - s' - 2c_1 - 2c) - W_1 y_{23}$$

$$M_A' = \frac{wf^2}{4}(+s_1' + s' - 2c_1 + 2c) + W_1 y_{23}$$

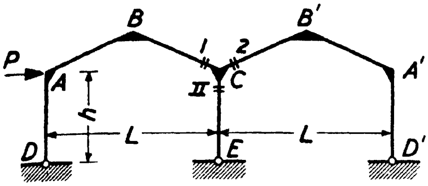
$$M_{C1} = \frac{wf^2}{4}(-s_2' - s' - 2c_2 - 2c) + W_1 y_{24}$$

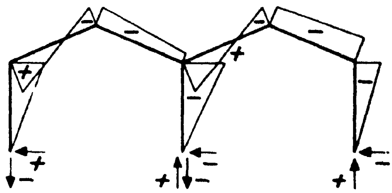
$$M_{C2} = \frac{wf^2}{4}(-s_2' + s' - 2c_2 + 2c) - W_1 y_{24}$$

$$M_{CII} = -\frac{wf^2}{2}(s' + 2c) + 2W_1 y_{24}$$

$$M_B = \frac{wf^2}{4} + \frac{M_A(1 + 2\phi) + M_{C1}}{2} \qquad M_B' = \frac{M_{C2} + M_A'(1 + 2\phi)}{2}$$

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$$M_A = M(+y_{11} + y_{13}) \qquad \text{where} \quad M = Ph$$

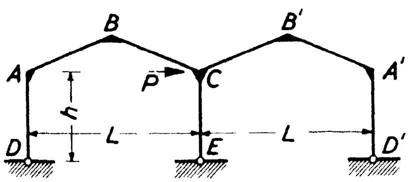
$$M'_A = M(+y_{11} - y_{13})$$

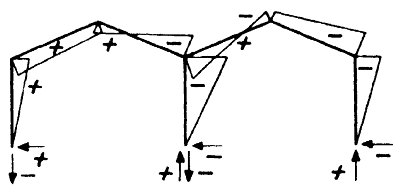
$$M_{C1} = M(+y_{12} - y_{14})$$

$$M_{C2} = M(+y_{12} + y_{14}) \qquad M_{CII} = -2My_{14}$$

$$M_B = \frac{M_A(1+2\phi) + M_{C1}}{2} - Pf$$

$$M'_B = \frac{M_{C2} + M'_A(1+2\phi)}{2}$$





$$M_A = +My_{23} \qquad \text{where} \quad M = Ph$$

$$M'_A = -My_{23}$$

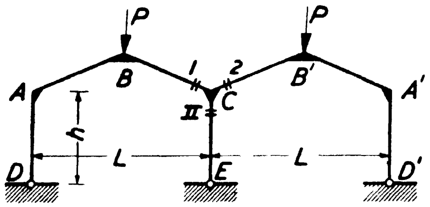
$$M_{C1} = -My_{24} \qquad M_{CII} = -2My_{24}$$

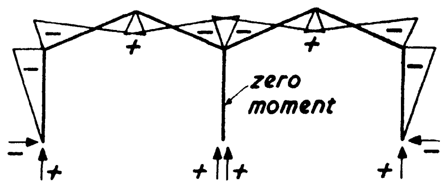
$$M_{C2} = +My_{24}$$

$$M_B = \frac{M_A(1+2\phi) + M_{C1}}{2}$$

$$M'_B = \frac{M_{C2} + M'_A(1+2\phi)}{2}$$

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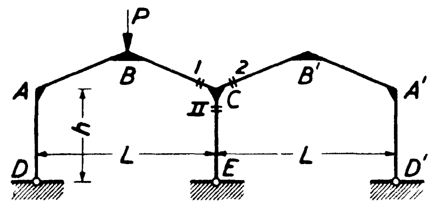


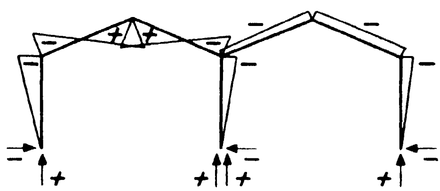
$$M_A = M'_A = -Mc_1$$

$$M_{C1} = M_{C2} = -Mc_2 \qquad \text{where} \quad M = PL$$

$$M_{CII} = 0$$

$$M_B = M'_B = \frac{PL}{4} + \frac{M_A(1+2\phi) + M_{C1}}{2}$$





$$M_A = \frac{M}{2}(-c_1 - c)$$

$$M'_A = \frac{M}{2}(-c_1 + c) \qquad \text{where} \quad M = PL$$

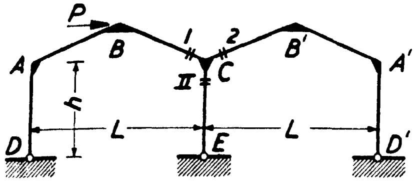
$$M_{C1} = \frac{M}{2}(-c_2 - c)$$

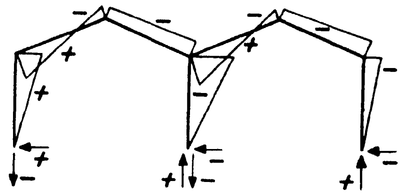
$$M_{C2} = \frac{M}{2}(-c_2 + c) \qquad M_{CII} = -Mc$$

$$M_B = \frac{PL}{4} + \frac{M_A(1+2\phi) + M_{C1}}{2}$$

$$M'_B = \frac{M_{C2} + M'_A(1+2\phi)}{2}$$

Extract. 'Kleinlogel, Mehrstielige Rahmen', Band I + II. Berlin—Verlag von Wilhelm Ernst & Sohn.





$$M_A = M(+c_1\phi + y_3)$$

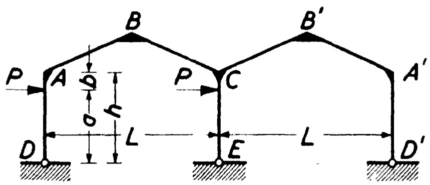
$$M'_A = M(+c_1\phi - y_3) \qquad \text{where} \quad M = Ph$$

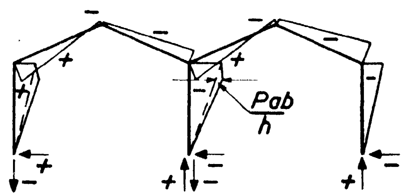
$$M_{C1} = M(+c_2\phi - y_4)$$

$$M_{C2} = M(+c_2\phi + y_4) \qquad M_{CII} = -2My_4$$

$$M_B = \frac{M_A(1+2\phi) + M_{C1} - Pf}{2}$$

$$M'_B = \frac{M_{C2} + M'_A(1+2\phi)}{2}$$





Constants: $a_1 = a/h \qquad b_1 = b/h$

$$R_I = Pba_1(1+a_1)x_I \qquad L_{II} = Pab_1(1+b_1)x_{II}$$

$$M_A = R_I(-n_{11} - n) + Pa(+y_{11} + y_{13}) + L_{II}r + Pay_{23}$$

$$M'_A = R_I(-n_{11} + n) + Pa(+y_{11} - y_{13}) - L_{II}r - Pay_{23}$$

$$M_{C1} = R_I(+n_{12} - n) + Pa(+y_{12} - y_{14}) + L_{II}r - Pay_{24}$$

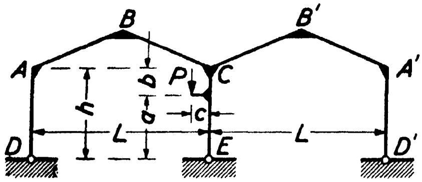
$$M_{C2} = R_I(+n_{12} + n) + Pa(+y_{12} + y_{14}) - L_{II}r + Pay_{24}$$

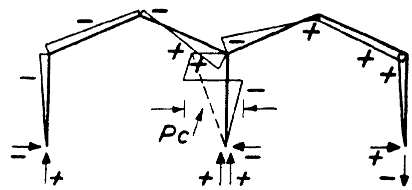
$$M_{CII} = -2(R_I n + Pay_{14} - L_{II}r + Pay_{24})$$

$$M_B = \frac{M_A(1+2\phi) + M_{C1}}{2} - Pa\phi$$

$$M'_B = \frac{M_{C2} + M'_A(1+2\phi)}{2}$$

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Constants: $a_1 = a/h \qquad b_1 = b/h$

$$L_{II} = Pc(1 - 3a_1^2)x_{II}$$

$$M_A = -L_{II}r - Pcy_{23}$$

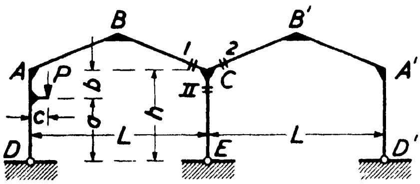
$$M'_A = +L_{II}r + Pcy_{23}$$

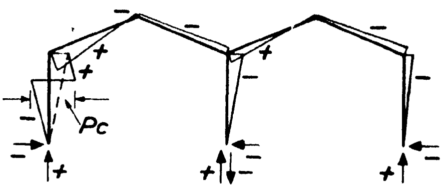
$$M_{C1} = -L_{II}r + Pcy_{24} \qquad M_{CII} = 2(-L_{II}r + Pcy_{24})$$

$$M_{C2} = +L_{II}r - Pcy_{24}$$

$$M_B = \frac{M_A(1+2\phi) + M_{C1}}{2}$$

$$M'_B = \frac{M_{C2} + M'_A(1+2\phi)}{2}$$





Constants: $a_1 = a/h \qquad b_1 = b/h$

$$R_I = Pc(1 - 3a_1^2)x_I$$

$$M_A = R_I(-n_{11} - n) + Pc(+y_{11} + y_{13})$$

$$M'_A = R_I(-n_{11} + n) + Pc(+y_{11} - y_{13})$$

$$M_{C1} = R_I(+n_{12} - n) + Pc(+y_{12} - y_{14})$$

$$M_{C2} = R_I(+n_{12} + n) + Pc(+y_{12} + y_{14})$$

$$M_{CII} = -2(R_I n + Pcy_{14})$$

$$M_B = \frac{M_A(1+2\phi) + M_{C1}}{2} - Pc\phi$$

$$M'_B = \frac{M_{C2} + M'_A(1+2\phi)}{2}$$

Extract: 'Kleinlogel, Mehrstielige Rahmen', Band I+II. Berlin—Verlag von Wilhelm Ernst & Sohn.

Formulæ for Support Reactions, Shear and Axial Forces.

Nomenclature:

V = vertical reaction or axial force in columns.

N = axial force in rafters or columns.

T = shear force at ends of rafters or columns.

H = horizontal reactions in columns.

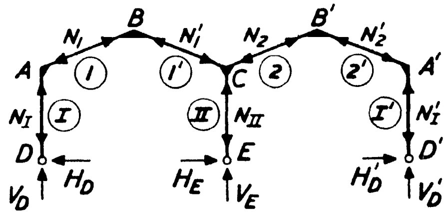
(a) For all loads:

$$M_{CII} = M_{C1} - M_{C2}$$

$$V_E = V_{C1} + V_{C2}$$

$$V_1 = \frac{-M_A + M_{C1}}{L}$$

$$V_2 = \frac{-M_{C2} + M'_A}{L}$$



All the values of H and V shown are positive

$$H_I = \frac{M_A}{h} \qquad H_{II} = \frac{M_{CII}}{h} \qquad H'_I = \frac{M'_A}{h}$$

The axial thrusts in the columns are:

$$N_I = V_D \qquad N_{II} = V_E \qquad N'_I = V'_D$$

The horizontal thrusts in unloaded columns are:

$$H_D = H_I \qquad H_E = H_{II} \qquad H'_D = H'_I$$

The shear forces in unloaded columns are:

$$T_I = +H_I \qquad T_{II} = -H_{II} \qquad T'_I = -H'_I$$

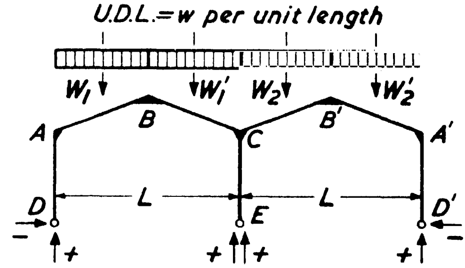
(b) For vertical U.D.L. over the whole frame:

$$V_D = +V_1 + \frac{wL}{2}$$

$$V_{C2} = +V_2 + \frac{wL}{2}$$

$$V_{C1} = -V_1 + \frac{wL}{2}$$

$$V'_D = -V_2 + \frac{wL}{2}$$



Reactions shown are those for total U.D.L

For vertical U.D.L. over the extreme left rafter:

$$V_D = +V_1 + \frac{3wL}{8} \qquad V_{C2} = +V_2$$

$$V_{C1} = -V_1 + \frac{wL}{8} \qquad V'_D = -V_2$$

Extract: 'Kleinlogel, Mehrstielige Rahmen', Band I+II. Berlin—Verlag von Wilhelm Ernst & Sohn.

For vertical U.D.L. over the second rafter from the left:

$$V_D = +V_1 + \frac{wL}{8} \qquad V_{C2} = +V_2$$

$$V_{C1} = -V_1 + \frac{3wL}{8} \qquad V'_D = -V_2$$

The shear forces in the rafters for *all* vertical U.D.L.s are:

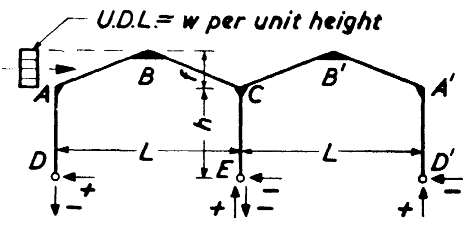
$$T_{L1} = +V_D \cos\theta + H_I \sin\theta \qquad T_{R1} = T_{L1} - W_1 \cos\theta$$
$$T'_{R1} = -V_{C1} \cos\theta - H_I \sin\theta \qquad T'_{L1} = T'_{R1} + W'_1 \cos\theta$$
$$T_{L2} = +V_{C2} \cos\theta + H'_I \sin\theta \qquad T_{R2} = T_{L2} - W_2 \cos\theta$$
$$T'_{R2} = -V'_D \cos\theta - H'_I \sin\theta \qquad T'_{L2} = T'_{R2} + W'_2 \cos\theta$$

where the suffix $L1$ refers to the left-hand end of the first rafter from the left, $R1$ refers to the right-hand end of the first rafter, and so on, and where W_1, W'_1, W_2 and W'_2 are the U.D.L.s on the first, second, third and fourth rafters from the left.

The axial thrusts for *all* vertical U.D.L.s are:

$$N_{1A} = +V_D \sin\theta - H_I \cos\theta \qquad N_{1B} = N_{1A} - W_1 \sin\theta$$
$$N'_{1C} = +V_{C1} \sin\theta - H_I \cos\theta \qquad N'_{1B} = N'_{1C} - W'_1 \sin\theta$$
$$N_{2C} = +V'_{C1} \sin\theta - H'_I \cos\theta \qquad N_{2B} = N_{2C} - W_2 \sin\theta$$
$$N'_{2A} = +V'_D \sin\theta - H'_I \cos\theta \qquad N'_{2B} = N'_{2A} - W'_2 \sin\theta$$

where W_1, W'_1, W_2 and W'_2 are the U.D.L.s on the first, second, third and fourth rafters respectively from the left.



(c) For horizontal U.D.L. applied to the extreme left rafter:

$$V_D = +V_1 - \frac{wf^2}{2L} = -V_{C1}$$

$$V_{C2} = +V_2 = -V'_D$$

$$H_{C1} = H_I - wf \qquad H_{C2} = H'_I$$

The shear forces in the rafters are:

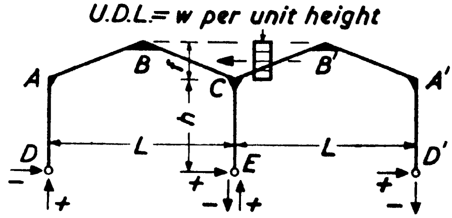
$$T_{L1} = +V_D \cos\theta + H_D \sin\theta \qquad T_{L2} = -V'_D \cos\theta + H_{C2} \sin\theta$$
$$T_{R1} = T_{L1} - wf \sin\theta \qquad T_{R2} = T_{L2}$$
$$T'_1 = +V_D \cos\theta - H_{C1} \sin\theta \qquad T'_2 = -V'_D \cos\theta - H'_D \sin\theta$$

The axial thrusts are:

$$N_{1A} = +V_D \sin\theta - H_D \cos\theta \qquad N_{2C} = -V'_D \sin\theta - H_{C2} \cos\theta$$
$$N_{1B} = N_{1A} + wf \cos\theta \qquad N_{2B} = N_{2C}$$
$$N'_1 = -V_D \sin\theta - H_{C1} \cos\theta \qquad N'_2 = +V'_D \sin\theta - H'_D \cos\theta$$

Extract: 'Kleinlogel, Mehrstielige Rahmen', Band I + II. Berlin—Verlag von Wilhelm Ernst & Sohn.

(d) For horizontal U.D.L. acting to the left on the second rafter from the left:



$$V_D = +V_1 + \frac{wf^2}{2L} = -V_{C1}$$

$$V_{C2} = +V_2 = -V'_D$$

$$H_{C1} = H_I + wf$$

$$H_{C2} = H'_I$$

The shear forces in the rafters are:

$$T'_{R1} = +V_D \cos\theta - H_{C1}\sin\theta \qquad T'_{R2} = -V'_D\cos\theta - H'_D\sin\theta$$

$$T'_{L1} = T'_{R1} + wf\sin\theta \qquad T'_{L2} = T'_{R2}$$

$$T_1 = +V_D\cos\theta + H_D\sin\theta \qquad T_2 = -V'_D\cos\theta + H_{C2}\sin\theta$$

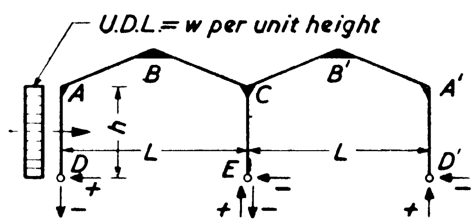
The axial thrusts are:

$$N'_{1C} = -V_D\sin\theta - H_{C1}\cos\theta \qquad N'_{2A} = +V'_D\sin\theta - H'_D\cos\theta$$

$$N'_{1B} = N'_{1C} + wf\cos\theta \qquad N'_{2B} = N_{2A}$$

$$N_1 = +V_D\sin\theta - H_D\cos\theta \qquad N_2 = -V'_D\sin\theta - H_{C2}\cos\theta$$

(e) For horizontal U.D.L. applied to extreme left column:



$$V_D = +V_1 \qquad V_{C2} = +V_2$$

$$V_{C1} = -V_1 \qquad V'_D = -V_2$$

$$H_D = H_I + \frac{wh}{2} \qquad H_E = H_{II}$$

$$H'_D = H'_I$$

The shear forces in the columns are:

$$T_D = +H_D \qquad T_E = -H_E \qquad T'_D = -H'_D$$

$$T_A = +H_D - wh \qquad T_C = -H_E \qquad T'_A = -H'_D$$

The shear forces in the rafters are:

$$T_1 = +V_1\cos\theta + T_A\sin\theta \qquad T_2 = +V_2\cos\theta - T'_A\sin\theta$$

$$T'_1 = +V_1\cos\theta - T_A\sin\theta \qquad T'_2 = +V_2\cos\theta + T'_A\sin\theta$$

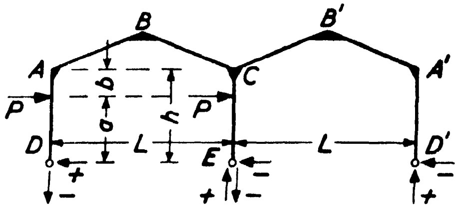
The axial thrusts in the rafters are:

$$N_1 = +V_1\sin\theta - T_A\cos\theta \qquad N_2 = +V_2\sin\theta + T'_A\cos\theta$$

$$N'_1 = -V_1\sin\theta - T_A\cos\theta \qquad N'_2 = -V_2\sin\theta + T'_A\cos\theta$$

Extract: 'Kleinlogel, Mehrstielige Rahmen', Band I+II. Berlin—Verlag von Wilhelm Ernst & Sohn.

(f) Gantry crane loads:



(1) Surge loads:

$V_D = +V_1 \qquad V_{C2} = +V_2$

$V_{C1} = -V_1 \qquad V'_D = -V_2$

$H_D = H_I + \frac{Pb}{h} \qquad H_E = H_{II} - \frac{Pb}{h}$

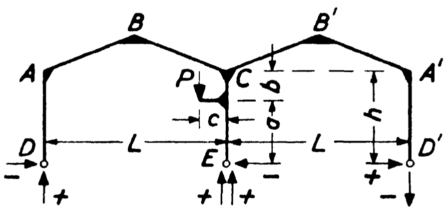
$H'_D = H'_I$

The shear forces in the column are:

$T_D = +H_D \qquad T_E = -H_E \qquad T'_D = -H'_D$

$T_A = +H_D - P \qquad T_C = -H_E - P \qquad T'_A = -H'_D$

(2) Bracket load on central column CE:



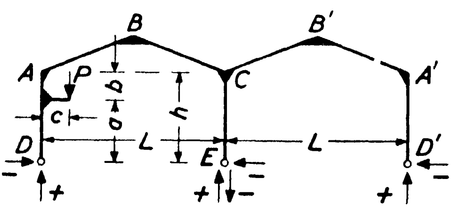
$V_D = +V_1 \qquad V_{C2} = +V_2$

$V_{C1} = P - V_1 \qquad V'_D = -V_2$

$H_D = H_I \qquad H_E = H_{II} - \frac{Pc}{h}$

$H'_D = H'_I$

(3) Bracket load on column AD:



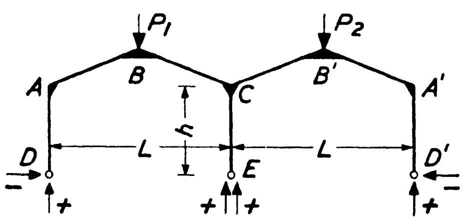
$V_D = P + V_1 \qquad V_{C2} = +V_2$

$V_{C1} = -V_1 \qquad V'_D = -V_2$

$H_D = H_I - \frac{Pc}{h} \qquad H_E = H_{II}$

$H'_D = H'_I$

(g) For vertical point loads on the ridges:



Reactions shown are applicable when $P_1 = P_2$

$V_D = +V_1 + \frac{P_1}{2}$

$V_{C1} = -V_1 + \frac{P_1}{2}$

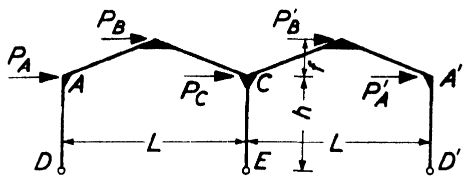
$V_{C2} = +V_2 + \frac{P_2}{2}$

$V'_D = -V_2 + \frac{P_2}{2}$

Extract: 'Kleinlogel, Mehrstielige Rahmen', Band I+II. Berlin—Verlag von Wilhelm Ernst & Sohn.

$$T_1 = +V_D \cos\theta + H_I \sin\theta \qquad N_1 = +V_D \sin\theta - H_I \cos\theta$$
$$T_1' = -V_{C1} \cos\theta - H_I \sin\theta \qquad N_1' = +V_{C1} \sin\theta - H_I \cos\theta$$
$$T_2 = +V_{C2} \cos\theta + H_I' \sin\theta \qquad N_2 = +V_{C2} \sin\theta - H_I' \cos\theta$$
$$T_2' = -V_D' \cos\theta - H_I' \sin\theta \qquad N_2' = +V_D' \sin\theta - H_I' \cos\theta$$

(h) For horizontal point loads applied at joints:

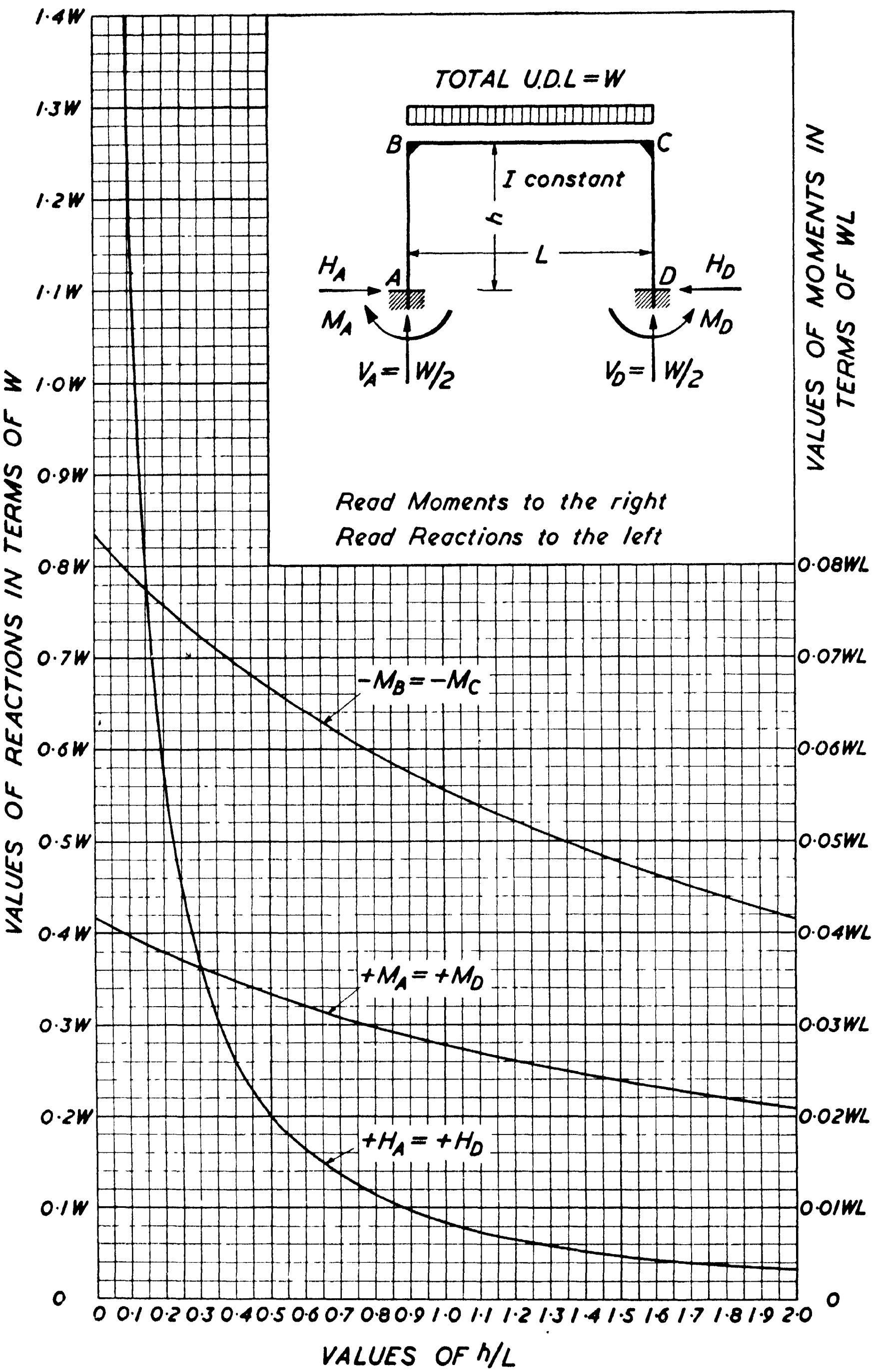


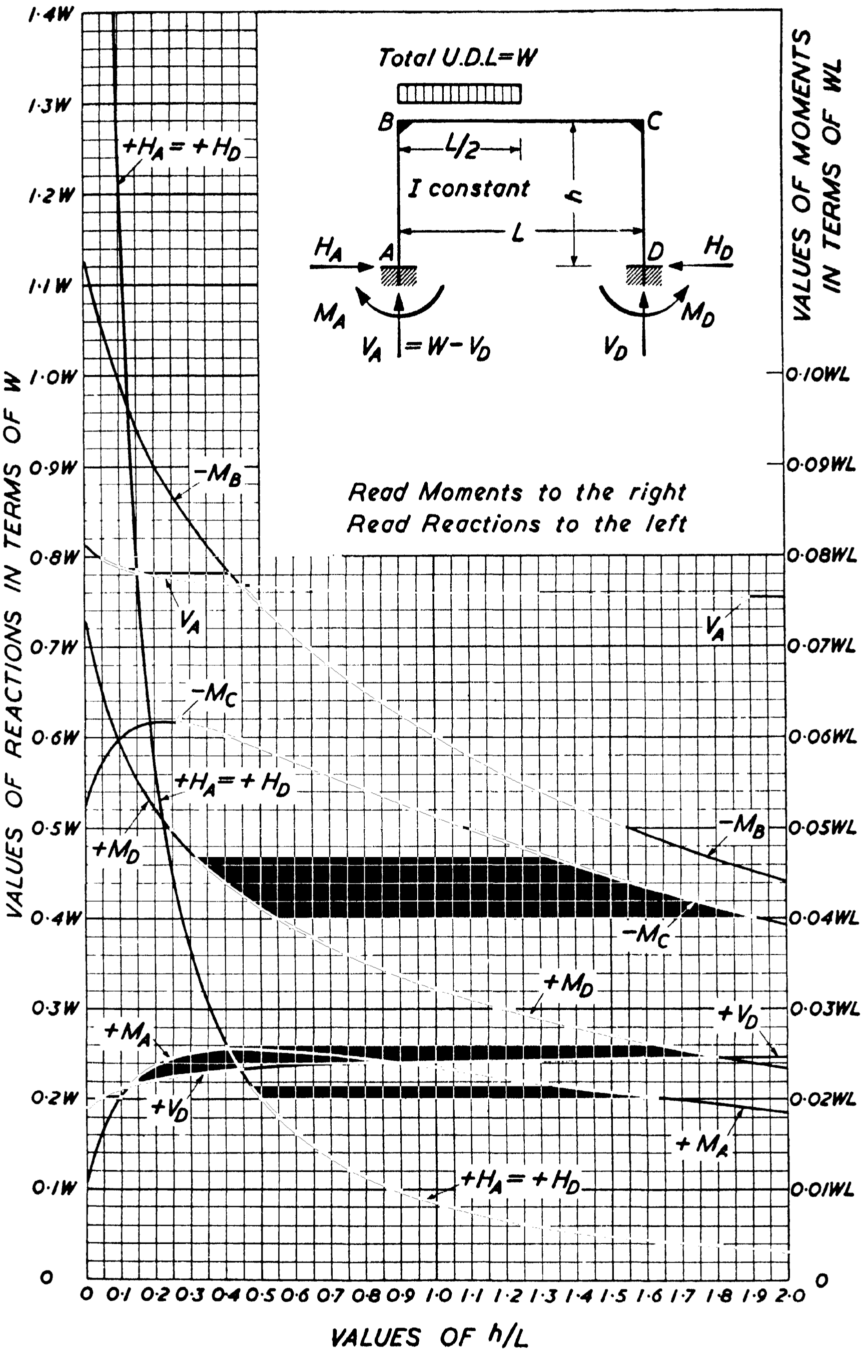
Reactions vary with the number and value of the loads

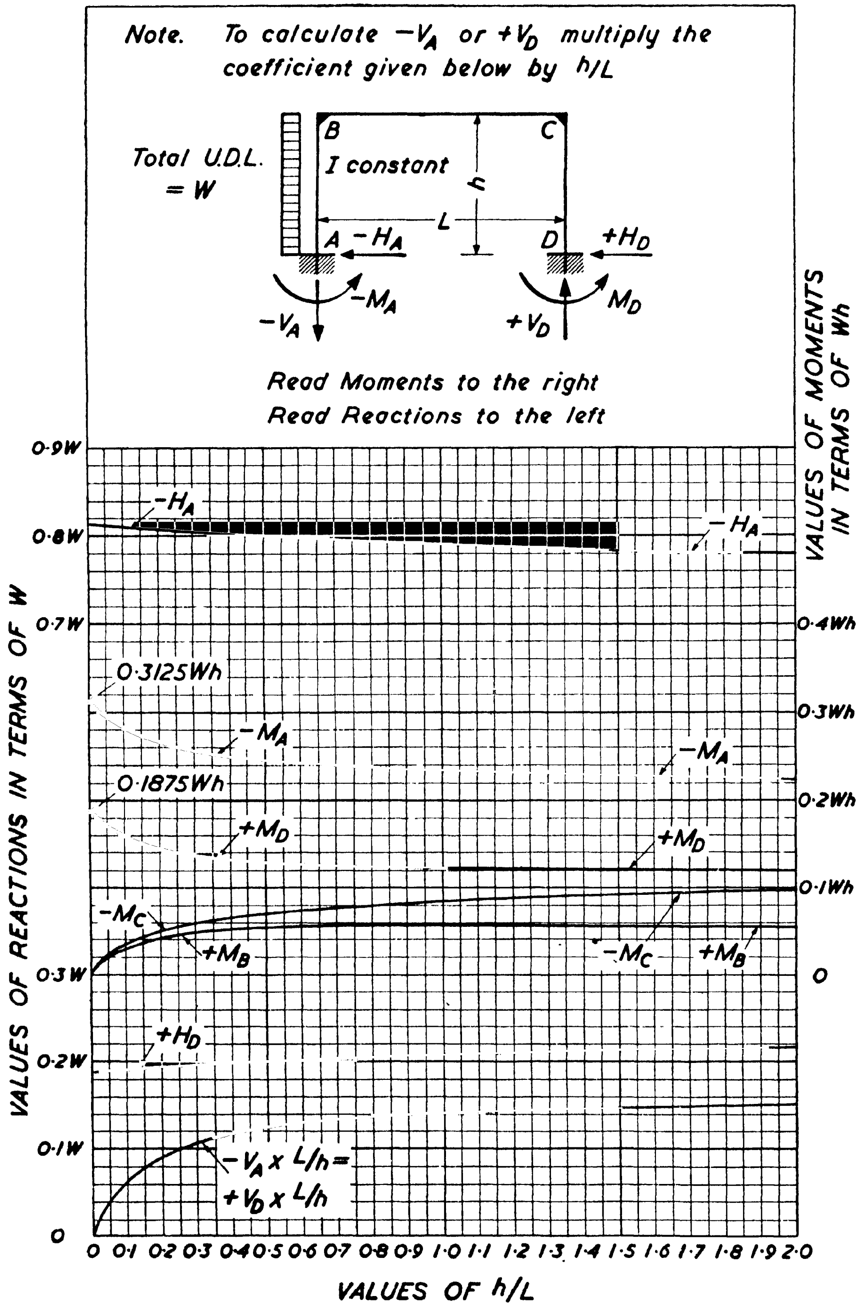
$$V_D = +V_1 - \frac{P_B f}{L}$$
$$V_{C2} = +V_2 - \frac{P'_B f}{L}$$
$$V_{C1} = -V_D$$
$$V_D' = -V_{C2}$$
$$H_{C1} = H_I - P_A - P_B$$
$$H_{C2} = H_I' + P_B' + P_A'$$

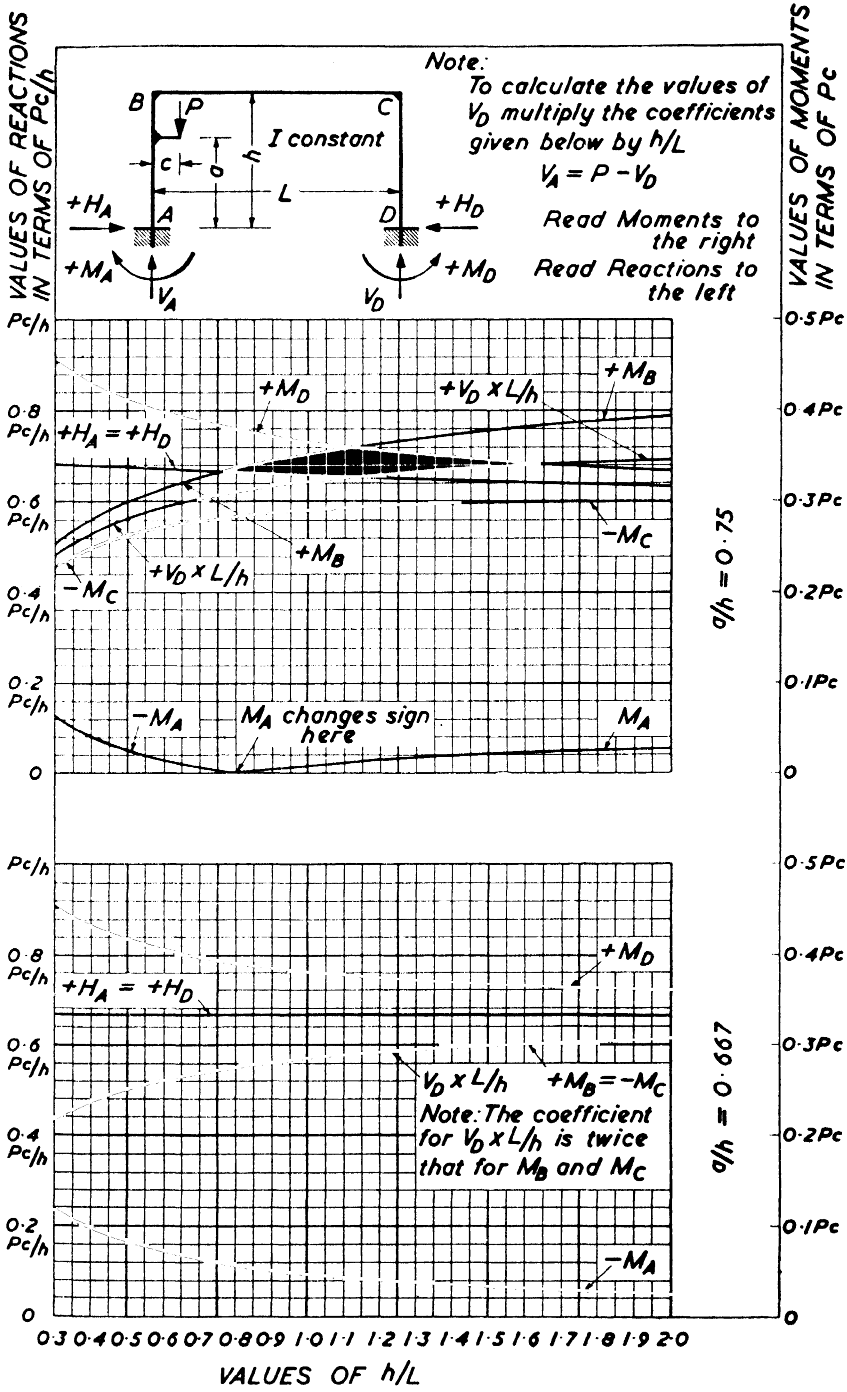
$$T_1 = +V_D \cos\theta + (H_I - P_A)\sin\theta \qquad N_1 = +V_D \sin\theta - (H_I - P_A)\cos\theta$$
$$T_1' = +V_D \cos\theta - H_{C1} \sin\theta \qquad N_1' = -V_D \sin\theta - H_{C1} \cos\theta$$
$$T_2 = -V_D' \cos\theta + H_{C2} \sin\theta \qquad N_2 = -V_D' \sin\theta - H_{C2} \cos\theta$$
$$T_2' = -V_D' \cos\theta - (H_I' + P_A')\sin\theta \qquad N_2' = +V_D' \sin\theta - (H_I' + P_A')\cos\theta$$

Extract: 'Kleinlogel, Mehrstielige Rahmen', Band I + II. Berlin—Verlag von Wilhelm Ernst & Sohn.

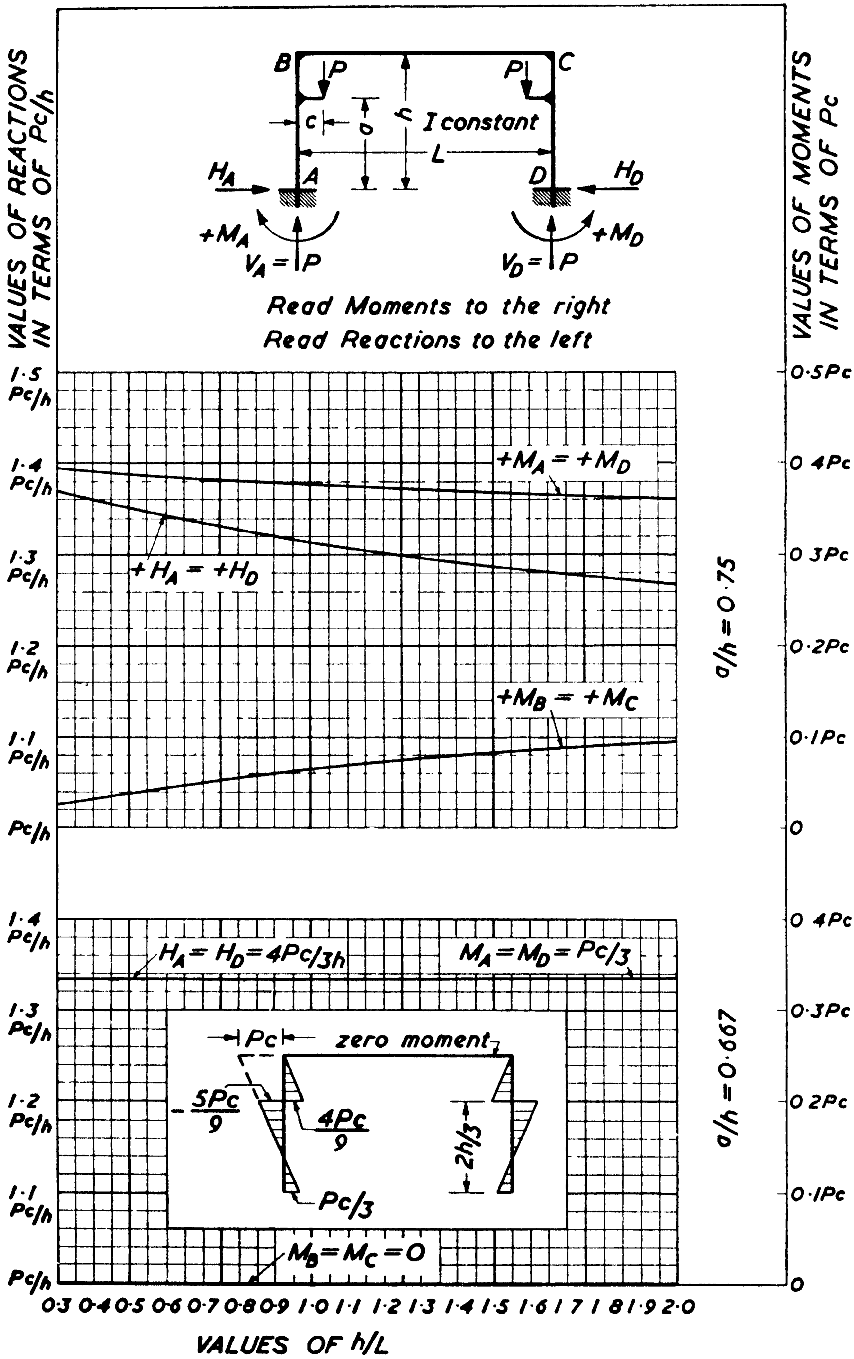

TOTAL U.D.L = W
B
C
I constant
h
L
H_A
A
D
H_D
M_A
M_D
$V_A = W/2$
$V_D = W/2$
Read Moments to the right
Read Reactions to the left
$-M_B = -M_C$
$+M_A = +M_D$
$+H_A = +H_D$
1·4W
1·3W
1·2W
1·1W
1·0W
0·9W
0·8W
0·7W
0·6W
0·5W
0·4W
0·3W
0·2W
0·1W
0
VALUES OF REACTIONS IN TERMS OF W
0·08WL
0·07WL
0·06WL
0·05WL
0·04WL
0·03WL
0·02WL
0·01WL
0
VALUES OF MOMENTS IN TERMS OF WL
0 0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2·0
VALUES OF h/L

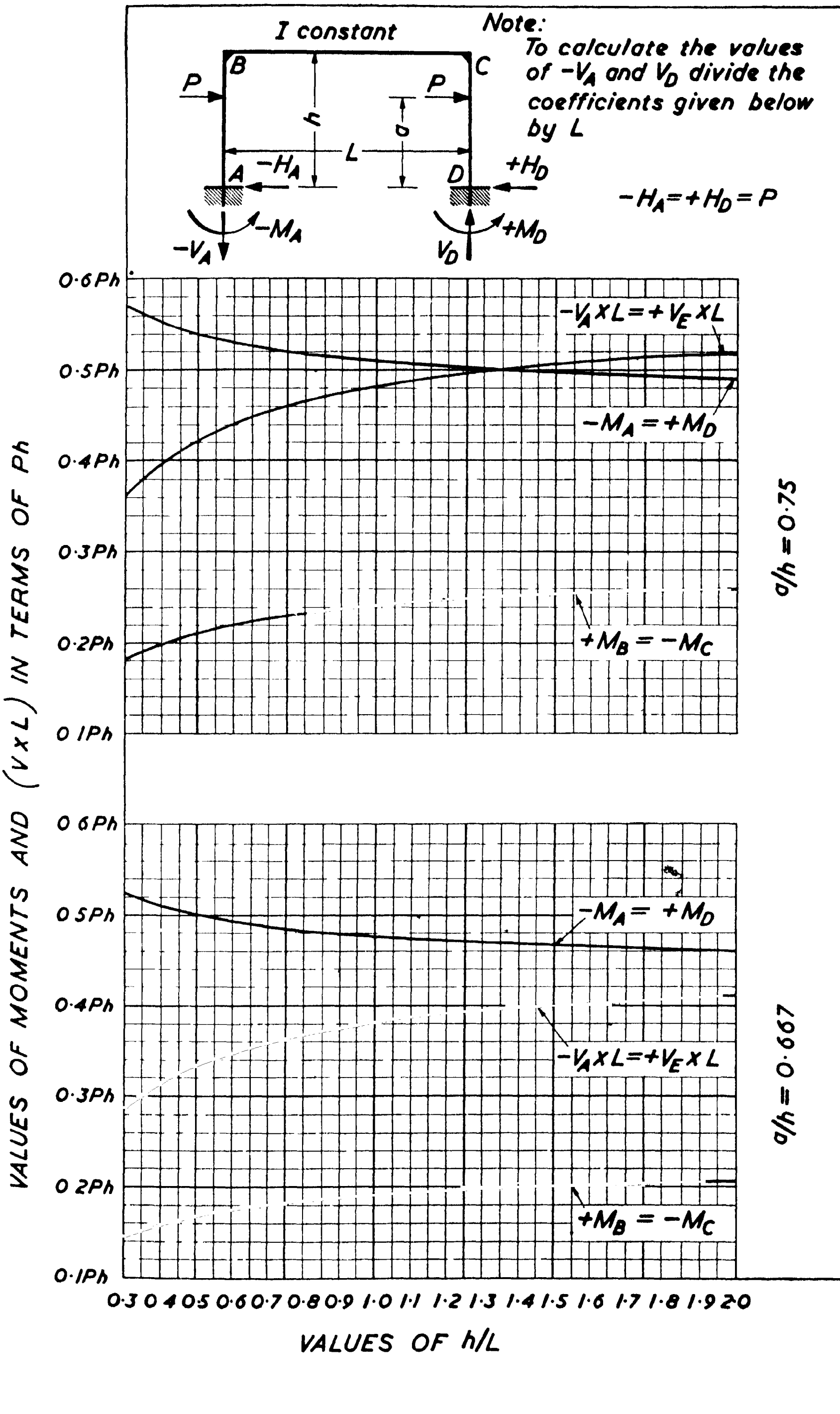

Total U.D.L=W
B
C
L/2
I constant
h
L
H_A
A
D
H_D
M_A
M_D
V_A = W − V_D
V_D
Read Moments to the right
Read Reactions to the left
+H_A = +H_D
−M_B
V_A
−M_C
+M_D
+M_A
+V_D
VALUES OF REACTIONS IN TERMS OF W
VALUES OF MOMENTS IN TERMS OF WL
1·4W
1·3W
1·2W
1·1W
1·0W
0·9W
0·8W
0·7W
0·6W
0·5W
0·4W
0·3W
0·2W
0·1W
0
0·10WL
0·09WL
0·08WL
0·07WL
0·06WL
0·05WL
0·04WL
0·03WL
0·02WL
0·01WL
0
0 0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2·0
VALUES OF h/L

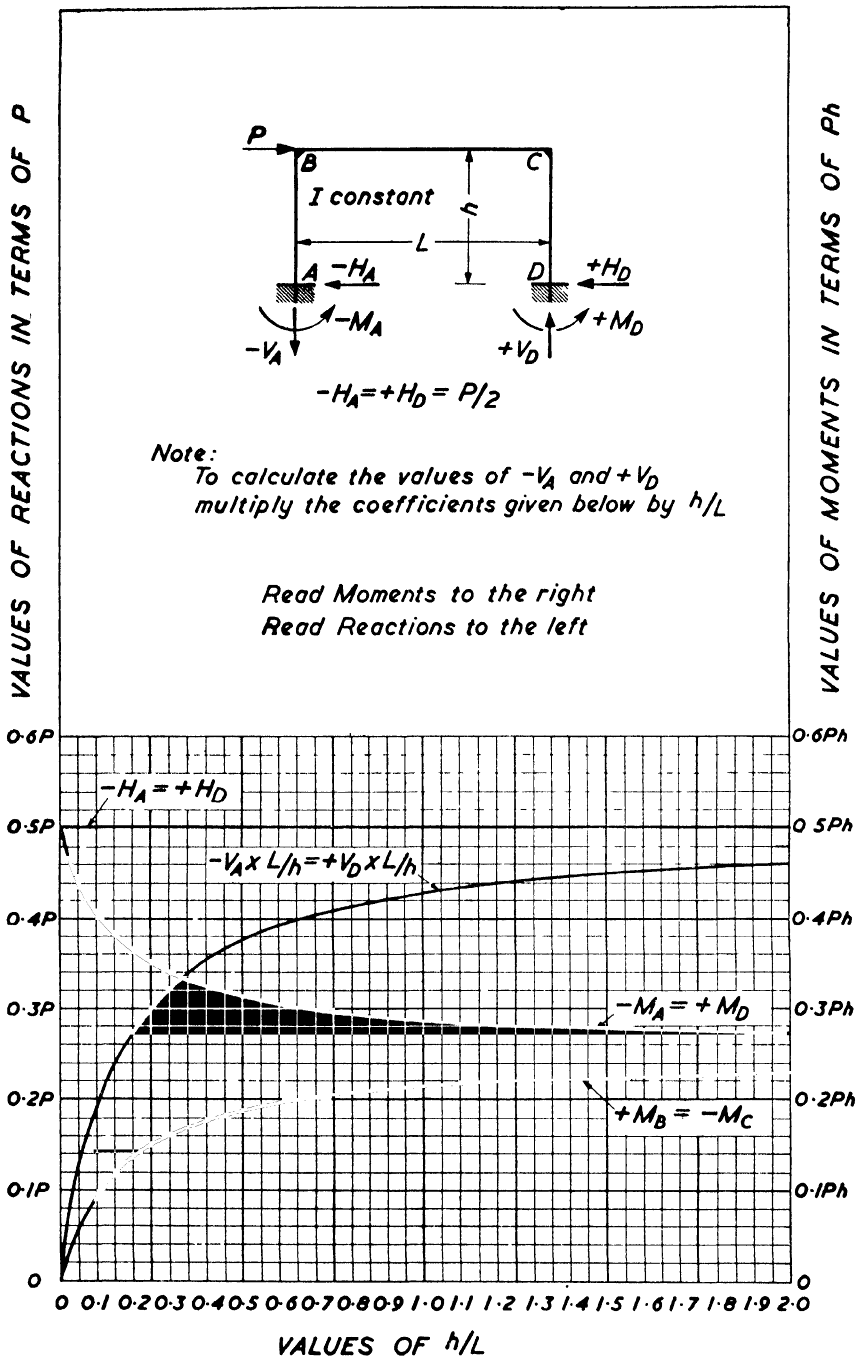

Note. To calculate $-V_A$ or $+V_D$ multiply the coefficient given below by h/L
Total U.D.L. = W
B
C
I constant
h
L
A
$-H_A$
D
$+H_D$
$-M_A$
M_D
$-V_A$
$+V_D$
Read Moments to the right
Read Reactions to the left
VALUES OF REACTIONS IN TERMS OF W
VALUES OF MOMENTS IN TERMS OF Wh
0·9W
0·8W
0·7W
0·3W
0·2W
0·1W
0
0·4Wh
0·3Wh
0·2Wh
0·1Wh
0
$-H_A$
0·3125Wh
$-M_A$
0·1875Wh
$+M_D$
$-M_C$
$+M_B$
$+H_D$
$-V_A \times L/h = +V_D \times L/h$
0 0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2·0
VALUES OF h/L

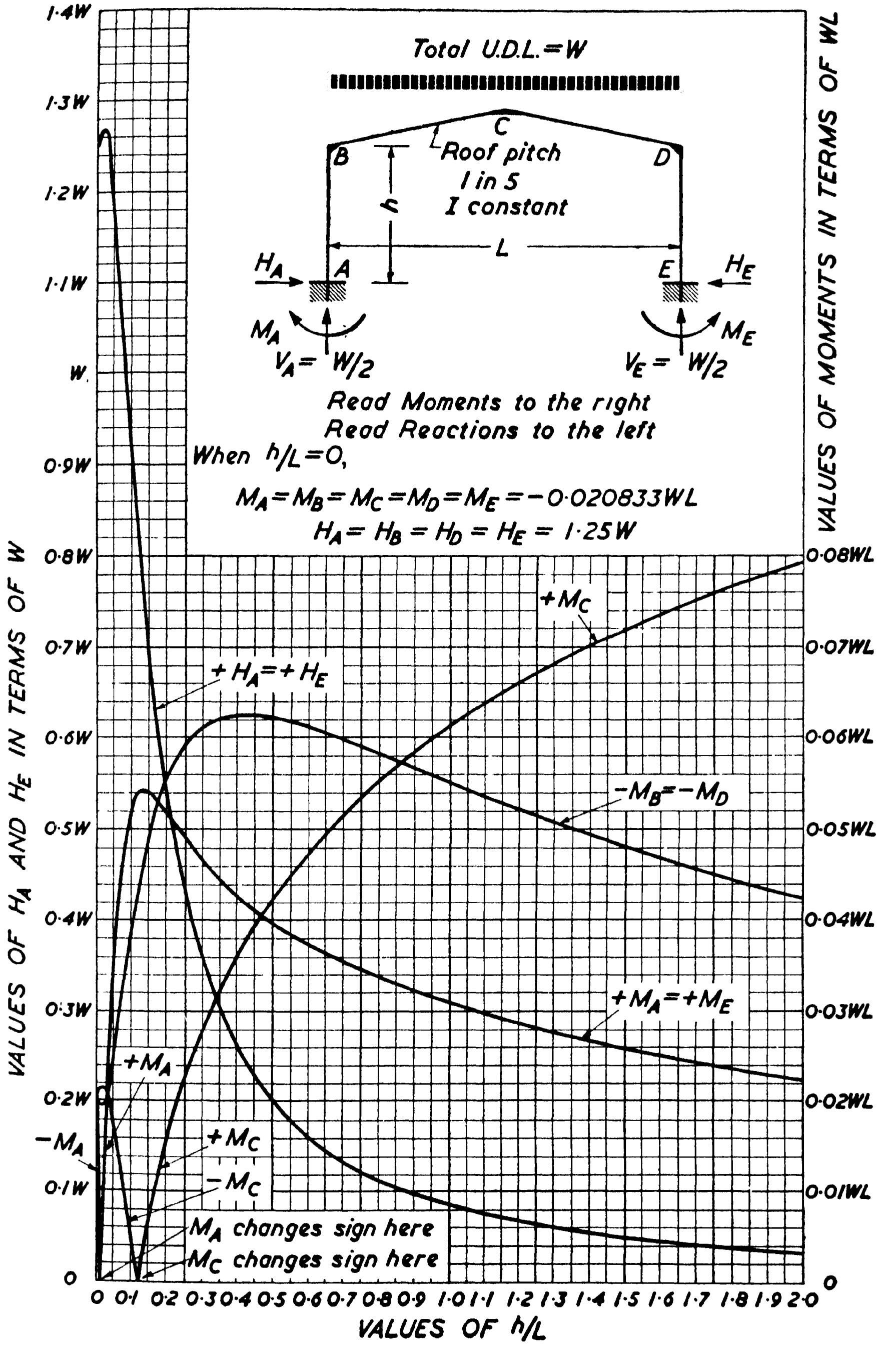


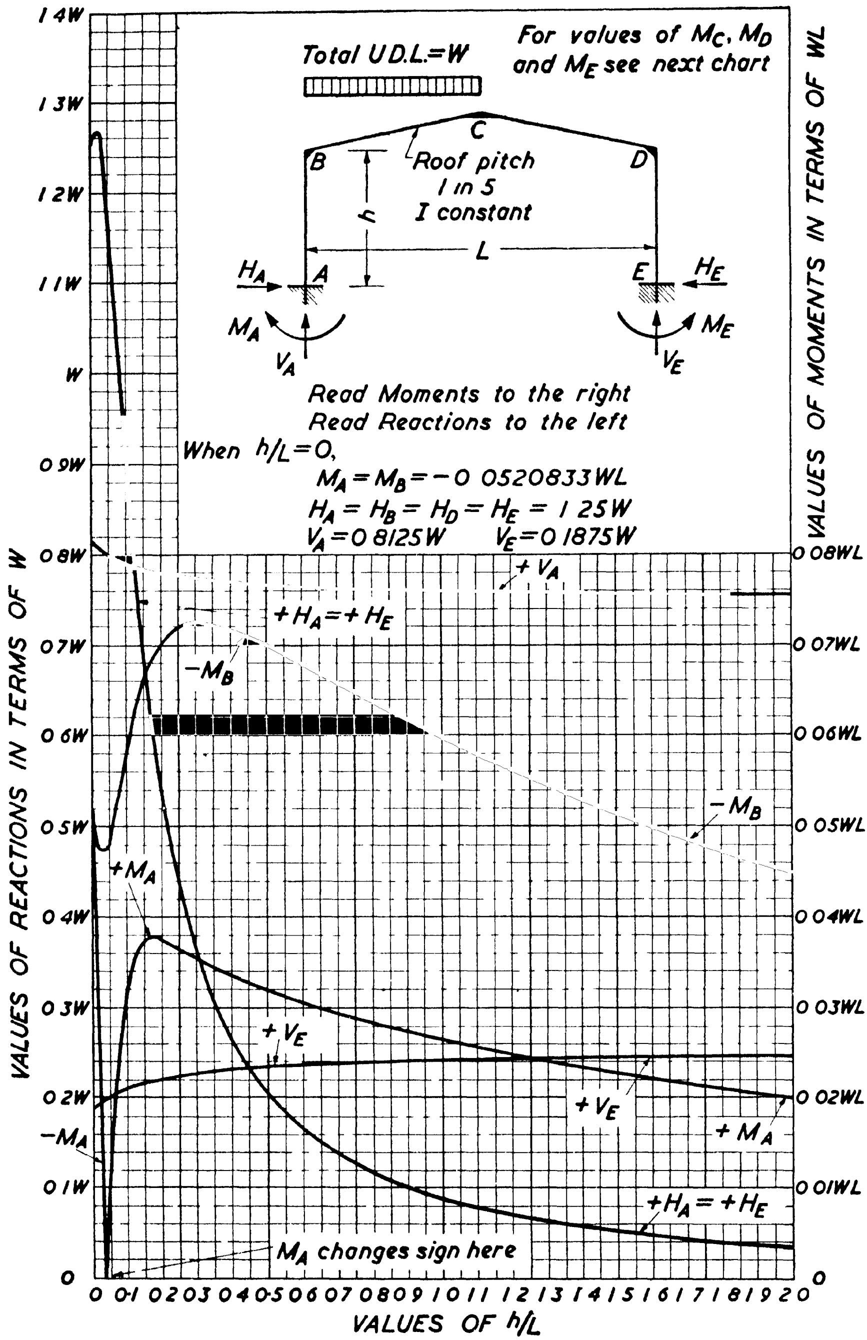
VALUES OF REACTIONS IN TERMS OF Pc/h
VALUES OF MOMENTS IN TERMS OF Pc
B
P
C
I constant
c
a
h
L
+HA
A
D
+HD
+MA
VA
VD
+MD
Note:
To calculate the values of VD multiply the coefficients given below by h/L
VA = P − VD
Read Moments to the right
Read Reactions to the left
+MD
+VD x L/h
+MB
+HA = +HD
−MC
+MB
+VD x L/h
−MC
−MA
MA changes sign here
MA
a/h = 0·75
+MD
+HA = +HD
VD x L/h
+MB = −MC
Note: The coefficient for VD x L/h is twice that for MB and MC
−MA
a/h = 0·667
Pc/h
0·8 Pc/h
0·6 Pc/h
0·4 Pc/h
0·2 Pc/h
0
0·5Pc
0·4Pc
0·3Pc
0·2Pc
0·1Pc
0
0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2·0
VALUES OF h/L

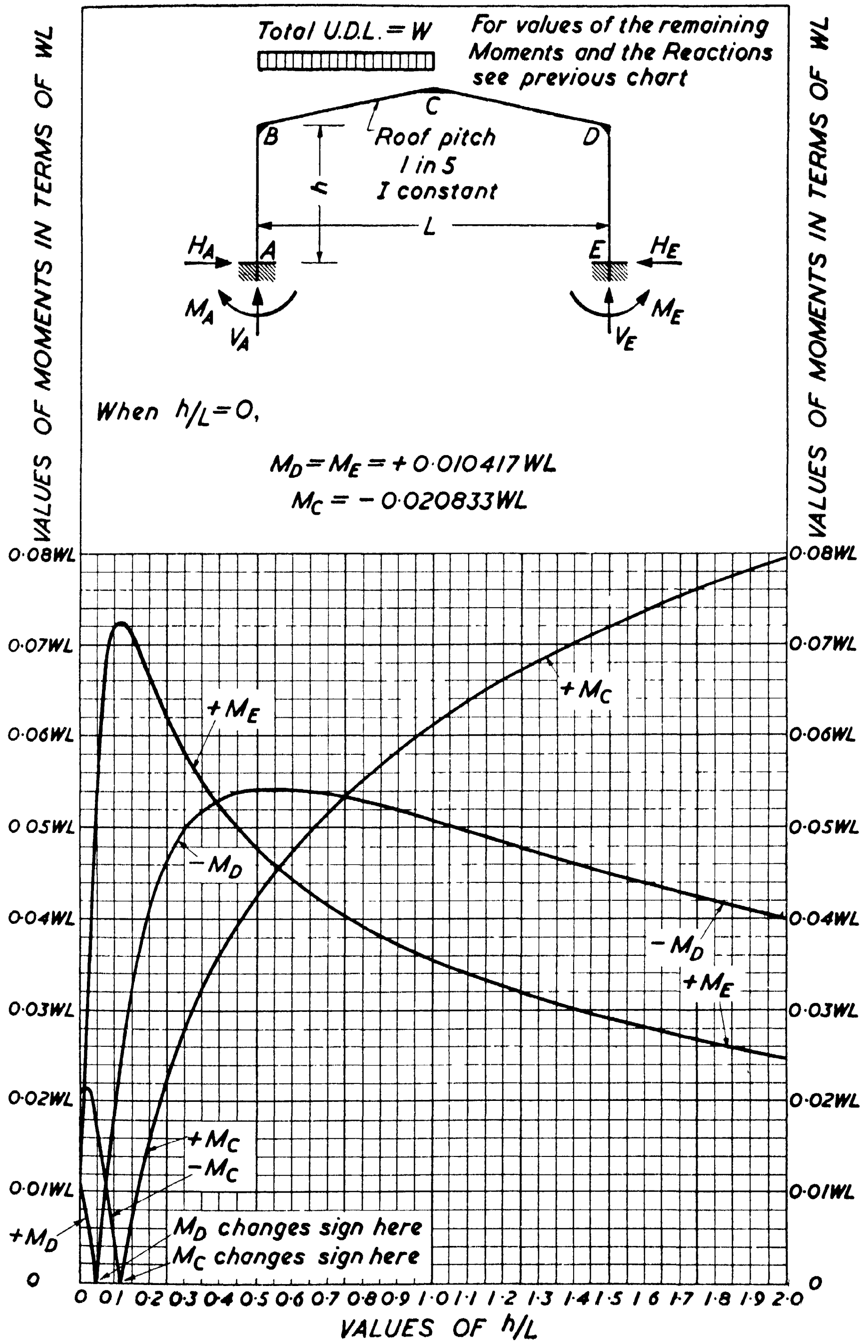

VALUES OF REACTIONS IN TERMS OF Pc/h
VALUES OF MOMENTS IN TERMS OF Pc
B
C
P
P
c
a
h
I constant
L
H_A
A
D
H_D
$+M_A$
$+M_D$
$V_A = P$
$V_D = P$
Read Moments to the right
Read Reactions to the left
1·5 Pc/h
1·4 Pc/h
1·3 Pc/h
1·2 Pc/h
1·1 Pc/h
Pc/h
0·5Pc
0 4Pc
0 3Pc
0·2Pc
0·1Pc
0
$+M_A = +M_D$
$+H_A = +H_D$
$+M_B = +M_C$
a/h = 0·75
1·4 Pc/h
1·3 Pc/h
1·2 Pc/h
1·1 Pc/h
Pc/h
0 4Pc
0·3Pc
0 2Pc
0·1Pc
0
$H_A = H_D = 4Pc/3h$
$M_A = M_D = Pc/3$
Pc
zero moment
$-\frac{5Pc}{9}$
$\frac{4Pc}{9}$
2h/3
Pc/3
$M_B = M_C = 0$
a/h = 0·667
0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2·0
VALUES OF h/L

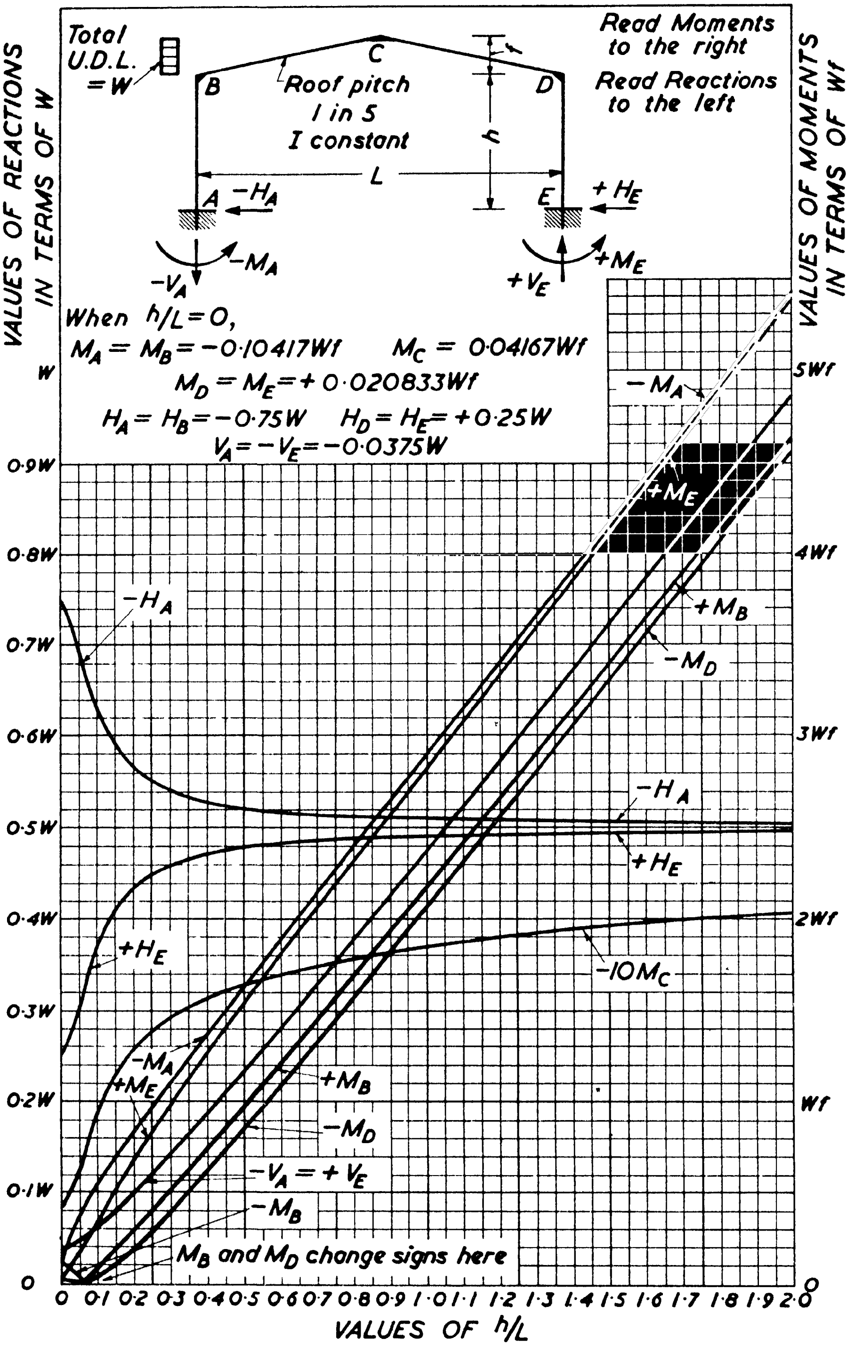

I constant
Note:
To calculate the values of −V_A and V_D divide the coefficients given below by L
−H_A = +H_D = P
−V_A × L = +V_E × L
−M_A = +M_D
+M_B = −M_C
a/h = 0·75
a/h = 0·667
VALUES OF MOMENTS AND (V × L) IN TERMS OF Ph
VALUES OF h/L

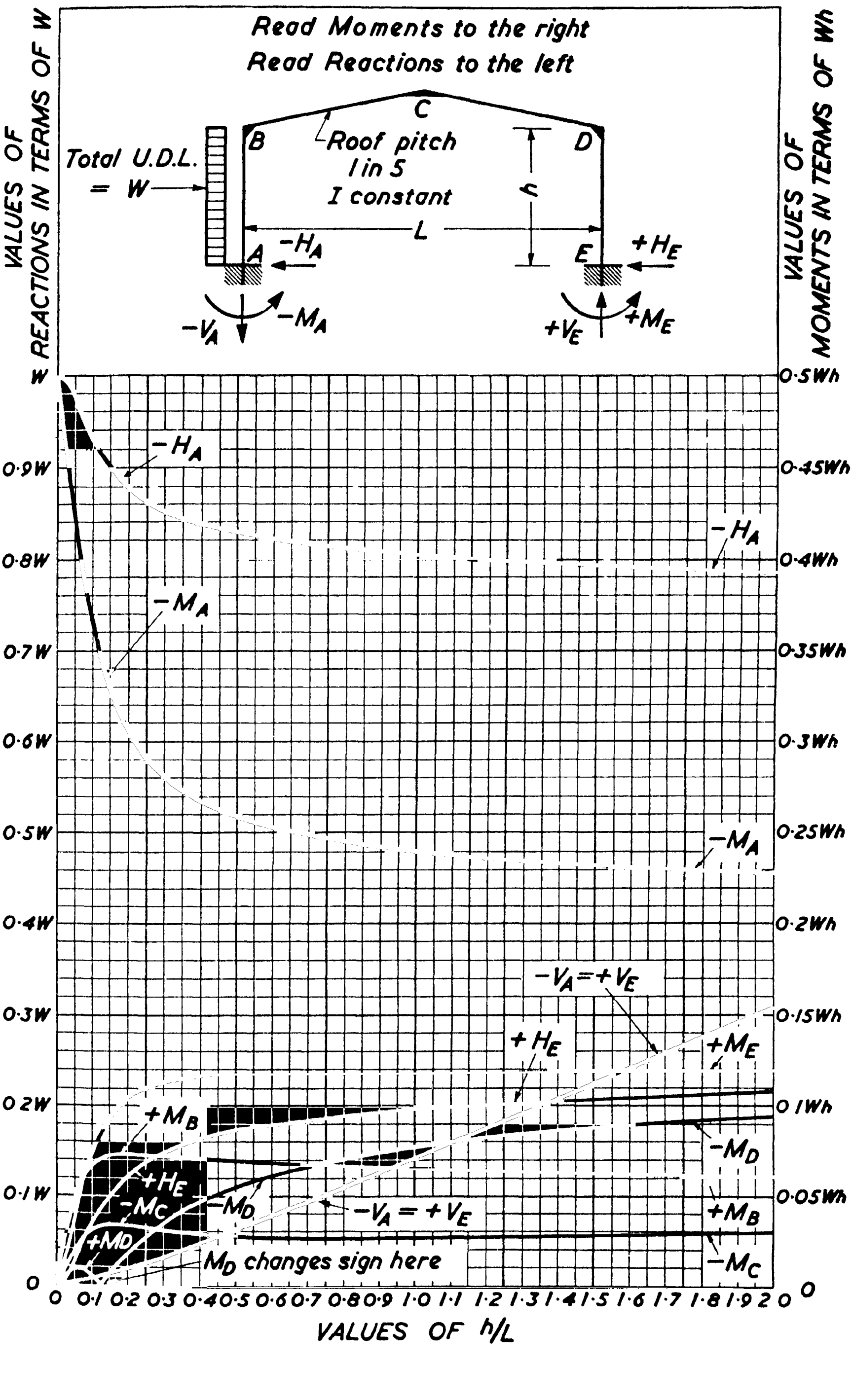

VALUES OF REACTIONS IN TERMS OF P
VALUES OF MOMENTS IN TERMS OF Ph
P
B
C
I constant
h
L
A
−H_A
D
+H_D
−M_A
+M_D
−V_A
+V_D
−H_A = +H_D = P/2
Note:
To calculate the values of −V_A and +V_D multiply the coefficients given below by h/L
Read Moments to the right
Read Reactions to the left
−H_A = +H_D
−V_A x L/h = +V_D x L/h
−M_A = +M_D
+M_B = −M_C
0·6P
0·5P
0·4P
0·3P
0·2P
0·1P
0
0·6Ph
0·5Ph
0·4Ph
0·3Ph
0·2Ph
0·1Ph
0
0 0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2·0
VALUES OF h/L

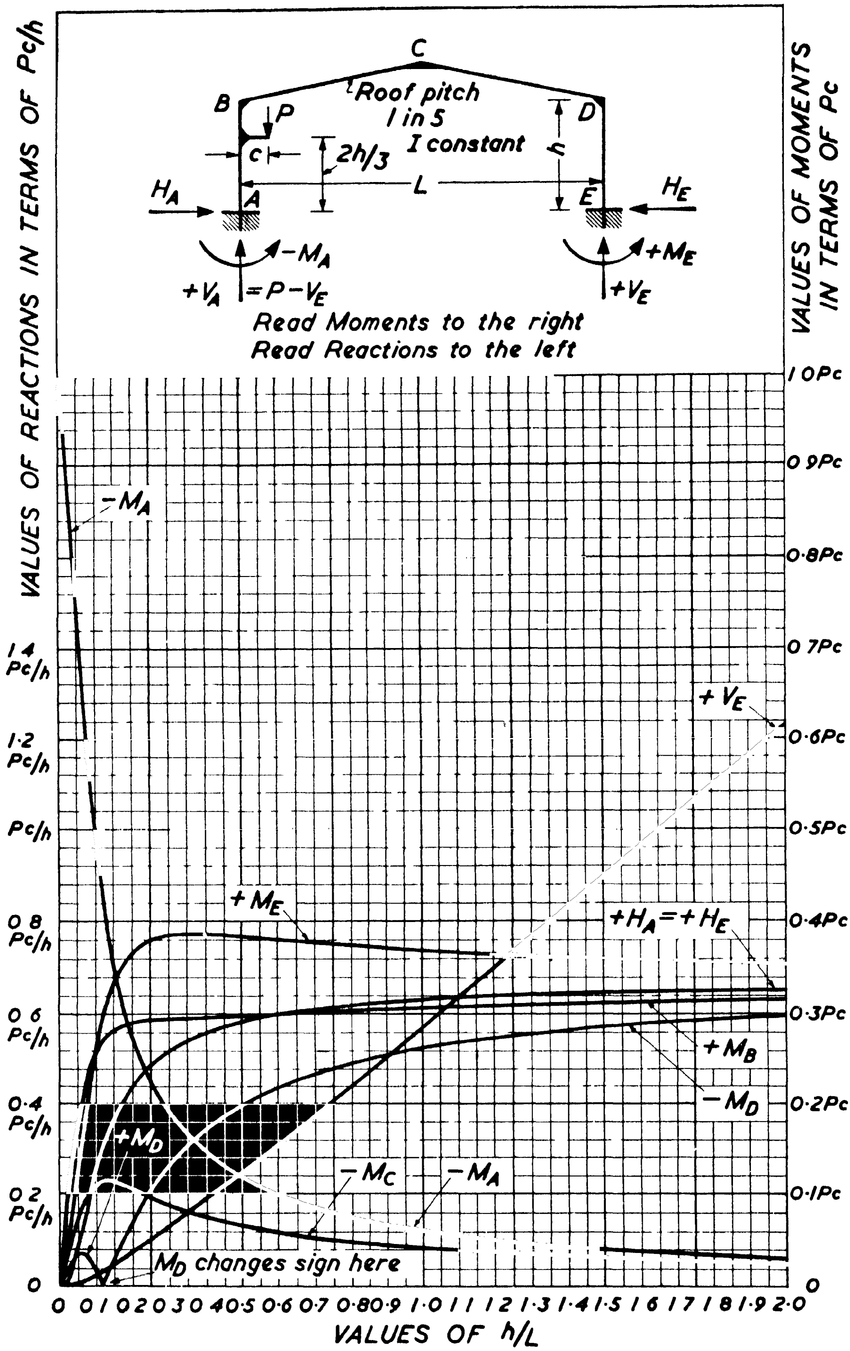

Total U.D.L.=W
C
B
D
Roof pitch
1 in 5
I constant
h
L
A
E
H_A
H_E
M_A
M_E
V_A = W/2
V_E = W/2
Read Moments to the right
Read Reactions to the left
When h/L=0,
$M_A=M_B=M_C=M_D=M_E=-0{\cdot}020833WL$
$H_A=H_B=H_D=H_E=1{\cdot}25W$
+M_C
+H_A=+H_E
−M_B=−M_D
+M_A=+M_E
+M_A
−M_A
+M_C
−M_C
M_A changes sign here
M_C changes sign here
VALUES OF H_A AND H_E IN TERMS OF W
VALUES OF MOMENTS IN TERMS OF WL
VALUES OF h/L
1·4W
1·3W
1·2W
1·1W
W
0·9W
0·8W
0·7W
0·6W
0·5W
0·4W
0·3W
0·2W
0·1W
0
0·08WL
0·07WL
0·06WL
0·05WL
0·04WL
0·03WL
0·02WL
0·01WL
0
0 0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2·0


Total U.D.L.=W
For values of M_C, M_D and M_E see next chart
Roof pitch 1 in 5
I constant
Read Moments to the right
Read Reactions to the left
When h/L=0,
$M_A = M_B = -0\ 0520833\,WL$
$H_A = H_B = H_D = H_E = 1\ 25W$
$V_A = 0\ 8125W$ $V_E = 0\ 1875W$
$+V_A$
$+H_A = +H_E$
$-M_B$
$+M_A$
$+V_E$
$-M_A$
M_A changes sign here
VALUES OF REACTIONS IN TERMS OF W
VALUES OF MOMENTS IN TERMS OF WL
VALUES OF h/L

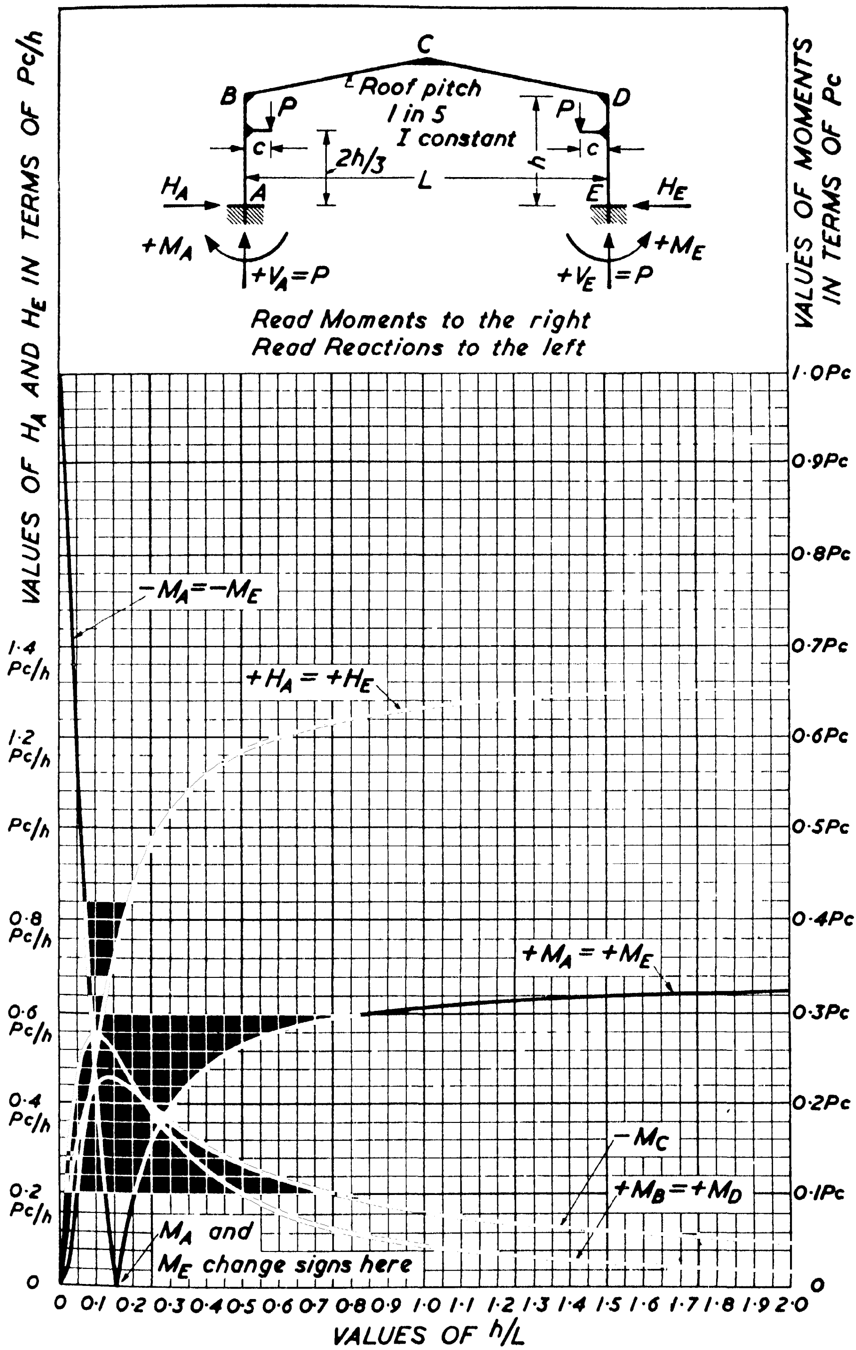

Total U.D.L. = W
For values of the remaining Moments and the Reactions see previous chart
C
B
D
Roof pitch 1 in 5
I constant
h
L
H_A
A
E
H_E
M_A
V_A
V_E
M_E
When h/L = 0,
$M_D = M_E = +0{\cdot}010417\,WL$
$M_C = -0{\cdot}020833\,WL$
VALUES OF MOMENTS IN TERMS OF WL
0·08WL
0·07WL
0·06WL
0·05WL
0·04WL
0·03WL
0·02WL
0·01WL
0
$+M_E$
$+M_C$
$-M_D$
$+M_D$
$-M_C$
M_D changes sign here
M_C changes sign here
0 0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2·0
VALUES OF h/L

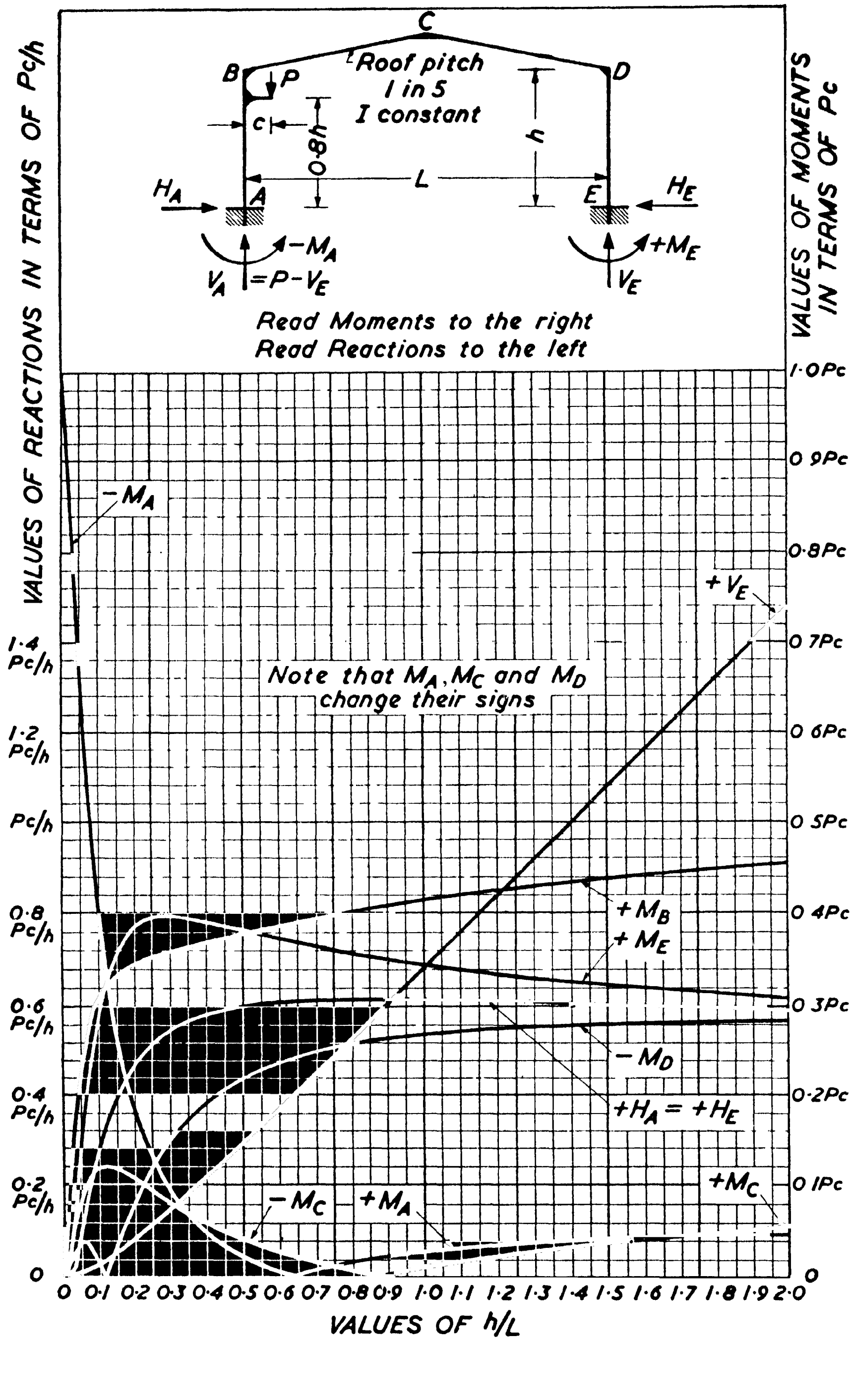

VALUES OF REACTIONS IN TERMS OF W
VALUES OF MOMENTS IN TERMS OF Wf
Total U.D.L. = W
Roof pitch 1 in 5
I constant
Read Moments to the right
Read Reactions to the left
B
C
D
A
E
L
h
−H_A
+H_E
−M_A
+M_E
−V_A
+V_E
When h/L = 0,
M_A = M_B = −0·10417Wf   M_C = 0·04167Wf
M_D = M_E = + 0·020833Wf
H_A = H_B = −0·75W   H_D = H_E = +0·25W
V_A = −V_E = −0·0375W
W
0·9W
0·8W
0·7W
0·6W
0·5W
0·4W
0·3W
0·2W
0·1W
0
5Wf
4Wf
3Wf
2Wf
Wf
0
−M_A
+M_E
+M_B
−M_D
−H_A
+H_E
−10M_C
−M_A
+M_E
+M_B
−M_D
−V_A = + V_E
−M_B
M_B and M_D change signs here
0 0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2·0
VALUES OF h/L

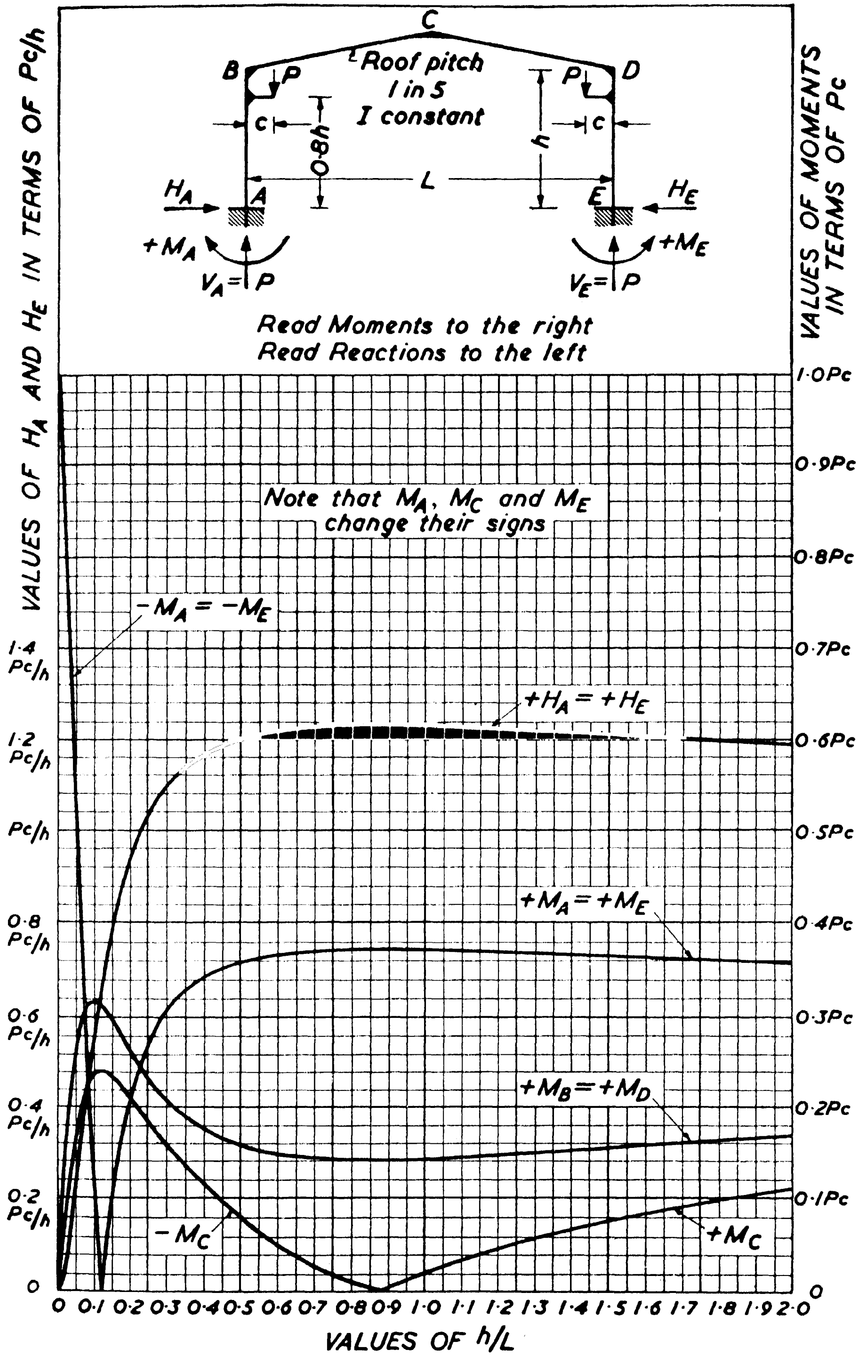

Read Moments to the right
Read Reactions to the left
Total U.D.L. = W
Roof pitch 1 in 5
I constant
A
B
C
D
E
h
L
−H_A
+H_E
−V_A
−M_A
+V_E
+M_E
VALUES OF REACTIONS IN TERMS OF W
VALUES OF MOMENTS IN TERMS OF Wh
W
0·9W
0·8W
0·7W
0·6W
0·5W
0·4W
0·3W
0·2W
0·1W
0
0·5Wh
0·45Wh
0·4Wh
0·35Wh
0·3Wh
0·25Wh
0·2Wh
0·15Wh
0·1Wh
0·05Wh
0
−H_A
−M_A
−V_A = +V_E
+H_E
+M_E
+M_B
−M_D
−M_C
+M_D
M_D
M_D changes sign here
0 0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2·0
VALUES OF h/L

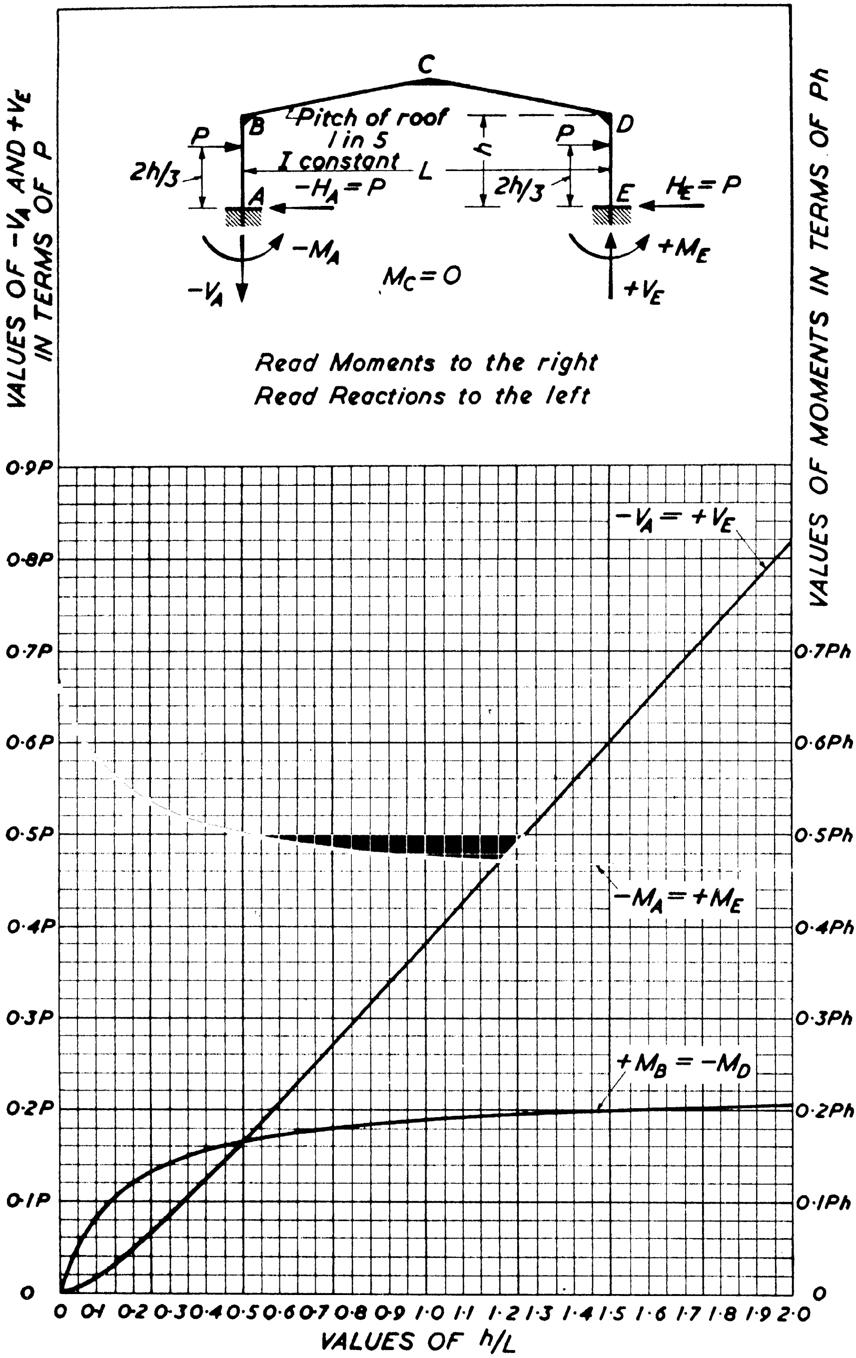


C
Roof pitch
1 in 5
I constant
B
P
D
c
2h/3
h
H_A
A
L
E
H_E
$-M_A$
$+M_E$
$+V_A$
$=P-V_E$
$+V_E$
Read Moments to the right
Read Reactions to the left
VALUES OF REACTIONS IN TERMS OF Pc/h
VALUES OF MOMENTS IN TERMS OF Pc
1·4 Pc/h
1·2 Pc/h
Pc/h
0·8 Pc/h
0·6 Pc/h
0·4 Pc/h
0·2 Pc/h
0
1·0Pc
0·9Pc
0·8Pc
0·7Pc
0·6Pc
0·5Pc
0·4Pc
0·3Pc
0·2Pc
0·1Pc
0
$-M_A$
$+V_E$
$+M_E$
$+H_A=+H_E$
$+M_B$
$-M_D$
$+M_D$
$-M_C$
$-M_A$
M_D changes sign here
0 0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2·0
VALUES OF h/L

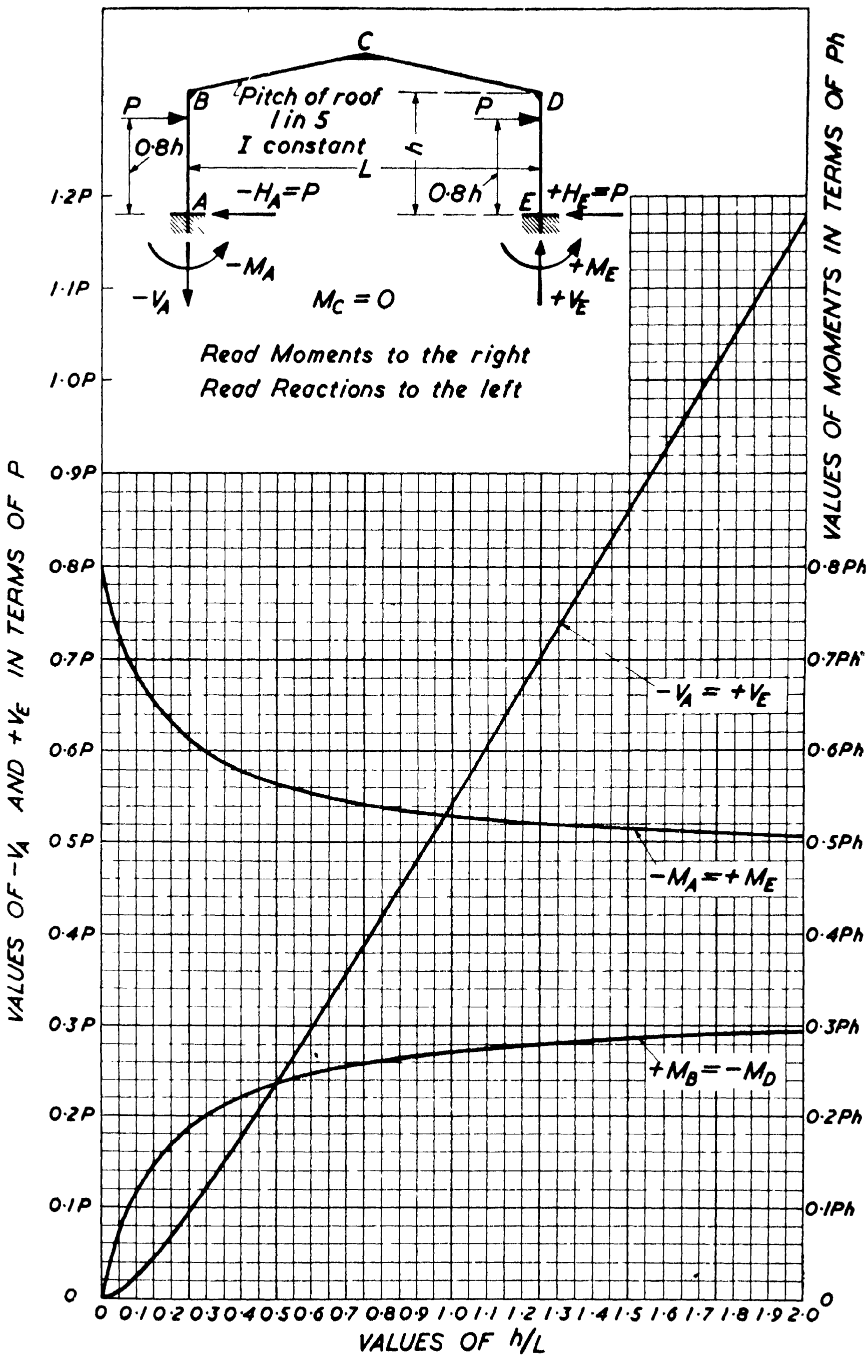

C
B
D
Roof pitch
1 in 5
I constant
P
P
c
c
2h/3
h
L
H_A
H_E
A
E
$+M_A$
$+M_E$
$+V_A=P$
$+V_E=P$
Read Moments to the right
Read Reactions to the left
VALUES OF H_A AND H_E IN TERMS OF Pc/h
VALUES OF MOMENTS IN TERMS OF Pc
$-M_A=-M_E$
$+H_A=+H_E$
$+M_A=+M_E$
$-M_C$
$+M_B=+M_D$
M_A and M_E change signs here
1·4 Pc/h
1·2 Pc/h
Pc/h
0·8 Pc/h
0·6 Pc/h
0·4 Pc/h
0·2 Pc/h
0
1·0Pc
0·9Pc
0·8Pc
0·7Pc
0·6Pc
0·5Pc
0·4Pc
0·3Pc
0·2Pc
0·1Pc
0
0 0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2·0
VALUES OF h/L

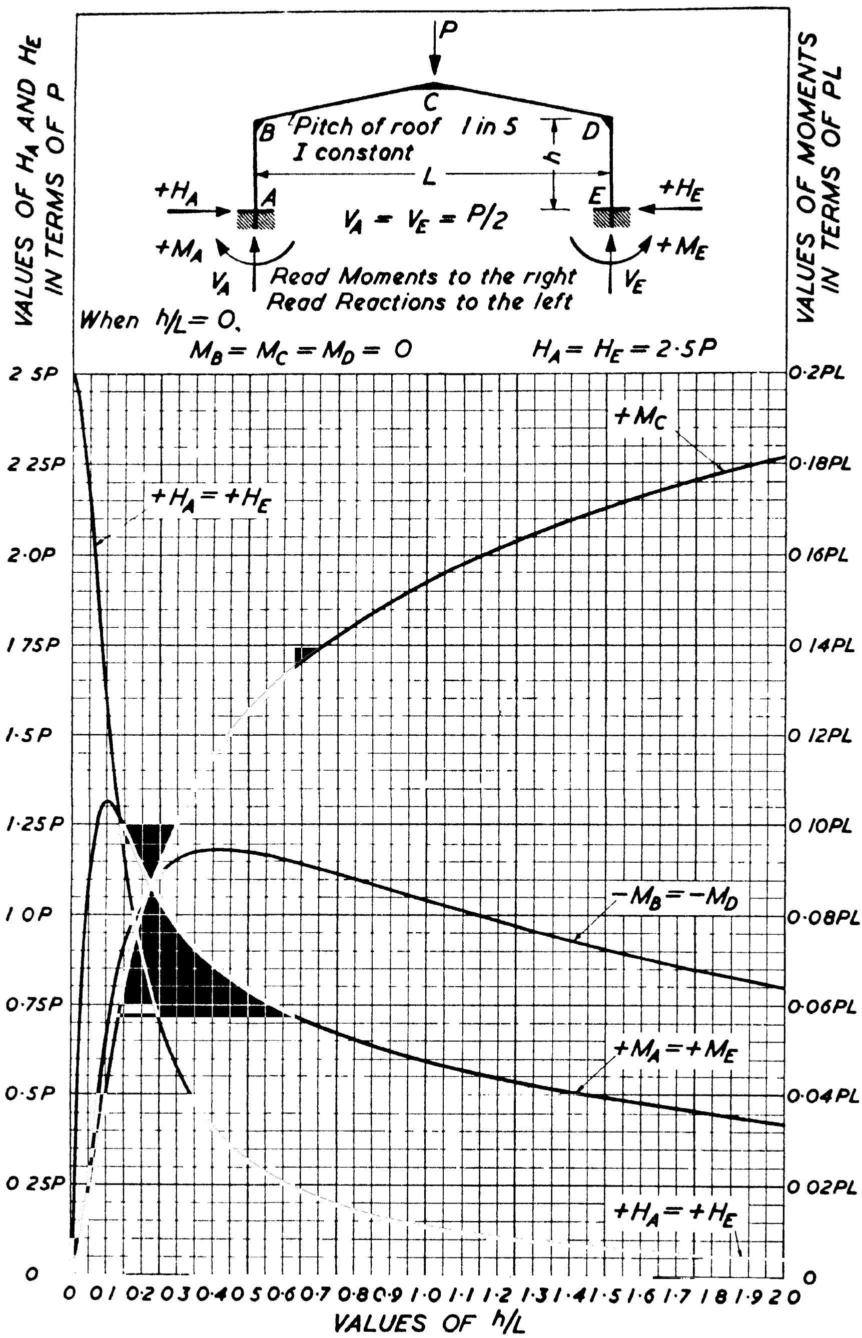

C
B
P
Roof pitch
1 in 5
I constant
D
c
0·8h
h
L
H_A
A
E
H_E
$-M_A$
$+M_E$
V_A $=P-V_E$
V_E
Read Moments to the right
Read Reactions to the left
VALUES OF REACTIONS IN TERMS OF Pc/h
VALUES OF MOMENTS IN TERMS OF Pc
$-M_A$
$+V_E$
Note that M_A, M_C and M_D change their signs
$+M_B$
$+M_E$
$-M_D$
$+H_A = +H_E$
$-M_C$
$+M_A$
$+M_C$
1·4 Pc/h
1·2 Pc/h
Pc/h
0·8 Pc/h
0·6 Pc/h
0·4 Pc/h
0·2 Pc/h
0
1·0Pc
0 9Pc
0·8Pc
0 7Pc
0 6Pc
0 5Pc
0 4Pc
0 3Pc
0·2Pc
0 1Pc
0
0 0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2·0
VALUES OF h/L

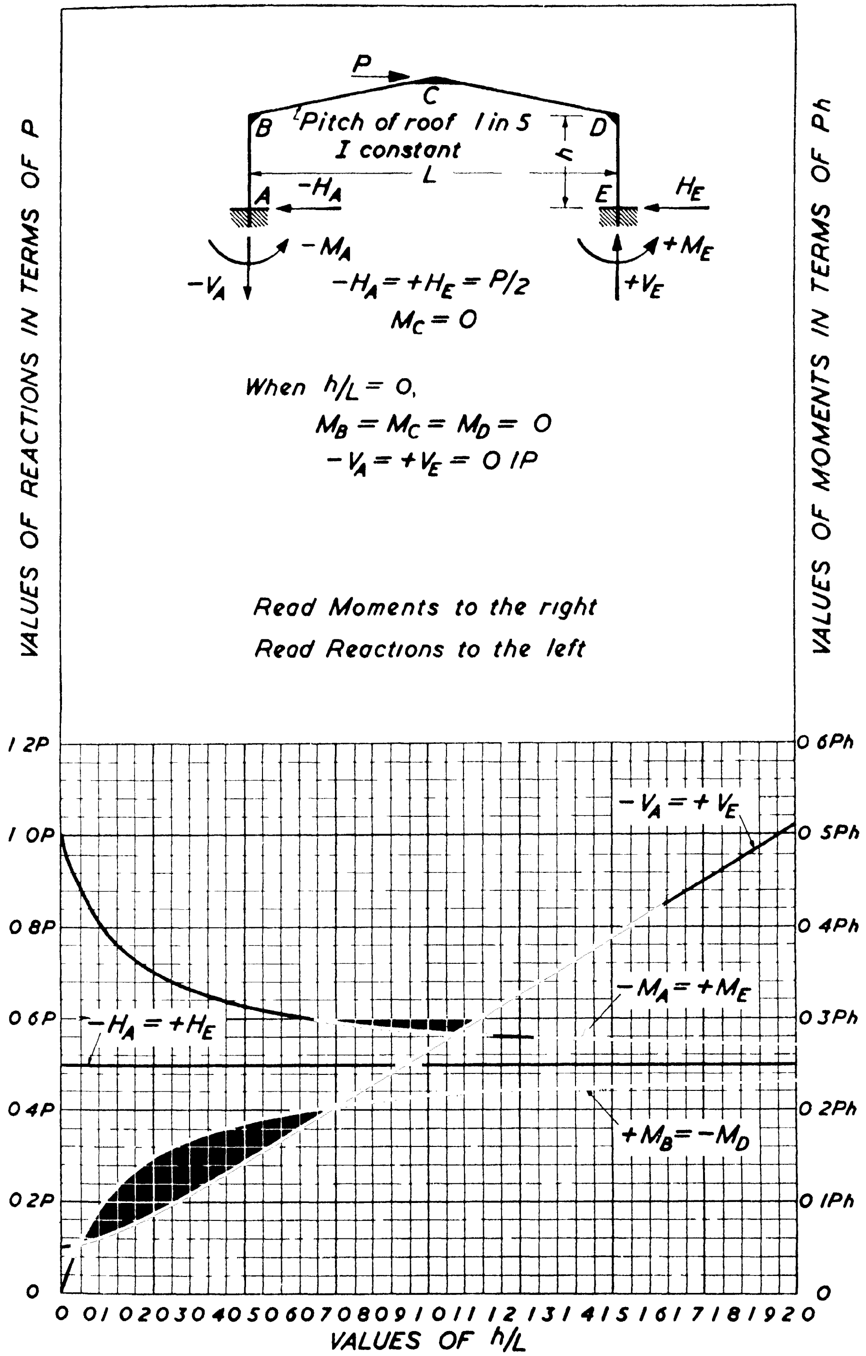

C
B
D
P
P
Roof pitch
1 in 5
I constant
c
c
0·8h
h
L
H_A
H_E
A
E
$+M_A$
$+M_E$
$V_A = P$
$V_E = P$
Read Moments to the right
Read Reactions to the left
Note that M_A, M_C and M_E change their signs
$-M_A = -M_E$
$+H_A = +H_E$
$+M_A = +M_E$
$+M_B = +M_D$
$-M_C$
$+M_C$
VALUES OF H_A AND H_E IN TERMS OF Pc/h
VALUES OF MOMENTS IN TERMS OF Pc
1·4 Pc/h
1·2 Pc/h
Pc/h
0·8 Pc/h
0·6 Pc/h
0·4 Pc/h
0·2 Pc/h
0
1·0Pc
0·9Pc
0·8Pc
0·7Pc
0·6Pc
0·5Pc
0·4Pc
0·3Pc
0·2Pc
0·1Pc
0
0 0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2·0
VALUES OF h/L

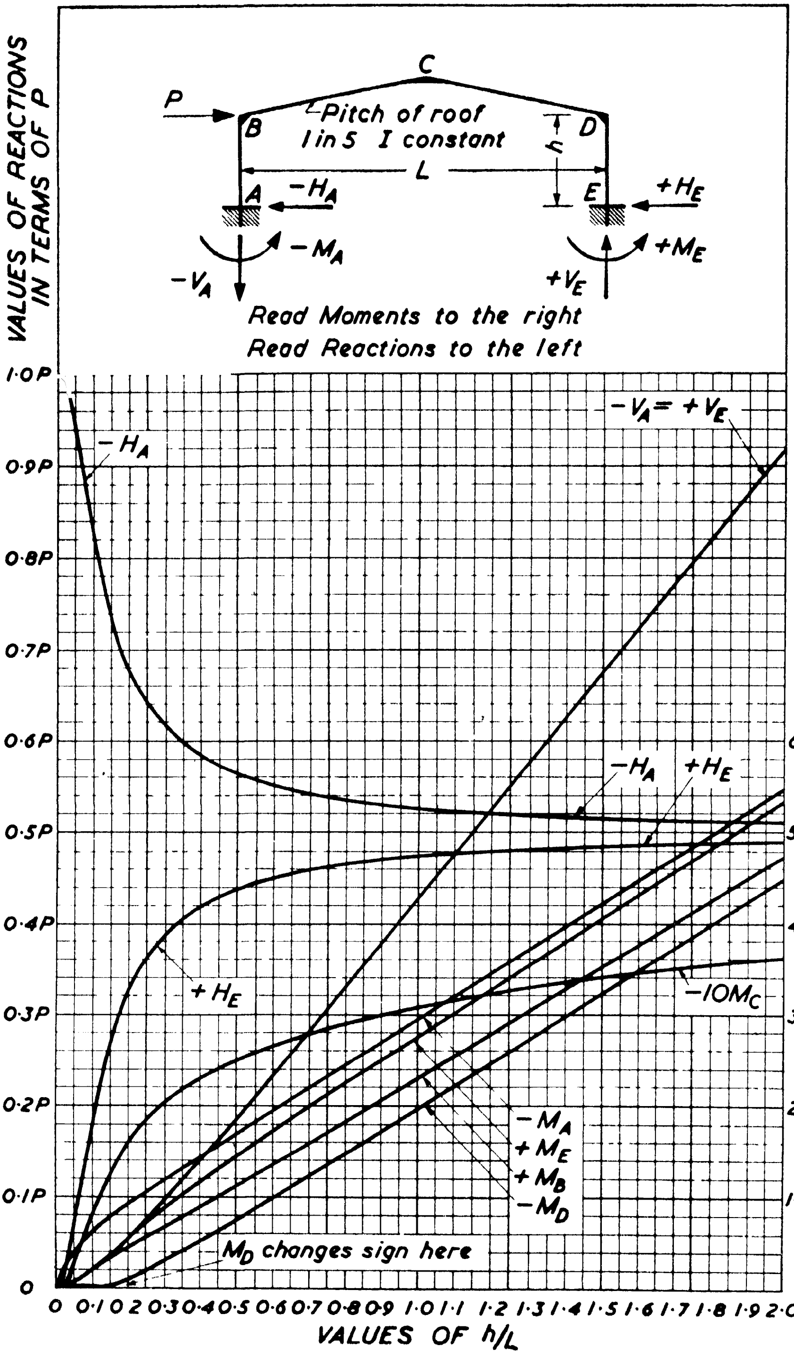


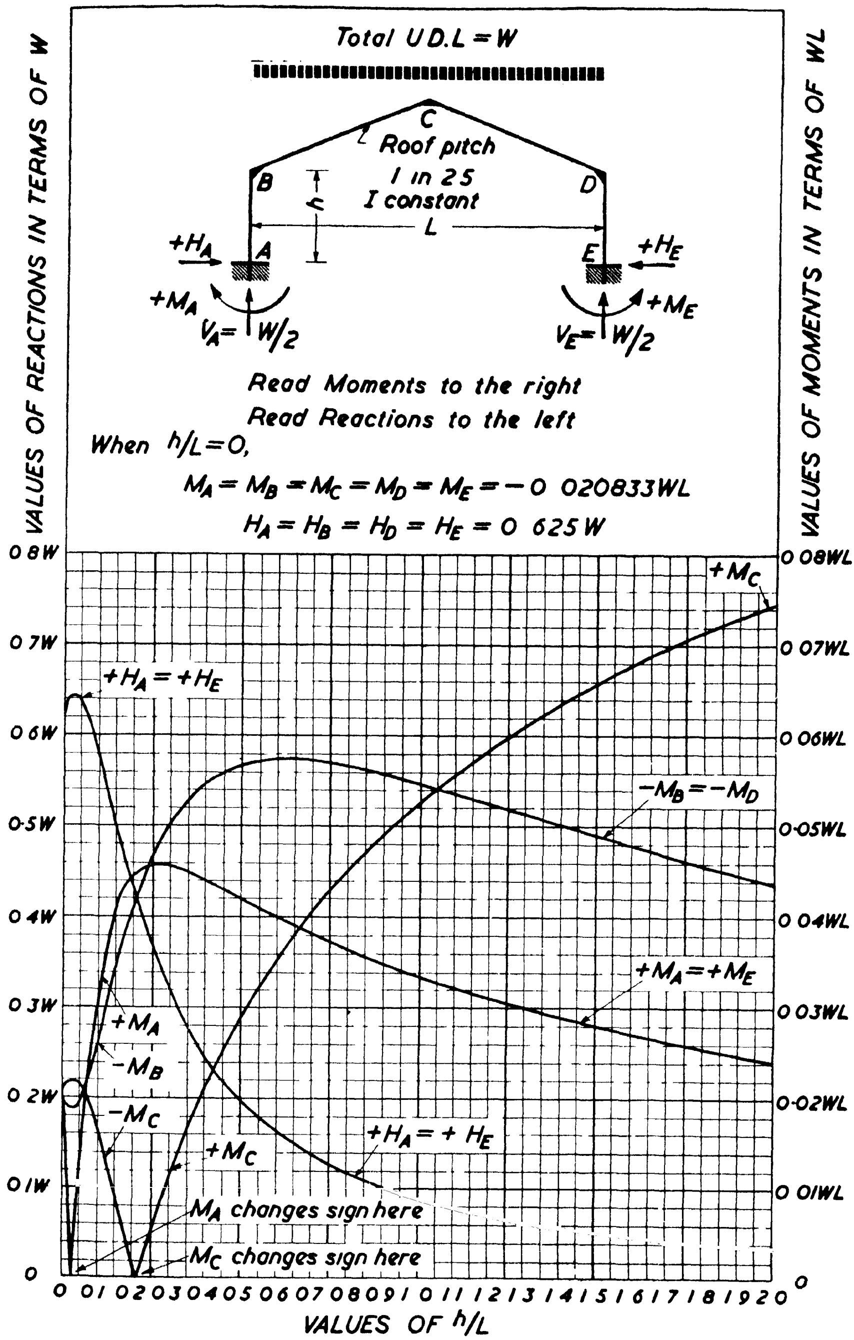
C
B
D
Pitch of roof
1 in 5
I constant
L
h
P
P
2h/3
2h/3
A
E
$-H_A = P$
$H_E = P$
$-M_A$
$+M_E$
$M_C = O$
$-V_A$
$+V_E$
Read Moments to the right
Read Reactions to the left
VALUES OF $-V_A$ AND $+V_E$ IN TERMS OF P
VALUES OF MOMENTS IN TERMS OF Ph
0·9P
0·8P
0·7P
0·6P
0·5P
0·4P
0·3P
0·2P
0·1P
0
0·7Ph
0·6Ph
0·5Ph
0·4Ph
0·3Ph
0·2Ph
0·1Ph
0
$-V_A = +V_E$
$-M_A = +M_E$
$+M_B = -M_D$
0 0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2·0
VALUES OF h/L


C
B
D
P
P
Pitch of roof
1 in 5
I constant
0·8h
h
L
−H_A=P
0·8h
+H_E=P
A
E
−M_A
+M_E
−V_A
+V_E
M_C = 0
Read Moments to the right
Read Reactions to the left
VALUES OF −V_A AND +V_E IN TERMS OF P
VALUES OF MOMENTS IN TERMS OF Ph
VALUES OF h/L
−V_A = +V_E
−M_A = +M_E
+M_B = −M_D
0
0·1P
0·2P
0·3P
0·4P
0·5P
0·6P
0·7P
0·8P
0·9P
1·0P
1·1P
1·2P
0·1Ph
0·2Ph
0·3Ph
0·4Ph
0·5Ph
0·6Ph
0·7Ph
0·8Ph
0 0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2·0

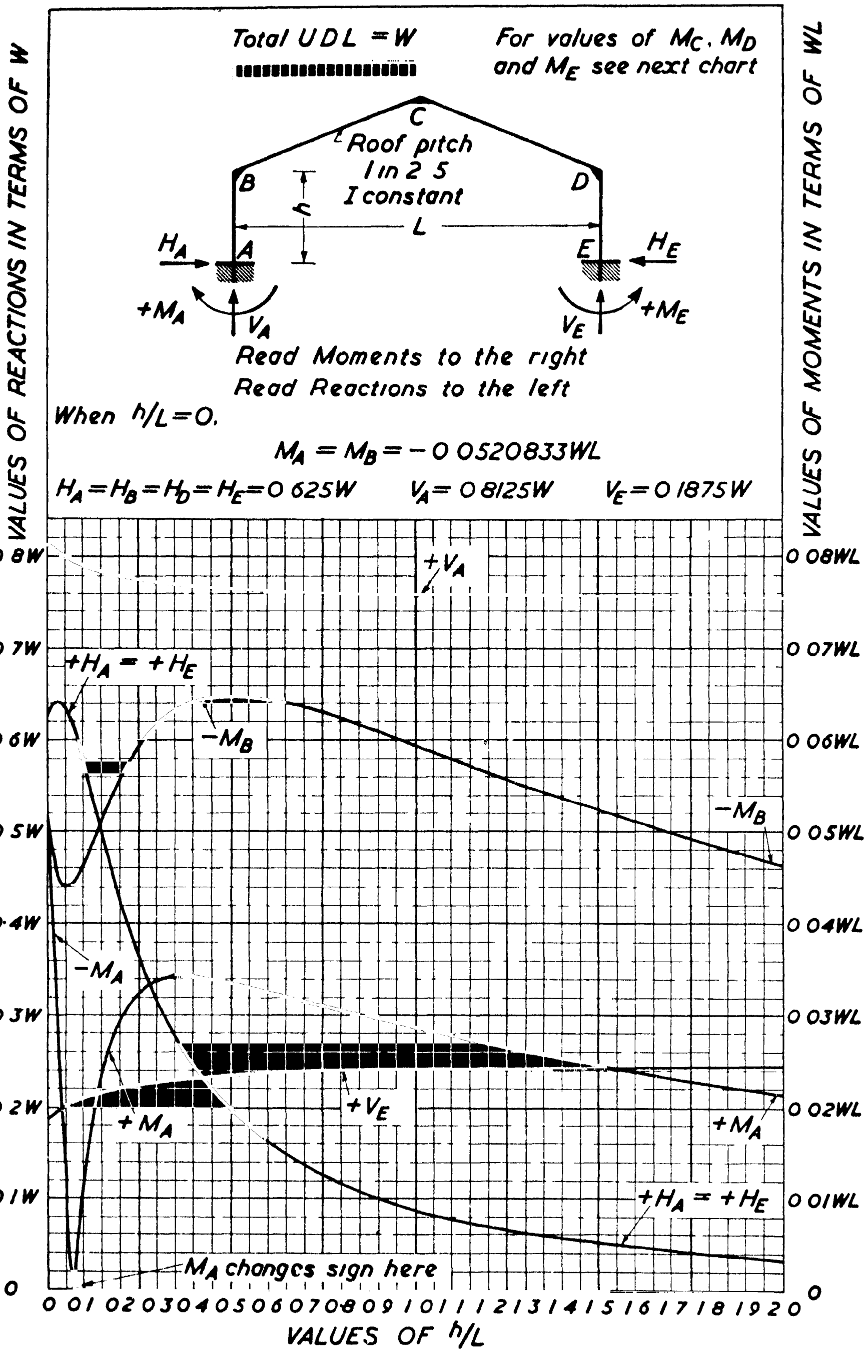

P
C
B
Pitch of roof 1 in 5
I constant
D
h
L
+H_A
A
E
+H_E
V_A = V_E = P/2
+M_A
+M_E
V_A
V_E
Read Moments to the right
Read Reactions to the left
When h/L = 0,
M_B = M_C = M_D = 0
H_A = H_E = 2·5P
VALUES OF H_A AND H_E IN TERMS OF P
VALUES OF MOMENTS IN TERMS OF PL
2 5P
2 25P
2·0P
1 75P
1·5P
1·25P
1 0P
0·75P
0·5P
0 25P
0
0·2PL
0·18PL
0 16PL
0 14PL
0 12PL
0 10PL
0·08PL
0·06PL
0·04PL
0 02PL
0
+M_C
+H_A = +H_E
−M_B = −M_D
+M_A = +M_E
+H_A = +H_E
0 0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2 0
VALUES OF h/L

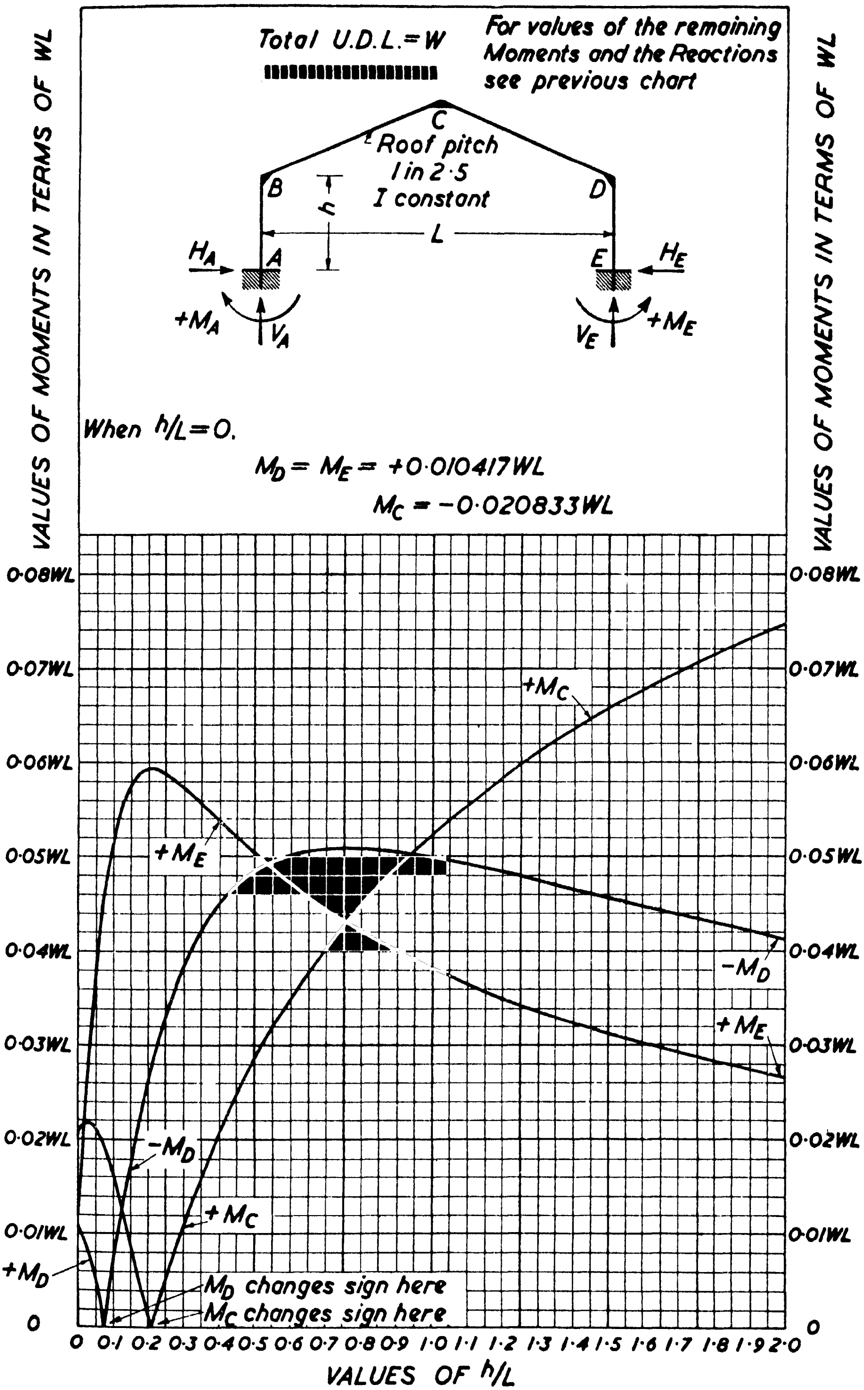

P
C
B
D
Pitch of roof 1 in 5
I constant
h
L
-H_A
A
E
H_E
-M_A
+M_E
-V_A
+V_E
-H_A = +H_E = P/2
M_C = O
When h/L = O,
M_B = M_C = M_D = O
-V_A = +V_E = O IP
Read Moments to the right
Read Reactions to the left
VALUES OF REACTIONS IN TERMS OF P
VALUES OF MOMENTS IN TERMS OF Ph
I 2P
I OP
O 8P
O 6P
O 4P
O 2P
O
O 6Ph
O 5Ph
O 4Ph
O 3Ph
O 2Ph
O IPh
O
-V_A = + V_E
-M_A = +M_E
-H_A = +H_E
+M_B = -M_D
O O1 O2 O3 O4 O5 O6 O7 O8 O9 I O I1 I2 I3 I4 I5 I6 I7 I8 I9 2 O
VALUES OF h/L

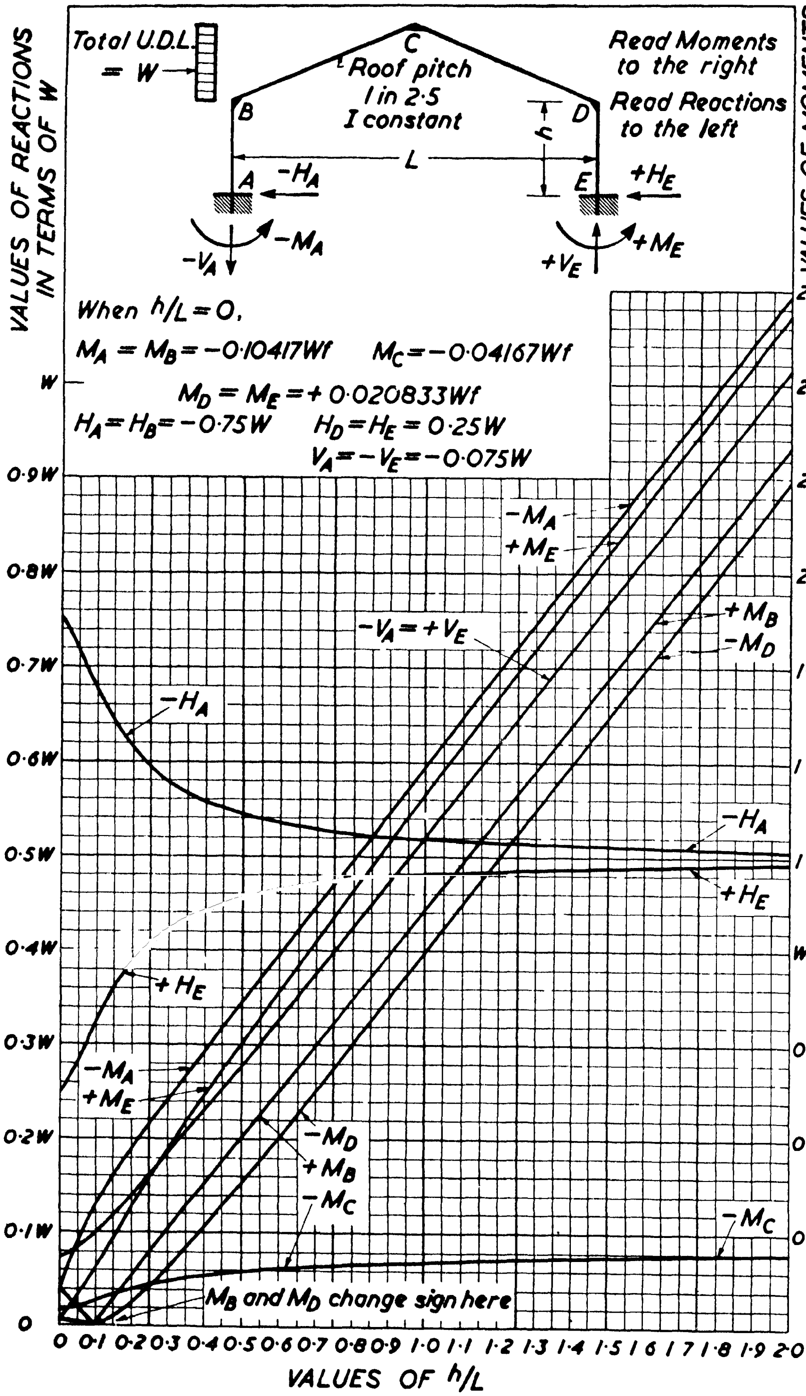

VALUES OF REACTIONS IN TERMS OF P
VALUES OF MOMENTS IN TERMS OF Pf
P
C
B
D
Pitch of roof
1 in 5 I constant
h
L
A
E
−H_A
+H_E
−M_A
+M_E
−V_A
+V_E
Read Moments to the right
Read Reactions to the left
1·0P
0·9P
0·8P
0·7P
0·6P
0·5P
0·4P
0·3P
0·2P
0·1P
0
6·0Pf
5·0Pf
4·0Pf
3·0Pf
2·0Pf
1·0Pf
0
−V_A = +V_E
−H_A
+H_E
−10M_C
−M_A
+M_E
+M_B
−M_D
M_D changes sign here
0 0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2·0
VALUES OF h/L

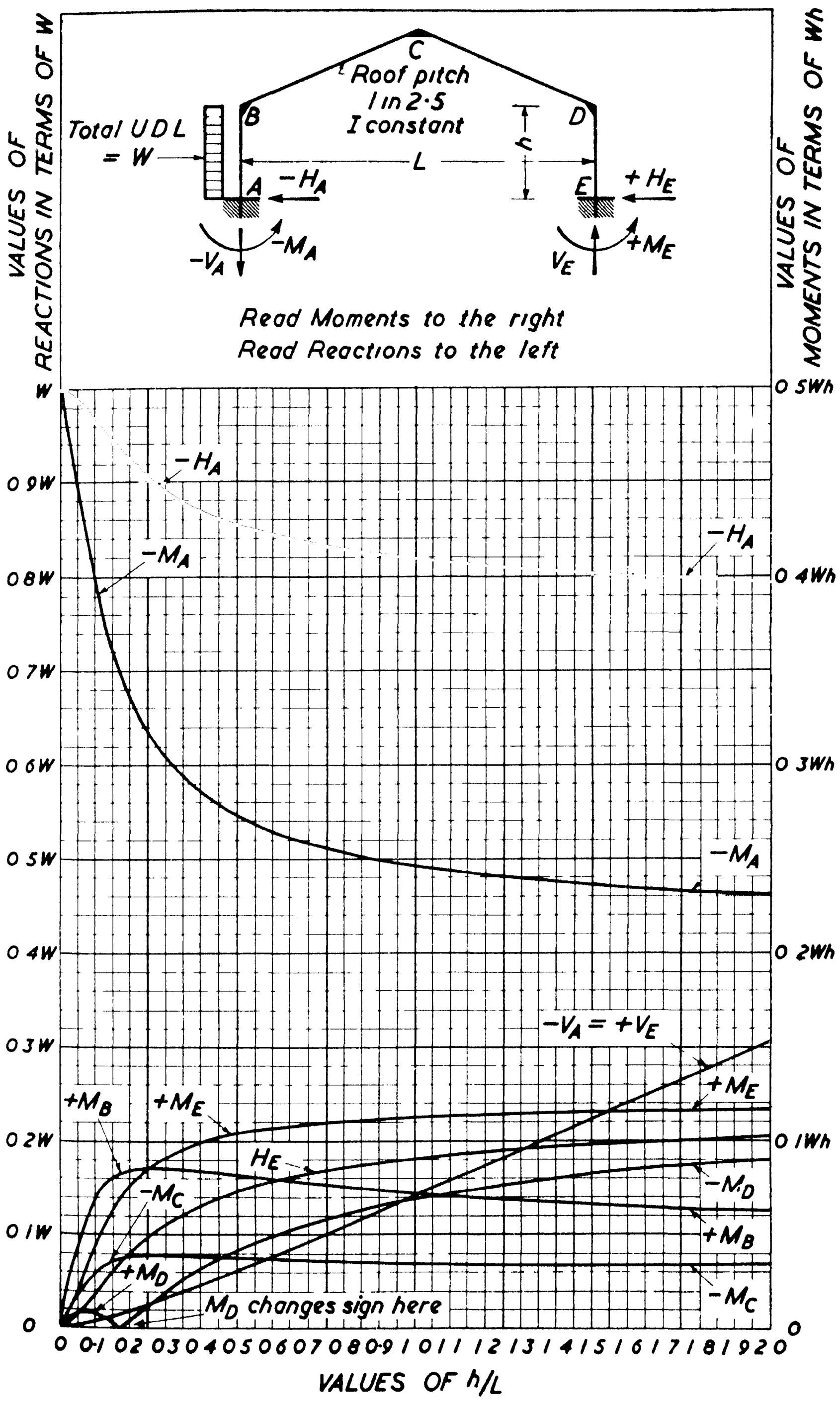


Total U.D.L = W
C
Roof pitch
1 in 25
I constant
B
D
h
L
+H_A
A
E
+H_E
+M_A
+M_E
V_A = W/2
V_E = W/2
Read Moments to the right
Read Reactions to the left
When h/L = 0,
M_A = M_B = M_C = M_D = M_E = − 0 020833WL
H_A = H_B = H_D = H_E = 0 625 W
VALUES OF REACTIONS IN TERMS OF W
VALUES OF MOMENTS IN TERMS OF WL
0 8W
0 7W
0 6W
0·5W
0 4W
0 3W
0 2W
0 1W
0
0 08WL
0 07WL
0 06WL
0·05WL
0 04WL
0 03WL
0·02WL
0 01WL
0
+M_C
+H_A = +H_E
−M_B = −M_D
+M_A = +M_E
+M_A
−M_B
−M_C
+M_C
+H_A = + H_E
M_A changes sign here
M_C changes sign here
0 01 02 03 04 05 06 07 08 09 1 0 1 1 1 2 1 3 1 4 1 5 1 6 1 7 1 8 1 9 2 0
VALUES OF h/L

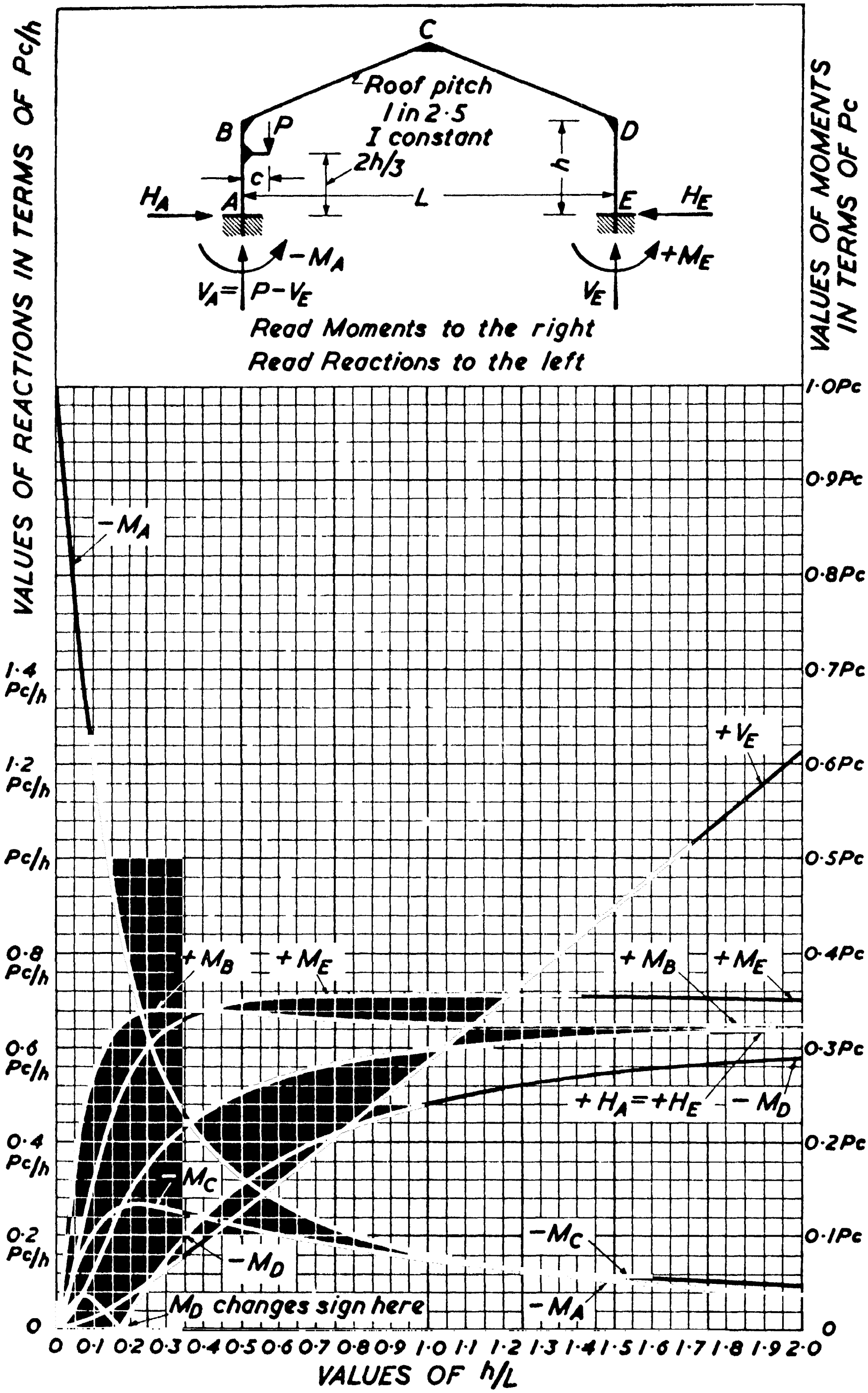

Total UDL = W
For values of M_C, M_D and M_E see next chart
C
Roof pitch 1 in 2 5
I constant
B
D
h
L
H_A
A
E
H_E
$+M_A$
V_A
V_E
$+M_E$
Read Moments to the right
Read Reactions to the left
When h/L=0,
$M_A = M_B = -0\ 0520833WL$
$H_A=H_B=H_D=H_E=0\ 625W$
$V_A= 0\ 8125W$
$V_E=0\ 1875W$
VALUES OF REACTIONS IN TERMS OF W
VALUES OF MOMENTS IN TERMS OF WL
0 8W
0 7W
0 6W
0 5W
0·4W
0 3W
0 2W
0 1W
0
0 08WL
0 07WL
0 06WL
0 05WL
0 04WL
0 03WL
0 02WL
0 01WL
0
$+V_A$
$+H_A = +H_E$
$-M_B$
$-M_A$
$+M_A$
$+V_E$
M_A changes sign here
0 01 02 03 04 05 06 07 08 09 1 0 1 1 1 2 1 3 1 4 1 5 1 6 1 7 1 8 1 9 2 0
VALUES OF h/L

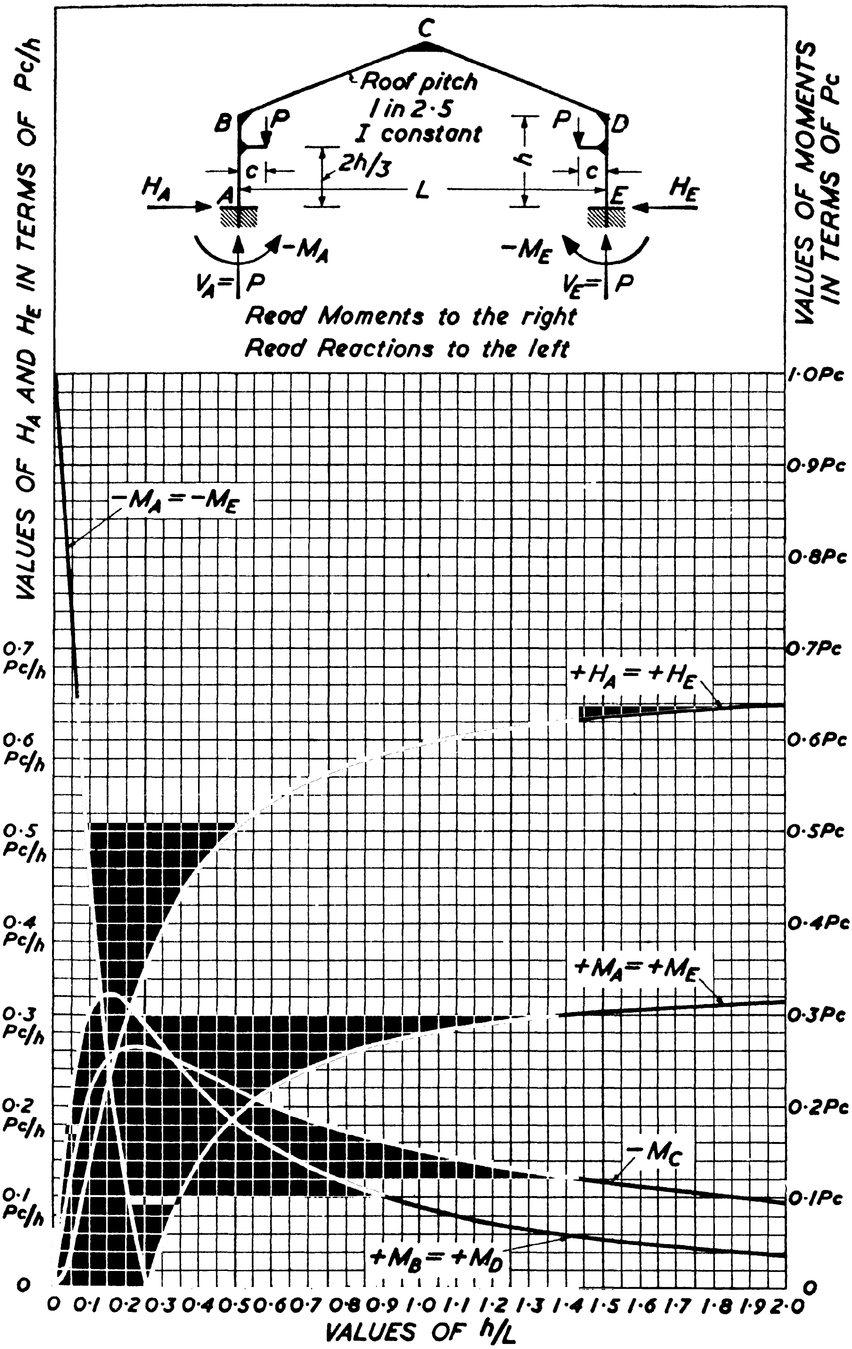

Total U.D.L.=W
For values of the remaining Moments and the Reactions see previous chart
C
Roof pitch 1 in 2·5
I constant
B
D
h
L
H_A
A
E
H_E
$+M_A$
V_A
V_E
$+M_E$
When h/L=0.
$M_D = M_E = +0{\cdot}010417WL$
$M_C = -0{\cdot}020833WL$
VALUES OF MOMENTS IN TERMS OF WL
0·08WL
0·07WL
0·06WL
0·05WL
0·04WL
0·03WL
0·02WL
0·01WL
0
$+M_C$
$+M_E$
$-M_D$
$+M_E$
$-M_D$
$+M_C$
$+M_D$
M_D changes sign here
M_C changes sign here
0 0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2·0
VALUES OF h/L

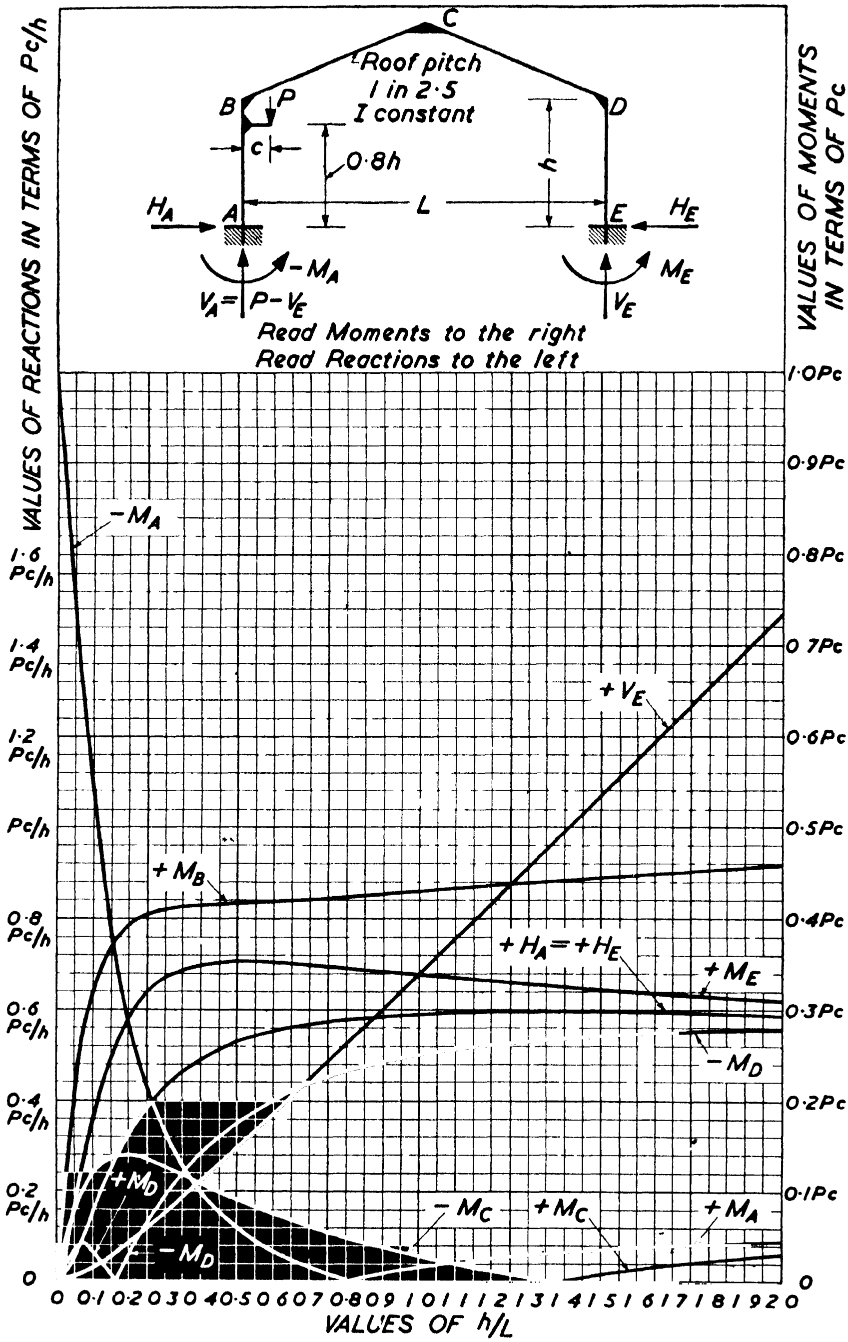

Total U.D.L. = W
C
Roof pitch 1 in 2·5
I constant
B
D
h
L
A
E
−H_A
+H_E
−M_A
+M_E
−V_A
+V_E
Read Moments to the right
Read Reactions to the left
When h/L = 0,
M_A = M_B = −0·10417Wf   M_C = −0·04167Wf
M_D = M_E = + 0·020833Wf
H_A = H_B = −0·75W   H_D = H_E = 0·25W
V_A = −V_E = −0·075W
VALUES OF REACTIONS IN TERMS OF W
VALUES OF MOMENTS IN TERMS OF Wf
W
0·9W
0·8W
0·7W
0·6W
0·5W
0·4W
0·3W
0·2W
0·1W
0
2·75Wf
2·5Wf
2·25Wf
2·0Wf
1·75Wf
1·5Wf
1·25Wf
Wf
0·75Wf
0·5Wf
0·25Wf
0
−M_A
+M_E
−V_A = +V_E
+M_B
−M_D
−H_A
+H_E
−M_D
+M_B
−M_C
M_B and M_D change sign here
0 0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2·0
VALUES OF h/L



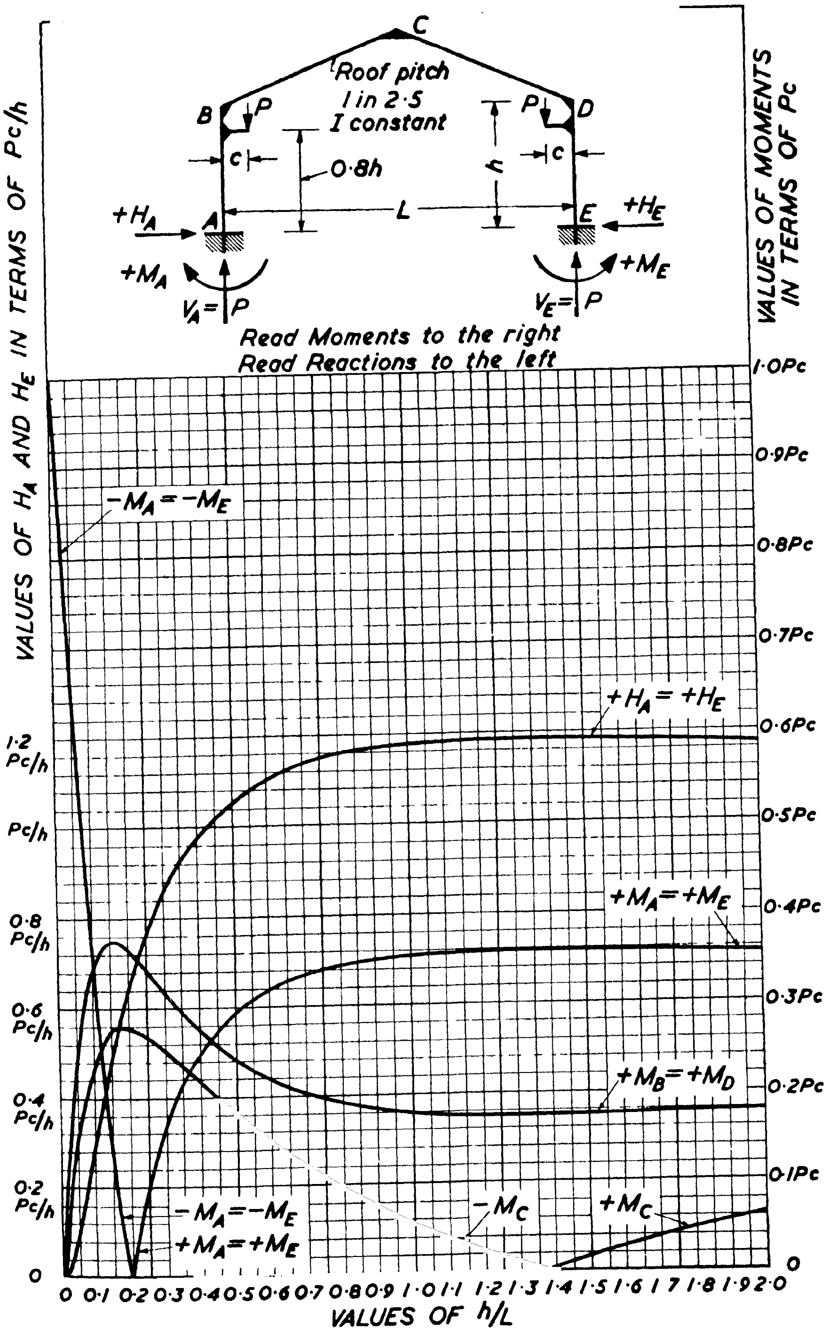
C
Roof pitch
1 in 2·5
I constant
B
D
Total UDL
= W
L
h
A
−H_A
E
+H_E
−M_A
+M_E
−V_A
V_E
Read Moments to the right
Read Reactions to the left
VALUES OF REACTIONS IN TERMS OF W
VALUES OF MOMENTS IN TERMS OF Wh
W
0 9W
0 8W
0 7W
0 6W
0 5W
0 4W
0 3W
0 2W
0 1W
0
0 5Wh
0 4Wh
0 3Wh
0 2Wh
0 1Wh
0
−H_A
−M_A
−V_A = +V_E
+M_B
+M_E
H_E
−M_C
+M_D
−M_D
M_D changes sign here
0 0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2·0
VALUES OF h/L

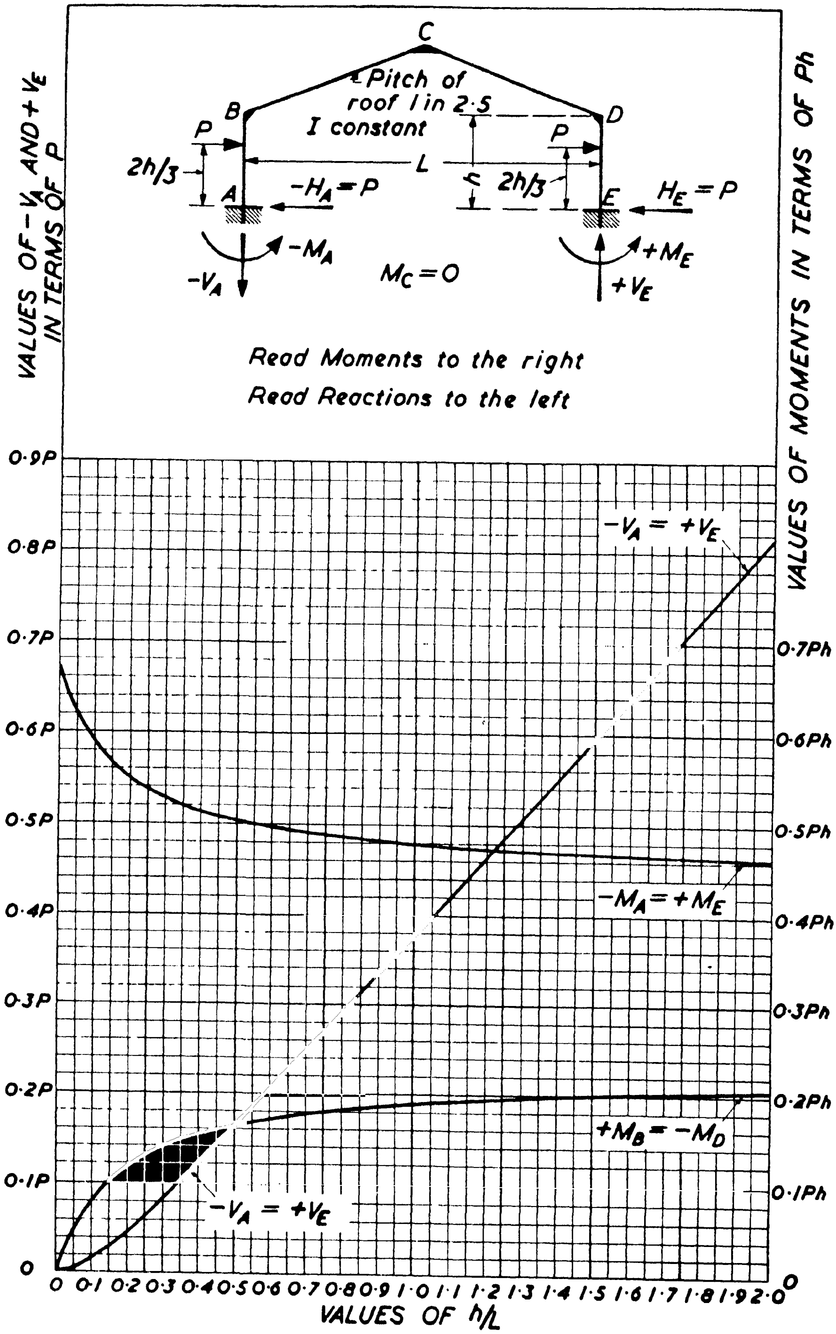

C
Roof pitch
1 in 2·5
I constant
B
P
D
c
2h/3
h
H_A
A
L
E
H_E
$-M_A$
$+M_E$
$V_A = P - V_E$
V_E
Read Moments to the right
Read Reactions to the left
VALUES OF REACTIONS IN TERMS OF Pc/h
VALUES OF MOMENTS IN TERMS OF Pc
1·0Pc
0·9Pc
0·8Pc
0·7Pc
0·6Pc
0·5Pc
0·4Pc
0·3Pc
0·2Pc
0·1Pc
0
1·4 Pc/h
1·2 Pc/h
Pc/h
0·8 Pc/h
0·6 Pc/h
0·4 Pc/h
0·2 Pc/h
0
$-M_A$
$+V_E$
$+M_B$
$+M_E$
$+M_B$
$+M_E$
$+H_A = +H_E$
$-M_D$
$-M_C$
$-M_D$
$-M_C$
$-M_A$
M_D changes sign here
0 0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2·0
VALUES OF h/L

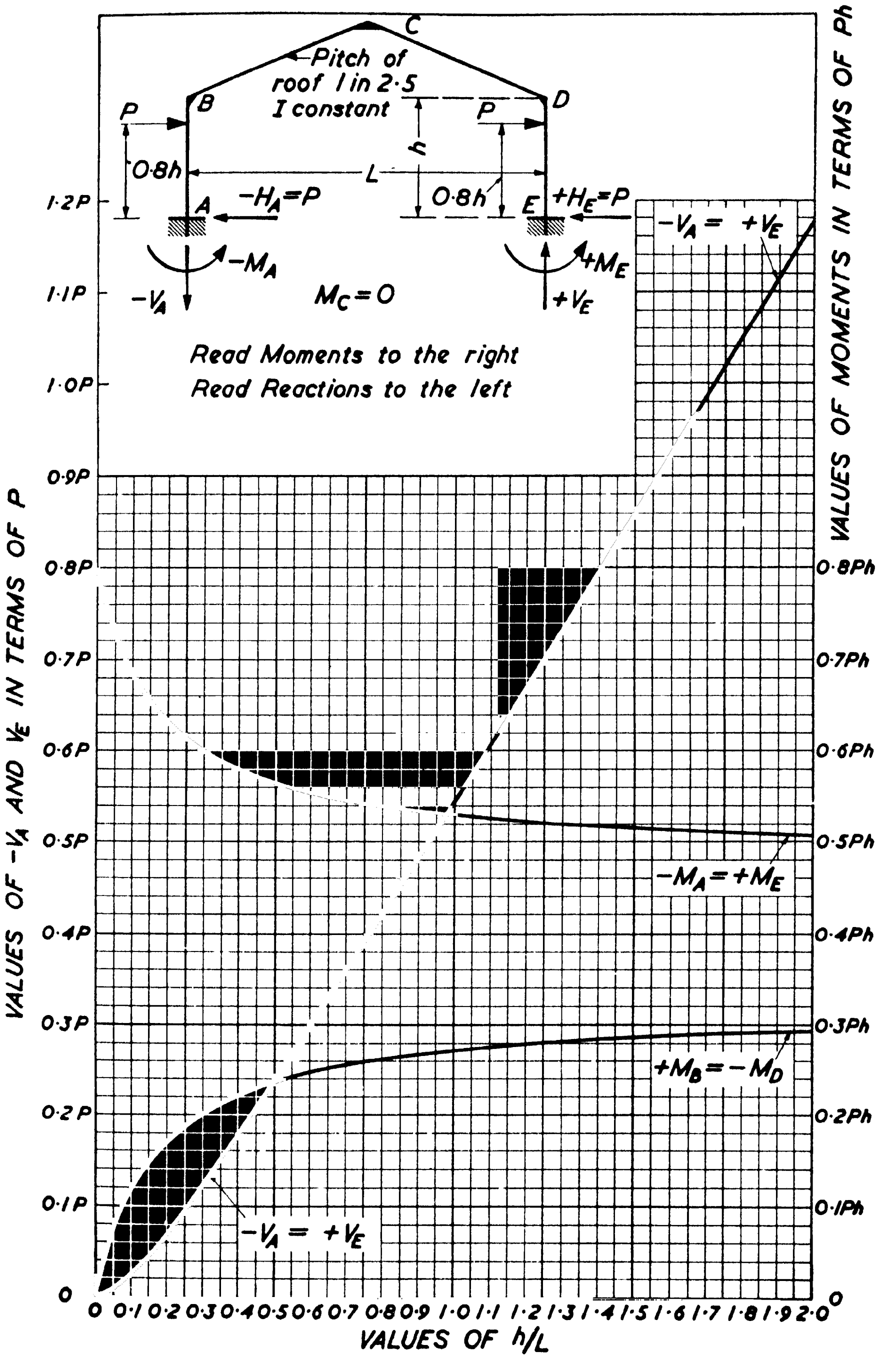

C
Roof pitch
1 in 2·5
I constant
B
P
P
D
2h/3
h
c
c
H_A
A
L
E
H_E
$-M_A$
$-M_E$
$V_A=P$
$V_E=P$
Read Moments to the right
Read Reactions to the left
VALUES OF H_A AND H_E IN TERMS OF Pc/h
VALUES OF MOMENTS IN TERMS OF Pc
$-M_A=-M_E$
$+H_A=+H_E$
$+M_A=+M_E$
$-M_C$
$+M_B=+M_D$
0·7 Pc/h
0·6 Pc/h
0·5 Pc/h
0·4 Pc/h
0·3 Pc/h
0·2 Pc/h
0·1 Pc/h
0
1·0Pc
0·9Pc
0·8Pc
0·7Pc
0·6Pc
0·5Pc
0·4Pc
0·3Pc
0·2Pc
0·1Pc
0
0 0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2·0
VALUES OF h/L

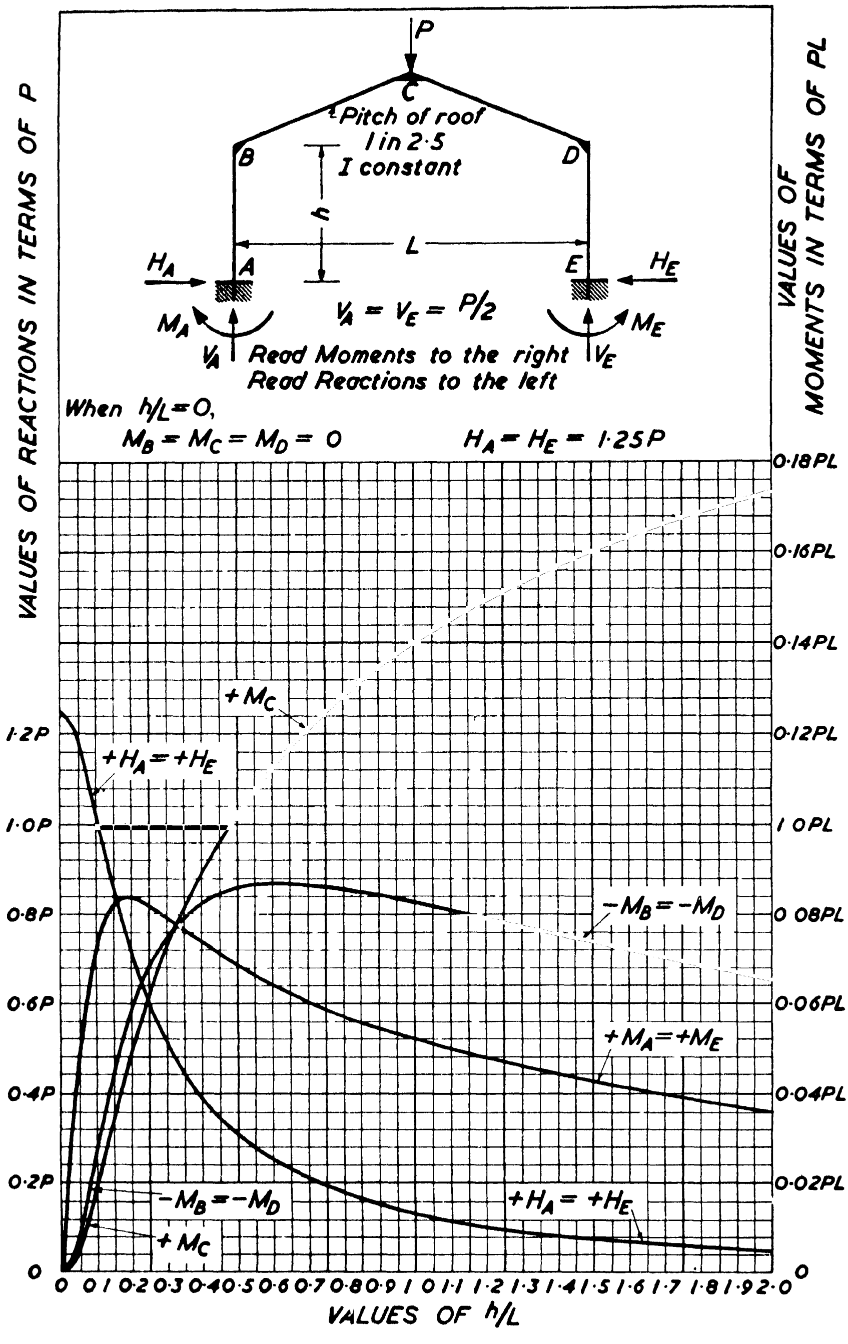


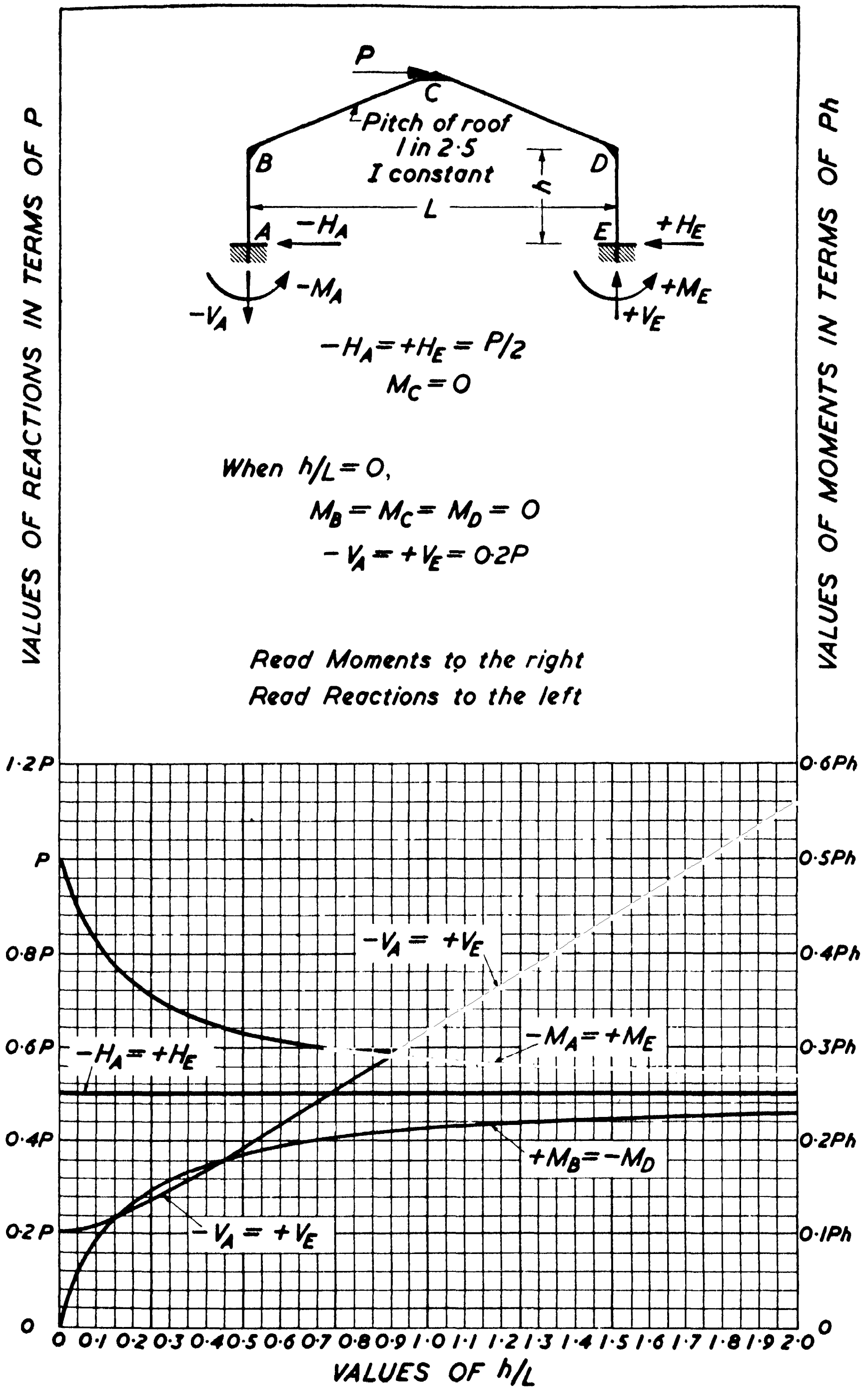
C
Roof pitch
1 in 2·5
I constant
B
P
D
c
0·8h
h
H_A
A
L
E
H_E
$-M_A$
M_E
$V_A = P - V_E$
V_E
Read Moments to the right
Read Reactions to the left
VALUES OF REACTIONS IN TERMS OF Pc/h
VALUES OF MOMENTS IN TERMS OF Pc
1·6 Pc/h
1·4 Pc/h
1·2 Pc/h
Pc/h
0·8 Pc/h
0·6 Pc/h
0·4 Pc/h
0·2 Pc/h
0
1·0Pc
0·9Pc
0·8Pc
0·7Pc
0·6Pc
0·5Pc
0·4Pc
0·3Pc
0·2Pc
0·1Pc
0
$-M_A$
$+V_E$
$+M_B$
$+H_A = +H_E$
$+M_E$
$-M_D$
$+M_D$
$-M_C$
$+M_C$
$+M_A$
0 0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2·0
VALUES OF h/L


C
Roof pitch
1 in 2·5
I constant
B
P
D
c
0·8h
h
+H_A
A
L
E
+H_E
+M_A
+M_E
V_A = P
V_E = P
Read Moments to the right
Read Reactions to the left
VALUES OF H_A AND H_E IN TERMS OF Pc/h
VALUES OF MOMENTS IN TERMS OF Pc
−M_A = −M_E
+H_A = +H_E
+M_A = +M_E
+M_B = +M_D
−M_C
+M_C
1·0Pc
0·9Pc
0·8Pc
0·7Pc
0·6Pc
0·5Pc
0·4Pc
0·3Pc
0·2Pc
0·1Pc
0
1·2 Pc/h
Pc/h
0·8 Pc/h
0·6 Pc/h
0·4 Pc/h
0·2 Pc/h
0
0 0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2·0
VALUES OF h/L

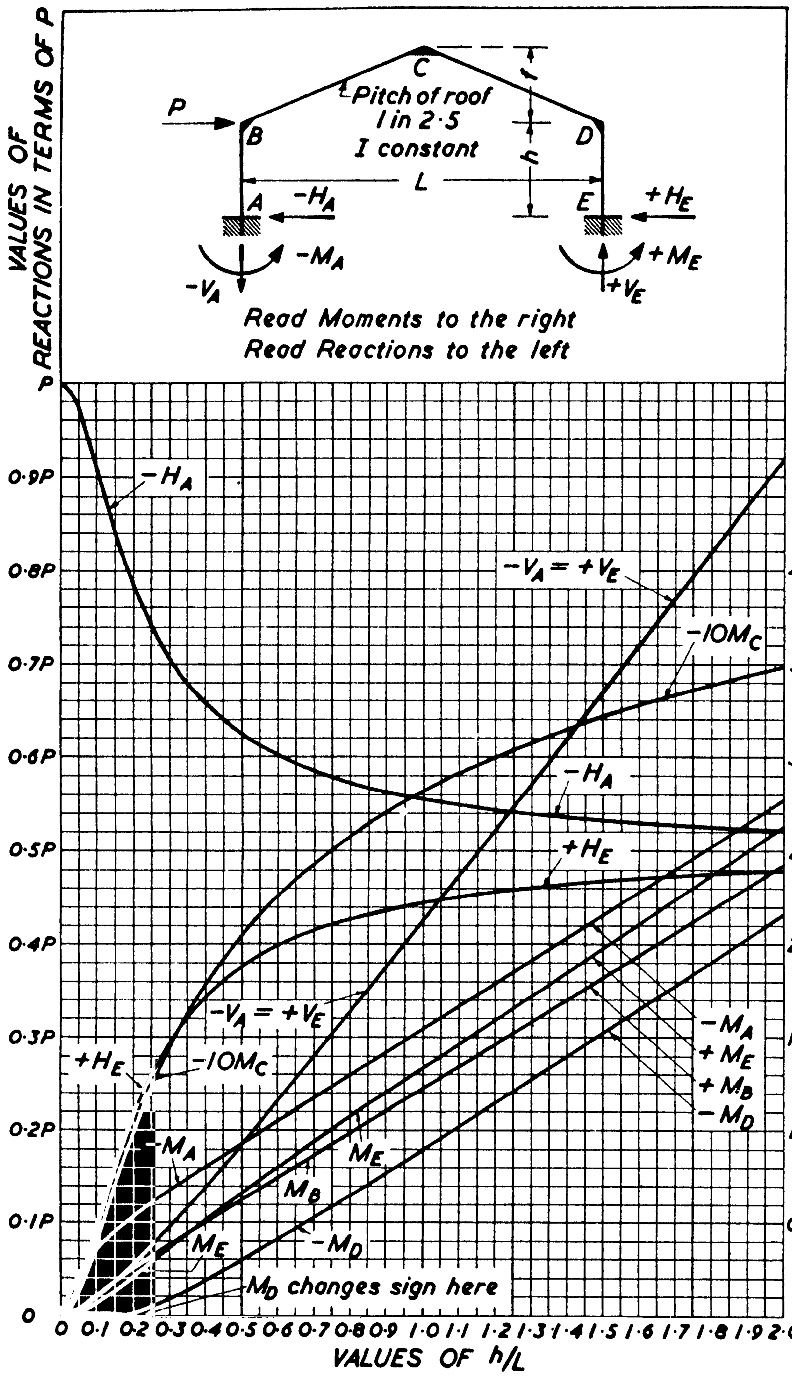

C
Pitch of roof 1 in 2·5
I constant
B
D
P
P
L
2h/3
2h/3
h
A
E
$-H_A = P$
$H_E = P$
$-M_A$
$+M_E$
$-V_A$
$+V_E$
$M_C = 0$
Read Moments to the right
Read Reactions to the left
VALUES OF $-V_A$ AND $+V_E$ IN TERMS OF P
VALUES OF MOMENTS IN TERMS OF Ph
0·9P
0·8P
0·7P
0·6P
0·5P
0·4P
0·3P
0·2P
0·1P
0
0·7Ph
0·6Ph
0·5Ph
0·4Ph
0·3Ph
0·2Ph
0·1Ph
0
$-V_A = +V_E$
$-M_A = +M_E$
$+M_B = -M_D$
$-V_A = +V_E$
0 0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2·0
VALUES OF h/L

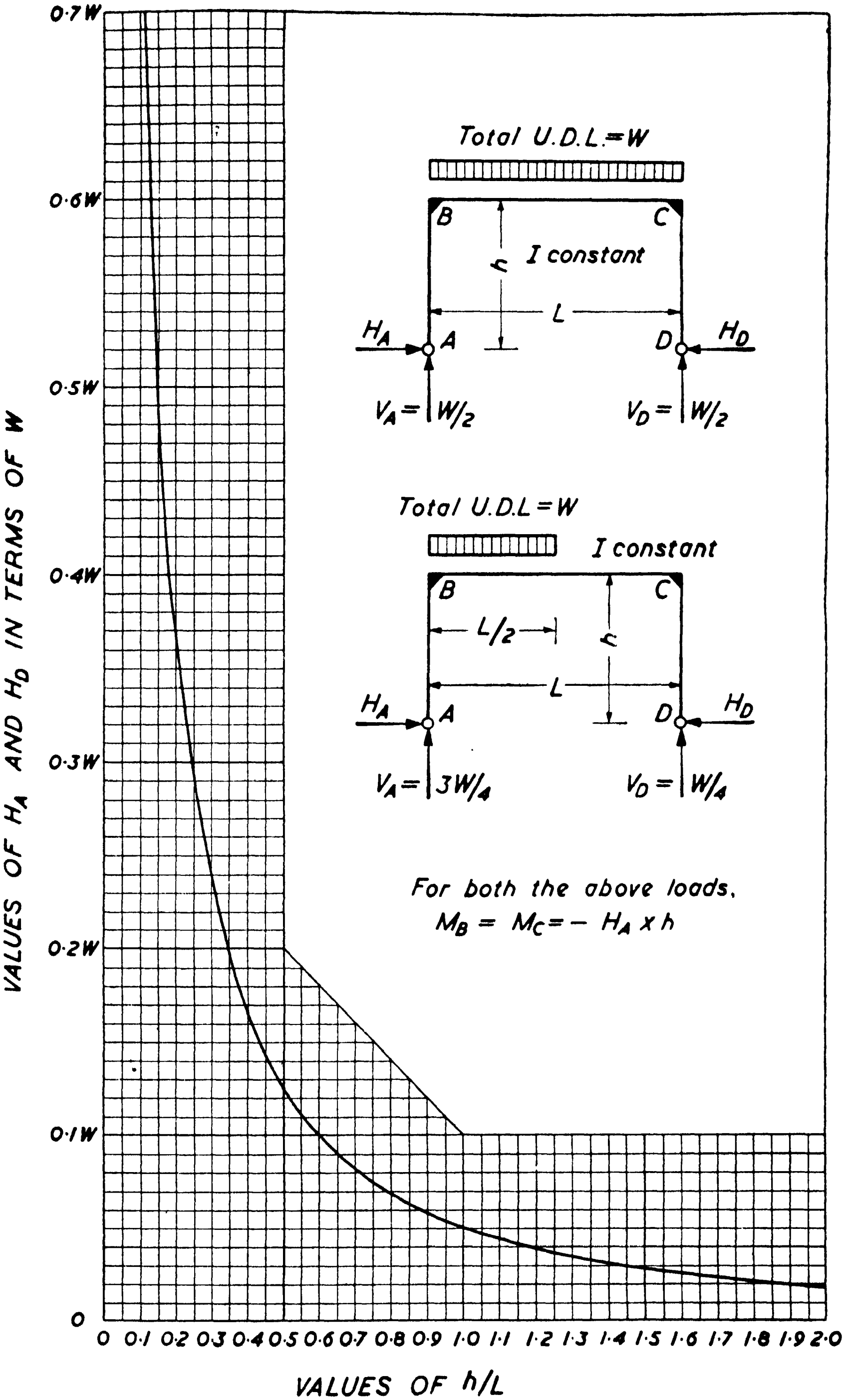

C
Pitch of roof 1 in 2·5
I constant
B
D
P
P
h
0·8h
L
0·8h
A
E
$-H_A=P$
$+H_E=P$
$-M_A$
$+M_E$
$-V_A$
$M_C=0$
$+V_E$
Read Moments to the right
Read Reactions to the left
$-V_A = +V_E$
$-M_A=+M_E$
$+M_B=-M_D$
$-V_A = +V_E$
VALUES OF $-V_A$ AND V_E IN TERMS OF P
VALUES OF MOMENTS IN TERMS OF Ph
VALUES OF h/L
0 0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2·0
0 0·1P 0·2P 0·3P 0·4P 0·5P 0·6P 0·7P 0·8P 0·9P 1·0P 1·1P 1·2P
0 0·1Ph 0·2Ph 0·3Ph 0·4Ph 0·5Ph 0·6Ph 0·7Ph 0·8Ph

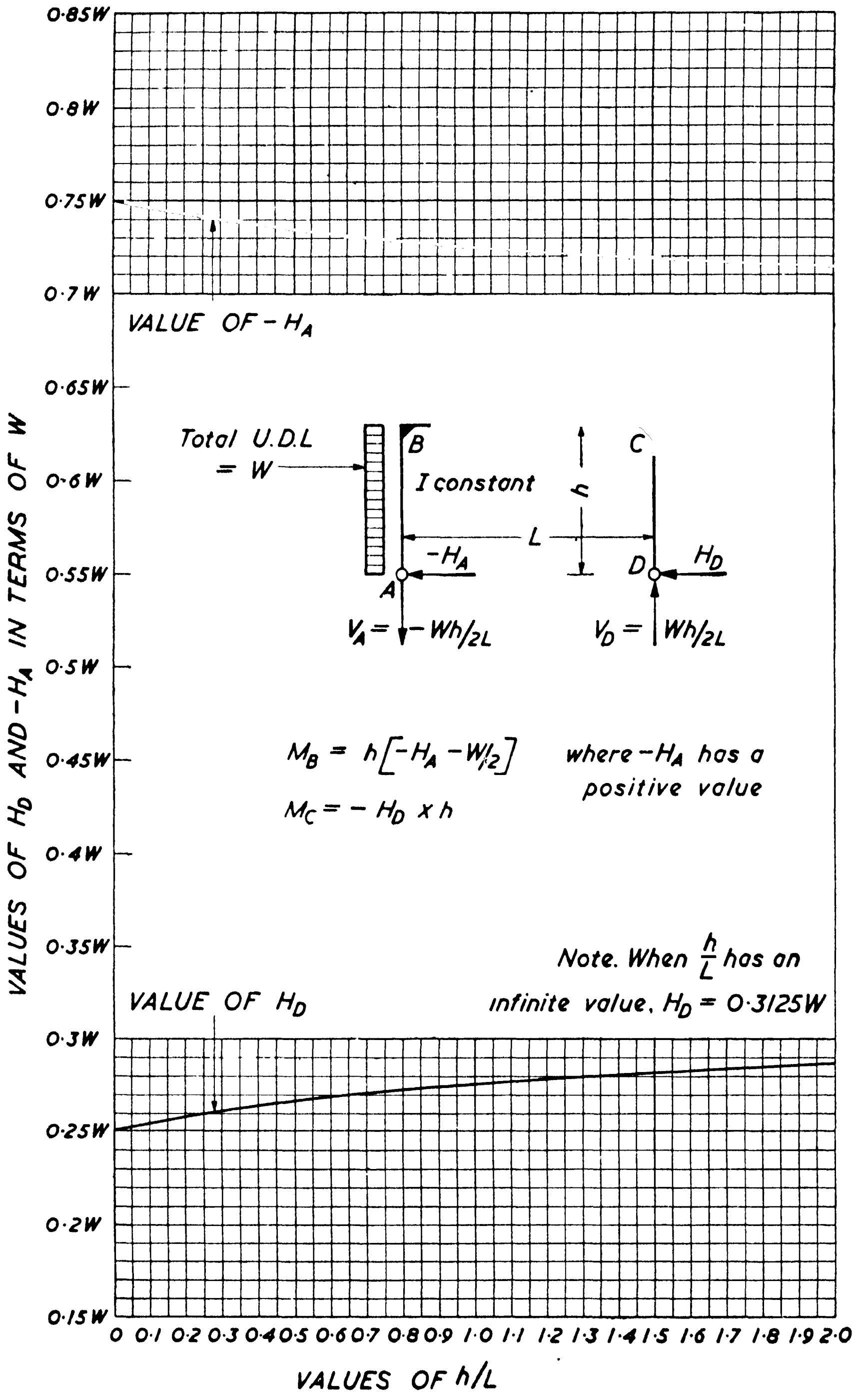

P
C
Pitch of roof
1 in 2·5
I constant
B
D
h
L
H_A
A
E
H_E
M_A
V_A = V_E = P/2
M_E
V_A
Read Moments to the right
Read Reactions to the left
V_E
When h/L = 0,
M_B = M_C = M_D = 0
H_A = H_E = 1·25P
VALUES OF REACTIONS IN TERMS OF P
VALUES OF MOMENTS IN TERMS OF PL
+M_C
+H_A = +H_E
−M_B = −M_D
+M_A = +M_E
−M_B = −M_D
+M_C
+H_A = +H_E
1·2P
1·0P
0·8P
0·6P
0·4P
0·2P
0
0·18PL
0·16PL
0·14PL
0·12PL
1 0PL
0 08PL
0·06PL
0·04PL
0·02PL
0
0 0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1 0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2·0
VALUES OF h/L

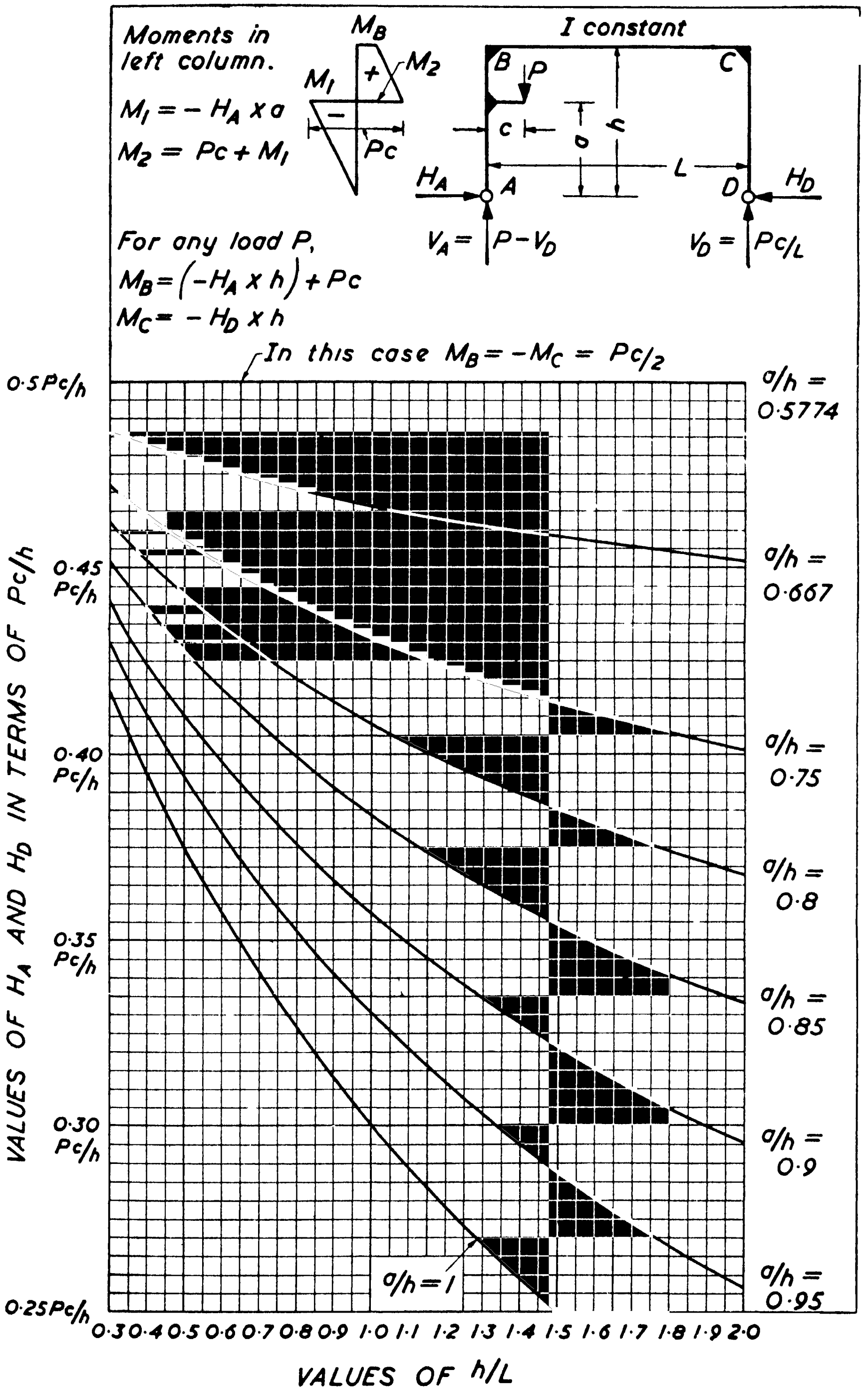


P
C
Pitch of roof
1 in 2·5
I constant
B
D
h
L
A
−H_A
E
+H_E
−M_A
+M_E
−V_A
+V_E
−H_A = +H_E = P/2
M_C = O
When h/L = O,
M_B = M_C = M_D = O
− V_A = + V_E = O·2P
Read Moments to the right
Read Reactions to the left
VALUES OF REACTIONS IN TERMS OF P
VALUES OF MOMENTS IN TERMS OF Ph
1·2P
P
0·8P
0·6P
0·4P
0·2P
0
0·6Ph
0·5Ph
0·4Ph
0·3Ph
0·2Ph
0·1Ph
0
−V_A = +V_E
−M_A = +M_E
−H_A = +H_E
+M_B = −M_D
−V_A = +V_E
0 0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2·0
VALUES OF h/L

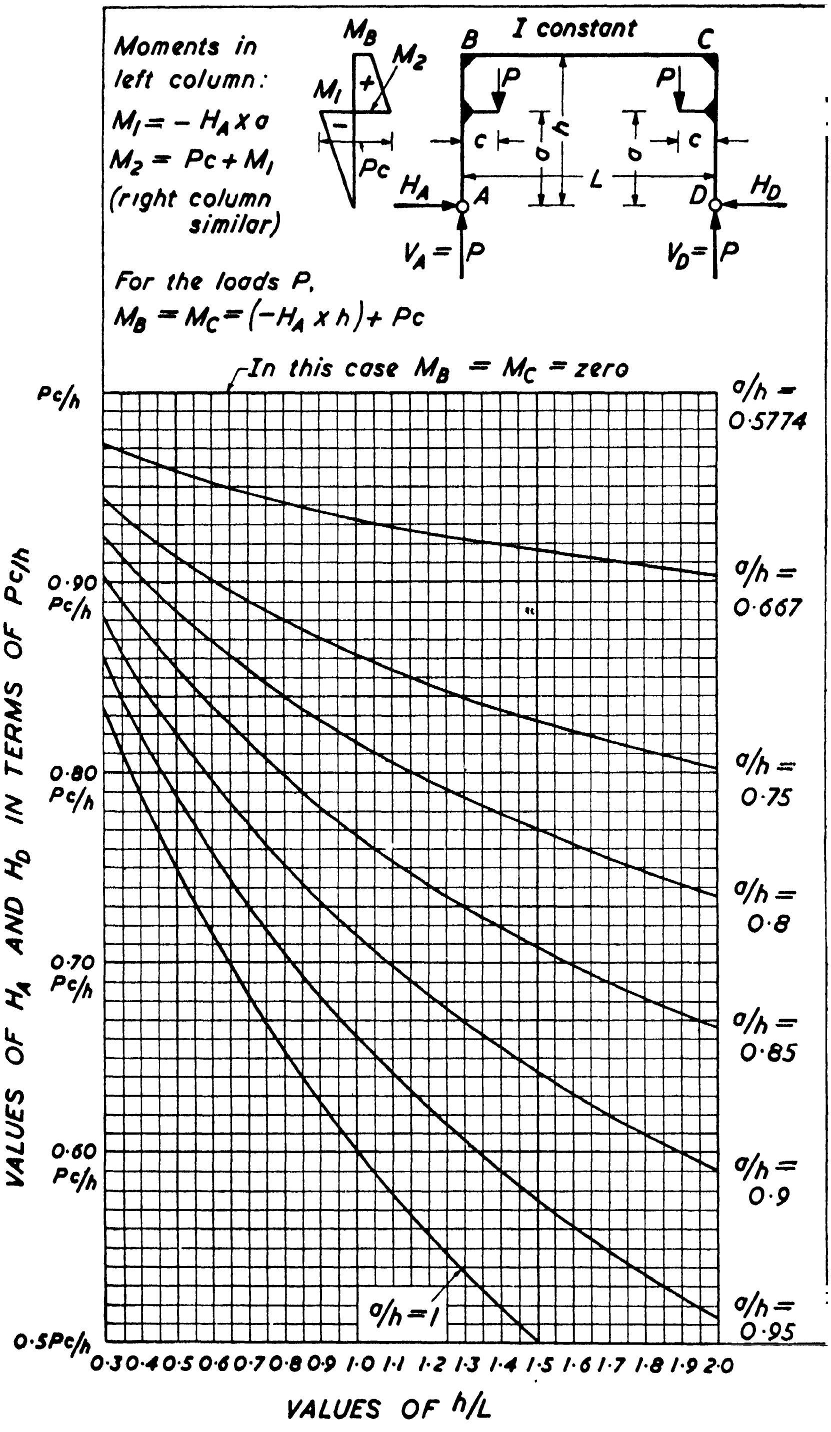

VALUES OF REACTIONS IN TERMS OF P
VALUES OF MOMENTS IN TERMS OF Pf
P
C
B
D
Pitch of roof 1 in 2·5
I constant
f
h
L
A
E
-H_A
+H_E
-M_A
+M_E
-V_A
+V_E
Read Moments to the right
Read Reactions to the left
0·9P
0·8P
0·7P
0·6P
0·5P
0·4P
0·3P
0·2P
0·1P
0
4·0Pf
3·5Pf
3·0Pf
2·5Pf
2·0Pf
1·5Pf
1·0Pf
0·5Pf
0
-V_A = +V_E
-10M_C
-H_A
+H_E
-M_A
+M_E
+M_B
-M_D
M_E
M_B
M_D changes sign here
0 0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2·0
VALUES OF h/L

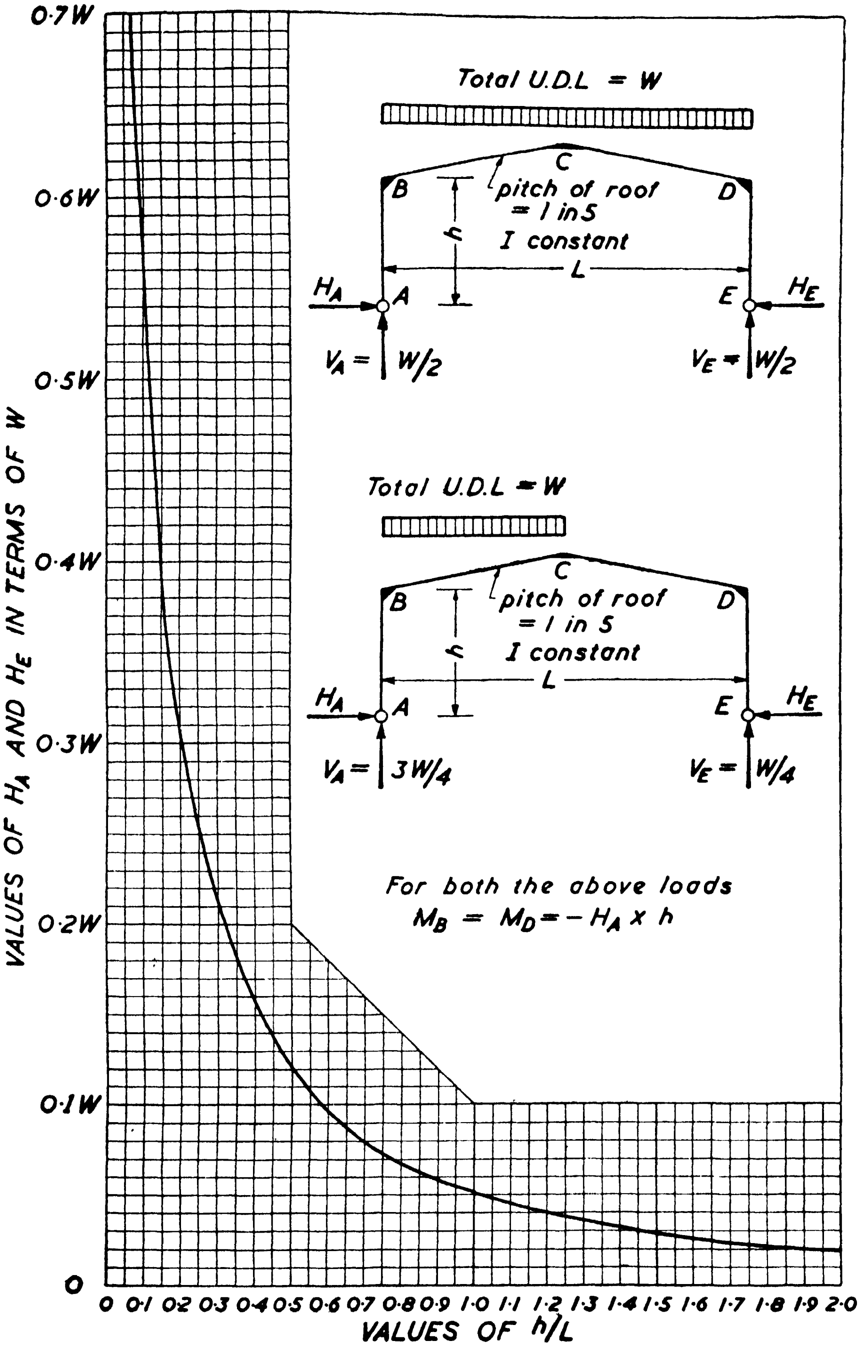

0·7W
0·6W
0·5W
0·4W
0·3W
0·2W
0·1W
0
VALUES OF H_A AND H_D IN TERMS OF W
Total U.D.L.=W
B
C
h
I constant
L
H_A
A
D
H_D
V_A = W/2
V_D = W/2
Total U.D.L = W
I constant
B
C
L/2
h
L
H_A
A
D
H_D
V_A = 3W/4
V_D = W/4
For both the above loads,
$M_B = M_C = - H_A \times h$
0 0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2·0
VALUES OF h/L

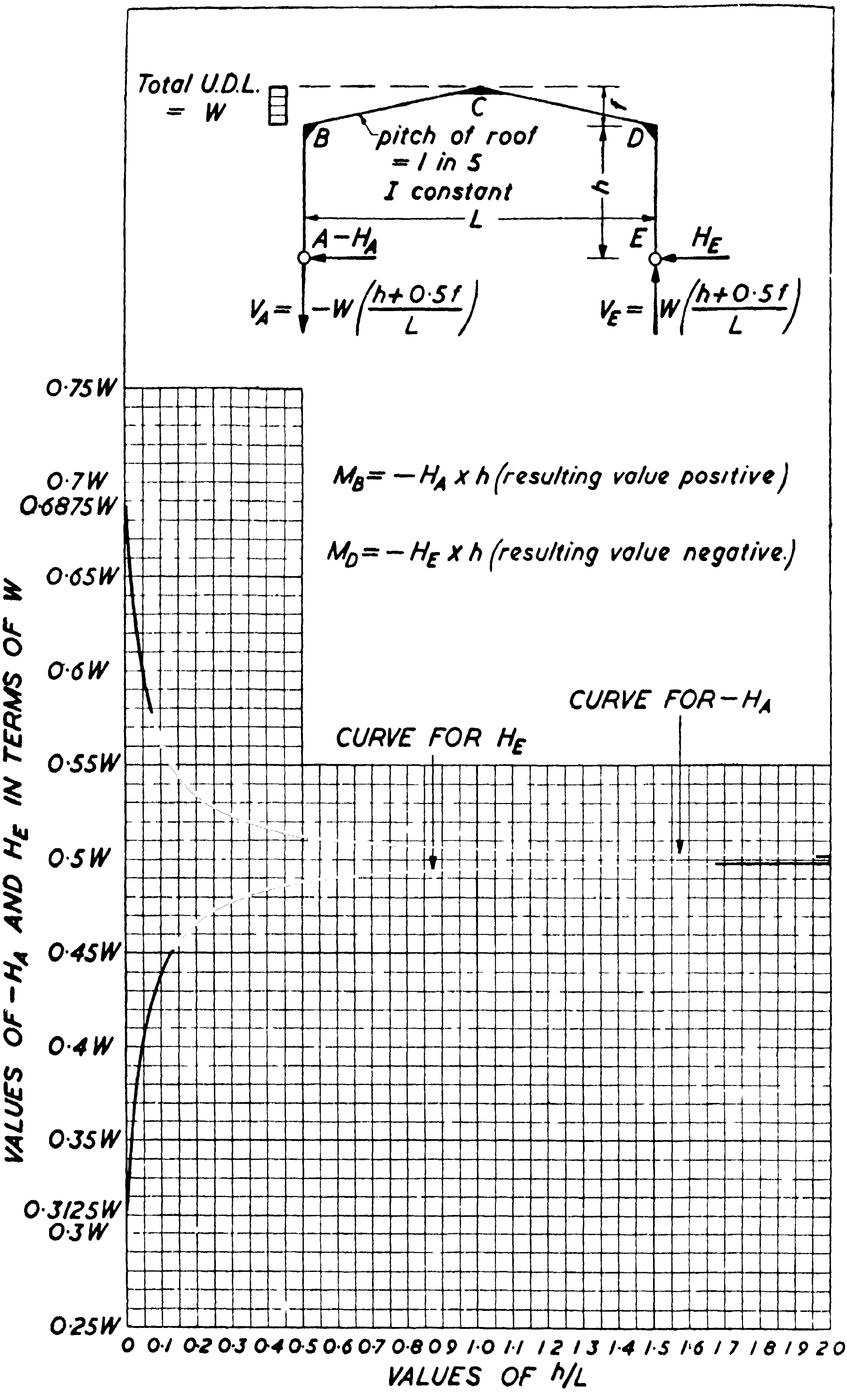

0·85W
0·8W
0·75W
0·7W
VALUE OF −H_A
0·65W
0·6W
0·55W
0·5W
0·45W
0·4W
0·35W
0·3W
0·25W
0·2W
0·15W
VALUES OF H_D AND −H_A IN TERMS OF W
Total U.D.L = W
B
C
I constant
h
L
−H_A
A
D
H_D
V_A = −Wh/2L
V_D = Wh/2L
$M_B = h[-H_A - W/2]$ where −H_A has a positive value
$M_C = -H_D \times h$
Note. When $\frac{h}{L}$ has an infinite value, H_D = 0·3125W
VALUE OF H_D
0 0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2·0
VALUES OF h/L

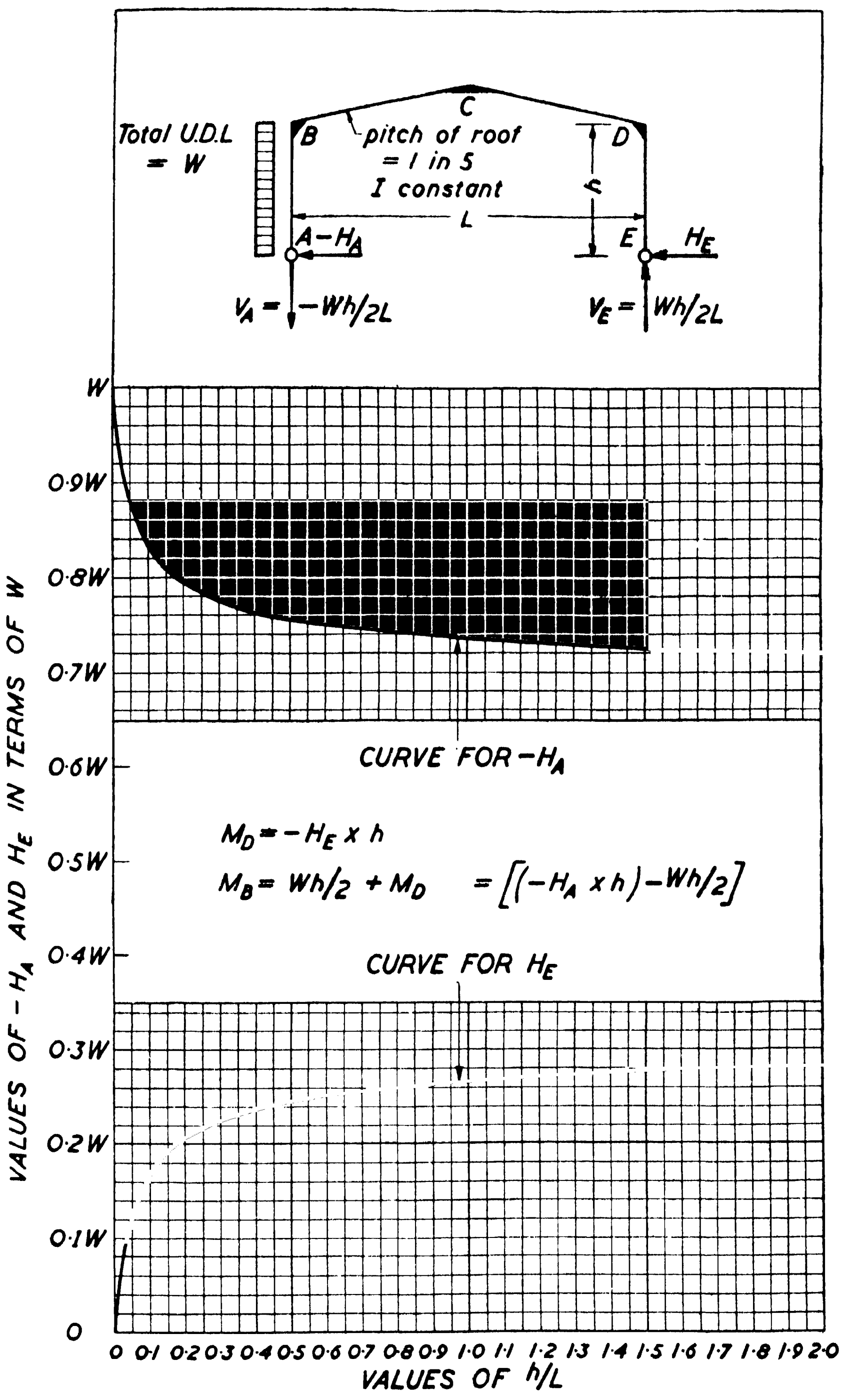


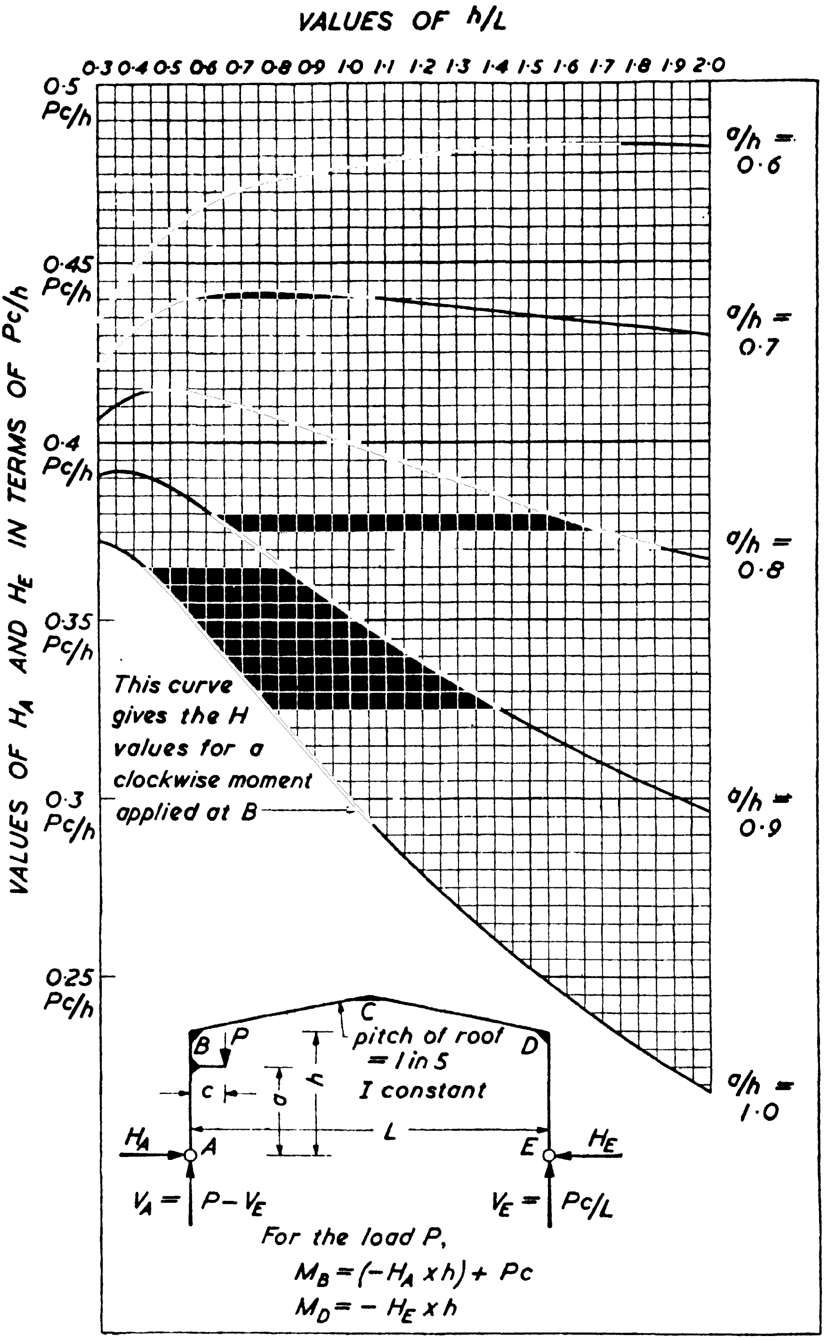
Moments in left column.
$M_1 = -H_A \times a$
$M_2 = Pc + M_1$
M_B
M_1
M_2
Pc
I constant
B
P
C
c
a
h
L
H_A
A
D
H_D
$V_A = P - V_D$
$V_D = Pc/L$
For any load P,
$M_B = (-H_A \times h) + Pc$
$M_C = -H_D \times h$
In this case $M_B = -M_C = Pc/2$
0·5Pc/h
0·45 Pc/h
0·40 Pc/h
0·35 Pc/h
0·30 Pc/h
0·25Pc/h
a/h = 0·5774
a/h = 0·667
a/h = 0·75
a/h = 0·8
a/h = 0·85
a/h = 0·9
a/h = 0·95
a/h = 1
VALUES OF H_A AND H_D IN TERMS OF Pc/h
0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2·0
VALUES OF h/L

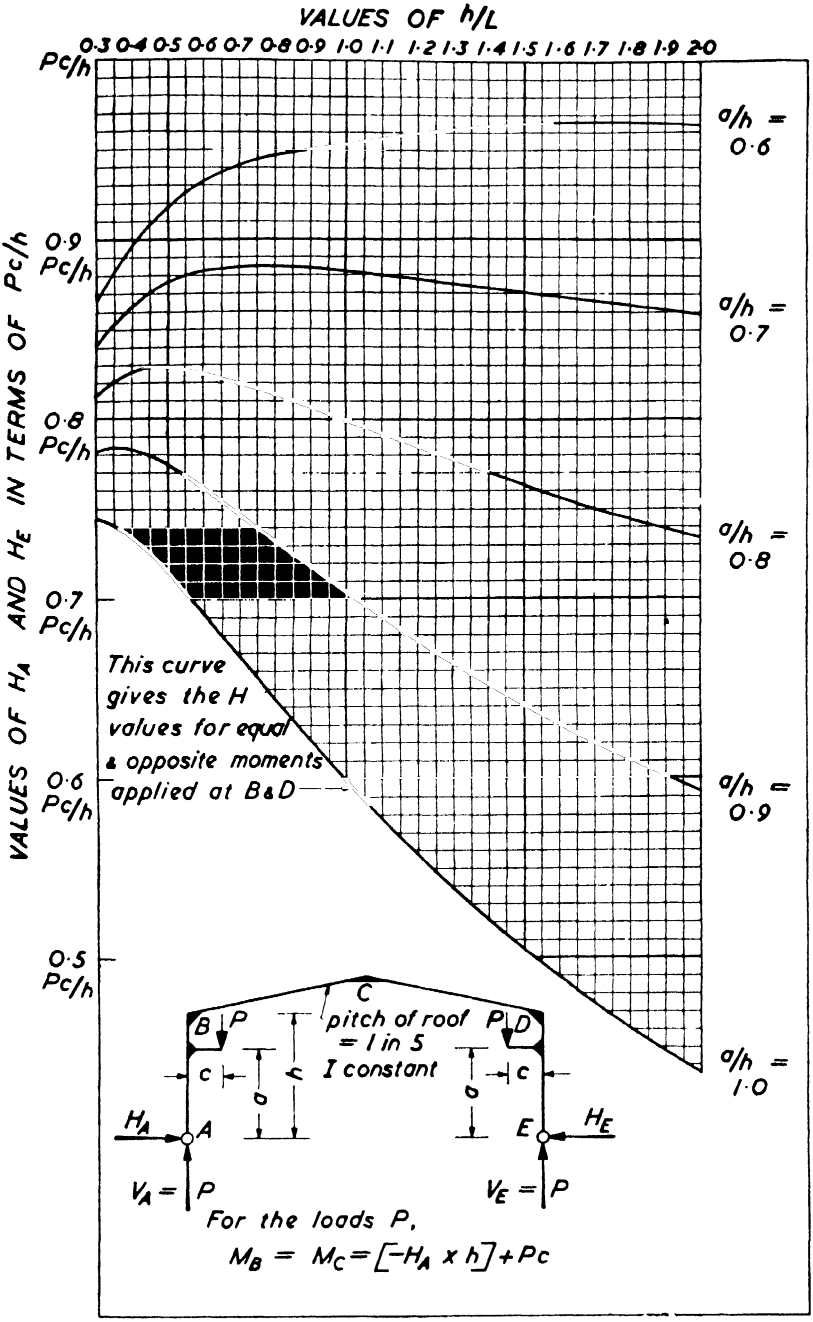

Moments in left column:
M1 = − HA x a
M2 = Pc + M1
(right column similar)
For the loads P,
MB = MC = (−HA x h) + Pc
MB
M2
M1
Pc
I constant
B
C
P
P
c
c
h
a
a
L
HA
HD
A
D
VA = P
VD = P
In this case MB = MC = zero
Pc/h
0·90 Pc/h
0·80 Pc/h
0·70 Pc/h
0·60 Pc/h
0·5Pc/h
VALUES OF HA AND HD IN TERMS OF Pc/h
a/h = 0·5774
a/h = 0·667
a/h = 0·75
a/h = 0·8
a/h = 0·85
a/h = 0·9
a/h = 0·95
a/h = 1
0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2·0
VALUES OF h/L

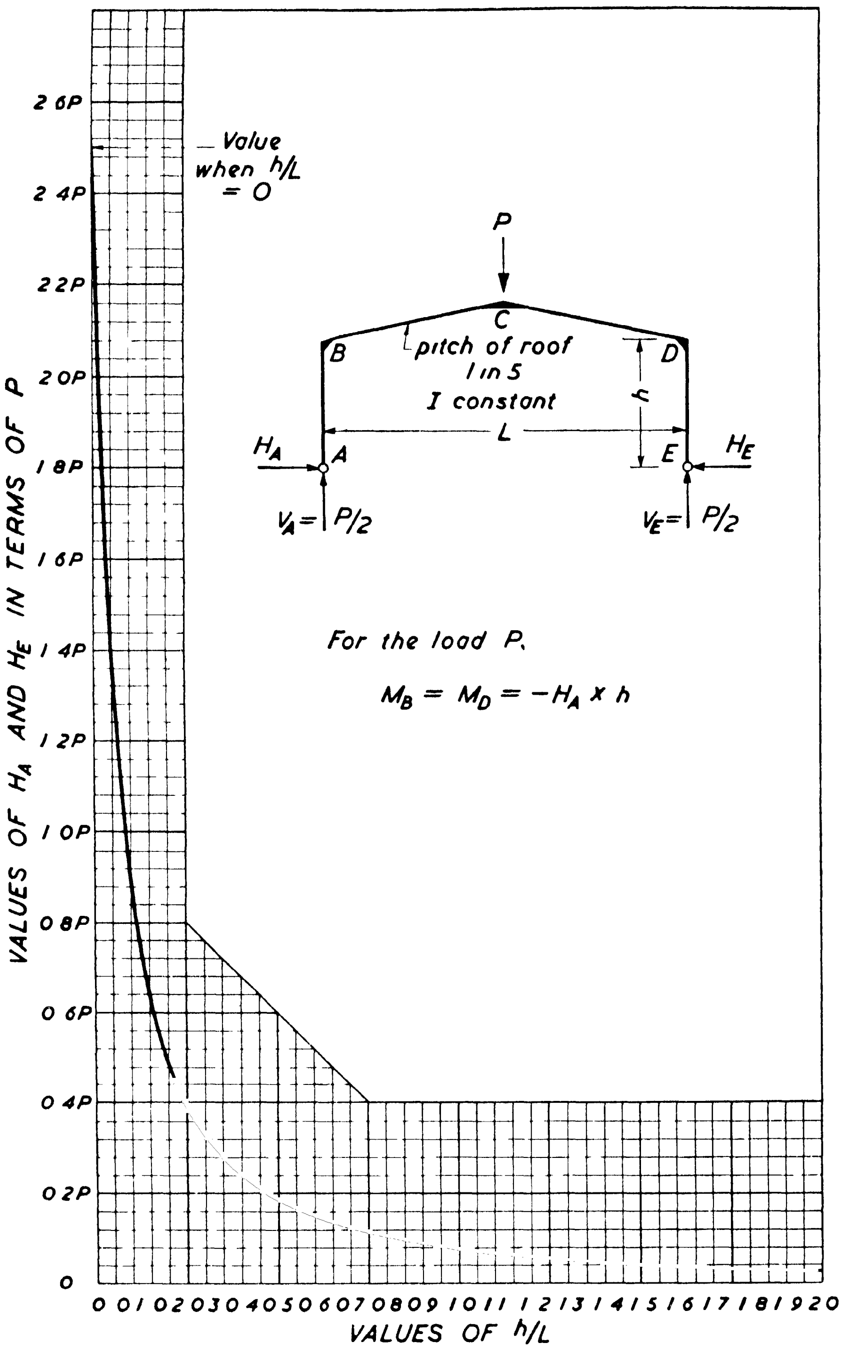

Total U.D.L = W
pitch of roof = 1 in 5
I constant
B C D A E h L
H_A H_E
V_A = W/2
V_E = W/2
Total U.D.L = W
pitch of roof = 1 in 5
I constant
V_A = 3W/4
V_E = W/4
For both the above loads
$M_B = M_D = -H_A \times h$
VALUES OF H_A AND H_E IN TERMS OF W
0 0·1W 0·2W 0·3W 0·4W 0·5W 0·6W 0·7W
0 0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2·0
VALUES OF h/L

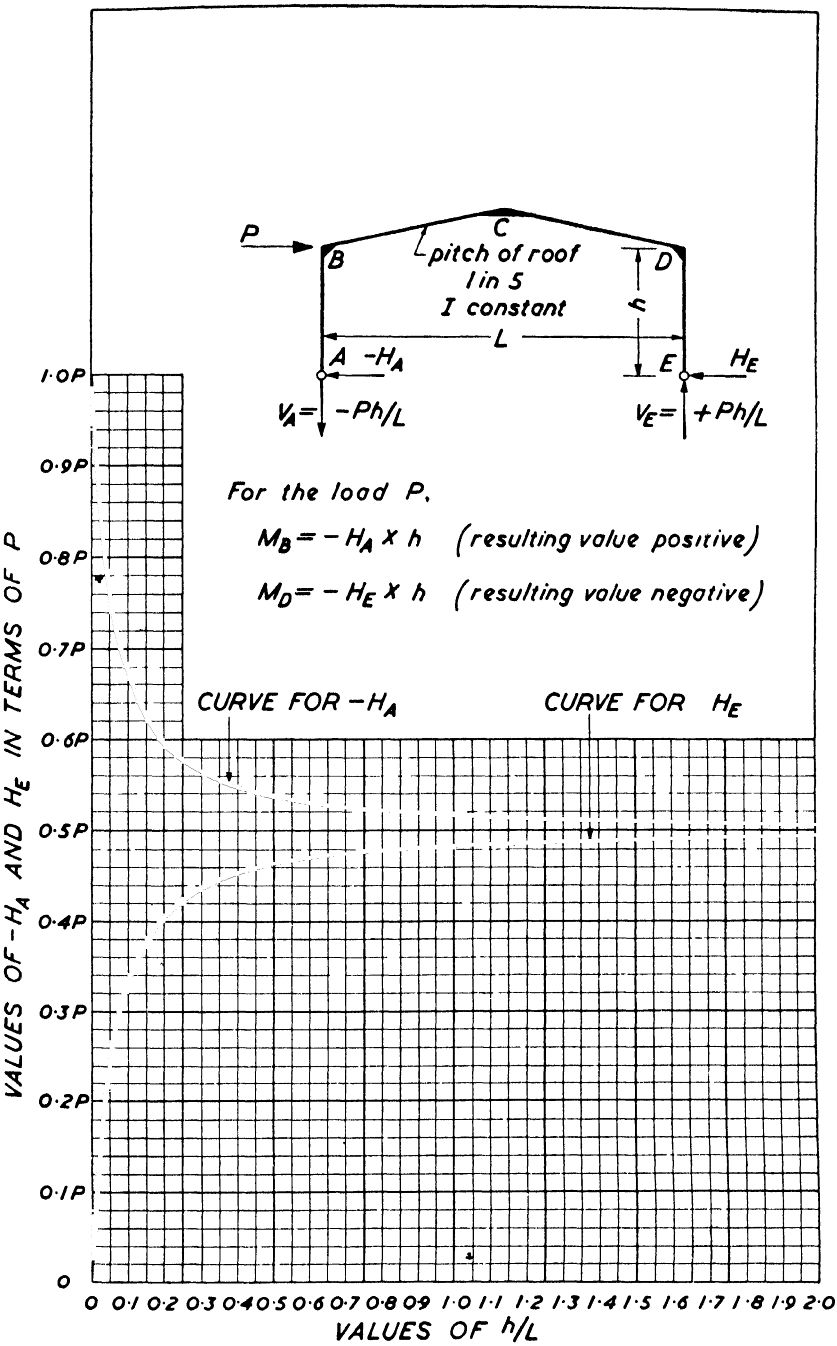

Total U.D.L. = W
C
B
D
pitch of roof = l in 5
I constant
h
L
A − H_A
E
H_E
$V_A = -W\left(\frac{h+0.5f}{L}\right)$
$V_E = W\left(\frac{h+0.5f}{L}\right)$
$M_B = -H_A \times h$ (resulting value positive)
$M_D = -H_E \times h$ (resulting value negative.)
CURVE FOR − H_A
CURVE FOR H_E
0·75W
0·7W
0·6875W
0·65W
0·6W
0·55W
0·5W
0·45W
0·4W
0·35W
0·3125W
0·3W
0·25W
VALUES OF − H_A AND H_E IN TERMS OF W
0 0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2·0
VALUES OF h/L

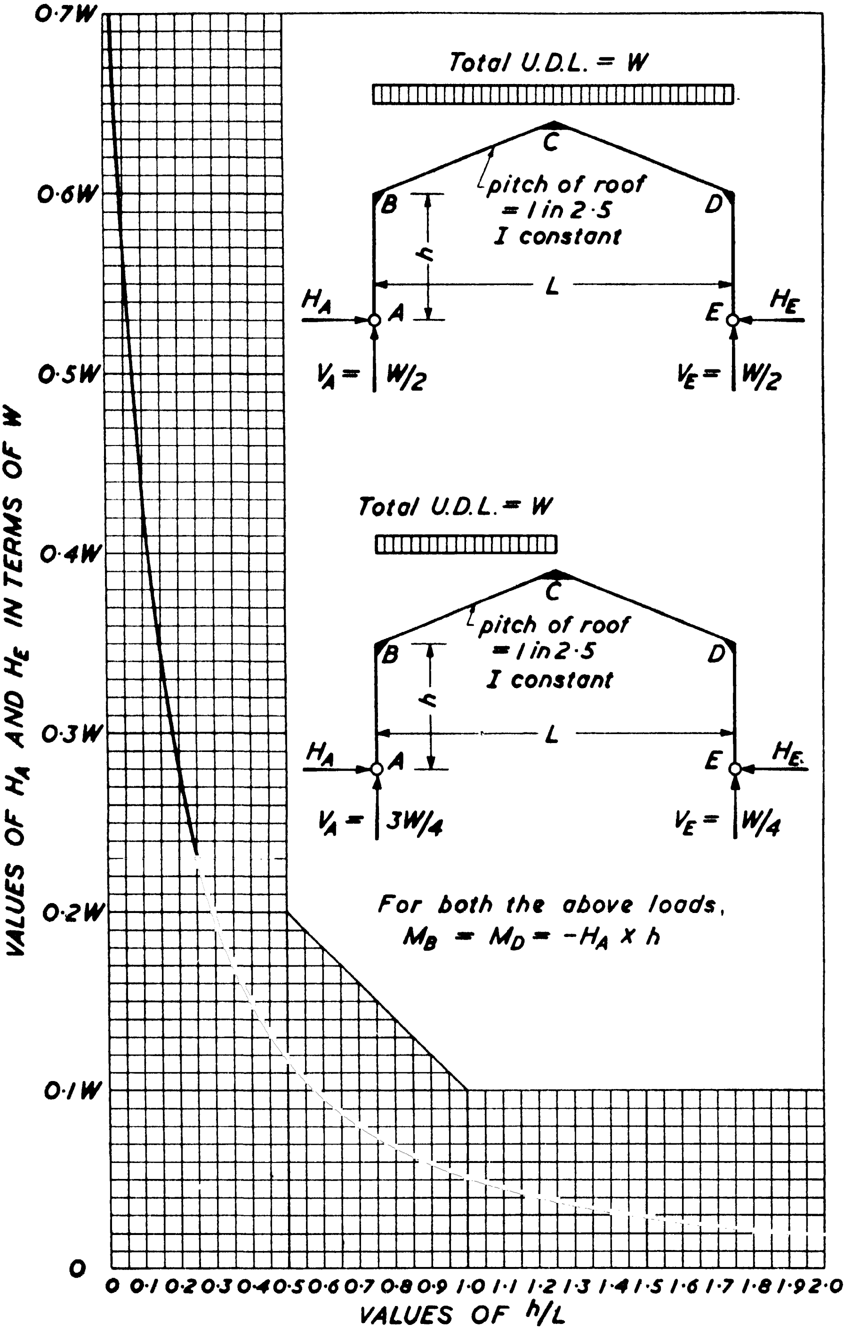

Total U.D.L = W
C
B
pitch of roof = 1 in 5
I constant
D
h
L
A −H_A
E
H_E
V_A = −Wh/2L
V_E = Wh/2L
W
0·9W
0·8W
0·7W
0·6W
0·5W
0·4W
0·3W
0·2W
0·1W
0
CURVE FOR −H_A
M_D = −H_E x h
M_B = Wh/2 + M_D = [(−H_A x h) − Wh/2]
CURVE FOR H_E
VALUES OF −H_A AND H_E IN TERMS OF W
0 0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2·0
VALUES OF h/L

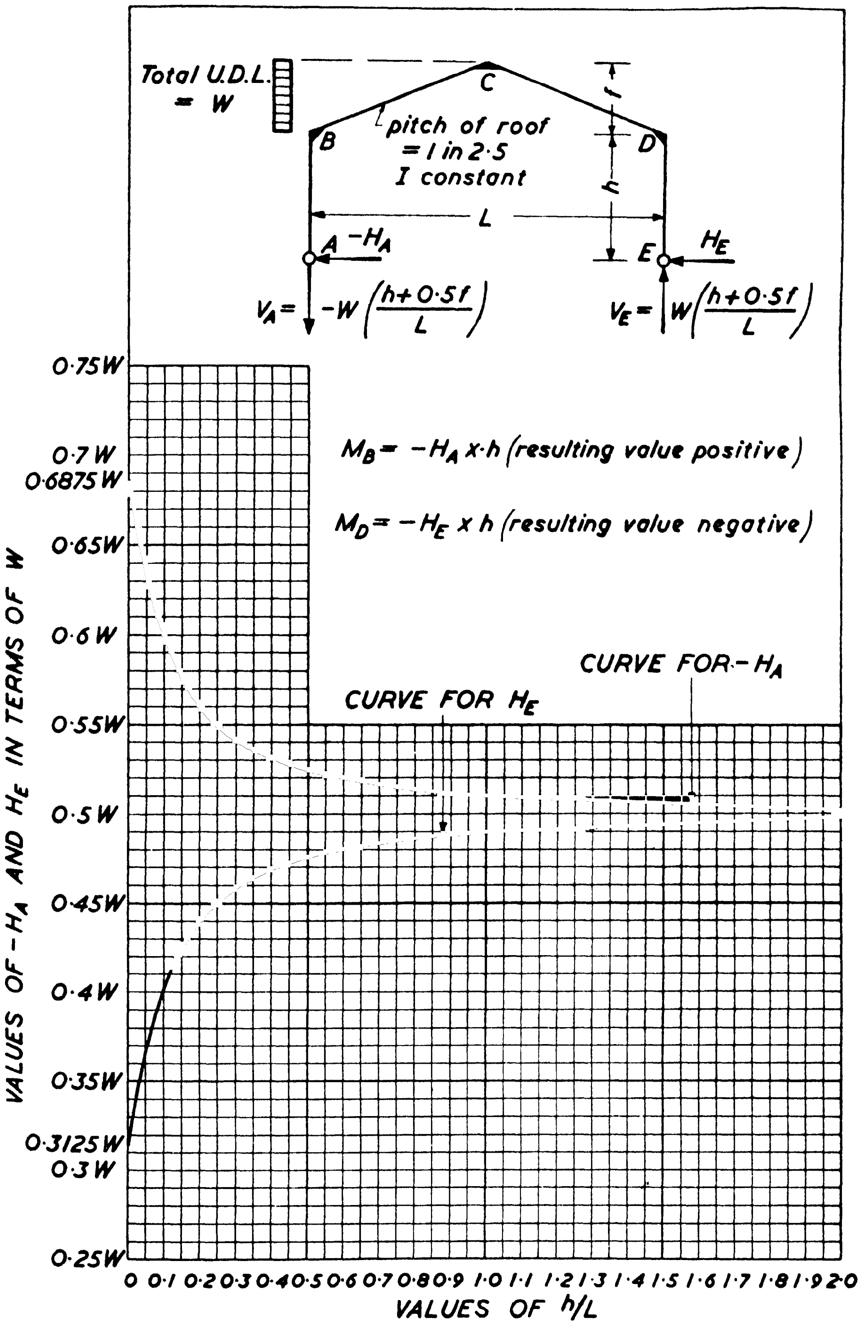

VALUES OF h/L
0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2·0
0·5 Pc/h
0·45 Pc/h
0·4 Pc/h
0·35 Pc/h
0·3 Pc/h
0·25 Pc/h
VALUES OF H_A AND H_E IN TERMS OF Pc/h
a/h = 0·6
a/h = 0·7
a/h = 0·8
a/h = 0·9
a/h = 1·0
This curve gives the H values for a clockwise moment applied at B
B P C D
pitch of roof = 1 in 5
I constant
c a h L
H_A A E H_E
V_A = P − V_E
V_E = Pc/L
For the load P,
$M_B = (-H_A \times h) + Pc$
$M_D = -H_E \times h$

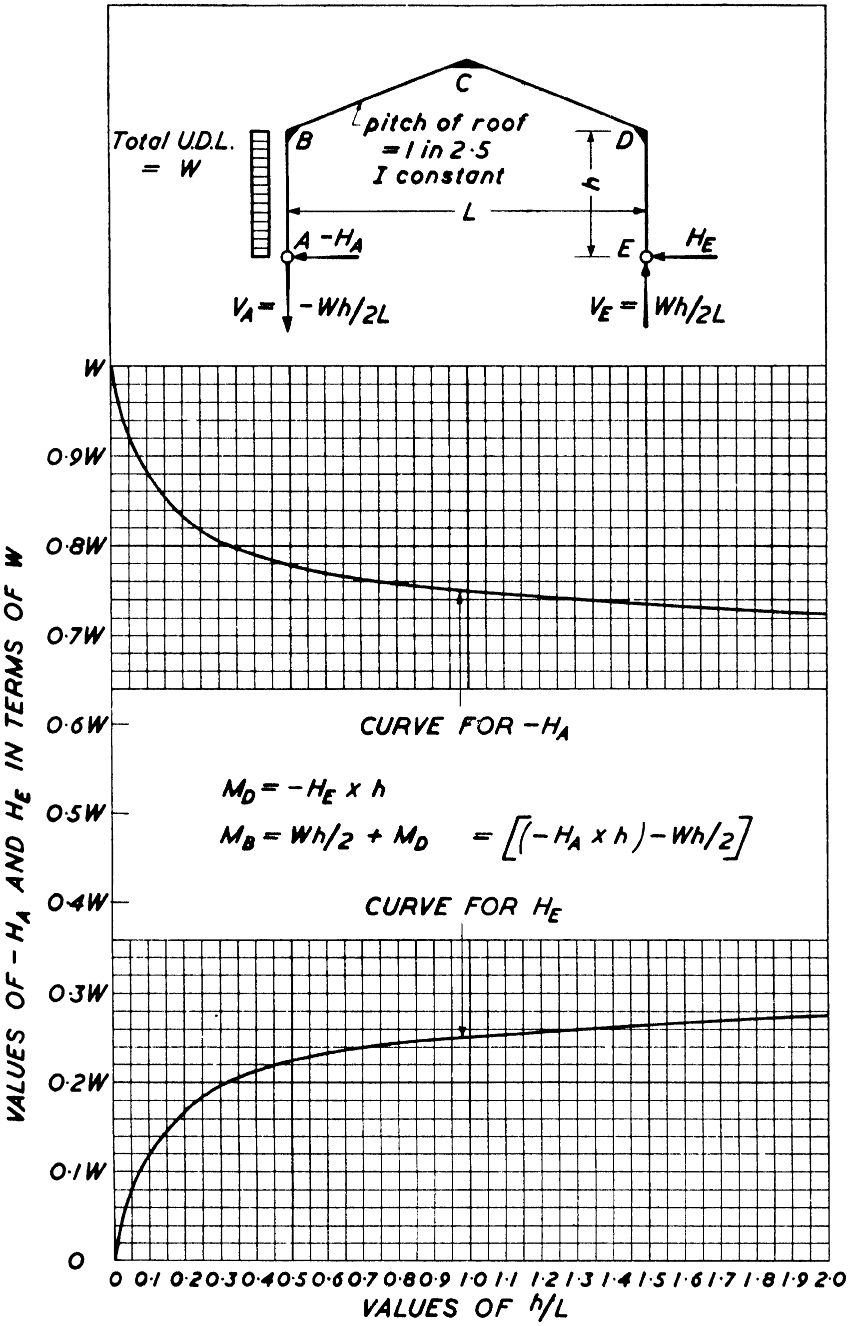

VALUES OF h/L
0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2·0
Pc/h
0·9 Pc/h
0·8 Pc/h
0·7 Pc/h
0·6 Pc/h
0·5 Pc/h
VALUES OF H_A AND H_E IN TERMS OF Pc/h
a/h = 0·6
a/h = 0·7
a/h = 0·8
a/h = 0·9
a/h = 1·0
This curve gives the H values for equal & opposite moments applied at B&D
C
B P
P D
pitch of roof = 1 in 5
I constant
c
h
a
H_A
A
E
H_E
V_A = P
V_E = P
For the loads P,
M_B = M_C = [−H_A x h] + Pc


Value
when h/L
= O
P
C
pitch of roof
1 in 5
I constant
B
D
h
L
H_A
A
E
H_E
V_A = P/2
V_E = P/2
For the load P,
M_B = M_D = −H_A x h
2 6P
2 4P
2 2P
2 0P
1 8P
1 6P
1 4P
1 2P
1 0P
0 8P
0 6P
0 4P
0 2P
0
VALUES OF H_A AND H_E IN TERMS OF P
0 01 02 03 04 05 06 07 08 09 10 11 12 13 14 15 16 17 18 19 20
VALUES OF h/L

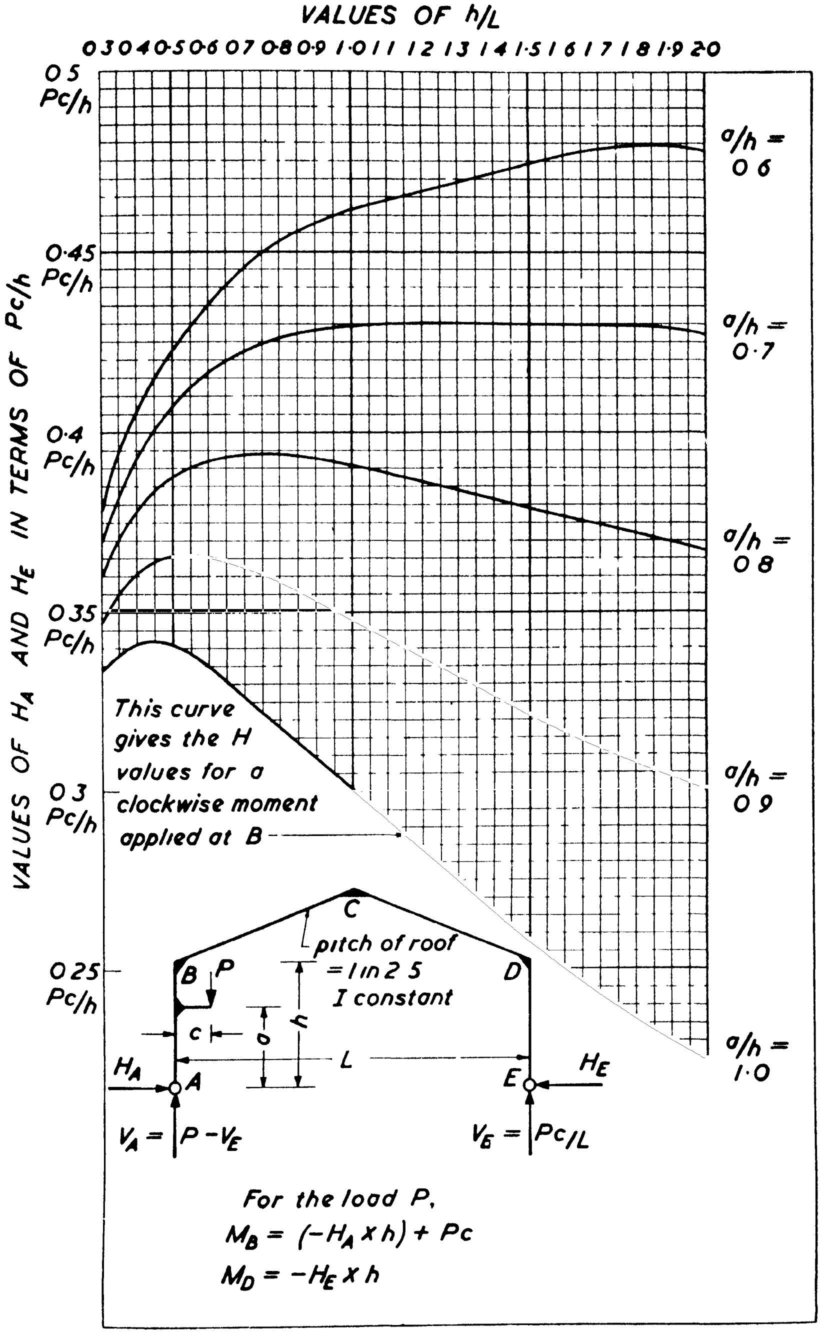

P
C
B
pitch of roof
1 in 5
I constant
D
h
L
A -H_A
E
H_E
V_A= -Ph/L
V_E= +Ph/L
For the load P,
M_B = - H_A × h (resulting value positive)
M_D = - H_E × h (resulting value negative)
CURVE FOR -H_A
CURVE FOR H_E
1·0P
0·9P
0·8P
0·7P
0·6P
0·5P
0·4P
0·3P
0·2P
0·1P
0
VALUES OF -H_A AND H_E IN TERMS OF P
0 0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2·0
VALUES OF h/L


Total U.D.L. = W
C
pitch of roof
= 1 in 2·5
I constant
B
D
h
L
H_A
A
E
H_E
V_A = W/2
V_E = W/2
Total U.D.L. = W
C
pitch of roof
= 1 in 2·5
I constant
B
D
h
L
H_A
A
E
H_E
V_A = 3W/4
V_E = W/4
For both the above loads,
$M_B = M_D = -H_A \times h$
0·7W
0·6W
0·5W
0·4W
0·3W
0·2W
0·1W
0
VALUES OF H_A AND H_E IN TERMS OF W
0 0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2·0
VALUES OF h/L

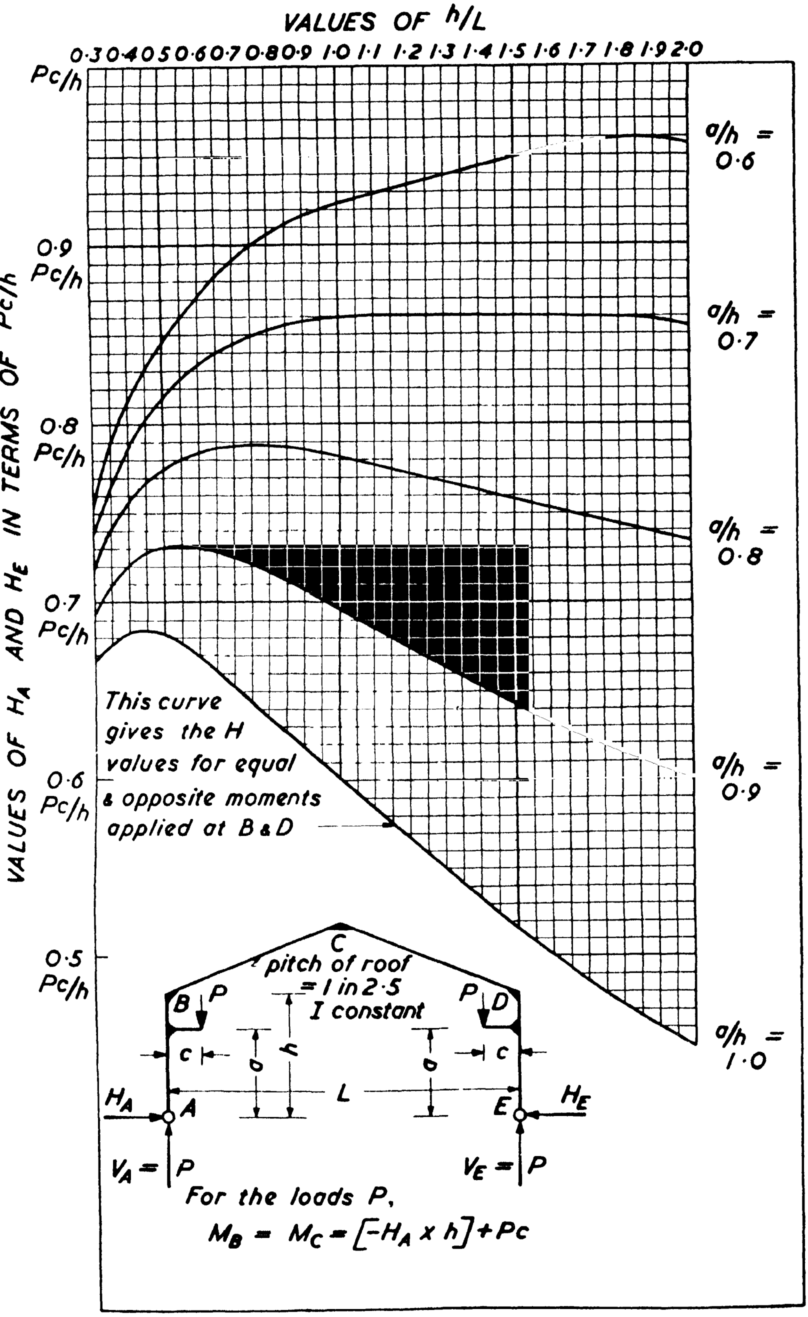

Total U.D.L. = W
C
B
D
pitch of roof = 1 in 2·5
I constant
f
h
L
A −H_A
E H_E
V_A = −W((h+0·5f)/L)
V_E = W((h+0·5f)/L)
M_B = −H_A x·h (resulting value positive)
M_D = −H_E x h (resulting value negative)
CURVE FOR −H_A
CURVE FOR H_E
0·75W
0·7W
0·6875W
0·65W
0·6W
0·55W
0·5W
0·45W
0·4W
0·35W
0·3125W
0·3W
0·25W
VALUES OF −H_A AND H_E IN TERMS OF W
0 0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2·0
VALUES OF h/L

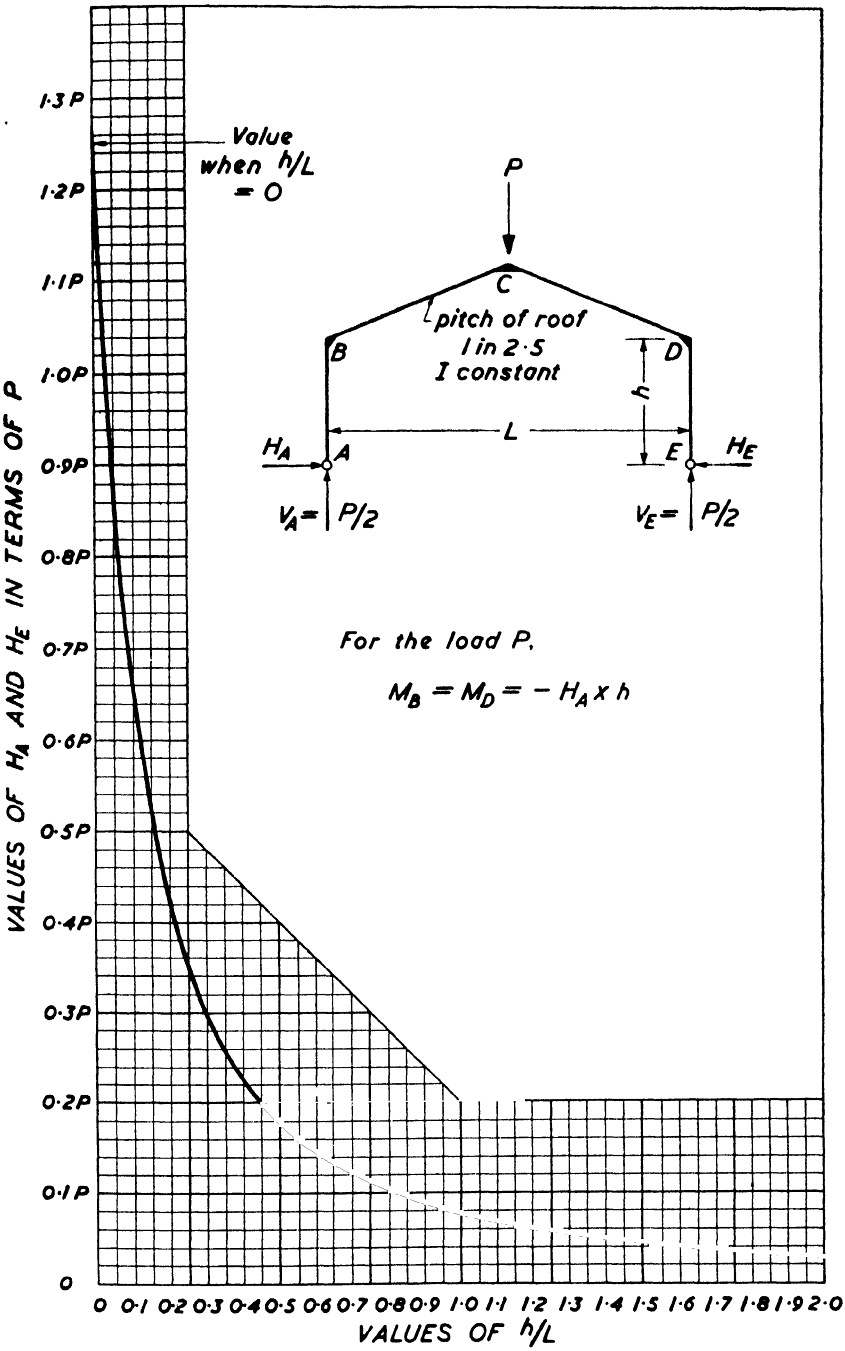


C
pitch of roof
= 1 in 2·5
I constant
Total U.D.L.
= W
B
D
h
L
A -H_A
E
H_E
V_A = -Wh/2L
V_E = Wh/2L
W
0·9W
0·8W
0·7W
0·6W
0·5W
0·4W
0·3W
0·2W
0·1W
0
CURVE FOR -H_A
M_D = -H_E x h
M_B = Wh/2 + M_D = [(-H_A x h) - Wh/2]
CURVE FOR H_E
VALUES OF -H_A AND H_E IN TERMS OF W
0 0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2·0
VALUES OF h/L

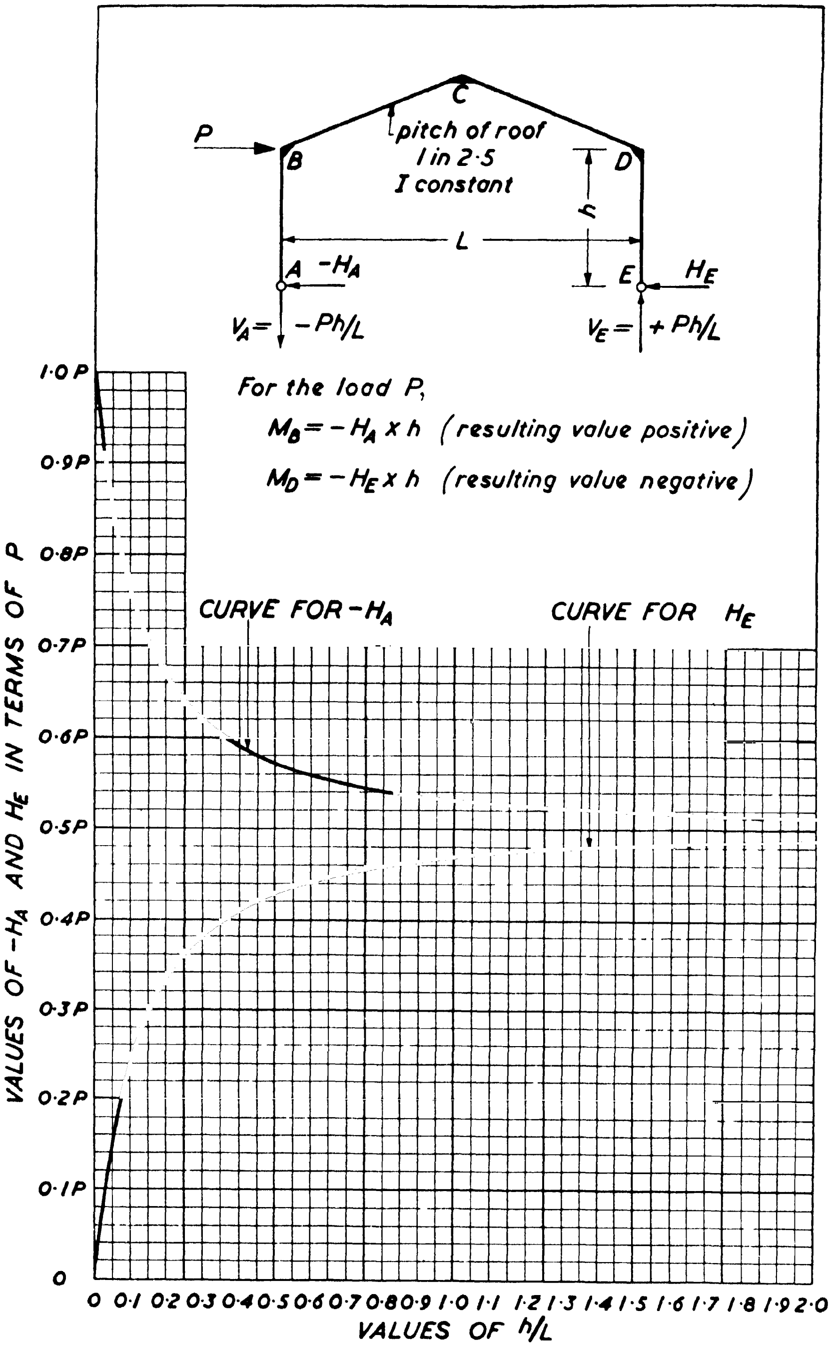


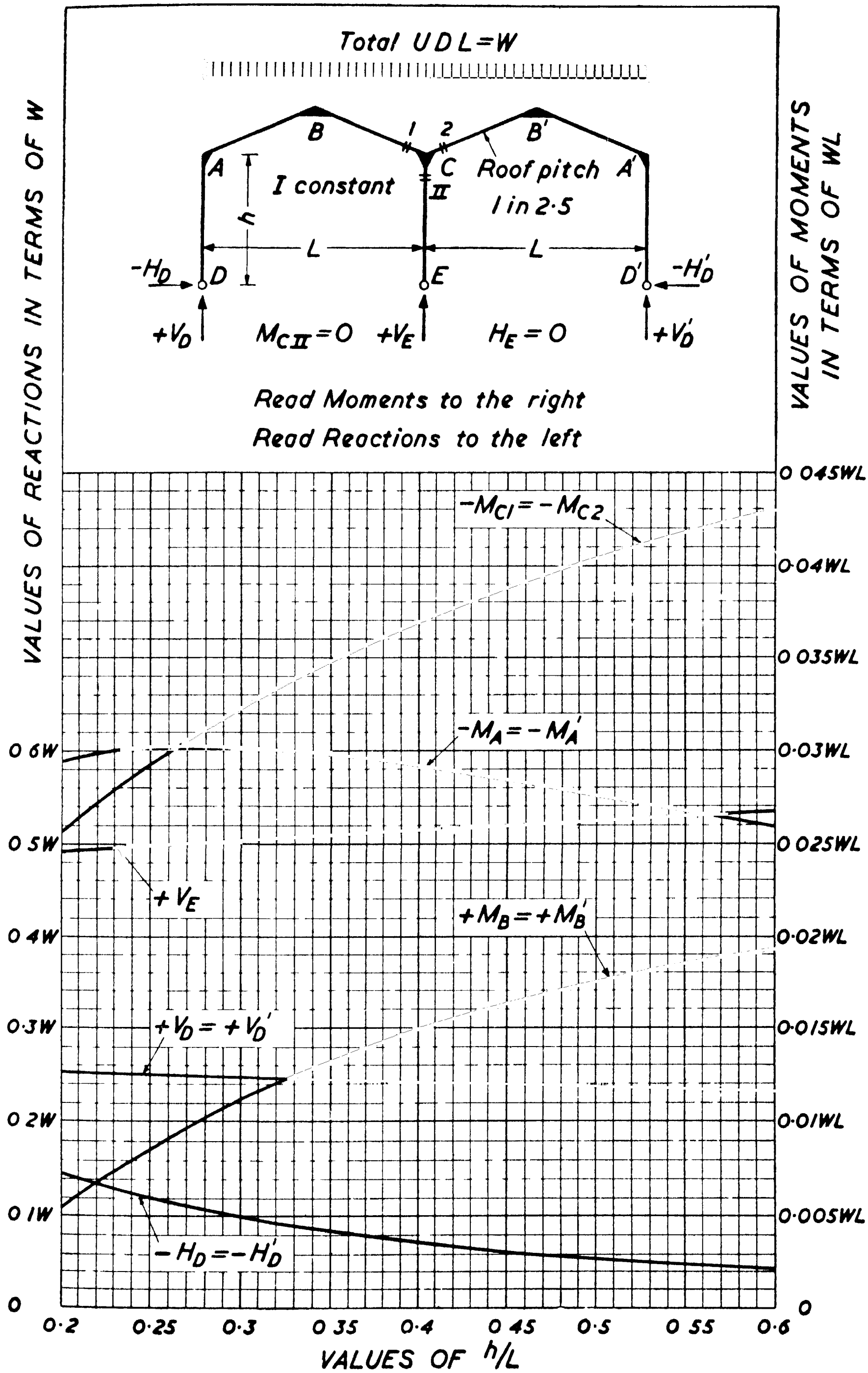
VALUES OF h/L
0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2·0
VALUES OF H_A AND H_E IN TERMS OF Pc/h
0·5 Pc/h
0·45 Pc/h
0·4 Pc/h
0·35 Pc/h
0·3 Pc/h
0·25 Pc/h
a/h = 0·6
a/h = 0·7
a/h = 0·8
a/h = 0·9
a/h = 1·0
This curve gives the H values for a clockwise moment applied at B
pitch of roof = 1 in 2·5
I constant
B
C
D
P
c
a
h
L
H_A
H_E
A
E
$V_A = P - V_E$
$V_E = Pc/L$
For the load P,
$M_B = (-H_A \times h) + Pc$
$M_D = -H_E \times h$

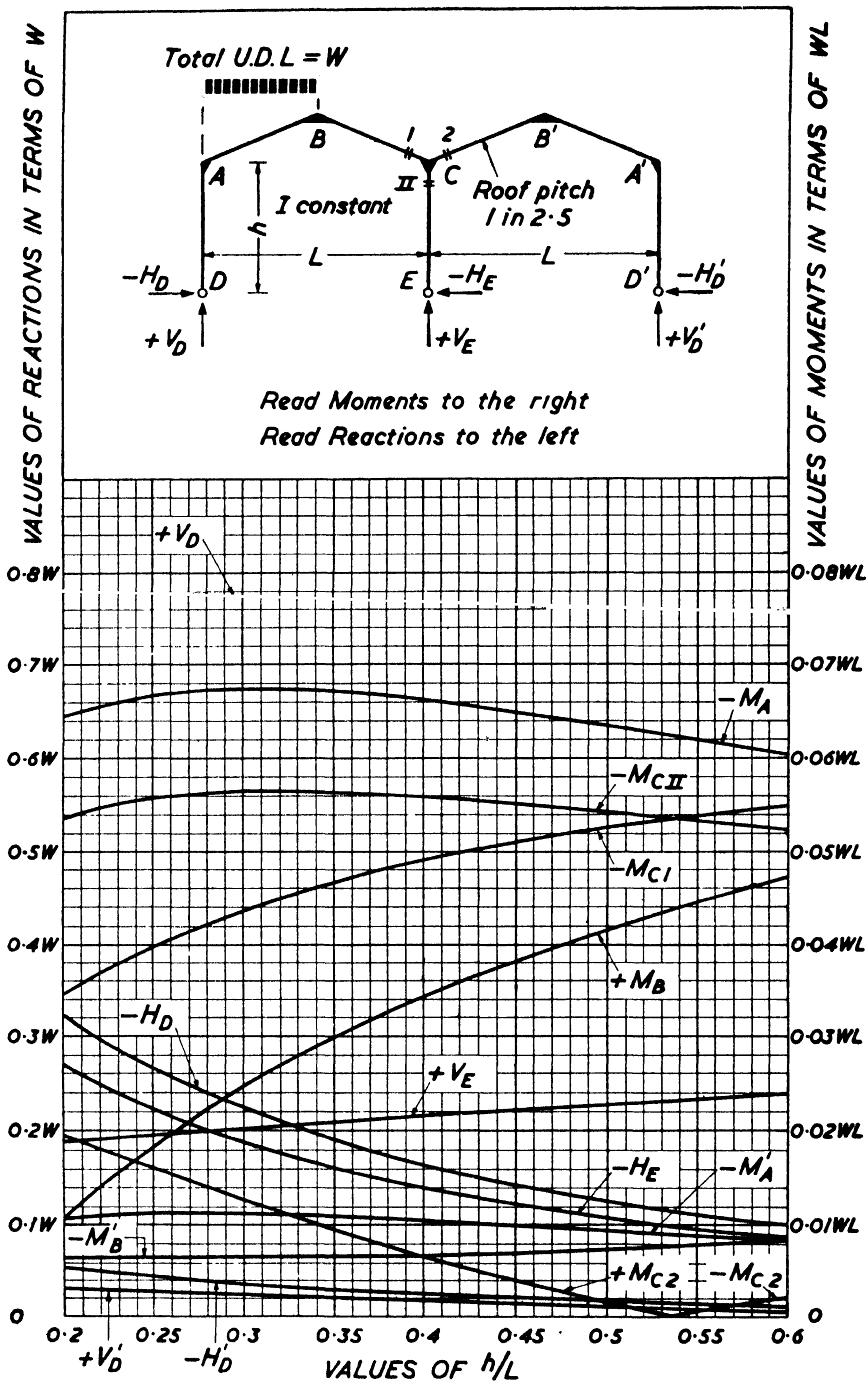

VALUES OF h/L
0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2·0
Pc/h
0·9 Pc/h
0·8 Pc/h
0·7 Pc/h
0·6 Pc/h
0·5 Pc/h
VALUES OF HA AND HE IN TERMS OF Pc/h
a/h = 0·6
a/h = 0·7
a/h = 0·8
a/h = 0·9
a/h = 1·0
This curve gives the H values for equal & opposite moments applied at B & D
C
pitch of roof = 1 in 2·5
I constant
B
D
P
P
c
c
a
h
a
L
HA
A
E
HE
VA = P
VE = P
For the loads P,
MB = MC = [−HA x h] + Pc

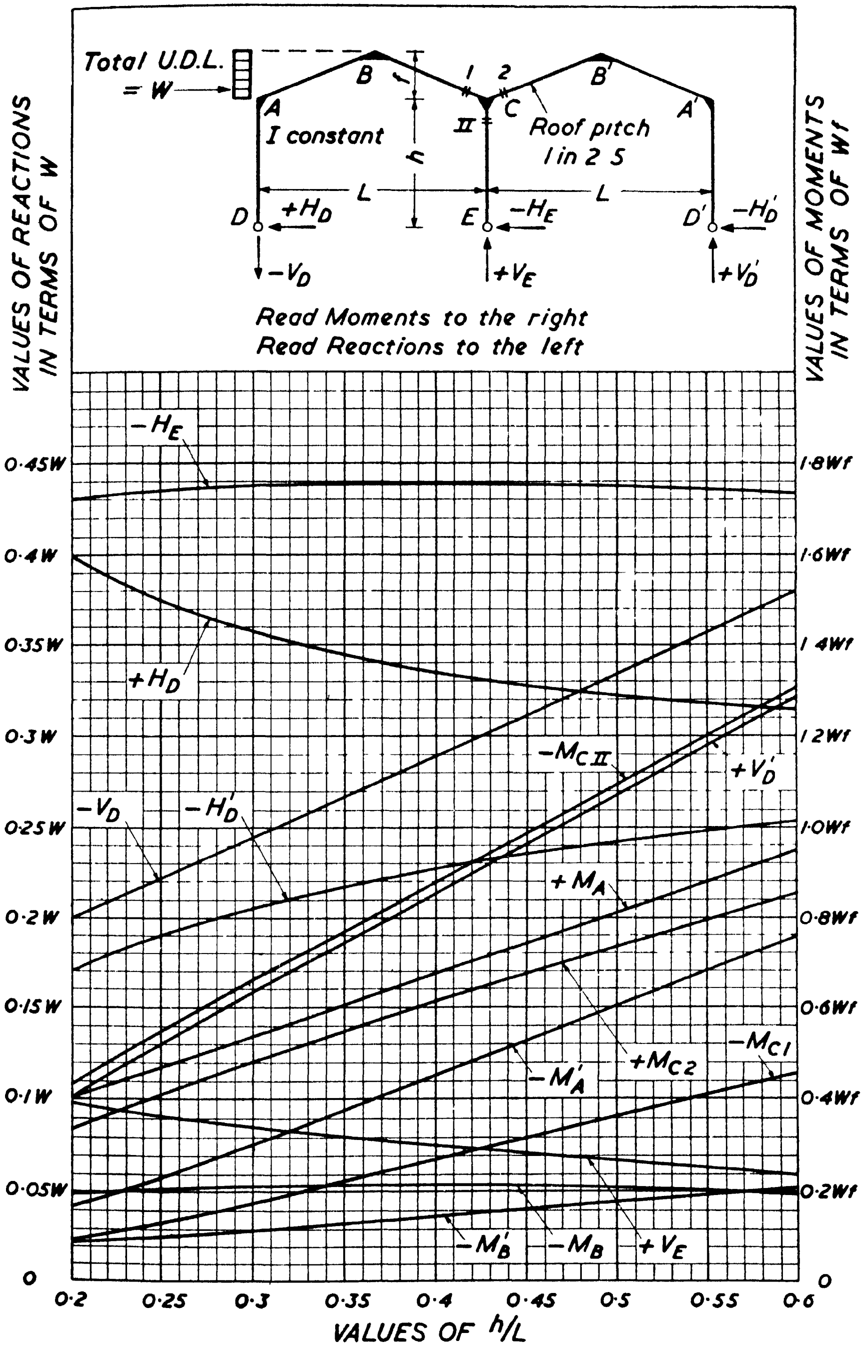


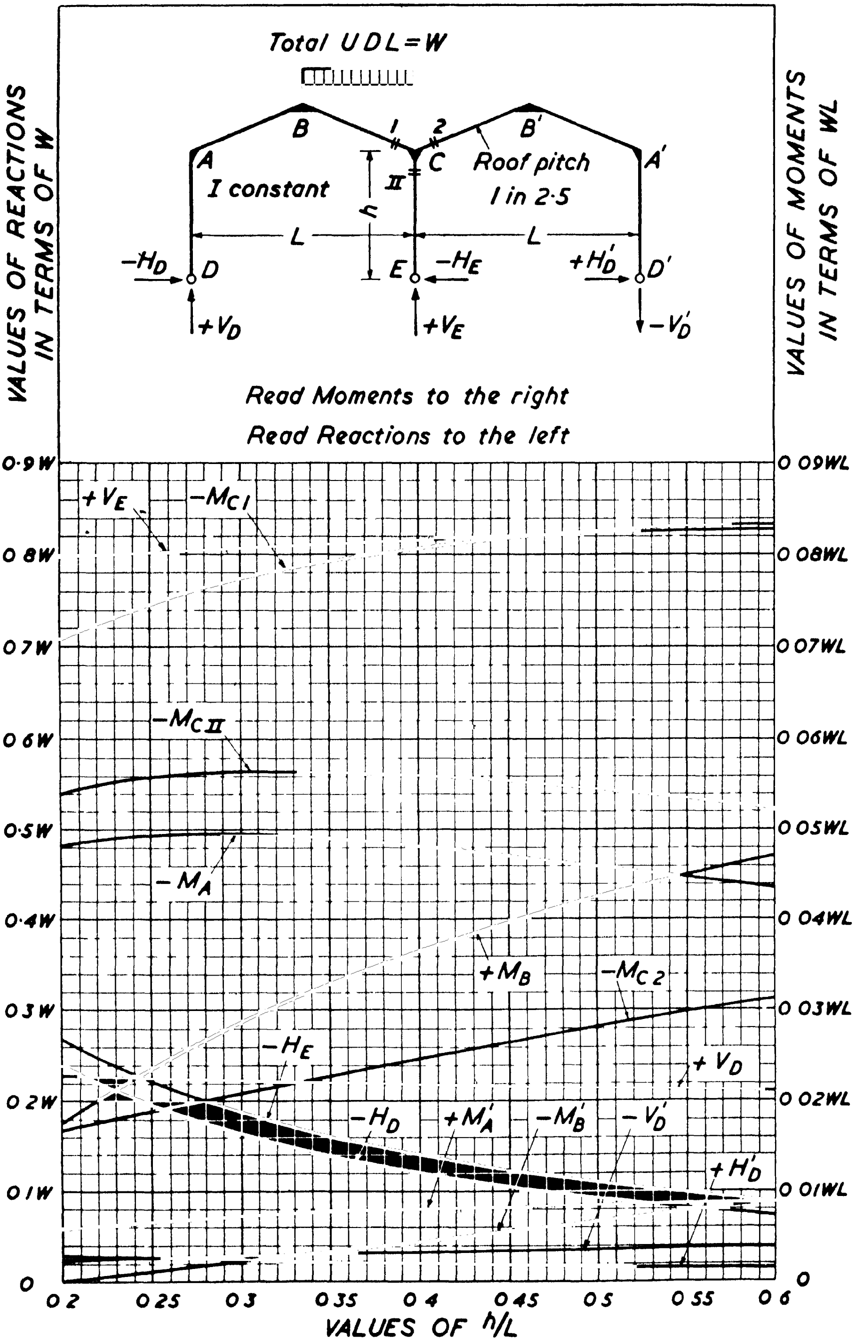
Value when h/L = 0
P
C
pitch of roof
1 in 2·5
I constant
B
D
h
L
H_A
A
E
H_E
V_A = P/2
V_E = P/2
For the load P,
$M_B = M_D = - H_A \times h$
1·3 P
1·2P
1·1P
1·0P
0·9P
0·8P
0·7P
0·6P
0·5P
0·4P
0·3P
0·2P
0·1P
0
VALUES OF H_A AND H_E IN TERMS OF P
0 0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2·0
VALUES OF h/L

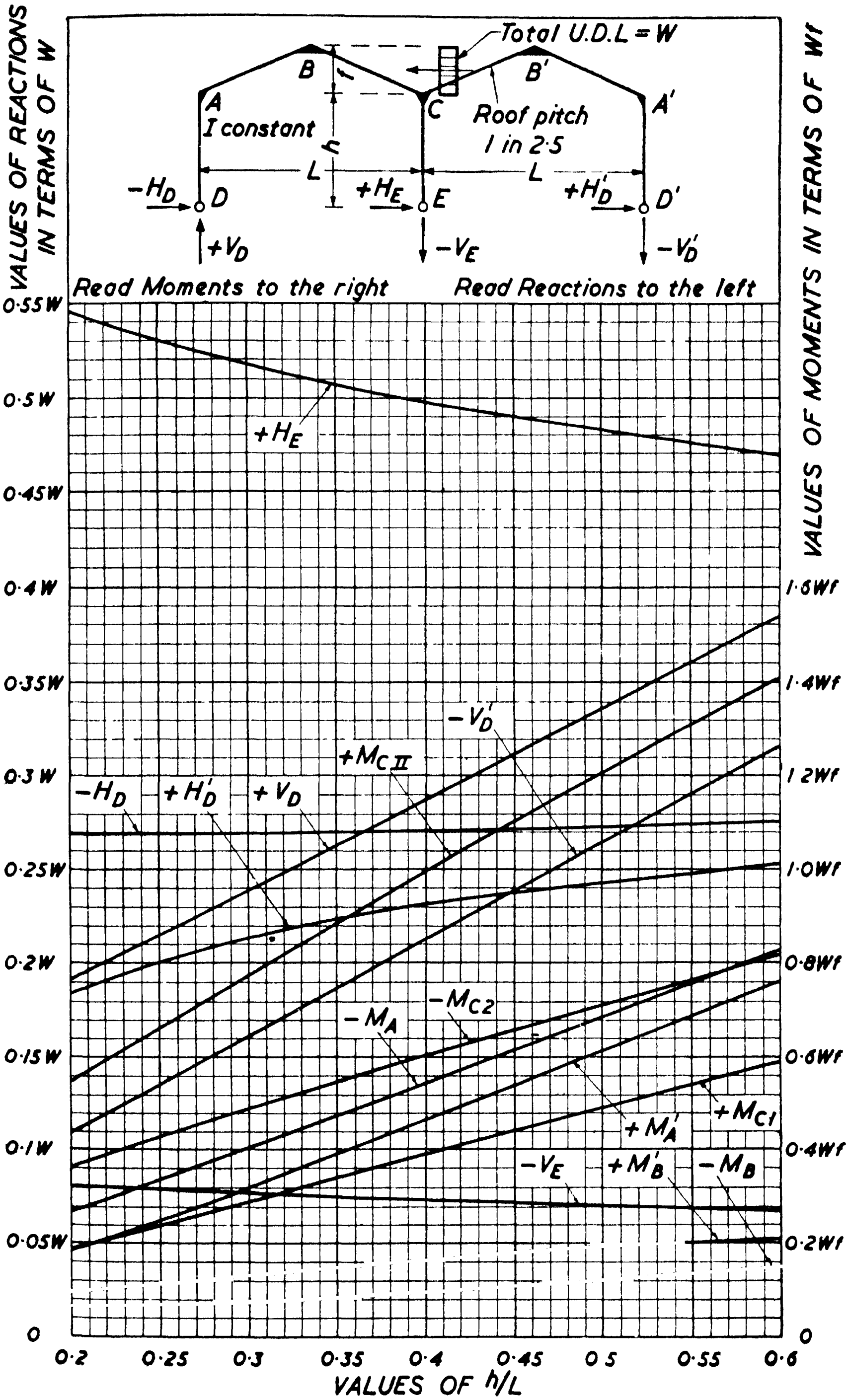

P
C
B
D
pitch of roof
1 in 2·5
I constant
h
L
A -H_A
E
H_E
V_A = -Ph/L
V_E = +Ph/L
For the load P,
M_B = -H_A x h (resulting value positive)
M_D = -H_E x h (resulting value negative)
CURVE FOR -H_A
CURVE FOR H_E
1·0 P
0·9P
0·8P
0·7P
0·6P
0·5P
0·4P
0·3P
0·2P
0·1P
0
VALUES OF -H_A AND H_E IN TERMS OF P
0 0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2·0
VALUES OF h/L

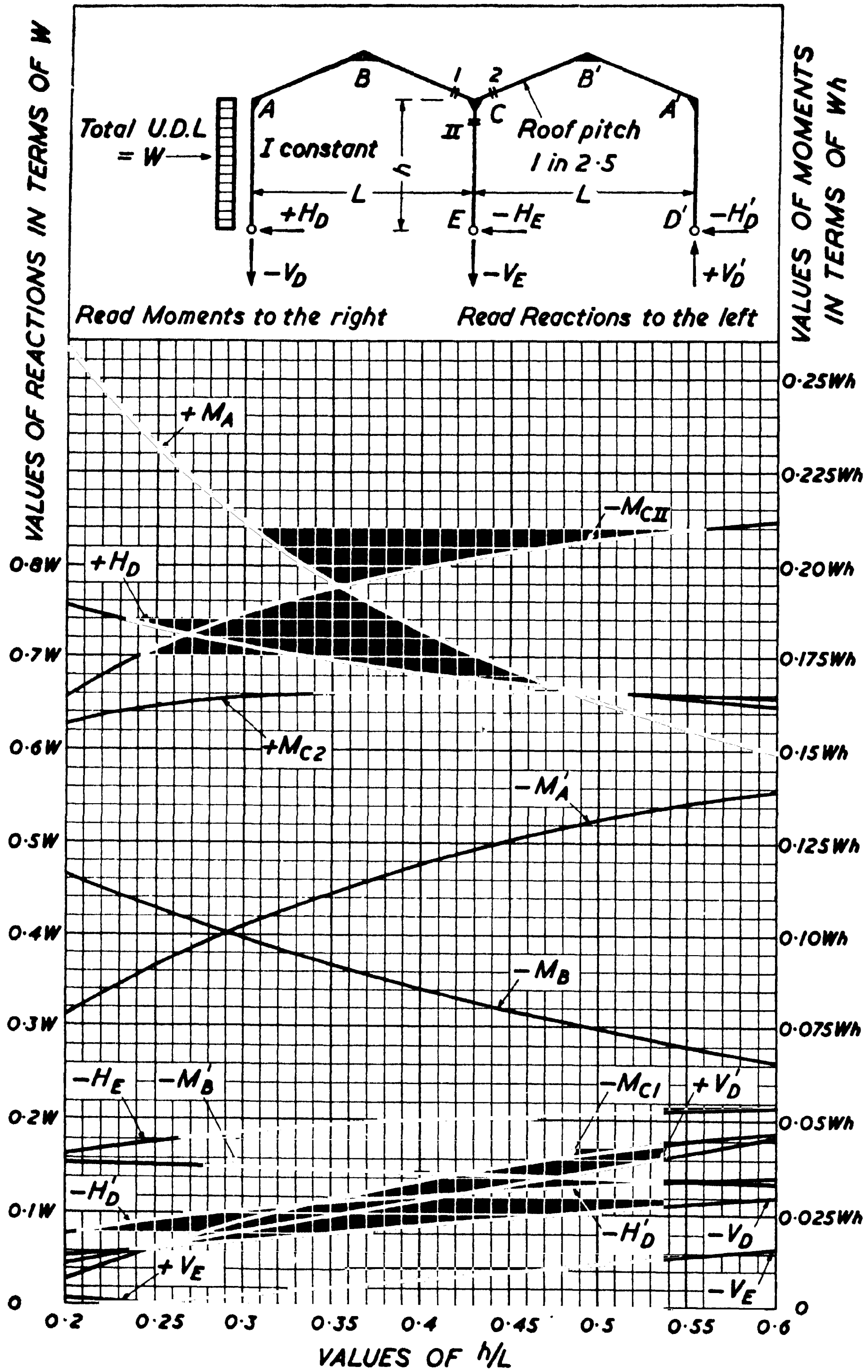

Total UDL=W
B
1
2
B'
A
C
Roof pitch
A'
I constant
II
1 in 2·5
h
L
L
-H_D
D
E
D'
-H'_D
+V_D
M_CII=0
+V_E
H_E=0
+V'_D
Read Moments to the right
Read Reactions to the left
VALUES OF REACTIONS IN TERMS OF W
VALUES OF MOMENTS IN TERMS OF WL
-M_C1=-M_C2
-M_A=-M'_A
+V_E
+M_B=+M'_B
+V_D=+V'_D
-H_D=-H'_D
0·6W
0·5W
0·4W
0·3W
0·2W
0·1W
0
0·045WL
0·04WL
0·035WL
0·03WL
0·025WL
0·02WL
0·015WL
0·01WL
0·005WL
0
0·2
0·25
0·3
0·35
0·4
0·45
0·5
0·55
0·6
VALUES OF h/L

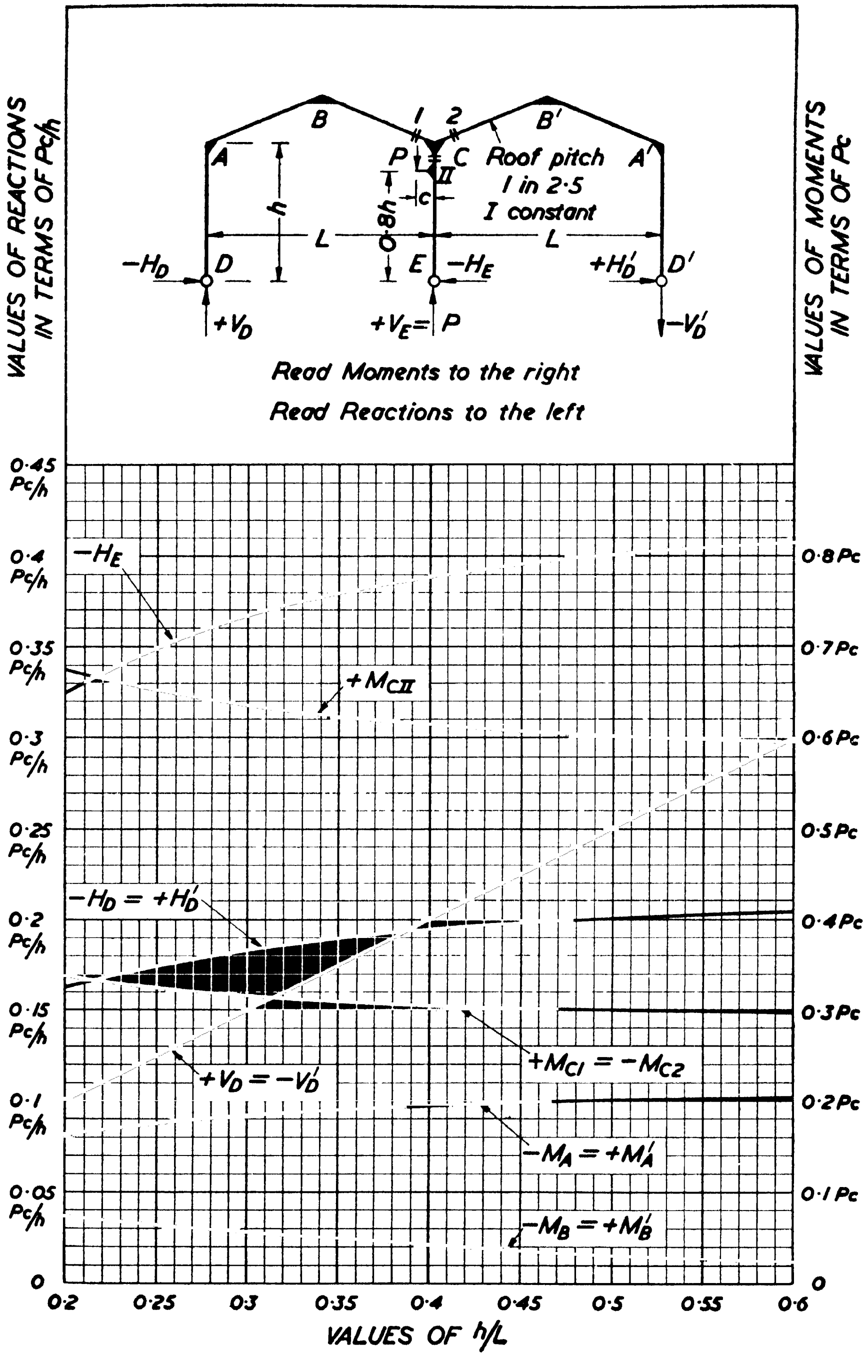

Total U.D.L = W
B
1
2
B'
A
II
C
Roof pitch
1 in 2·5
A'
I constant
h
L
L
$-H_D$
D
E
$-H_E$
D'
$-H'_D$
$+V_D$
$+V_E$
$+V'_D$
Read Moments to the right
Read Reactions to the left
VALUES OF REACTIONS IN TERMS OF W
VALUES OF MOMENTS IN TERMS OF WL
0·8W
0·7W
0·6W
0·5W
0·4W
0·3W
0·2W
0·1W
0
0·08WL
0·07WL
0·06WL
0·05WL
0·04WL
0·03WL
0·02WL
0·01WL
0
$+V_D$
$-M_A$
$-M_{CII}$
$-M_{C1}$
$+M_B$
$-H_D$
$+V_E$
$-H_E$
$-M'_A$
$-M'_B$
$+M_{C2}$
$-M_{C2}$
0·2
0·25
0·3
0·35
0·4
0·45
0·5
0·55
0·6
$+V'_D$
$-H'_D$
VALUES OF h/L

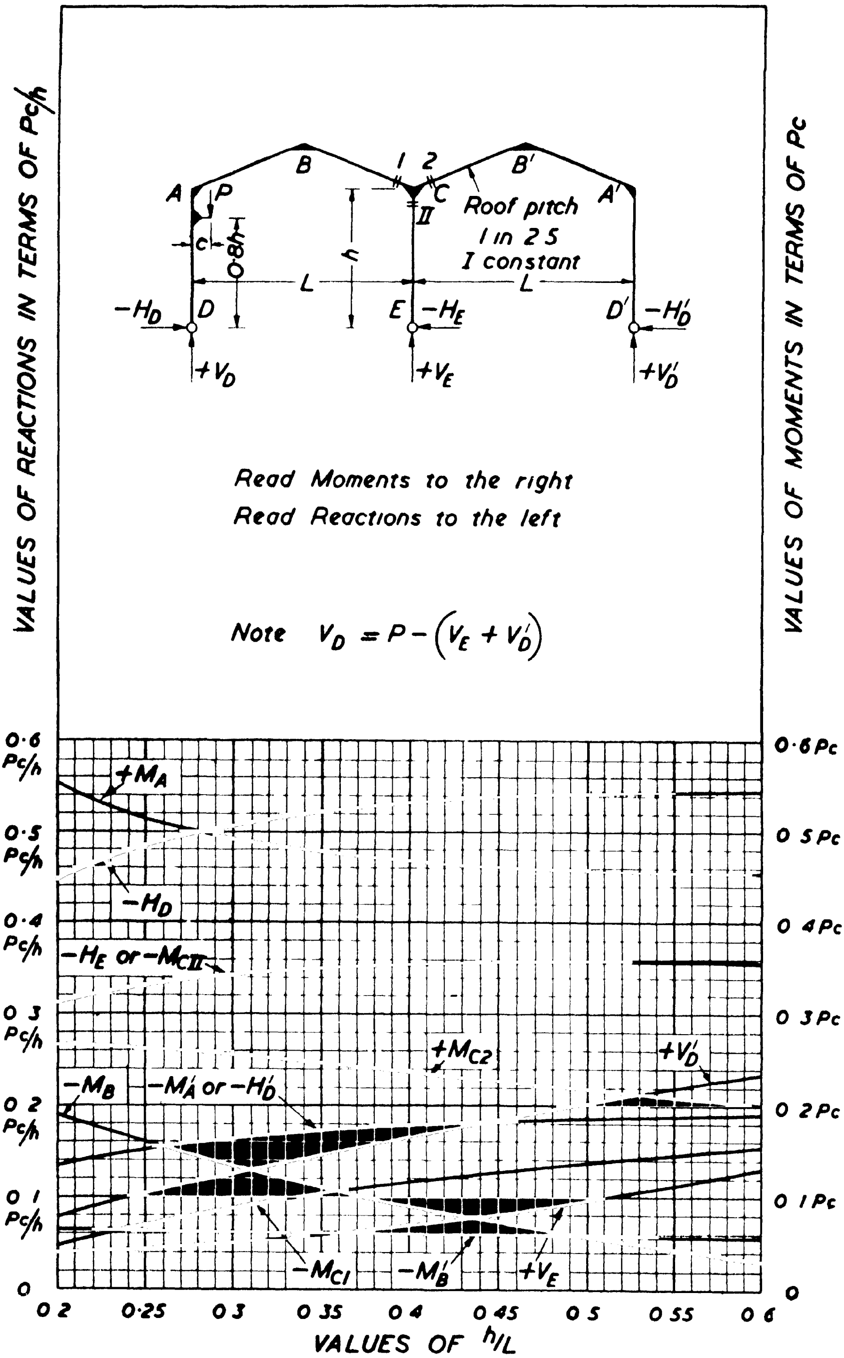

Total U.D.L. = W
B
B'
1
2
A
C
A'
II
I constant
Roof pitch 1 in 2.5
f
h
L
L
D
+H_D
E
-H_E
D'
-H'_D
-V_D
+V_E
+V'_D
Read Moments to the right
Read Reactions to the left
VALUES OF REACTIONS IN TERMS OF W
VALUES OF MOMENTS IN TERMS OF Wf
-H_E
+H_D
-M_C II
+V'_D
-V_D
-H'_D
+M_A
+M_C2
-M'_A
-M_C1
-M'_B
-M_B
+V_E
0·45W
0·4W
0·35W
0·3W
0·25W
0·2W
0·15W
0·1W
0·05W
0
1·8Wf
1·6Wf
1·4Wf
1·2Wf
1·0Wf
0·8Wf
0·6Wf
0·4Wf
0·2Wf
0
0·2
0·25
0·3
0·35
0·4
0·45
0·5
0·55
0·6
VALUES OF h/L

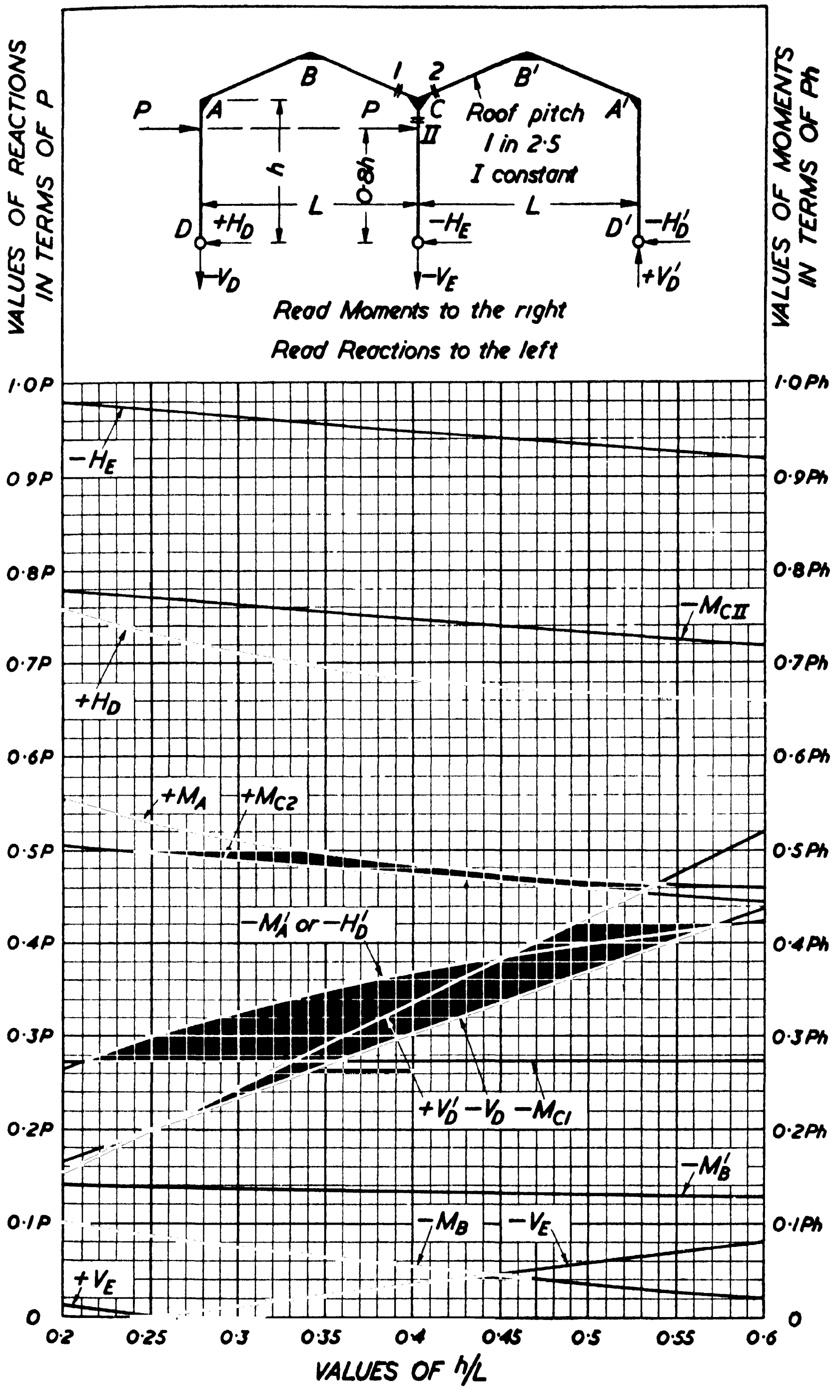

Total UDL = W
A
B
C
B'
A'
D
E
D'
I constant
Roof pitch
1 in 2·5
L
L
h
−H_D
+V_D
+V_E
−H_E
+H'_D
−V'_D
Read Moments to the right
Read Reactions to the left
VALUES OF REACTIONS IN TERMS OF W
VALUES OF MOMENTS IN TERMS OF WL
0·9W
0·8W
0·7W
0·6W
0·5W
0·4W
0·3W
0·2W
0·1W
0
0·09WL
0·08WL
0·07WL
0·06WL
0·05WL
0·04WL
0·03WL
0·02WL
0·01WL
0
+V_E
−M_C1
−M_CII
−M_A
+M_B
−M_C2
−H_E
+V_D
−H_D
+M'_A
−M'_B
−V'_D
+H'_D
0·2
0·25
0·3
0·35
0·4
0·45
0·5
0·55
0·6
VALUES OF h/L

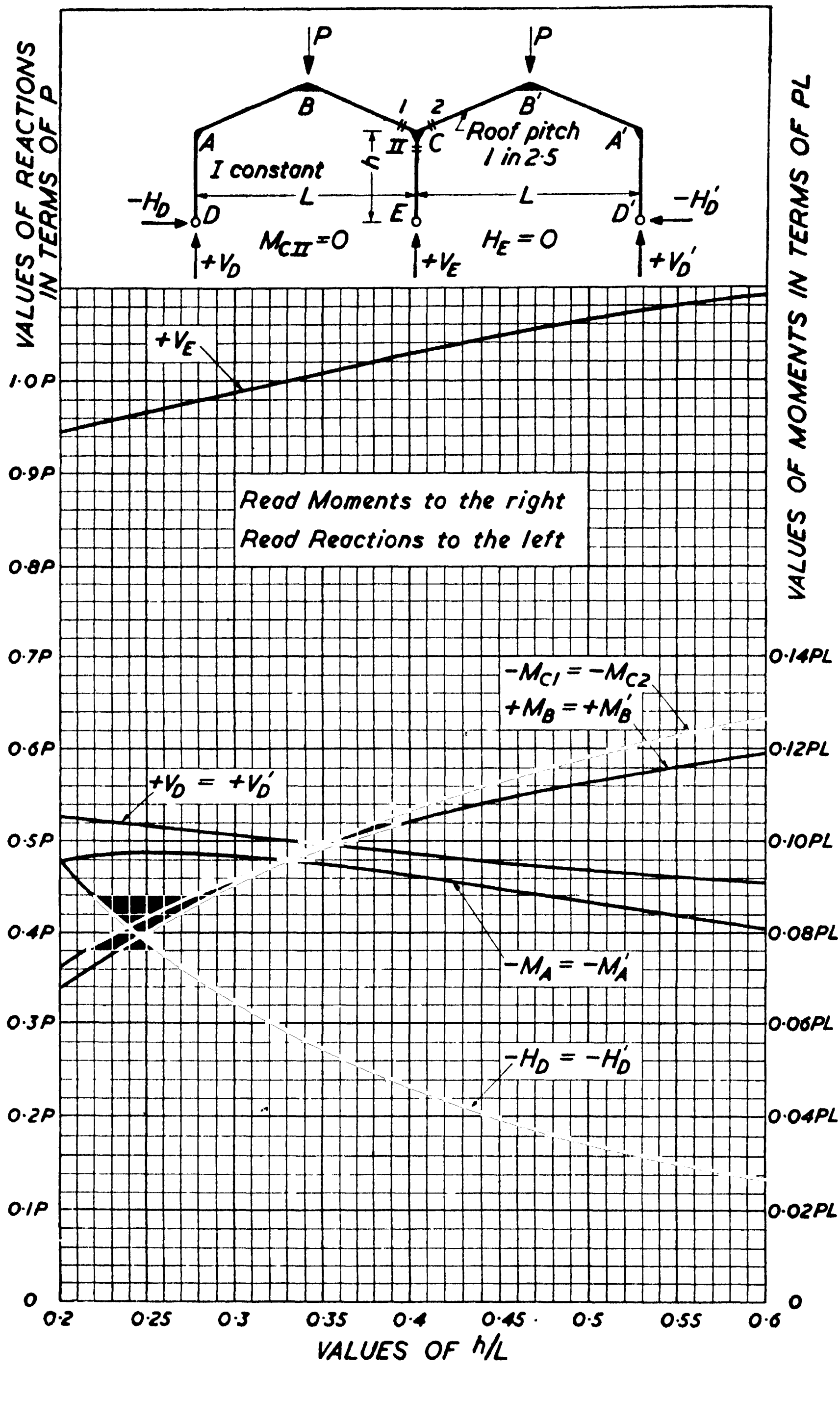

Total U.D.L = W
B
B'
A
A'
C
I constant
Roof pitch
1 in 2·5
f
h
L
L
−H_D
D
+H_E
E
+H'_D
D'
+V_D
−V_E
−V'_D
Read Moments to the right
Read Reactions to the left
VALUES OF REACTIONS IN TERMS OF W
VALUES OF MOMENTS IN TERMS OF Wf
0·55W
0·5W
0·45W
0·4W
0·35W
0·3W
0·25W
0·2W
0·15W
0·1W
0·05W
0
1·6Wf
1·4Wf
1·2Wf
1·0Wf
0·8Wf
0·6Wf
0·4Wf
0·2Wf
0
+H_E
−V'_D
+M_C II
−H_D
+H'_D
+V_D
−M_C2
−M_A
+M'_A
+M_C1
−V_E
+M'_B
−M_B
0·2
0·25
0·3
0·35
0·4
0·45
0·5
0·55
0·6
VALUES OF h/L

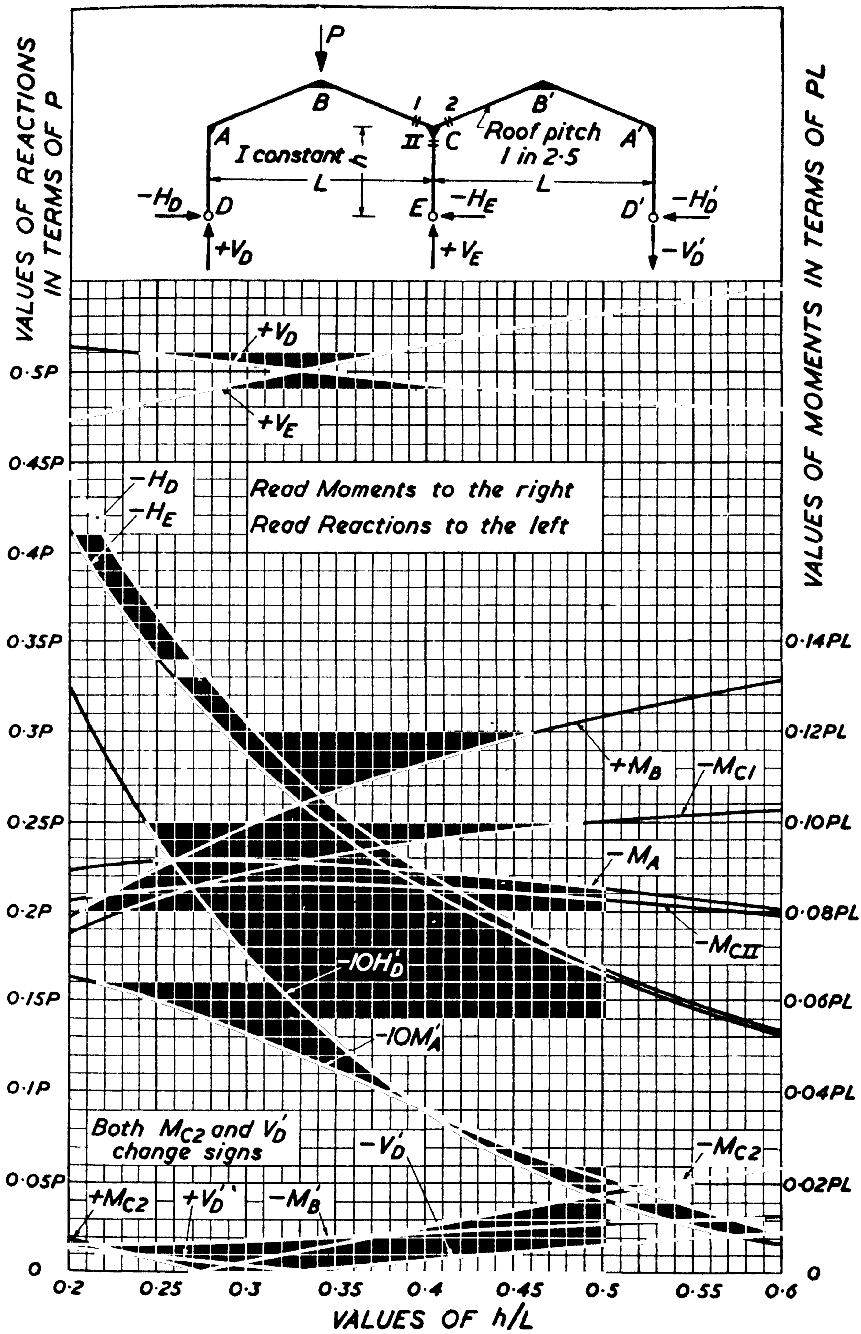

Total U.D.L = W
I constant
Roof pitch 1 in 2·5
A B C B' A'
I 2 II
h L L
+H_D −H_E −H'_D
E D'
−V_D −V_E +V'_D
Read Moments to the right
Read Reactions to the left
VALUES OF REACTIONS IN TERMS OF W
VALUES OF MOMENTS IN TERMS OF Wh
+M_A
−M_CII
+H_D
+M_C2
−M'_A
−M_B
−H_E
−M'_B
−M_C1
+V'_D
−H'_D
+V_E
−H'_D
−V_D
−V_E
0·8W
0·7W
0·6W
0·5W
0·4W
0·3W
0·2W
0·1W
0
0·25Wh
0·225Wh
0·20Wh
0·175Wh
0·15Wh
0·125Wh
0·10Wh
0·075Wh
0·05Wh
0·025Wh
0
0·2 0·25 0·3 0·35 0·4 0·45 0·5 0·55 0·6
VALUES OF h/L

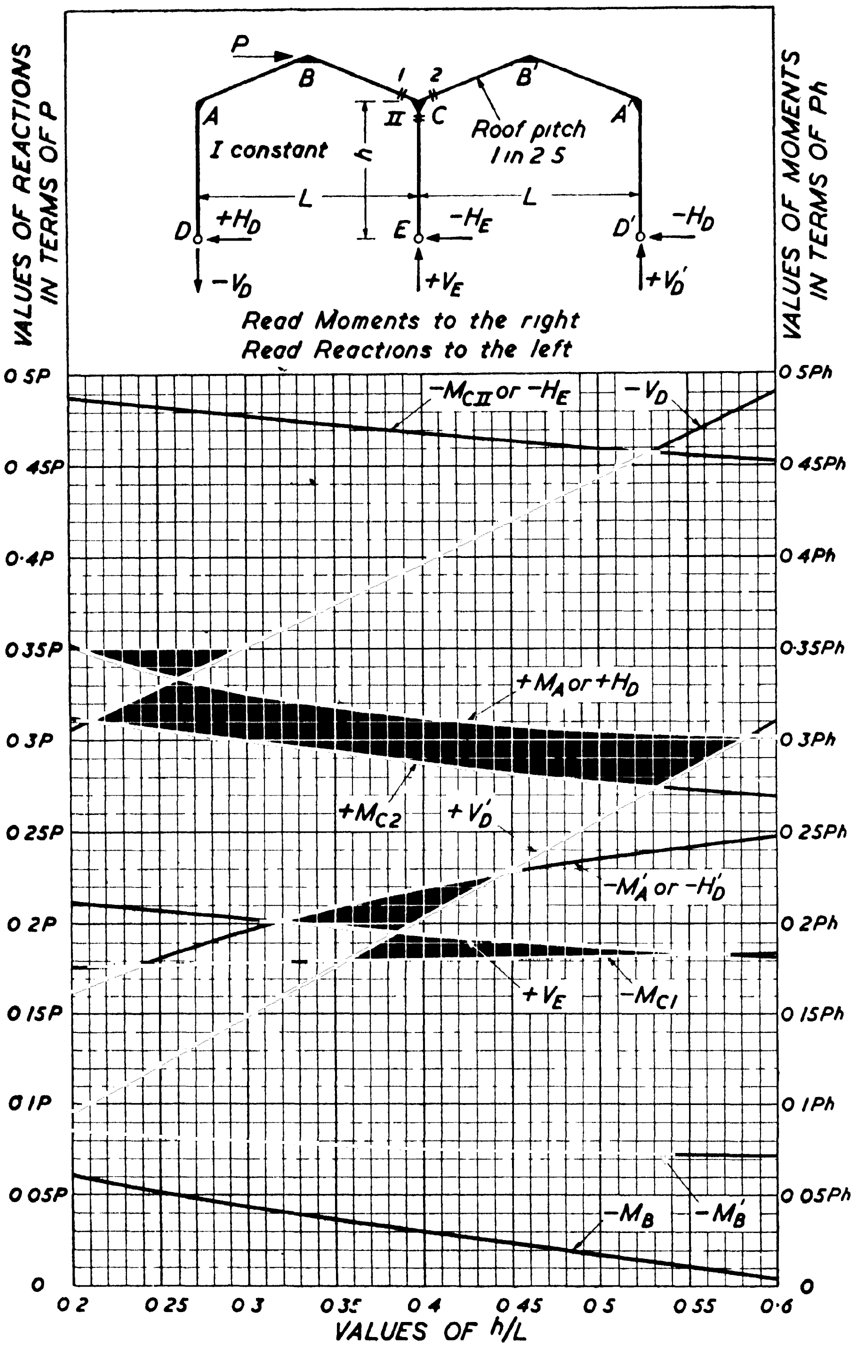

VALUES OF REACTIONS IN TERMS OF Pc/h
VALUES OF MOMENTS IN TERMS OF Pc
B
B'
1
2
A
P
C
Roof pitch
A'
II
1 in 2·5
h
c
I constant
0·8h
L
L
−H_D
D
E
−H_E
+H_D'
D'
+V_D
+V_E = P
−V_D'
Read Moments to the right
Read Reactions to the left
0·45 Pc/h
0·4 Pc/h
0·35 Pc/h
0·3 Pc/h
0·25 Pc/h
0·2 Pc/h
0·15 Pc/h
0·1 Pc/h
0·05 Pc/h
0
0·8 Pc
0·7 Pc
0·6 Pc
0·5 Pc
0·4 Pc
0·3 Pc
0·2 Pc
0·1 Pc
0
−H_E
+M_CII
−H_D = +H_D'
+M_C1 = −M_C2
+V_D = −V_D'
−M_A = +M_A'
−M_B = +M_B'
0·2
0·25
0·3
0·35
0·4
0·45
0·5
0·55
0·6
VALUES OF h/L


VALUES OF REACTIONS IN TERMS OF Pc/h
VALUES OF MOMENTS IN TERMS OF Pc
A
P
B
1 2
B'
C
Roof pitch
1 in 2.5
I constant
A'
II
c
0.8h
h
L
L
-H_D
D
E
-H_E
D'
-H'_D
+V_D
+V_E
+V'_D
Read Moments to the right
Read Reactions to the left
Note V_D = P - (V_E + V'_D)
0.6 Pc/h
0.5 Pc/h
0.4 Pc/h
0.3 Pc/h
0.2 Pc/h
0.1 Pc/h
0
0.6 Pc
0.5 Pc
0.4 Pc
0.3 Pc
0.2 Pc
0.1 Pc
0
+M_A
-H_D
-H_E or -M_CII
+M_C2
+V'_D
-M_B
-M'_A or -H'_D
-M_C1
-M'_B
+V_E
0.2
0.25
0.3
0.35
0.4
0.45
0.5
0.55
0.6
VALUES OF h/L

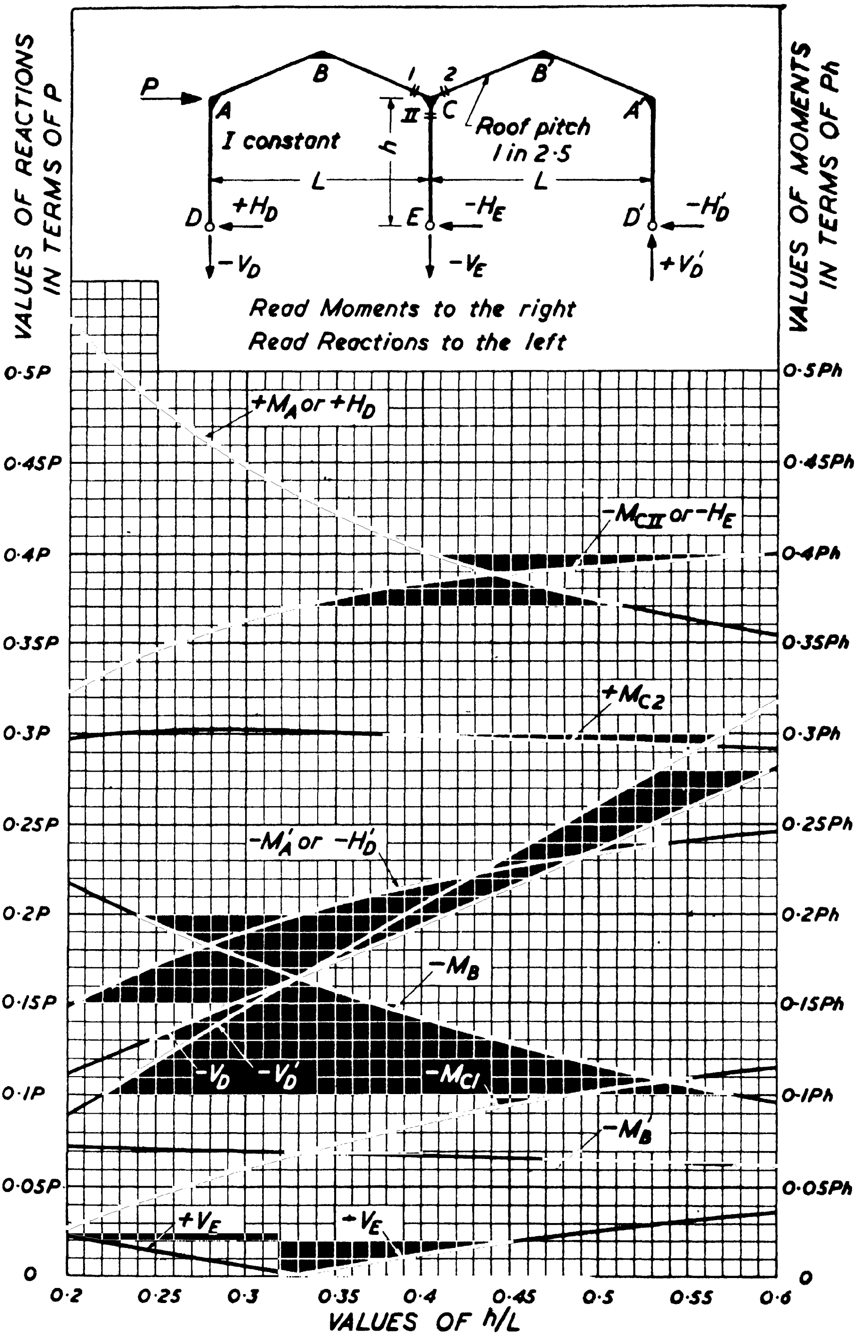

VALUES OF REACTIONS IN TERMS OF P
VALUES OF MOMENTS IN TERMS OF Ph
P
A
B
1
2
C
B′
A′
II
Roof pitch 1 in 2·5
I constant
h
0·8h
L
D
+H_D
−H_E
D′
−H′_D
−V_D
−V_E
+V′_D
Read Moments to the right
Read Reactions to the left
1·0P
0·9P
0·8P
0·7P
0·6P
0·5P
0·4P
0·3P
0·2P
0·1P
0
1·0Ph
0·9Ph
0·8Ph
0·7Ph
0·6Ph
0·5Ph
0·4Ph
0·3Ph
0·2Ph
0·1Ph
0
−H_E
+H_D
−M_CII
+M_A
+M_C2
−M′_A or −H′_D
+V′_D
−V_D
−M_C1
−M′_B
−M_B
−V_E
+V_E
0·2
0·25
0·3
0·35
0·4
0·45
0·5
0·55
0·6
VALUES OF h/L

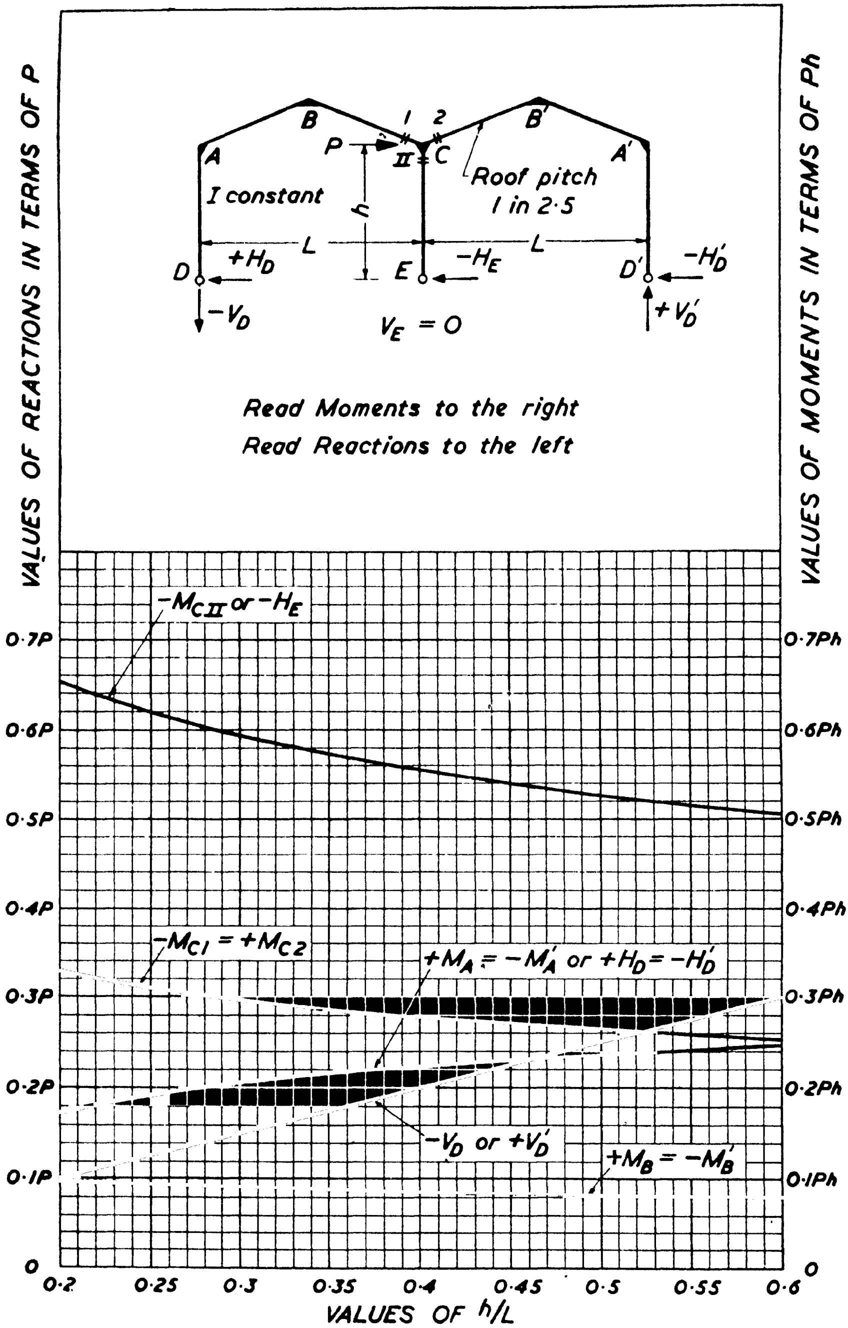

P
P
B
B'
1
2
A
II
C
Roof pitch
1 in 2·5
A'
h
I constant
-H_D
L
L
-H'_D
D
E
D'
M_CII = O
H_E = O
+V_D
+V_E
+V'_D
VALUES OF REACTIONS IN TERMS OF P
VALUES OF MOMENTS IN TERMS OF PL
+V_E
1·0P
0·9P
0·8P
0·7P
0·6P
0·5P
0·4P
0·3P
0·2P
0·1P
0
Read Moments to the right
Read Reactions to the left
-M_C1 = -M_C2
+M_B = +M'_B
+V_D = +V'_D
-M_A = -M'_A
-H_D = -H'_D
0·14PL
0·12PL
0·10PL
0·08PL
0·06PL
0·04PL
0·02PL
0
0·2
0·25
0·3
0·35
0·4
0·45
0·5
0·55
0·6
VALUES OF h/L


P
B
B'
1
2
A
II
C
Roof pitch
1 in 2·5
A'
I constant
h
L
L
$-H_D$
D
E
$-H_E$
D'
$-H_D'$
$+V_D$
$+V_E$
$-V_D'$
VALUES OF REACTIONS IN TERMS OF P
VALUES OF MOMENTS IN TERMS OF PL
$+V_D$
$+V_E$
0·5P
0·45P
0·4P
0·35P
0·3P
0·25P
0·2P
0·15P
0·1P
0·05P
0
$-H_D$
$-H_E$
Read Moments to the right
Read Reactions to the left
0·14PL
0·12PL
0·10PL
0·08PL
0·06PL
0·04PL
0·02PL
0
$+M_B$
$-M_{C1}$
$-M_A$
$-M_{CII}$
$-10H_D'$
$-10M_A'$
Both M_{C2} and V_D' change signs
$-V_D'$
$-M_{C2}$
$+M_{C2}$
$+V_D'$
$-M_B'$
0·2
0·25
0·3
0·35
0·4
0·45
0·5
0·55
0·6
VALUES OF h/L



VALUES OF REACTIONS IN TERMS OF P
VALUES OF MOMENTS IN TERMS OF Ph
P
B
B'
1
2
A
II
C
A'
I constant
h
Roof pitch 1 in 2 5
L
L
D
+H_D
E
-H_E
D'
-H_D
-V_D
+V_E
+V_D'
Read Moments to the right
Read Reactions to the left
-M_CII or -H_E
-V_D
+M_A or +H_D
+M_C2
+V_D'
-M_A' or -H_D'
+V_E
-M_C1
-M_B
-M_B'
0 5P
0 45P
0·4P
0 35P
0 3P
0 25P
0 2P
0 15P
0 1P
0 05P
0
0 5Ph
0 45Ph
0 4Ph
0·35Ph
0 3Ph
0 25Ph
0 2Ph
0 15Ph
0 1Ph
0 05Ph
0
0 2
0 25
0 3
0 35
0 4
0 45
0 5
0 55
0·6
VALUES OF h/L


P
A
B
1
2
II
C
B'
A'
Roof pitch
1 in 2·5
I constant
h
L
L
D
+H_D
E
-H_E
D'
-H'_D
-V_D
-V_E
+V'_D
Read Moments to the right
Read Reactions to the left
VALUES OF REACTIONS IN TERMS OF P
VALUES OF MOMENTS IN TERMS OF Ph
0·5P
0·45P
0·4P
0·35P
0·3P
0·25P
0·2P
0·15P
0·1P
0·05P
0
0·5Ph
0·45Ph
0·4Ph
0·35Ph
0·3Ph
0·25Ph
0·2Ph
0·15Ph
0·1Ph
0·05Ph
0
+M_A or +H_D
-M_CII or -H_E
+M_C2
-M'_A or -H'_D
-M_B
-V_D
-V'_D
-M_C1
-M'_B
+V_E
-V_E
0·2
0·25
0·3
0·35
0·4
0·45
0·5
0·55
0·6
VALUES OF h/L


B
1 2
B'
P
A
II
C
A'
Roof pitch
1 in 2·5
I constant
h
L
L
+H_D
$-H_E$
$-H_D'$
D
E
D'
$-V_D$
$+V_D'$
$V_E = 0$
Read Moments to the right
Read Reactions to the left
VALUES OF REACTIONS IN TERMS OF P
VALUES OF MOMENTS IN TERMS OF Ph
$-M_{CII}$ or $-H_E$
$-M_{C1} = +M_{C2}$
$+M_A = -M_A'$ or $+H_D = -H_D'$
$-V_D$ or $+V_D'$
$+M_B = -M_B'$
0·7P
0·6P
0·5P
0·4P
0·3P
0·2P
0·1P
0
0·7Ph
0·6Ph
0·5Ph
0·4Ph
0·3Ph
0·2Ph
0·1Ph
0
0·2
0·25
0·3
0·35
0·4
0·45
0·5
0·55
0·6
VALUES OF h/L

16. GEOMETRICAL PROPERTIES OF SECTIONS

BRIEF references to geometrical properties of sections were made in earlier chapters in order to provide some background for the calculations and derivation of the formulæ for bending moments, shearing forces and deflections in beams. In this chapter tables are provided giving the formulæ for common geometrical shapes, and descriptions are given of the methods of calculating not only these formulæ but the properties of more unusual sections.

First and Second Moments of Area

The axes *xx* and *yy* of the irregular area *A* shown in Fig. 245 pass through the centroid *O* and are at right angles to each other.

Consider an element of area *dA* whose co-ordinates are *x* and *y*. Then the moments of *dA* about the *xx* and *yy* axes are $y \cdot dA$ and $x \cdot dA$ respectively. Now, as the axes pass through the centroid, the sum of the moments about these axes of all the elements of area must be zero, i.e. $\int y \cdot dA = 0$ and $\int x \cdot dA = 0$. In other words, the total area moment of any figure taken about any line through its centroid is zero.

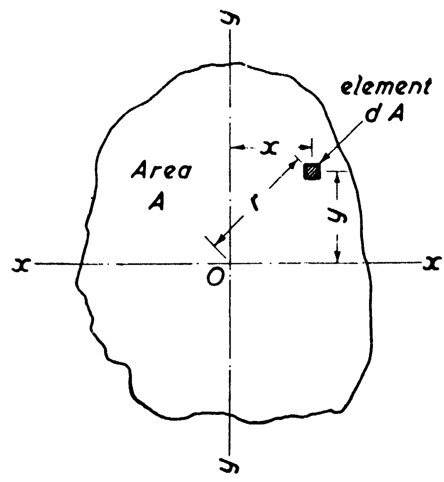


FIG. 245

The terms $y \cdot dA$ and $x \cdot dA$ are known as the first moments of area. When the element of area is multiplied by y^2 or x^2 the resulting values are called second moments of area or, more commonly, moments of inertia, I (although this term is correctly applied only to masses and not to areas). Hence, the moments of inertia of the area *A* about the *xx* and *yy* axes have the following values:

$$I_{xx} = \int y^2 \cdot dA$$

$$I_{yy} = \int x^2 \cdot dA.$$

The approximate moment of inertia of any area may be found by dividing the area into a number of small areas and summating the second moments of all the small areas about the axis being considered. Where, however, the area is regular in shape, the moments of inertia are more easily calculated by simple integration.

Example 1. Calculate the value I_{xx} for the square shown in Fig. 246.

Let L be the length of the diagonal AC and h the height OB. Consider an element of length l and thickness δy parallel to AC and distance y from it.

Then
$$\frac{l}{L}=\frac{h-y}{h}$$

and
$$l=\frac{L}{h}(h-y).$$

Hence, the moment of inertia of the element about AC, i.e. the axis xx,

$$=l\,.\,\delta y\times y^2$$
$$=\frac{L}{h}(h-y)y^2\,.\,\delta y$$
$$=\frac{L}{h}(hy^2-y^3)\delta y.$$

Therefore the moment of inertia of the whole triangle ABC about the xx axis

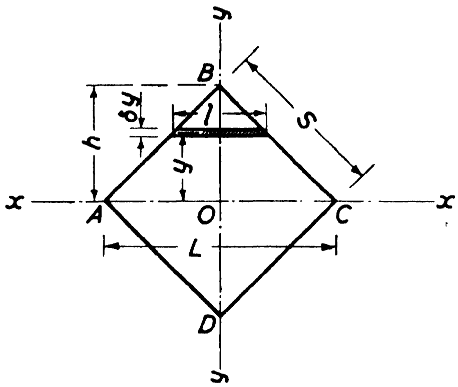


Fig. 246

$$=\frac{L}{h}\int_0^h (hy^2-y^3)\,dy.$$
$$=\frac{L}{h}\left[\frac{hy^3}{3}-\frac{y^4}{4}\right]_0^h$$
$$=\frac{L}{h}\left[\frac{h^4}{3}-\frac{h^4}{4}\right]$$
$$=\frac{Lh^3}{12}$$

Consequently, I_{xx} for the whole square $ABCD$
$$=\frac{Lh^3}{6}$$

Let the length of a side of the square be S.

Then
$$S^2=\left(\frac{L}{2}\right)^2+h^2.$$

But $L/2=h$. Therefore,
$$S^2=2h^2$$

and
$$\frac{Lh^3}{6}=\frac{2h\times h^3}{6}=\frac{h^4}{3}=\frac{S^4}{12}$$

Hence,
$$I_{xx}=\frac{S^4}{12}$$

Similarly
$$I_{yy}=\frac{S^4}{12}.$$

Product of Inertia

If the element of area dA in Fig. 245 is multiplied by the product xy the resulting value is called the product of inertia. Hence, the product of inertia for the whole area

$$=I_{xy}=\int xy\,.\,dA.$$

Principal Axes

Principal axes are two axes at right angles passing through the centroid and positioned so that the product of inertia is zero.

In the case of sections with an axis of symmetry, that axis will be a principal axis.

For regular figures which are symmetrical about one or more axes, such as squares, circles, 'I'- and channel sections, the positions of the principal axes are easy to locate. For irregular sections, such as angles with unequal legs, the positions have to be calculated.

To find the principal axes when there is no axis of symmetry, proceed by finding the moment of inertia about any two axes at right angles passing through the centroid, say I_{xx} and I_{yy}, and also the product moment I_{xy} about the centroid.

Then the angle of inclination θ of the principal axes to XX and YY is given by:

$$\text{Tan } 2\theta=-\frac{2I_{XY}}{I_{XX}-I_{YY}}.$$

Polar Axis

For any area there is one further important axis, i.e. the polar axis, perpendicular to the plane of the area and passing through the centroid.

In Fig. 245 the polar axis passes through O and is situated at a distance r from the element of area dA. Now the polar moment of inertia of the whole area, which is known either as I_{zz} or, in torsion problems, as K, $=\int r^2\,.\,dA$. But $r^2=x^2+y^2$.

Therefore, $$\int y^2\,.\,dA+\int x^2\,.\,dA=\int r^2\,.\,dA$$

or $$I_{xx}+I_{yy}=I_{zz}.$$

It will be realised that the three axes are all perpendicular to one another. Now, I_{zz} is a fixed quantity for any given area. Hence, if the axes xx and yy are replaced by two other axes, say, mm and nn, making angles θ respectively with them, then

$$I_{mm}+I_{nn}=I_{zz}$$

therefore $$I_{mm}+I_{nn}=I_{xx}+I_{yy}.$$

This fact may be expressed in the following theorem:

> The polar moment of inertia of any area is equal to the sum of the moments of inertia about any two axes at right angles to each other, lying in the plane of the area, and intersecting at the point where the polar axis passes through the area.

The Principle of Parallel Axes

Fig. 247 shows an area A whose centroid O is situated distance d from the XX axis shown and distance b from the YY axis.

Then the moment of inertia of the element of area dA about the XX axis $=(y+d)^2dA$, while the value for the whole area A

$$\begin{aligned} I_{XX} &= \int(y+d)^2dA \\ &= \int(y^2+2yd+d^2)dA \\ &= \int y^2dA + 2d\int ydA + d^2\int dA. \end{aligned}$$

As the xx axis passes through the centroid,

$$\int y \,.\, dA = 0.$$

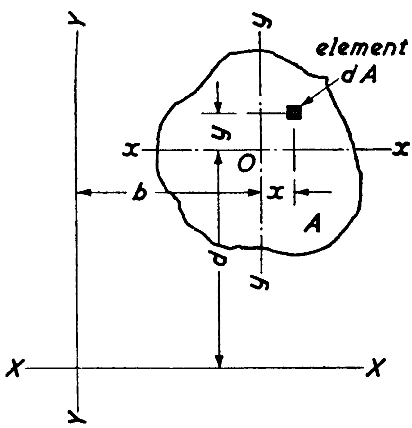


FIG. 247

Therefore,

$$\begin{aligned} I_{XX} &= \int y^2dA + d^2\int dA \\ &= I_{xx} + Ad^2. \end{aligned}$$

Similarly,

$$I_{YY} = I_{yy} + Ab^2.$$

These results may be expressed in the following theorem:

The moment of inertia of a figure about any axis is equal to the moment of inertia of the figure about a parallel axis through the centroid plus the product of the area of the figure and the square of the perpendicular distance between the axes.

It follows that the moment of inertia of a figure about an axis through its centroid is always smaller than the moment of inertia about any other axis parallel to the first.

Moments of Inertia of Compound Sections

The foregoing principles may be used in a number of ways to calculate the moments of inertia of compound sections, the methods employed being best described by working out some examples.

Example 2. Calculate the moments of inertia about the XX and YY axes of the welded compression member shown in Fig. 248 which is composed of four plates 1 in. in thickness.

The section may be considered as a large rectangle, B broad and D deep, from which a rectangle, b broad and d deep, is removed.

Then

$$\begin{aligned} I_{XX} &= \frac{BD^3}{12} - \frac{bd^3}{12} \\ &= \frac{10 \times 12^3}{12} - \frac{8 \times 10^3}{12} \\ &= 773{\cdot}33 \text{ in.}^4. \end{aligned}$$

and
$$I_{YY}=\frac{DB^3}{12}-\frac{db^3}{12}$$
$$=\frac{12\times 10^3}{12}-\frac{10\times 8^3}{12}$$
$$=573{\cdot}33 \text{ in.}^4.$$

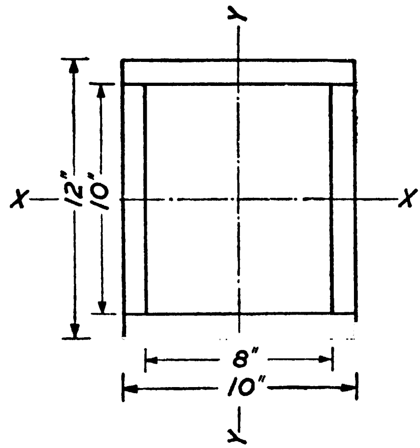


FIG. 248

Alternatively, the principle of parallel axes may be employed, the calculations being done in tabular form, thus:

Members	*Area* A	*Lever Arm* x	*Lever Arm* y	*Value* Ax^2	*Value* Ay^2	I_{xx}	I_{yy}
Two horizontal plates	2×10	0	$\pm 5{\cdot}5$	0	605	1·66	166·67
Two vertical plates	2×10	$\pm 4{\cdot}5$	0	405	0	166·67	1·66
			Totals	405	605	168·33	168·33

$$I_{XX}=\Sigma I_{xx}+\Sigma Ay^2$$
$$=168{\cdot}33+605$$
$$=773{\cdot}33 \text{ in.}^4$$
$$I_{YY}=\Sigma I_{yy}+\Sigma Ax^2$$
$$=168{\cdot}33+405$$
$$=573{\cdot}33 \text{ in.}^4.$$

It should be noted that the value of the moment of inertia of a plate about its longitudinal axis is small compared with the value Ay^2 or Ax^2. Consequently, such values are usually ignored.

Example 3. Calculate the value I_{XX} for the welded beam shown in Fig. 249.

As the beam is asymmetrical about the XX axis the first step is to find the position of the axis. However, by employing a table and choosing a suitable origin for moments the whole of the computations can flow from one stage

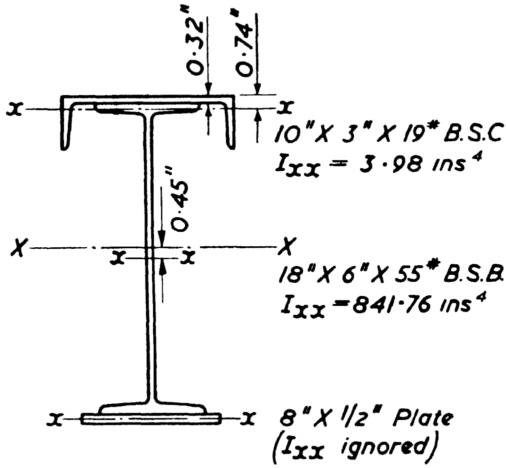


FIG. 249

to the next. If the centre of the B.S.B. is selected as origin, then the table is as follows:

Component	*Cross-Sectional Area* A	*Lever Arm* y	*First Moment* Ay	*Second Moment* Ay^2	*Value of* I_{xx} *for Component*
Channel	5·67	8·58	48·65	417	4
B.S.B.	16·18	—	—	—	842
Plate	4·00	-9·25	-37·00	342	—
$\Sigma A =$	25·85	$\Sigma Ay =$	11·65	759	846
				846	
			$\Sigma I =$	1605	

Distance from the origin to the XX axis

$$=\bar{y}=\frac{\Sigma Ay}{\Sigma A}=\frac{11\cdot65}{25\cdot85}$$

$$=0\cdot45 \text{ in.}$$

Therefore, the moment of inertia,

$$I_{XX}=\Sigma I - A\bar{y}^2$$

$$=1605-(25\cdot85\times0\cdot45^2)$$

$$=1600 \text{ in.}^4.$$

Section Modulus

The section modulus Z of a member about any axis is the value obtained when the moment of inertia I about that axis is divided by the distance y to the outermost fibre of the section.

For members symmetrical about both principal axes there is one value of Z for each of these axes as the distance y to the outermost fibre is the same on either side of a neutral axis. When, however, an axis is not symmetrically situated the distance y varies on either side of it and two values of Z result.

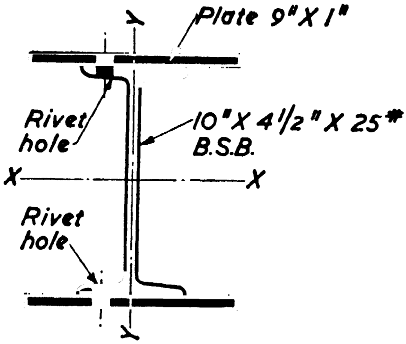


FIG. 250

Example 4. Calculate the value of Z about the XX axis for the compound girder shown in Fig. 250 which consists of a 10-in. × 4½-in. × 25-lb. R.S.J. with 9-in. × ½-in. plates riveted to the top and bottom flanges with ¾-in. diameter rivets.

If it is assumed that one $\frac{13}{16}$-in. rivet hole is deducted from each flange then a table can be made up as follows:

Component	A	y	Ay^2	I_{xx}
R.S.J.	—	—	—	122·34
Two 9-in. × ½-in. plates	9	5·25	248·06	—
Two rivet holes	−1·63	5·0	−40·75	—
		$\Sigma Ay^2 =$	207·31	122·34
		$\Sigma I_{xx} =$	122·34	
		$I_{XX} =$	329·65	

Now $$Z_{XX} = \frac{I_{XX}}{y} = \frac{329{\cdot}65}{5{\cdot}5} = 60 \text{ in.}^3.$$

This value for Z_{XX} is given in section tables.

The moment of inertia of the material removed from the rivet hole is assumed to be Ay^2 only, as I_{xx} is so small that it may be ignored. From section tables it will be found that the average thickness of the flange of a 10-in. × 4½-in. R.S.J. is 0·505 in. Hence the thickness of flange and plate = 1·005 in., and A for two $\frac{13}{16}$-in. diameter rivet holes = 2 × 0·8125 × 1·005 = 1·63 sq. in.

Radius of Gyration

The property known as the radius of gyration r is used in column problems and is equal to $\sqrt{(I/A)}$, where I is the moment of inertia about the axis being considered and A is the cross-sectional area.

Example 5. Fig. 251 shows a stanchion composed of two 12-in. × 4-in. × 31-lb. B.S. Channels connected together by bracing. Ignoring the bracing, calculate (a) the distance between the outer faces of the webs of the channels so that the radii of gyration about the XX and YY axes shall be equal, and (b) find the value of the radii of gyration.

For *each* channel,
$$A = 9{\cdot}21 \text{ sq. in.}$$
$$I_{xx} = 200{\cdot}09 \text{ in.}^4$$
$$I_{yy} = 12{\cdot}12 \text{ in.}^4.$$

The position of the I_{yy} axis is shown in Fig. 251.

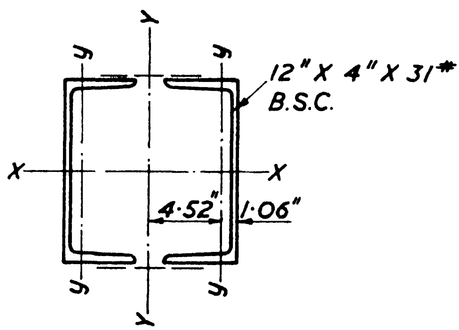


FIG. 251

The radii of gyration will be equal if $I_{XX} = I_{YY}$.

Now, I_{xx} for two channels

$$= I_{XX} = 2 \times 200{\cdot}09$$
$$= 400{\cdot}18 \text{ in.}^4.$$
$$I_{YY} = 2I_{yy} + 2Ax^2.$$
$$= (2 \times 12{\cdot}12) + (2 \times 9{\cdot}21x^2)$$
$$= 24{\cdot}24 + 18{\cdot}42x^2.$$

I_{YY} must equal $400{\cdot}18$ in.^{4}.

Therefore,
$$24{\cdot}24 + 18{\cdot}42x^2 = 400{\cdot}18$$

and
$$x^2 = \frac{400{\cdot}18 - 24{\cdot}24}{18{\cdot}42}$$
$$x = 4{\cdot}52 \text{ in.}$$

The distance from the YY axis to the outside face of the web of a channel $= 4{\cdot}52 + 1{\cdot}06$

$$= 5{\cdot}58 \text{ in.}$$

Hence, the distance over outside faces of the webs

$$= 11{\cdot}16 \text{ in.}$$

Now,
$$r_{XX} = r_{YY} = \sqrt{\frac{I}{A}}.$$

Therefore the radii of gyration

$$=\sqrt{\frac{400 \cdot 18}{18 \cdot 42}}$$

$$=4 \cdot 66 \text{ in.}$$

Example 8. Fig. 252 shows a section through a strut which consists of one $6\frac{1}{2}$-in. external diameter tube and two $4\frac{1}{2}$-in. external diameter tubes, each being $\frac{1}{4}$ in. thick, adequately braced together. Ignoring the bracing, calculate the radii of gyration about the principal axes.

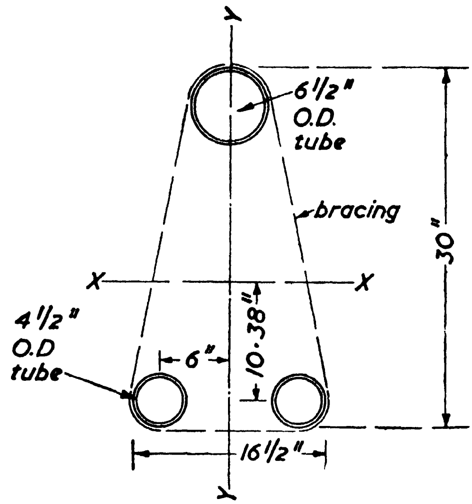


FIG. 252

For the $6\frac{1}{2}$-in. tube,

$$A = 4 \cdot 91 \text{ sq. in.}$$

$$I_{xx} = 24 \cdot 00 \text{ in.}^4.$$

For each $4\frac{1}{2}$-in. tube,

$$A = 3 \cdot 34 \text{ sq. in.}$$

$$I_{xx} = 7 \cdot 56 \text{ in.}^4.$$

The values of I_{XX} and I_{YY} can be found as follows, considering the xx axis of the smaller tubes as origin for the calculations for XX and the yy axis of the large tube as origin for the YY calculations.

Components	A	x	y	Ax	Ay	Ax^2	Ay^2	I_{xx}	I_{yy}
$6\frac{1}{2}$-in. diam. tube	4·91	—	24·5	—	120·29	—	2947	24	24
Two $4\frac{1}{2}$-in. diam. tubes	6·68	6	—	40·08	—	241	—	15	15
$\Sigma A=$	11·59			$\Sigma Ay=$	120·29	241	2947	39	39
					$\Sigma I_{yy}=$	39	39	$=\Sigma I_{xx}$	
					$I_{YY}=$	280	2986		

$$\bar{y} = \frac{\Sigma Ay}{\Sigma A} = \frac{120{\cdot}29}{11{\cdot}59}$$

$$= 10{\cdot}38 \text{ in.}$$

$$I_{XX} = \Sigma I - Ay^2$$

$$= 2986 - (11{\cdot}59 \times 10{\cdot}38^2)$$

$$= 1738 \text{ in.}^4$$

$$I_{YY} = \Sigma I$$

$$= 280 \text{ in.}^4.$$

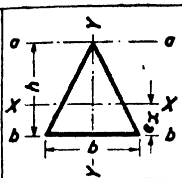
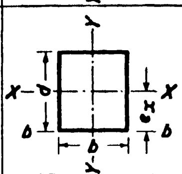
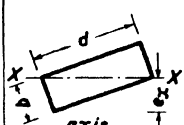
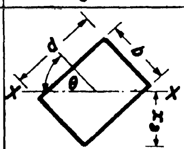
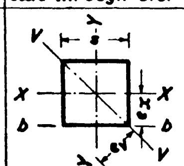
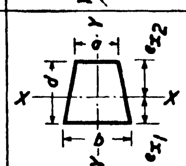
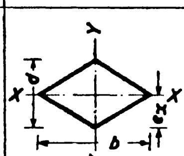
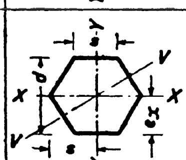
Now

$$r_{XX} = \sqrt{\frac{I_{XX}}{A}} = \sqrt{\frac{1738}{11{\cdot}59}}$$

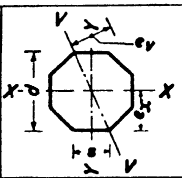
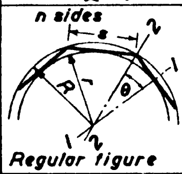
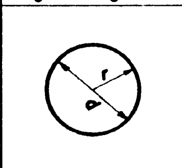
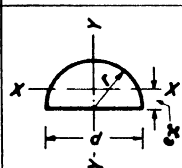
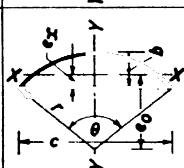
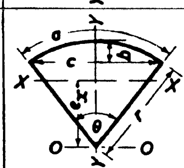
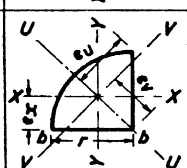
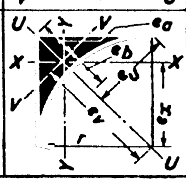
$$= 12{\cdot}25 \text{ in.}$$

$$r_{YY} = \sqrt{\frac{I_{YY}}{A}} = \sqrt{\frac{280}{11{\cdot}59}}$$

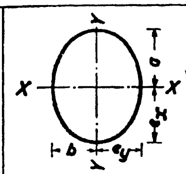
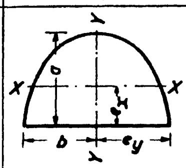
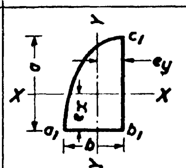
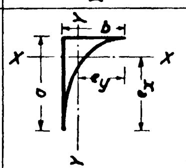
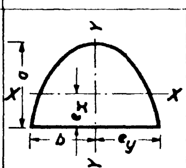
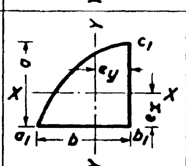
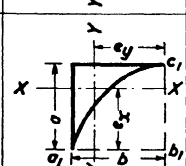
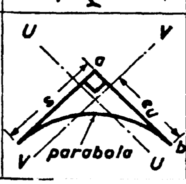
$$4{\cdot}91 \text{ in.}$$

GEOMETRICAL PROPERTIES OF PLANE SECTIONS					
Section		Area	Position of Centroid	Moments of Inertia	Section Moduli
TRIANGLE		$A = \frac{bh}{2}$	$e_x = \frac{h}{3}$	$I_{XX} = bh^3/36$ $I_{YY} = hb^3/48$ $I_{aa} = bh^3/4$ $I_{bb} = bh^3/12$	Z_{XX} base $= bh^2/12$ apex $= bh^2/24$ $Z_{YY} = bh^2/24$
RECTANGLE		$A = bd$	$e_x = \frac{h}{2}$	$I_{XX} = bd^3/12$ $I_{YY} = db^3/12$ $I_{bb} = bd^3/3$	$Z_{XX} = bd^2/6$ $Z_{YY} = db^2/6$
RECTANGLE		$A = bd$	$e_x = \frac{bd}{\sqrt{b^2+d^2}}$	$I_{XX} = \frac{b^3 d^3}{6(b^2+d^2)}$	$Z_{XX} = \frac{b^2 d^2}{6\sqrt{b^2+d^2}}$
RECTANGLE		$A = bd$	$e_x = \frac{b.\sin\theta + d.\cos\theta}{2}$	$I_{XX} = \frac{bd(b^2\sin^2\theta + d^2\cos^2\theta)}{12}$	$Z_{XX} = \frac{bd(b^2\sin^2\theta + d^2\cos^2\theta)}{6(b.\sin\theta + d.\cos\theta)}$
SQUARE		$A = s^2$	$e_x = \frac{s}{2}$ $e_v = \frac{s}{\sqrt{2}}$	$I_{XX} = I_{YY} = s^4/12$ $I_{bb} = s^4/3$ $I_{VV} = s^4/12$	$Z_{XX} = Z_{YY} = \frac{s^3}{6}$ $Z_{VV} = \frac{s^3}{6\sqrt{2}}$
TRAPEZIUM		$A = \frac{d(a+b)}{2}$	$e_{x_1} = \frac{d(2a+b)}{3(a+b)}$	$I_{XX} = \frac{d^3(a^2+4ab+b^2)}{36(a+b)}$ $I_{YY} = \frac{d(a^3+a^2b+ab^2+b^3)}{48}$	$Z_{XX} = \frac{I_{XX}}{d-e_x}$ (two values) $Z_{YY} = \frac{2I_{YY}}{b}$
DIAMOND		$A = \frac{bd}{2}$	$e_x = \frac{d}{2}$	$I_{XX} = \frac{bd^3}{48}$ $I_{YY} = \frac{db^3}{48}$	$Z_{XX} = \frac{bd^2}{24}$ $Z_{YY} = \frac{db^2}{24}$
HEXAGON		$A = 0{\cdot}866d^2$	$e_x = 0{\cdot}866s$ $= d/2$	$I_{XX} = I_{YY} = I_{VV}$ $= 0{\cdot}0601d^4$	$Z_{XX} = 0{\cdot}1203d^3$ $Z_{YY} = Z_{VV}$ $= 0{\cdot}1042d^3$

GEOMETRICAL PROPERTIES OF PLANE SECTIONS

Section		Area	Position of Centroid	Moments of Inertia	Section Moduli
OCTAGON		$A=0.8284d^2$ $s=0.4142d$	$e_x=\frac{d}{2}$ $e_V=0.541d$	$I_{XX}=I_{YY}=I_{VV}$ $=0.0547d^4$	$Z_{XX}=Z_{YY}$ $=0.1095d^3$ $Z_{VV}=0.1011d^3$
POLYGON		$A=\frac{ns^2\cot\theta}{4}$ $A=nr^2\tan\theta$ $A=\frac{nR^2\sin 2\theta}{2}$	$e=r$ or R depending on the axis and value of n	$I_1=I_2$ $=\frac{A(6R^2-s^2)}{24}$ $=\frac{A(12r^2+s^2)}{48}$	$Z=\frac{I}{e}$
CIRCLE		$A=\pi r^2$ $A=0.7854d^2$	$e=r=\frac{d}{2}$	$I=\frac{\pi d^4}{64}$ $I=0.7854r^4$	$Z=\frac{\pi d^3}{32}$ $Z=0.7854r^3$
SEMI-CIRCLE		$A=1.5708r^2$	$e_x=0.424r$	$I_{XX}=0.1098r^4$ $I_{YY}=0.3927r^4$	Z_{XX} base $=0.2587r^3$ crown $=0.1907r^3$ $Z_{YY}=0.3927r^3$
SEGMENT		$A=\frac{r^2}{2}\left(\frac{\pi\theta^\circ}{180^\circ}-\sin\theta\right)$	$e_o=\frac{c^3}{12A}$ $e_x=e_o-r.\cos\frac{\theta}{2}$	$I_{XX}=\frac{r^4}{16}\left(\frac{\pi\theta^\circ}{90^\circ}-\sin 2\theta\right)-\frac{20r^4(1-\cos\theta)^3}{\pi\theta^\circ-180^\circ\sin\theta}$ $I_{YY}=\frac{r^4}{48}\left(\frac{\pi\theta^\circ}{30^\circ}-8\sin.\theta+\sin.2\theta\right)$	Z_{XX} base $=I_{XX}/e_x$ crown $=\frac{I_{XX}}{b-e_x}$ $Z_{YY}=\frac{2I_{YY}}{c}$
SECTOR		$A=\frac{\theta^\circ}{360^\circ}\pi r^2$	$e_x=\frac{2}{3}r\frac{c}{a}$ $e_x=\frac{r^2c}{3A}$	$I_{XX}=I_o-\frac{360^\circ}{\theta^\circ\pi}\sin^2\frac{\theta}{2}\cdot\frac{4r^4}{9}$ $I_{YY}=\frac{r^4}{8}\left(\frac{\pi\theta^\circ}{180^\circ}-\sin.\theta\right)$ $I_o=\frac{r^4}{8}\left(\frac{\pi\theta^\circ}{180^\circ}+\sin.\theta\right)$	Z_{XX} centre $=I_{XX}/e_x$ crown $=\frac{I_{XX}}{r-e_x}$ $Z_{YY}=\frac{2I_{YY}}{c}$
QUADRANT		$A=\frac{\pi r^2}{4}$	$e_x=0.424r$ $e_V=0.6r$ $e_U=0.707r$	$I_{XX}=I_{YY}=0.0549r^4$ $I_{bb}=0.1963r^4$ $I_{UU}=0.0714r^4$ $I_{VV}=0.0384r^4$	Minimum Values $Z_{XX}=Z_{YY}$ $=0.0953r^3$ $Z_{UU}=0.1009r^3$ $Z_{VV}=0.064r^3$
COMPLEMENT		$A=0.2146r^2$	$e_x=0.777r$ $e_V=1.098r$ $e_U=0.707r$ $e_a=0.316r$ $e_b=0.391r$	$I_{XX}=I_{YY}=0.0076r^4$ $I_{UU}=0.012r^4$ $I_{VV}=0.0031r^4$	Minimum Values $Z_{XX}=Z_{YY}$ $=0.0097r^3$ $Z_{UU}=0.017r^3$ $Z_{VV}=0.0079r$

GEOMETRICAL PROPERTIES OF PLANE SECTIONS

	Section	Area	Position of Centroid	Moments of Inertia	Section Moduli
ELLIPSE		$A = \pi ab$	$e_x = a$ $e_y = b$	$I_{XX} = 0.7854ba^3$ $I_{YY} = 0.7854ab^3$	$Z_{XX} = 0.7854ba^2$ $Z_{YY} = 0.7854ab^2$
SEMI-ELLIPSE		$A = \frac{\pi ab}{2}$	$e_x = 0.424a$ $e_y = b$	$I_{XX} = 0.1098ba^3$ $I_{YY} = 0.3927ab^3$ $I_{base} = 0.3927ba^3$	Z_{XX} - base $= 0.2587ba^2$ Z_{XX} - crown $= 0.1907ba^2$ $Z_{YY} = 0.3927ab^2$
1/4 ELLIPSE		$A = 0.7854ab$	$e_x = 0.424a$ $e_y = 0.424b$	$I_{XX} = 0.0549ba^3$ $I_{YY} = 0.0549ab^3$ $I_{b_1a_1} = 0.1963ba^3$ $I_{b_1c_1} = 0.1963ab^3$	Z_{XX} - base $= 0.1293ba^2$ Z_{XX} - crown $= 0.0953ba^2$ Z_{YY} - base $= 0.1293ab^2$ Z_{YY} - crown $= 0.0953ab^2$
COMPLEMENT		$A = 0.2146ab$	$e_x = 0.777a$ $e_y = 0.777b$	$I_{XX} = 0.0076ba^3$ $I_{YY} = 0.0076ab^3$	Z_{XX} - base $= 0.0338ba^2$ Z_{XX} - apex $= 0.0097ba^2$ Z_{YY} - base $= 0.0338ab^2$ Z_{YY} - apex $= 0.0097ab^2$
FULL PARABOLA		$A = \frac{4ab}{3}$	$e_x = \frac{2a}{5}$ $e_y = b$	$I_{XX} = 0.0914ba^3$ $I_{YY} = 0.2666ab^3$ $I_{base} = 0.3048ba^3$	Z_{XX} - base $= 0.2286ba^2$ Z_{XX} - crown $= 0.1524ba^2$ $Z_{YY} = 0.2666ab^2$
SEMI-PARABOLA		$A = \frac{2ab}{3}$	$e_x = \frac{2a}{5}$ $e_y = \frac{3b}{8}$	$I_{XX} = 0.0457ba^3$ $I_{YY} = 0.0396ab^3$ $I_{b_1a_1} = 0.1524ba^3$ $I_{b_1c_1} = 0.1333ab^3$	Z_{XX} - base $= 0.1143ba^2$ Z_{XX} - crown $= 0.076ba^2$ Z_{YY} - base $= 0.1055ab^2$ Z_{YY} - crown $= 0.0633ab^2$
COMPLEMENT		$A = \frac{ab}{3}$	$e_x = \frac{7a}{10}$ $e_y = \frac{3b}{4}$	$I_{XX} = 0.0176ba^3$ $I_{YY} = 0.0125ab^3$ $I_{a_1b_1} = 0.181ba^3$ $I_{b_1c_1} = 0.2ab^3$	Z_{XX} - base $= 0.0587ba^2$ Z_{XX} - apex $= 0.0252ba^2$ Z_{YY} - base $= 0.05ab^2$ Z_{YY} - apex $= 0.0167ab^2$
FILLET		$A = \frac{s^2}{6}$	$e_u = e_v = \frac{4s}{5}$	$I_{UU} = I_{VV} = 0.00524s^4$ $I_{ab} = 0.1119a^4$	$Z_{UU} = Z_{VV}$ base $= 0.0262a^3$ apex $= 0.0066a^3$

17. BENDING AND AXIAL STRESSES

Pure Bending

In this section it is intended to elaborate upon bending stresses to which passing reference was made on page 1.

Consider Fig. 253 which shows the elevation and section of a homogeneous

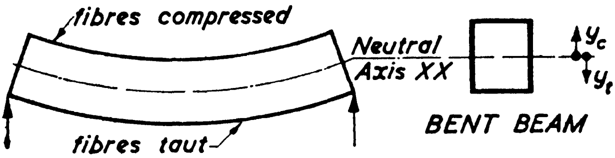


FIG. 253

beam which is subjected to pure bending. Then the stress f at any fibre $= M.y/I$ where y is the distance from the neutral axis to the fibre.

The maximum stress, which occurs either at the top or bottom of this particular beam can be found from the formula:

$$f_{\max} = \frac{M}{Z} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where Z is the section modulus.

If, however, a member is considered which is not symmetrical about its xx axis, say, a 'T'-bar, then two values of Z will exist and the maximum compressive stress will be different from the maximum tensile stress.

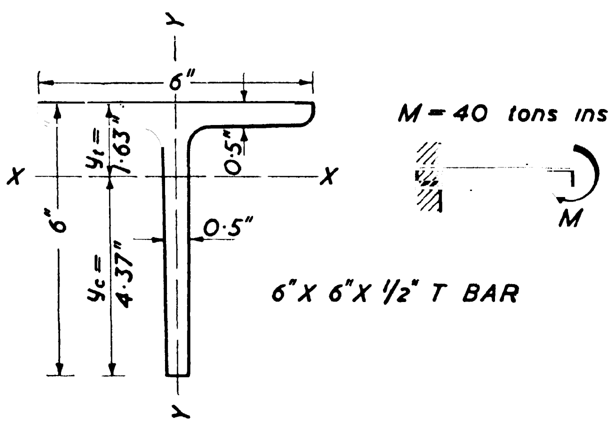


FIG. 254

Example 1. The 'T'-bar shown in Fig. 254 is subjected to a B.M. of 40 tons in. Calculate the maximum compressive and tensile stresses.

Z_t, the section modulus in terms of tension,

$$=\frac{I_{xx}}{y_t}=\frac{19{\cdot}04}{1{\cdot}63}=11{\cdot}68 \text{ in.}^3.$$

Z_c, the section modulus in terms of compression.

$$=\frac{I_{xx}}{y_c}=\frac{19{\cdot}04}{4{\cdot}37}=4{\cdot}36 \text{ in.}^3.$$

Now $f=M/Z$.

Therefore, the maximum tensile stress,

$$f_t=\frac{M}{Z_t}=\frac{40}{11{\cdot}68}=3{\cdot}43 \text{ tons per sq. in.}$$

while the maximum compressive stress,

$$f_c=\frac{M}{Z_c}=\frac{40}{4{\cdot}37}=9{\cdot}15 \text{ tons per sq. in.}$$

Unsymmetrical Bending

Fig. 255 shows the section of a rolled-steel beam which is subjected to an oblique force P acting through the C.G. of the beam.

The resulting B.M. M may be resolved into two components along the *principal axes* xx and yy, so that

$$\left.\begin{aligned} M_{xx}&=M\,.\,\sin\theta \\ M_{yy}&=M\,.\,\cos\theta \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (2)$$

Consequently, the stress at any fibre in the beam may be found from the expression

$$f=M\left(\pm\frac{y}{I_{xx}}\sin\theta\pm\frac{x}{I_{yy}}\cos\theta\right). \quad . \quad (3)$$

where the positive values represent compressive stresses and the negative, tensile. In another form, the expression becomes

$$f=\pm\frac{M_{xx}}{Z_{xx}}\pm\frac{M_{yy}}{Z_{yy}}. \quad . \quad . \quad . \quad . \quad (4)$$

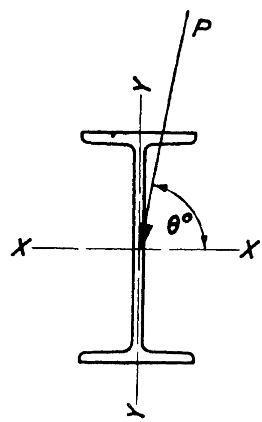


FIG. 255

The same expressions may be employed for other symmetrical sections provided that the force is assumed to act at the C.G. of the section.

It should be noted that for most British Standard Beams the resultant stress f increases rapidly as the angle θ (Fig. 255) departs from 90°. However, this effect is not so marked with Broad Flange Beams. The charts on pages 441 and 442 have been prepared to enable designers to choose suitable sections for beams which are subjected to unsymmetrical bending.

Z-polygons

In the expression

$$f=M\left(\frac{y}{I_{xx}}\sin\theta+\frac{x}{I_{yy}}\cos\theta\right),$$

the section modulus,

$$Z=\frac{I_{xx}}{y\,.\,\sin\theta}+\frac{I_{yy}}{x\,.\,\cos\theta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Hence, for any given direction of loading and for any given point in a section Z is a constant. Thus the value Z is a measure of the strength of a section for bending in any direction.

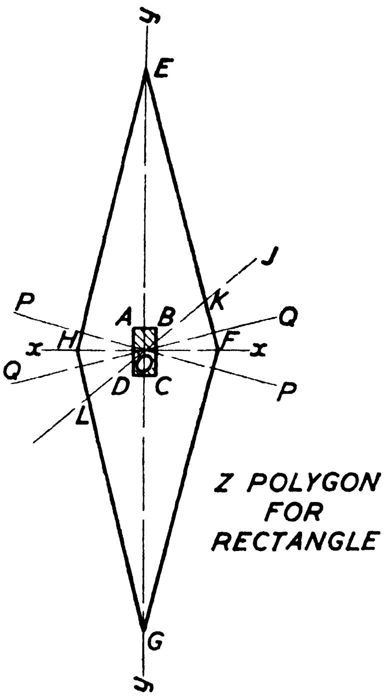


FIG. 256

Equation (5) is a 'straight line' equation, so that the variation in Z for the critical points in any section may be easily represented on a figure known as a Z-polygon.

Consider Fig. 256, which shows a rectangle $ABCD$ with its xx and yy axes produced in both directions.

With O, the centroid of the section, as origin, set off to some scale OE and OG equal to the maximum value of Z_{xx}, i.e. with y relative to AB or CD. Similarly, set off OF and OH equal to the maximum value of Z_{yy}, i.e. with x relative to BC or DA. Then EF is the Z-line for the point B FG the Z-line for C, and so on, while the full figure $EFGH$ is known as the Z-polygon for the rectangle.

Now, if a moment M is applied to the rectangle in the plane JO, then KO, which is intercepted by the Z-polygon, is the value of Z appropriate to the particular plane of the moment. If this value $=Z_{KO}$, then the maximum stress in the section may be found from the expression

$$f=\frac{M}{Z_{KO}}.$$

It will be appreciated that the value of the intercept LO could equally well be used, as LO is equal to KO.

The Z-polygon provides several sources of information. For example, it is obvious that the plane of maximum strength is along the axis yy. There are two planes of minimum strength, i.e. PP and QQ, along which the distance from the origin to the polygon is least.

Fig. 257 shows Z-polygons for a 12-in. × 5-in. × 32-lb. B.S.B., and a 12-in. × 12-in. × 75-lb. Broad Flange Beam.

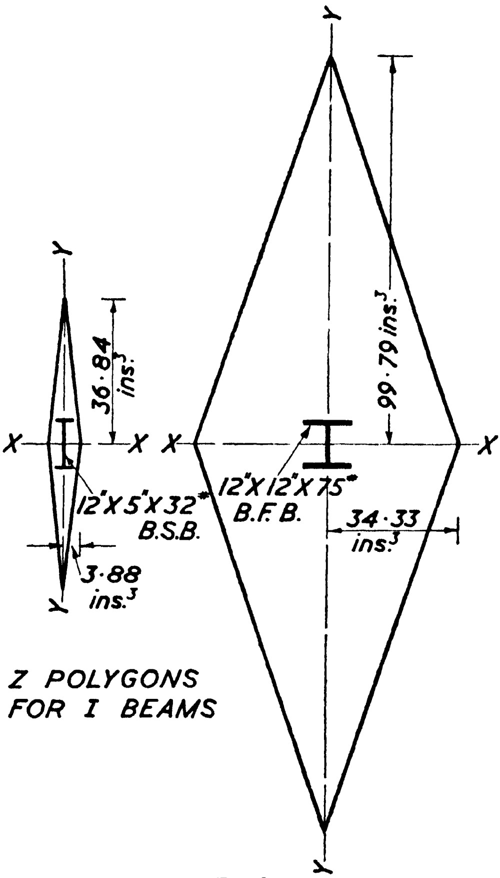


FIG. 257

Pure Tension or Compression

When a tensile force P is applied axially to a member the resultant stress in any fibre is uniform and is given by the expression

$$f = -\frac{P}{A} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6A)$$

where A = the cross-sectional area of the member.

Similarly, when a compressive force is applied the resultant stress in any fibre,

$$f = +\frac{P}{A} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6B)$$

Bending and Axial Stresses

When a member is subjected to bending and axial forces, as shown in Fig. 258, the resulting maximum fibre stresses

$$f_{\max} = \pm\frac{P}{A} \pm \frac{M_{xx}}{Z_{xx}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

A similar effect may be obtained in an eccentrically loaded column. Consider Fig. 259, which shows the cross-section of a column which is symmetrical about both the *xx* and *yy* axes. If a compressive force *P* is applied

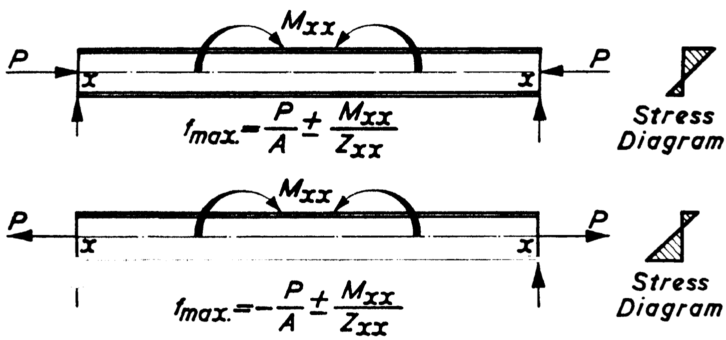


FIG. 258

on the *yy* axis at a distance $\pm e_x$ from the *xx* axis then the resultant fibre stress at any distance *y* from the *xx* axis

$$f = +\frac{P}{A} \pm \frac{P.e_x \times y}{I_{xx}} \quad \ldots\ldots\ldots \quad (8)$$

$$= +\frac{P}{A} \pm \frac{M_{xx} . y}{I_{xx}}$$

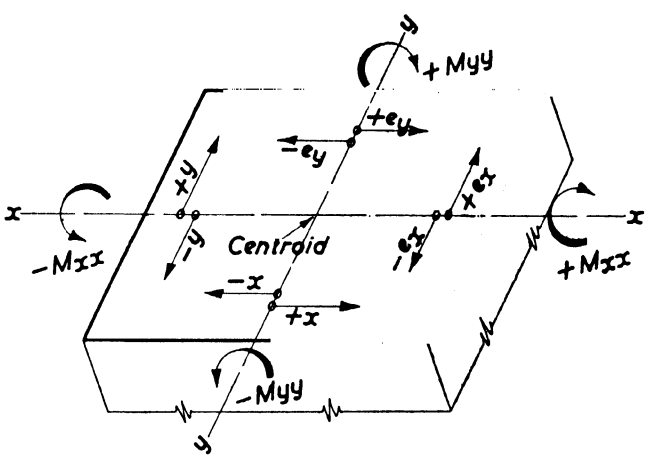


FIG. 259

If the force *P* is applied on the *xx* axis at a distance $\pm e_y$ from the *yy* axis then the resultant stress,

$$f = \frac{P}{A} \pm \frac{M_{yy} . x}{I_{yy}}$$

When the force may be compressive or tensile and eccentric about both axes, then the fibre stress at any point

$$f = \pm\frac{P}{A} \pm \frac{M_{xx} \cdot y}{I_{xx}} \pm \frac{M_{yy} \cdot x}{I_{yy}} \quad \ldots \ldots \quad (9)$$

When y and x are maxima, equation (9) becomes

$$f_{\max} = \pm\frac{P}{A} \pm \frac{M_{xx}}{Z_{xx}} \pm \frac{M_{yy}}{Z_{yy}} \quad \ldots \ldots \quad (10)$$

Having given the formulæ for a symmetrical section, it is convenient to consider the effect of any type of force acting eccentrically on any cross-section.

General Expression for Stress

Consider the cross-section of a compression member shown in Fig. 260, xx and yy being any rectangular axes through the centroid.

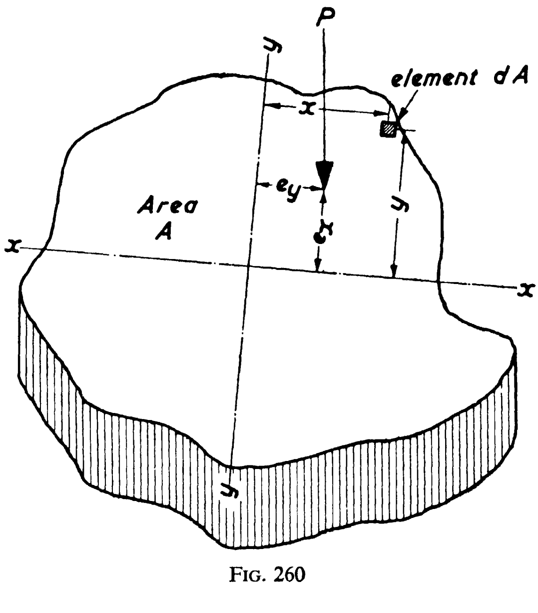


FIG. 260

Suppose that a force P is applied longitudinally to the member, its eccentricity about the xx and yy axes being e_x and e_y respectively. Then the effect upon the member is the same as if P is applied at the centroid and moments M_{xx} and M_{yy}, equal to $P \cdot e_x$ and $P \cdot e_y$ respectively, are applied about the xx and yy axes respectively.

If it is assumed that the member is not stressed beyond the elastic limit, then the unit stress f at any point $= a + bx + cy$, where x and y are the co-ordinates of the point.

Consider an element of area dA, having a total stress of $f \cdot dA$.

Then $\quad P = \int f \cdot dA = a\int dA + b\int x \cdot dA + c\int y \cdot dA$

Since xx and yy pass through the centroid

$$\int x \, . \, dA = 0 \quad \text{and} \quad \int y \, . \, dA = 0.$$

Hence

$$P = a\int dA = aA$$

or

$$a = \frac{P}{A} \quad \ldots \ldots \ldots \ldots \quad \text{(a)}$$

Similarly,

$$M_{xx} = \int f \, . \, dA \, . \, y = a\int y \, . \, dA + b\int xy \, . \, dA + c\int y^2 \, . \, dA$$

$$= bI_{xy} + cI_{xx} \quad \ldots \ldots \ldots \ldots \quad \text{(b)}$$

$$M_{yy} = \int f \, . \, dA \, . \, x = a\int x \, . \, dA + b\int x^2 \, . \, dA + c\int xy \, . \, dA$$

$$= cI_{xy} + bI_{yy} \quad \ldots \ldots \ldots \ldots \quad \text{(c)}$$

Now,

$$M_{yy}I_{xx} = bI_{xx}I_{yy} + cI_{xx}I_{xy} \quad \ldots \ldots \quad (c \times I_{xx})$$

$$M_{xx}I_{xy} = bI_{xy}^2 + cI_{xx}I_{xy} \quad \ldots \ldots \quad (b \times I_{xy})$$

Subtracting,

$$M_{yy}I_{xx} - M_{xx}I_{xy} = b(I_{xx}I_{yy} - I_{xy}^2)$$

whence

$$b = \frac{M_{yy}I_{xx} - M_{xx}I_{xy}}{I_{xx}I_{yy} - I_{xy}^2} \quad \ldots \ldots \ldots \quad \text{(d)}$$

Similarly,

$$c = \frac{M_{xx}I_{yy} - M_{yy}I_{xy}}{I_{xx}I_{yy} - I_{xy}^2} \quad \ldots \ldots \ldots \quad \text{(e)}$$

Employing equations (a), (d) and (e),

$$f = a + bx + cy$$

$$= \frac{P}{A} + \frac{M_{yy}I_{xx} - M_{xx}I_{xy}}{I_{xx}I_{yy} - I_{xy}^2} \, . \, x + \frac{M_{xx}I_{yy} - M_{yy}I_{xy}}{I_{xx}I_{yy} - I_{xy}^2} \, . \, y \quad . \quad \text{(11A)}$$

If the numerator and denominator of the coefficient of x are divided by I_{xx} and those for y are divided by I_{yy}, the expression becomes

$$f = \frac{P}{A} + \frac{M_{yy} - M_{xx}(I_{xy}/I_{xx})}{I_{yy} - (I_{xy}^2/I_{xx})} \, . \, x + \frac{M_{xx} - M_{yy}(I_{xy}/I_{yy})}{I_{xx} - (I_{xy}^2/I_{yy})} \, . \, y \quad . \quad \text{(11B)}$$

Either formula may be used but the latter avoids the differences of large quantities which in slide-rule calculations may lead to appreciable errors.

Bending about Principal Axes

It will be noted that equations (11A) and (11B) are related to any pair of rectangular axes xx and yy which are chosen for convenience of calculation. Some designers prefer to work from principal axes, especially if the values of I are known for these axes, but normally there is very little to choose between the amount of calculation for each method.

Suppose that uu and vv are the appropriate principal axes, then equation (9) becomes

$$f = \pm\frac{P}{A} \pm \frac{M_{uu} \, . \, v}{I_{uu}} \pm \frac{M_{vv} \, . \, u}{I_{vv}} \quad \ldots \ldots \quad (12)$$

where v and u are measured normal to the uu and vv axes respectively.

Example 2. The 9-in. × 4-in. × $\frac{7}{8}$-in. angle shown in Fig. 261 is subjected to a B.M. of 100 tons in. in the plane of the yy axis and it is free to deflect

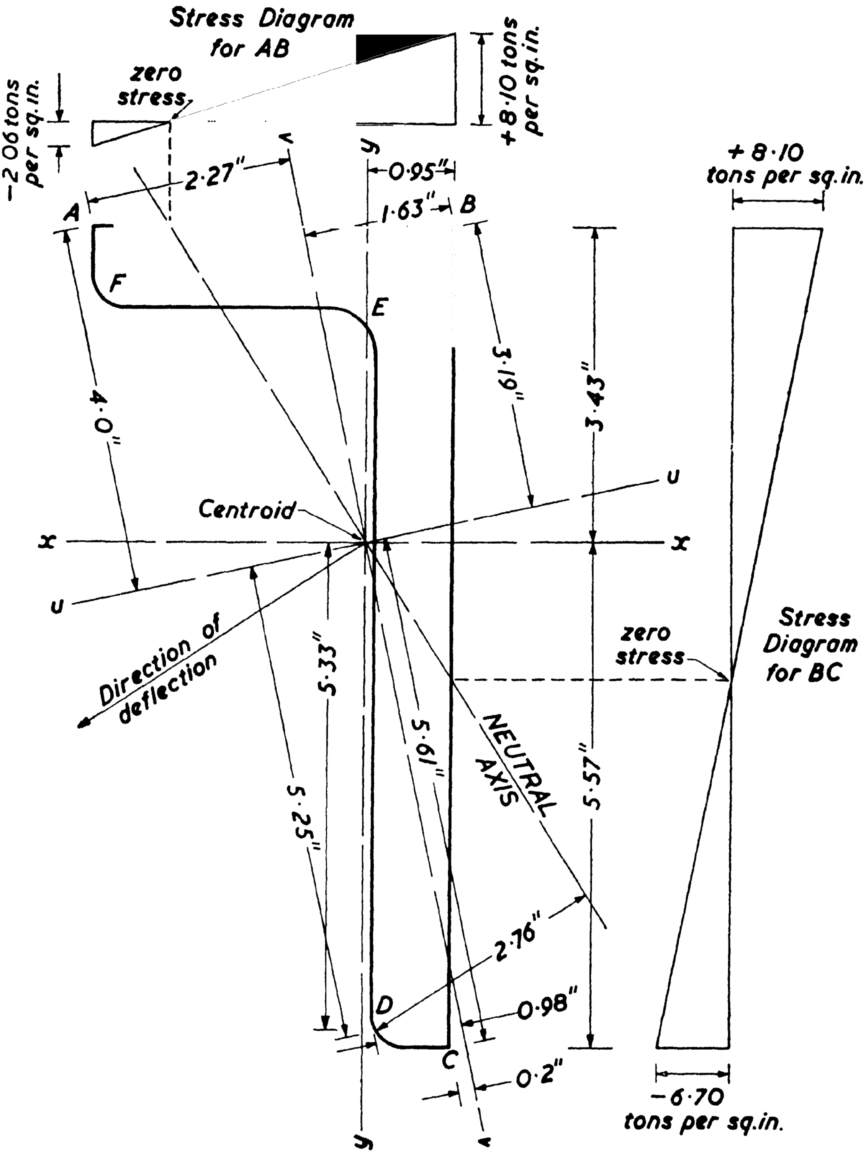


FIG. 261

both downwards and laterally although the legs are restrained so that they remain parallel to their original position. Calculate the maximum stress.

The properties of the section given in B.S. No. 4A are as follows:

$$I_{xx}=86{\cdot}13 \text{ in.}^4 \qquad I_{yy}=10{\cdot}60 \text{ in.}^4$$
$$I_{uu}=89{\cdot}48 \text{ in.}^4 \qquad I_{vv}=7{\cdot}25 \text{ in.}^4$$
$$c_x=3{\cdot}43 \text{ in.} \qquad c_y=0{\cdot}95 \text{ in.}$$
$$\text{Tan } \theta=0{\cdot}21, \quad \text{i.e. } \theta=11°52'$$

Consequently,

$$\sin \theta=0{\cdot}2056$$
$$\cos \theta=0{\cdot}9786$$
$$M_{uu}=M_{xx} \cos 11° \, 52'=97{\cdot}86 \text{ tons in.}$$
$$M_{vv}=M_{xx} \sin 11° \, 52'=20{\cdot}56 \text{ tons in.}$$

The principal problem is to find the point of maximum stress. Now the stress in any fibre of a member subjected to bending is directly proportional to the distance of that fibre from the neutral axis. Therefore, if the neutral axis is located the most highly stressed fibre is that most remote from it.

Consider point A, the co-ordinates of which, with respect to the uu and vv axes, are v and u respectively.

$$f_A=\frac{M_{uu} \, . \, v}{I_{uu}}+\frac{M_{vv} \, . \, u}{I_{vv}}$$
$$=\frac{97{\cdot}86 \times 4{\cdot}0}{89{\cdot}48}-\frac{20{\cdot}56 \times 2{\cdot}27}{7{\cdot}25}$$
$$=-2{\cdot}06 \text{ tons per sq. in.}$$

Considering point B,

$$f_B=\frac{97{\cdot}86 \times 3{\cdot}19}{89{\cdot}48}+\frac{20{\cdot}56 \times 1{\cdot}63}{7{\cdot}25}$$
$$=+8{\cdot}10 \text{ tons per sq. in.}$$

Considering point C,

$$f_C=-\frac{97{\cdot}86 \times 5{\cdot}61}{89{\cdot}48}-\frac{20{\cdot}56 \times 0{\cdot}2}{7{\cdot}25}$$
$$=-6{\cdot}70 \text{ tons per sq. in.}$$

Now there is a constant variation of stress from A to B and from B to C. Consequently, the neutral axis is located by joining the points of zero stress in AB and BC. It is evident that the point most remote from the neutral axis, and therefore most highly stressed, is D.

$$f_D=-\frac{97{\cdot}86 \times 5{\cdot}25}{89{\cdot}48}-\frac{20{\cdot}56 \times 0{\cdot}98}{7{\cdot}25}$$
$$=-8{\cdot}53 \text{ tons per sq. in.}$$

i.e. the stress is tensile.

It is worth noting that although the plane of the applied B.M. is in the YY axis, the plane of bending is at right angles to the neutral axis. Further, it will be observed that had the neutral axis been assumed to be located at right angles to the plane of bending, as in the case of symmetrical sections,

the resulting stresses at A, B and C would have been incorrect numerically and, in addition, the stress at A would have been given as compressive whereas it is actually tensile.

Nevertheless, it must be repeated that the stresses have been computed on the assumption that the section is free to deflect in any manner but not to twist. If, however, lateral deflection is prevented, owing to restraint offered by the load or any other medium, and vertical deflection only is permitted, then the stresses will be correctly given by using the applied moment in conjunction with the appropriate section moduli for the XX axis.

Example 3. As an alternative solution the angle section in Example 2 will be analysed using equation (11B).

First calculate the value of I_{xy}. It was stated in the chapter dealing with the Geometrical Properties of Sections that:

$$\tan 2\theta = -\frac{2I_{xy}}{I_{xx}-I_{yy}}$$

$$=\frac{2I_{xy}}{I_{yy}-I_{xx}}.$$

Therefore $$2I_{xy}=\tan 2\theta(I_{yy}-I_{xx})$$

$$= +0{\cdot}4397(10{\cdot}60-86{\cdot}13)$$

and $$I_{xy} = -16{\cdot}61 \text{ in.}^4.$$

M acts in the plane of the yy axis.

Hence $M_{xx}=100$ tons in. $M_{yy}=0$.

For point D, the co-ordinates are: $x=0{\cdot}125$, $y=-5{\cdot}33$.

Therefore,

$$f=-\frac{M_{xx}(I_{xy}/I_{xx})}{I_{yy}-(I_{xy}{}^2/I_{xx})}\cdot x+\frac{M_{xx}}{I_{xx}-(I_{xy}{}^2/I_{yy})}\cdot y$$

$$=\frac{-100\times-16{\cdot}61/86{\cdot}13}{10{\cdot}60-(-16{\cdot}61^2/86{\cdot}13)}\cdot 0{\cdot}125+\frac{100\times-5{\cdot}33}{86{\cdot}13-(-16{\cdot}61^2/10{\cdot}60)}$$

$$= +0{\cdot}33-8.87$$

$$= -8{\cdot}54 \text{ tons per sq. in.}$$

which agrees very closely with the previous result.

Location of Neutral Axis

In Example 2 a method of locating the neutral axis was given. Now the neutral axis is the line running through points of zero stress. In sections subjected to bending only, i.e. with no longitudinal load P, one point of zero stress is the centroid of the section. If equation (11A) or (11B) is equated to zero, omitting the term P/A, the ratio y/x is obtained which gives the tangent of the angle ϕ which the neutral axis makes with the xx axis of the section.

Hence, if equation (11A) is used,

$$\tan\phi = -\frac{M_{yy}I_{xx}-M_{xx}I_{xy}}{M_{xx}I_{yy}-M_{yy}I_{xy}} \quad . \; . \; . \; . \; . \quad (13)$$

Now the stress in any fibre of a member subjected to bending is directly proportional to the distance of that fibre from the neutral axis. Therefore, provided a member is subjected to bending only, the stress at any fibre,

$$f=\frac{M_{NA}}{I_{NA}}\cdot y_{NA} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

where M and I have the usual significance and y_{NA} is the lever arm about the neutral axis NA.

When, in addition to bending, a member is subjected to an axial force P, the stress at any fibre, including the centroid, due to $P=P/A$. Hence, the neutral axis shifts parallel to the position it occupies when there is only bending, the perpendicular distance travelled being

$$\frac{P}{A}\times\frac{I_{NA}}{M_{NA}}.$$

The neutral axis may also be located with reference to the principal axis *uu* by employing equation (12) without the term P/A. If the angle between the neutral and principal axes is θ, then

$$\tan\theta=\frac{M_{vv}\,.\,I_{uu}}{M_{uu}\,.\,I_{vv}}.$$

Example 4. Calculate the maximum stress in the angle section of Examples 2 and 3 using equations (13) and (14). See Fig. 261.

Now
$$\tan\phi=-\frac{M_{yy}I_{xx}-M_{xx}I_{xy}}{M_{xx}I_{yy}-M_{yy}I_{xy}}$$
$$=-\frac{0-(100\times-16{\cdot}61)}{(100\times10{\cdot}60)-0}$$
$$=-\frac{16{\cdot}61}{10{\cdot}60}=-1{\cdot}567.$$

Therefore
$$\phi=-57^\circ\,27'$$
$$\sin\phi=-0{\cdot}8429$$
$$\sin 2\phi=-0{\cdot}9070$$
$$\cos\phi=+0{\cdot}5380$$

and
$$M_{NA}=M\,.\,\cos\phi$$
$$=100\times0{\cdot}5380$$
$$=53{\cdot}8 \text{ tons in.}$$

y_{NA} for this point is found to be $-2{\cdot}76$ in., either by calculations or scaling from Fig. 261.

It may be shown that

$$I_{NA}=\cos^2\phi I_{xx}+\sin^2\phi I_{yy}-\sin 2\phi I_{xy}$$
$$=(0{\cdot}538)^2\times86{\cdot}13+(-0{\cdot}8429)^2\times10{\cdot}60+0{\cdot}907\times(-16{\cdot}61)$$
$$=24{\cdot}93+7{\cdot}53-15{\cdot}07$$
$$=17{\cdot}39 \text{ in.}^4.$$

Then $$f=\frac{M_{NA}}{I_{NA}}\cdot y_{NA}=\frac{53{\cdot}8\times(-2{\cdot}76)}{17{\cdot}39}$$
$$= -8{\cdot}54 \text{ tons per sq. in. as before.}$$

The Circle of Inertia

Unsymmetrical sections subjected to bending may be analysed graphically by using a construction known as the Circle of Inertia. The method of analysis is best described by taking a practical example.

Example 5. The member shown in Fig. 262, which consists of two 9-in. × $3\frac{1}{2}$-in. × 22-lb. B.S. Channels, is subjected to a B.M. of 90 tons in. in the plane of the YY axis and 135 tons in. in the plane of the XX axis.

The properties of each channel section, given in B.S. No. 4, are as follows:

$$I_{xx}=82{\cdot}62 \text{ in.}^4 \qquad I_{yy}=6{\cdot}90 \text{ in.}^4$$
$$c_x=0 \qquad c_y=1{\cdot}00 \text{ in.}^4$$
$$\text{Cross-sectional area } A=6{\cdot}55 \text{ sq. in.}$$

The centroid of the cross-section of the member could be calculated in the usual way, but, obviously, it lies midway between the centroids of the individual channel sections.

Therefore, for the whole member, mentioning the left-hand channel first:

$$I_{XX}=82{\cdot}62+(6{\cdot}55\times1{\cdot}75^2)+6{\cdot}90+(6{\cdot}55\times1{\cdot}75^2)$$
$$=129{\cdot}64 \text{ in.}^4$$
$$I_{YY}=6{\cdot}90+(6{\cdot}55\times2{\cdot}75^2)+82{\cdot}62+(6{\cdot}55\times2{\cdot}75^2)$$
$$=188{\cdot}59 \text{ in.}^4.$$
$$I_{XY}=(A\times -x\times -y)+(A\times x\times y)$$
$$=2\times6{\cdot}55\times2{\cdot}75\times1{\cdot}75$$
$$=63{\cdot}04 \text{ in.}^4.$$

Note that a symmetrical section such as a channel has no value I_{xy} about its own centroid, as the xx and yy axes are principal axes.

Now the Circle of Inertia may be drawn.

From the centroid of the member, O, set off vertically, i.e. along the YY axis, Oa and ab to some scale to represent I_{XX} and I_{YY} respectively.

Let Ob be a diameter of the circle of inertia, c being the centre. Then any diameter can represent any pair of rectangular axes in the plane of the cross-section of the member. From a, and scaling horizontally, set off ad equal to I_{XY}, scaling to the right as the product of inertia is positive.

Draw the diameter $ecdf$ through d. Then fd and de represent the values of the moments of inertia I_{UU} and I_{VV} about the principal axes. When scaled, $I_{UU}=89$ ins.^{4} and $I_{VV}=229$ ins.^{4}.

Draw lines through f and O and through e and O. Then these lines are respectively the principal axes UU and VV.

Now set off the values of M_{XX} and M_{YY} to some scale on the YY and XX axes respectively. Then gO is the resultant of these moments and also the resultant of M_{vv} and M_{uu} which are represented by gh and gj respectively.

Therefore,

$$M_{VV} = 66 \text{ tons in.}$$

and

$$M_{UU} = 149 \text{ tons in.}$$

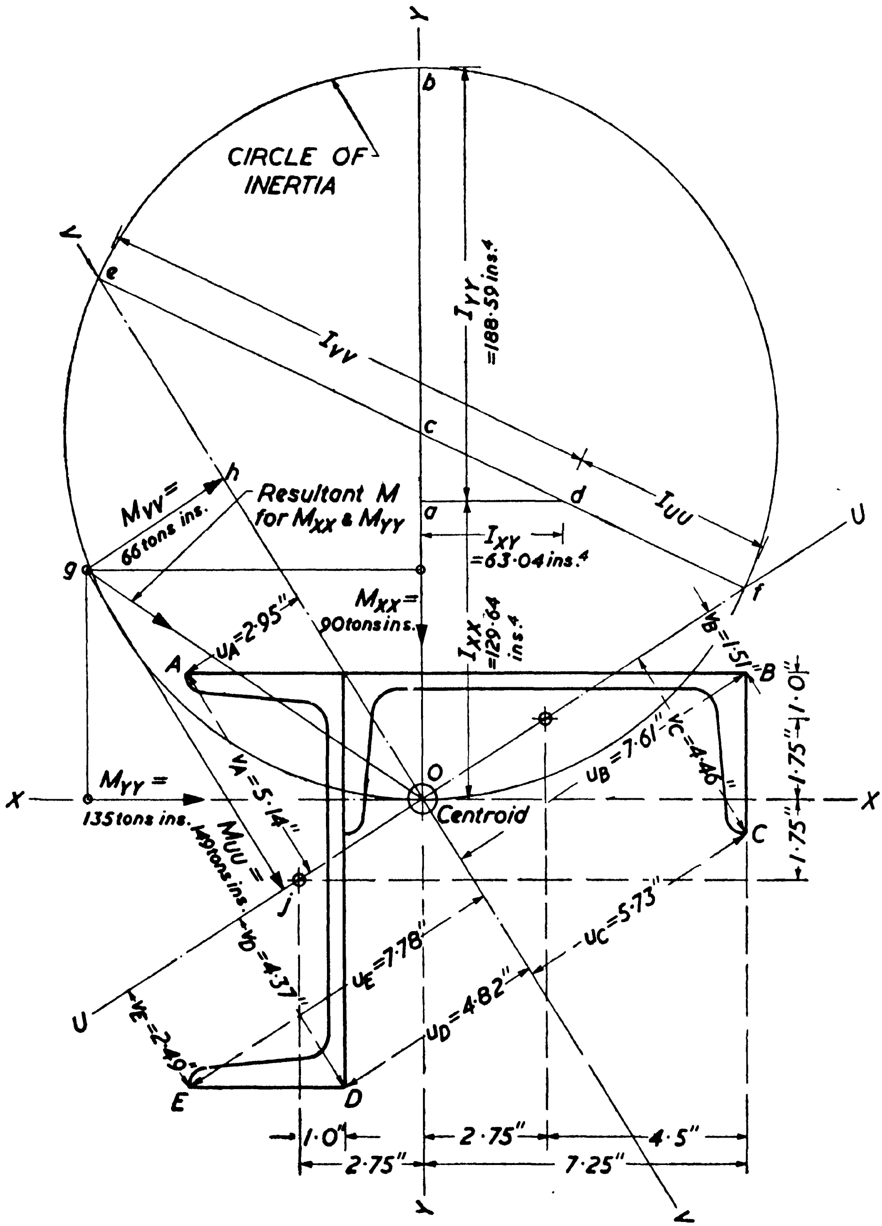


FIG. 262

When the values of v and u have been scaled for each corner of the member, the fibre stresses may be calculated.

Now
$$f = \pm\frac{M_{UU} \cdot v}{I_{UU}} \pm \frac{M_{VV} \cdot u}{I_{VV}}$$

Hence,

$$f_A = +\frac{149 \times 5{\cdot}14}{89} + \frac{66 \times 2{\cdot}95}{229}$$
$$= +9{\cdot}45 \text{ tons per sq. in.}$$
$$f_B = -\frac{149 \times 1{\cdot}51}{89} - \frac{66 \times 7{\cdot}61}{229}$$
$$= -4{\cdot}72 \text{ tons per sq. in.}$$
$$f_C = -\frac{149 \times 4{\cdot}46}{89} - \frac{66 \times 5{\cdot}73}{229}$$
$$= -9{\cdot}12 \text{ tons per sq. in.}$$
$$f_D = -\frac{149 \times 4{\cdot}37}{89} + \frac{66 \times 4{\cdot}82}{229}$$
$$= -5{\cdot}93 \text{ tons per sq. in.}$$
$$f_E = -\frac{149 \times 2{\cdot}49}{89} + \frac{66 \times 7{\cdot}78}{229}$$
$$= -1{\cdot}93 \text{ tons per sq. in.}$$

Deflection

The value and direction of the deflection of members with unsymmetrical sections may be obtained by calculating the resultant of the deflections about the principal axes. As an alternative, a direct calculation, normal to the neutral axis, may be made when the neutral axis has been located. However, the position of the neutral axis varies with the plane of loading whereas the principal axes are functions of the section, and therefore are constant for any loading.

Example 6. Calculate the deflection in the 9-in. × 4-in. × $\frac{7}{8}$-in. angle employed for Examples 2 to 4 assuming that the moment M_{xx} of 100 tons in. is caused by a U.D.L. of 8 tons over a span of 8 ft. 4 in., i.e. 100 in.

Now for a U.D.L.

$$d = \frac{5WL^3}{384EI} = \frac{WL}{8} \cdot \frac{5L^2}{48EI}$$
$$= M \cdot \frac{5L^2}{48EI}$$

Assume that $E = 13{,}000$ tons per sq. in.

Considering the uu axis,

$$d_{uu} = M_{uu} \cdot \frac{5 \times 100^2}{48 \times 13{,}000 \times I_{uu}}$$ (see p. 434)
$$= 0{\cdot}088 \text{ in.}$$

Considering the vv axis,

$$d_{vv} = M_{vv} \cdot \frac{5 \times 100^2}{48 \times 13{,}000 \times I_{vv}}$$

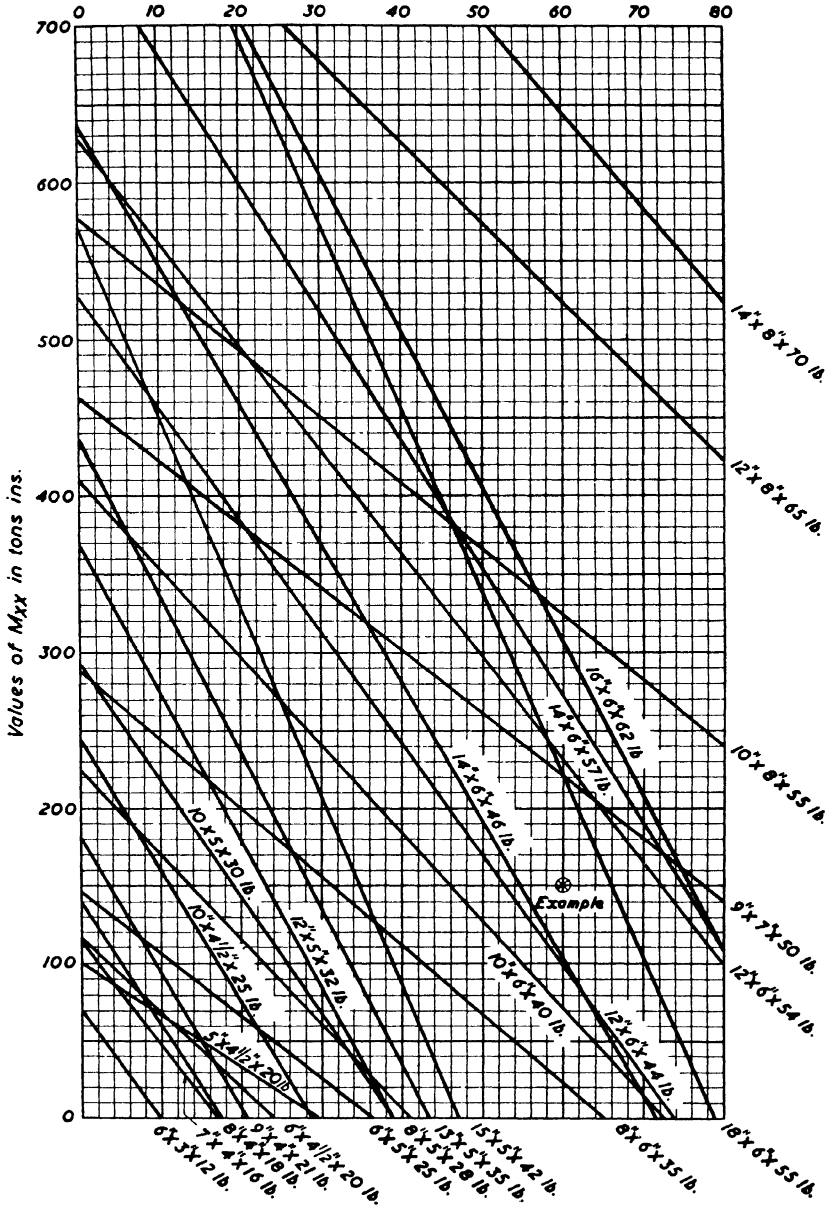
$$= 0{\cdot}227 \text{ in.}$$

The resultant deflection,

$$d_{NA} = \sqrt{(d_{uu}^2 + d_{vv}^2)}$$

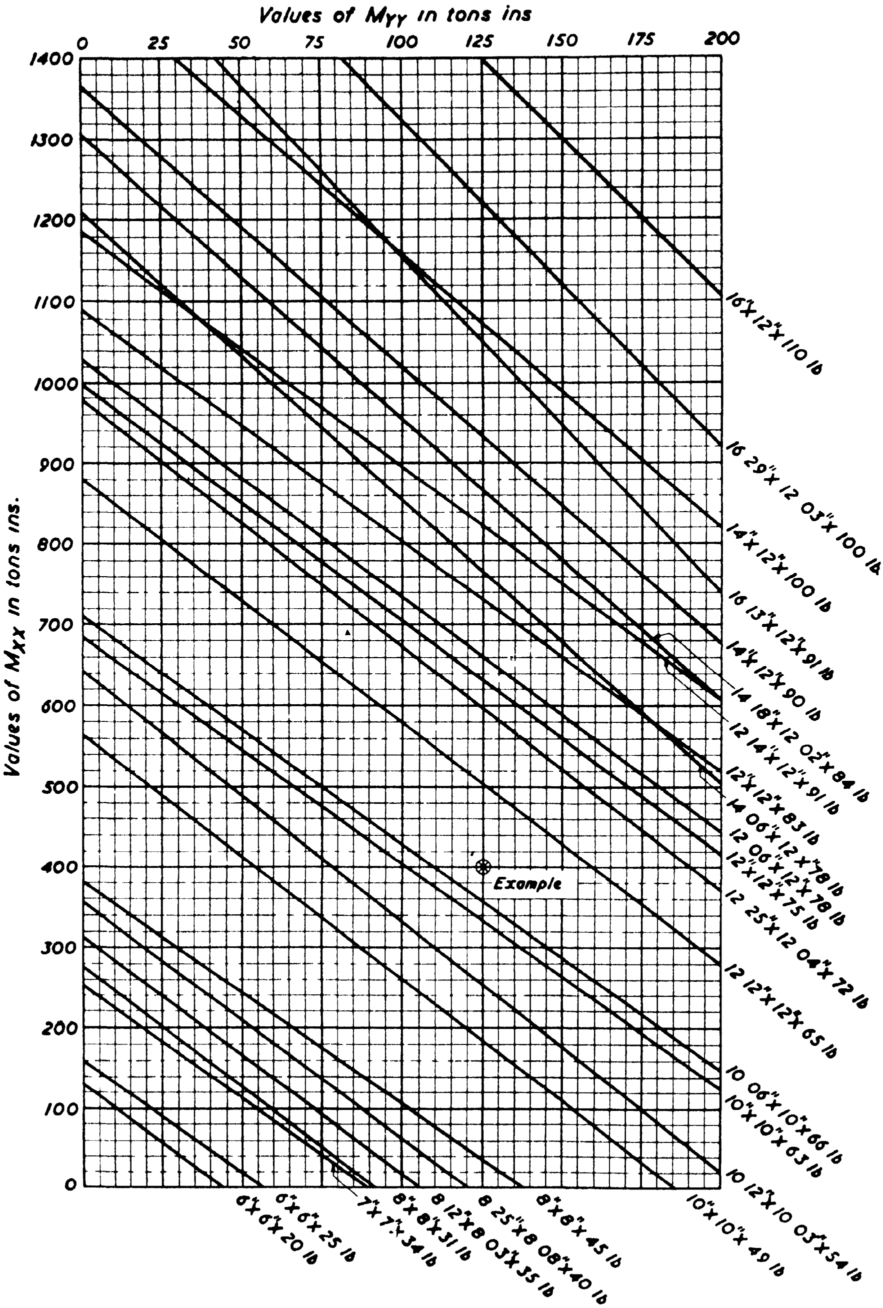
$$= 0{\cdot}244 \text{ in.}$$

OBLIQUE LOADING ON B.S. BEAMS
MAXIMUM STRESS 10 TONS/SQ.IN.



Choose lightest beam to the right of the point considered
Example : $M_{XX} = 150$ *tons ins.* $M_{YY} = 60$ *tons ins.*
Section required = 9"x 7"x 50 lb.

OBLIQUE LOADING ON BROAD FLANGE BEAMS
MAXIMUM STRESS 10 TONS/SQ. IN.



Choose lightest beam to the right of the point considered
Example M_{XX} *= 400 tons ins* M_{YY} *= 125 tons ins*
Section required = 12 12"x 12"x 65 lb

18. FRAMING FOR SINGLE-STOREY SHEDS

THE framing for single-span single-storey sheds can be arranged in a number of ways dependent upon the design assumptions made and those in general use are shown in Figs. 263 to 268. Typical B.M. diagrams for the stanchions are shown in the Figures and, in addition, the dotted lines indicate, in an exaggerated manner, the deflected form of the structure under side-wind loading.

The cross-section shown in Fig. 263 (a) is perhaps more widely used than any other and consists basically of roof trusses and stanchions. The trusses may be carried directly by stanchions as shown in Fig. 263 (b) or there may be in addition intermediate trusses supported by eaves beams, Fig. 263 (c). In each case the stanchions must be fixed at their bases and will act as vertical

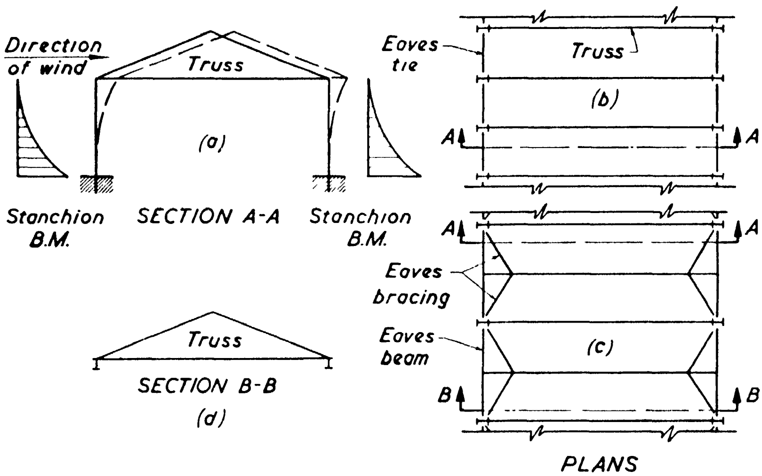


FIG. 263

cantilevers under the action of side-wind loading. The wind B.M. in the stanchions must be taken up by the foundations. For the design of suitable stanchion bases and foundations see pages 554 and 769. Where intermediate trusses are carried by eaves beams, lateral bracing, as shown in Fig. 263, will be required to transmit to the stanchions the horizontal reactions of the trusses unless the beams are designed for the combined vertical and horizontal loads. Bracing is normally the more economical.

Approximate weights of single-span trusses both welded and riveted are given in Figs. 291 and 292.

The imposition of moments on the foundations can be avoided by the use of knee braces between the trusses and stanchions as shown in cross-section and plan in Figs. 264 (a) and (b). Where eaves beams are used, as in Figs. 264 (c) and (d), the intermediate trusses will be of simple form and, again,

lateral bracing at eaves level will be required to transmit the horizontal loads to the stanchion caps. For a shed otherwise identical with that shown in Fig. 263 the maximum B.M. in the stanchions will be reduced and, in addition its position will be transferred from the stanchion base to the junction of knee brace and stanchion.

If head room or other conditions so dictate, the braces may be on one side only, the other end of the truss being simply supported, in which case the

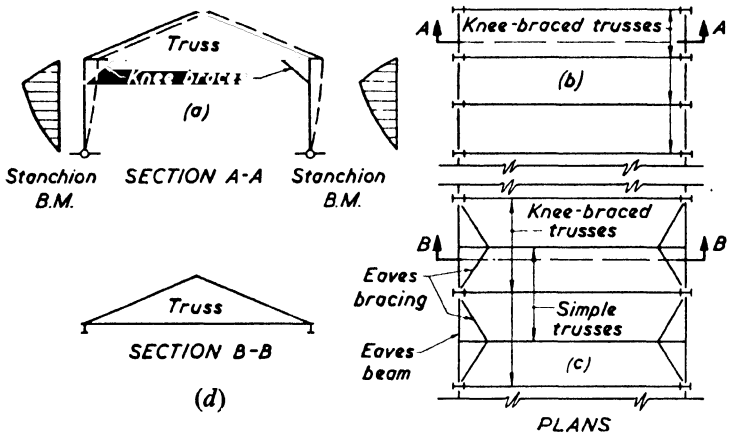


FIG. 264

knee-braced stanchion takes up all the moment due to side wind on the frame as a whole whilst the other stanchion acts as a vertical post carrying direct load but subjected to a wind B.M. as a beam simply supported at base and cap levels. (See Fig. 265.)

A common variant is that shown in Fig. 266, which still further reduces the stanchion bending moments, other conditions remaining unaltered, but

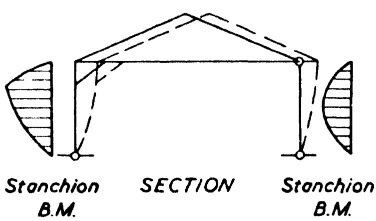


FIG. 265

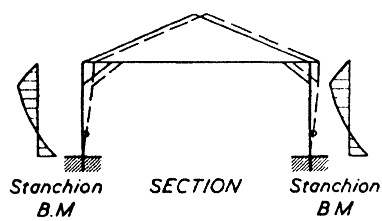


FIG. 266

at the expense of imposing moments on the foundations approximately half those shown in Fig. 263.

Another form of construction, shown in Fig. 267, employs horizontal eaves girders of lattice construction to transmit all horizontal loads from trusses and intermediate posts to the ends of the building and thence via bracing to the foundations. This type of construction is particularly economical when the building is short enough to enable the wind loads to be

taken to the end frames by eaves girders of reasonable proportions, as the additional bracing in these girders will be relatively light. The trusses may be supported directly by stanchions as shown in plan at Fig. 267 (b) or by eaves beams and stanchions as depicted in Fig. 267 (c). Alternatively, if the building is too long to give reasonably light eaves girders, the spans of these

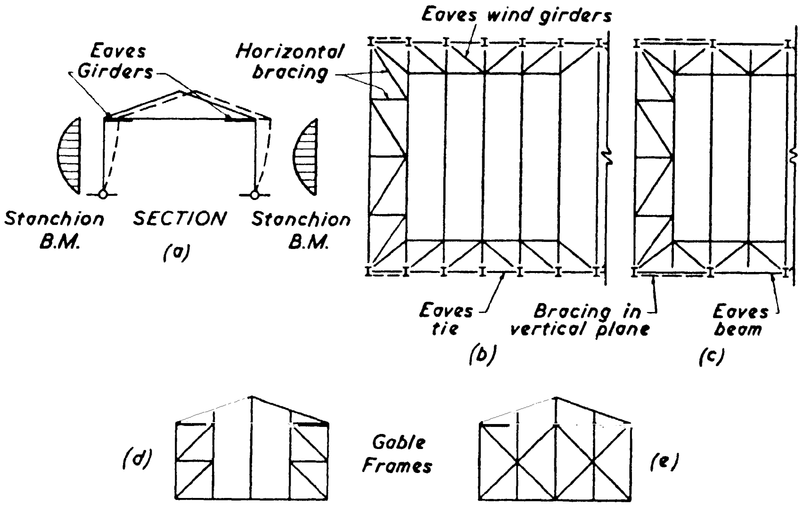


FIG. 267

girders may be reduced by the insertion, at appropriate intermediate points, of frames designed to give the necessary horizontal resistance.

The horizontal eaves girders may be replaced by girders in the planes of the truss rafters and in which the purlins can be made to serve as the chord members of the lattice girders.

Alternative arrangements of the gable frames are shown in Figs. 267 (d)

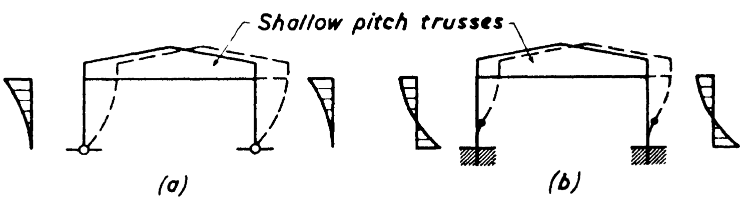


FIG. 268

and (e). In Fig. 267 (d) stanchions are braced together in pairs with diagonals in tension only, and hence each pair is assumed to act in resisting the wind effect on one side only, whilst in Fig. 267 (e) stiff bracing is used, capable of acting both in tension and compression so that all the stanchions in the gable act in conjunction.

If it is required to use a roof of shallow pitch and in consequence there is a reasonable depth of truss at the shoe, the effect of a knee brace will be simulated as shown in Figs. 268 (a) and (b), showing stanchions hinged and fixed

at their bases respectively. The horizontal eaves lattice girders referred to previously may be used with this type of roof if required.

Arched or portal construction may also be used, the principal types being shown in Figs. 269; they are respectively three-hinged, two-hinged and hingeless or fixed arches. The first has the advantage of being statically determinate although the moments and crown deflection when loaded are greater than for the other two types. The moments in the fixed-ended arch or portal are the least of the three. All three types have the disadvantage of imposing horizontal thrusts of some magnitude upon the foundations, whilst the hingeless type also causes moments on the foundations. The calculations for the two-hinged and hingeless portals are necessarily based upon the normal arch theory, which presupposes the existence of the appropriate horizontal resistance at the feet of the portals to provide the arch thrust and it should be noted that comparatively small horizontal displacements at the footings will cause considerable redistribution of the moments. These types of framing should not therefore be used in cases where the existence of adequate horizontal resistance is suspect unless the effect of arch spread is

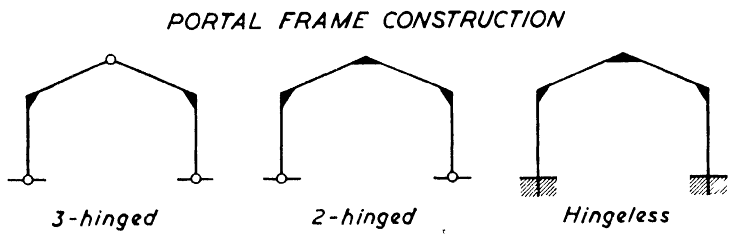


FIG. 269

included in the design calculations or unless positive means are taken to determine the extent of the lateral movement of the portal feet, for example, by the insertion of ties at base level.

The calculations for a three-pinned frame also call for the provision of an adequate horizontal thrust, but the effect of footing spread in this case is of little importance: all forces and moments in the frame are increased in inverse ratio to that of the reduced crown height compared with the original crown height, but no redistribution effects occur.

It is generally found that the weight of a portal frame designed on the elastic theory is greater than that of the lightest comparable construction utilising trusses and columns, but this effect may be offset by savings and advantages in other directions.

The portal frames may be of solid I-section (joist or welded plate) or of lattice type. Formulæ dealing with two-hinged and hingeless frames are given on pages 287–311. These may be used to calculate moments, shears and thrusts when a solid section is used, but if lattice work is employed the vertical and horizontal reactions can be calculated from the formulæ given and a stress diagram can be drawn in the normal manner utilising the reactions so found. In the case of the three-pinned arch the vertical reactions at the feet may be found by taking moments about them in the normal manner

whilst the horizontal reactions can be found by utilising the fact that the moment at the crown hinge is zero. For examples of the calculations for a three-pinned arch see pages 106–109.

Multi-span Ridged Roof Sheds

Multi-span sheds may be constructed of a series of simple truss or portal spans provided that a relatively large number of internal columns can be permitted. The principal advantage of multi-bay construction in truss work is, however, the ease with which large floor areas may be covered with a minimum number of supports. Fig. 270 (a) indicates a conventional layout

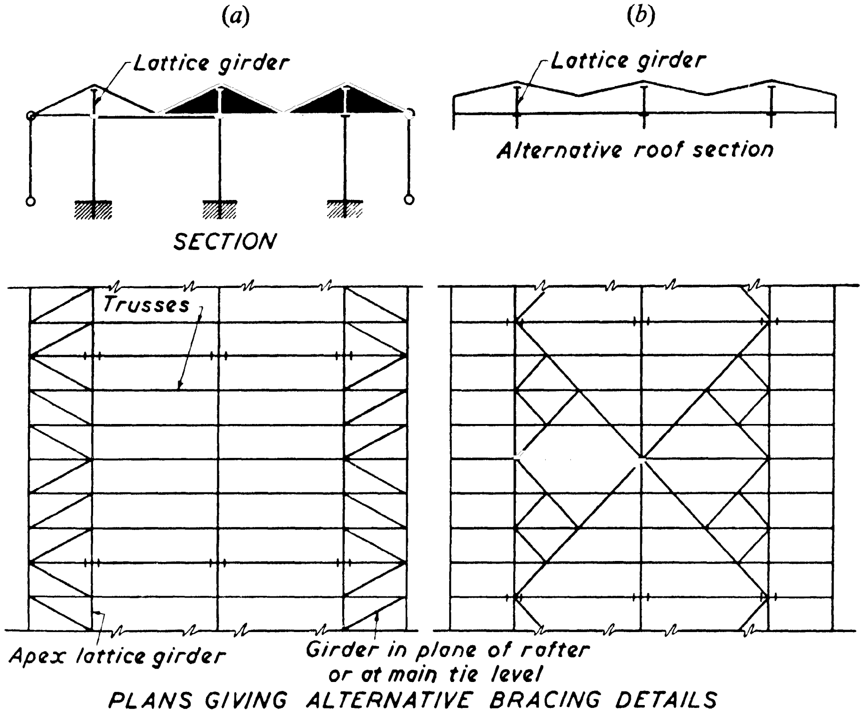


FIG. 270

in which apex lattice girders support umbrella-type roof trusses. The lattice girders are normally designed as simple spans between columns but may also be designed as continuous girders. The construction is economical for lattice girders with spans up to 120–150 ft. at spacings not exceeding 60–70 ft. Approximate weights of single-span lattice girders are given in Fig. 293. A stiffer structural frame results from the use of trusses having appreciable depth at the valley as indicated in the alternative section, Fig. 270 (b). The same type of construction may be adopted with a north-light truss outline, and it may be convenient to replace the vertical apex lattice girder by an inclined girder in the plane of the north slopes of the slung trusses.

Should it be necessary still further to reduce the number of columns, the

apex lattice girders may be carried by main lattice girders simply supported at their ends, or continuous over intermediate supports if desired, as shown in Fig. 271. Such girders will project through the roof covering, and weathering details are apt to be somewhat complicated.

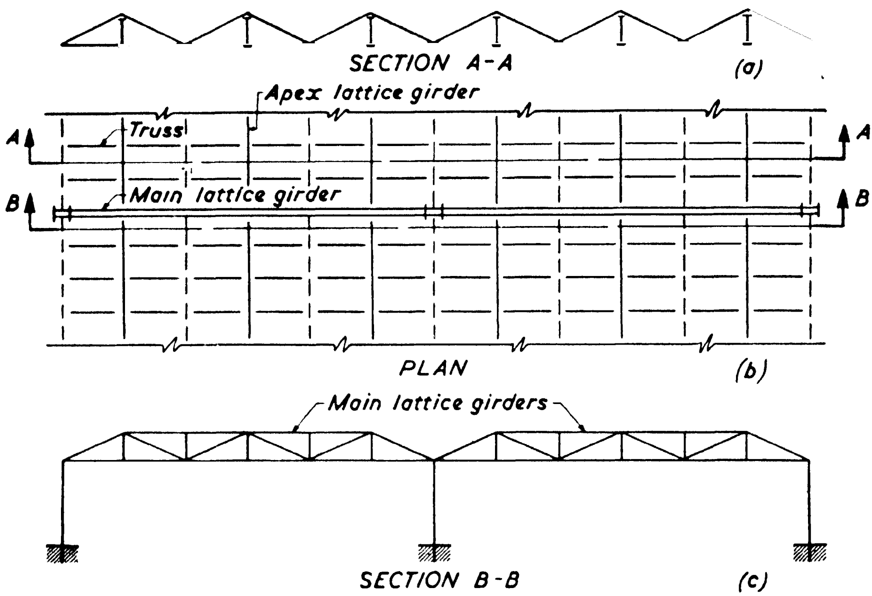


FIG. 271

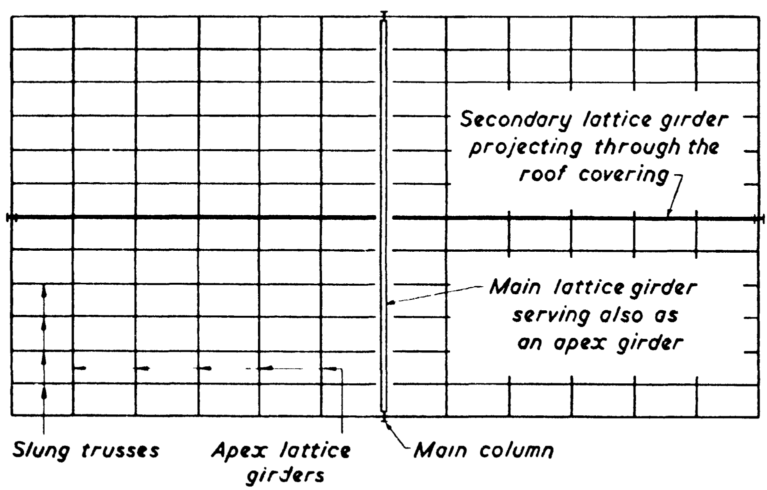


FIG. 272

The sub-division of framing may be carried still further, as shown in Fig. 272, which gives a layout which dispenses entirely with internal columns.

A form of construction which has also found favour is shown in Fig. 273. It consists of lattice girders carrying valley beams which, in turn, support north-light trusses running parallel to the lattice girders. The top chords

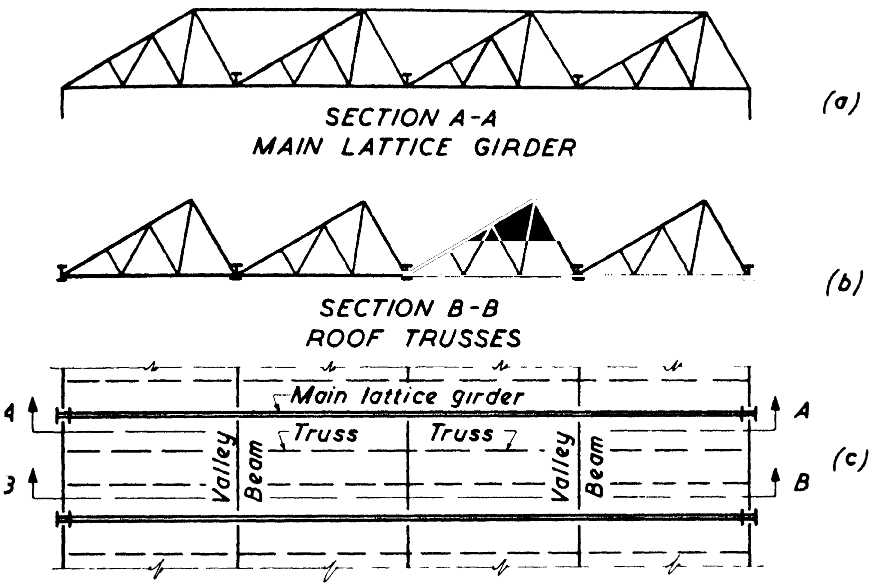


FIG. 273

of these girders are thus above the roof covering and elaborate flashing arrangements are required.

Flat-roofed Single-storey Sheds

These roofs may be of simple joist construction for moderate spans, but if required with monitor lights advantage may be taken of the upstand of the monitor to incorporate lattice girders inside the vertical glazing to reduce the number of supporting columns as shown in Fig. 274. Alternatively the upper and lower roof levels together with the monitor upstand can be designed as a cranked beam as shown in Fig. 275.

Stressed-skin Construction

Stressed-skin construction may be defined as that which utilises the stiffness of the skin of the structure as part of the basis of design. Such an assumption is made in the design of certain reinforced concrete structures and can be utilised in steel-frame construction wherein latticed frames in surface planes simulate the solid skin. The underlying idea is that all forces which act at angles to the skin or its equivalent are resolved into components acting in the planes of the skin, the shape of which is arranged so that this can be done.

To demonstrate the principle in its simplest form consider the single-span north-light roof, shown in Fig. 276, in each slope of which there is a lattice girder. The 'verticals' or struts in these girders act as rafters spanning from

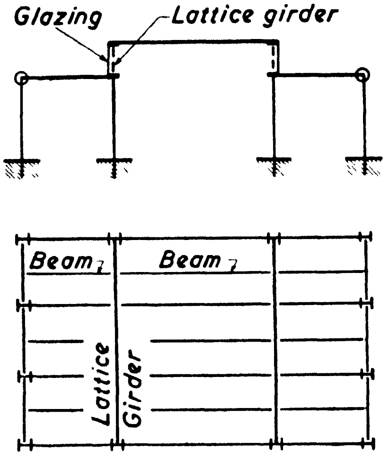


FIG. 274

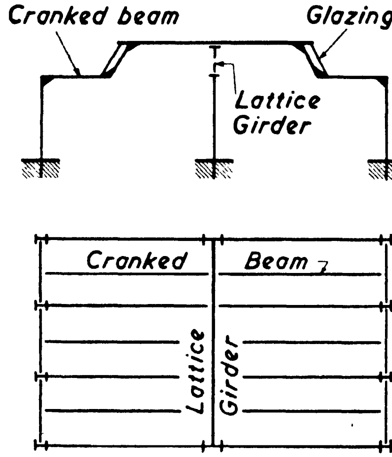


FIG. 275

eaves to apex levels, as shown in Fig. 276 (a) and (c), and receive the dead and wind loads from the purlins, which have been omitted from the Figure for the sake of clarity.

The eaves reaction of each rafter is postulated to be vertical and this is

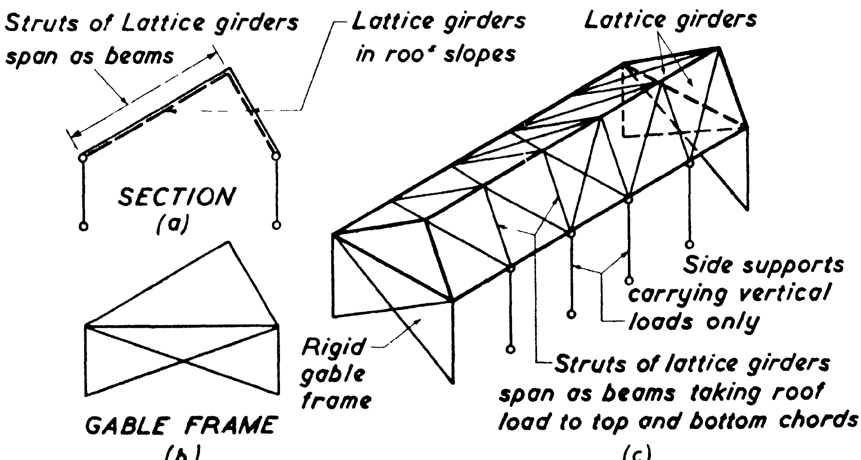


FIG. 276

shown provided by side posts. From this it follows that the direction of the reaction at the apex end under dead load, wind pressure or wind suction can be found as shown in Fig. 277 (a), (b) and (c).

The load on each rafter is then resolved into components in the lines of

these reactions, and the original load on each is replaced by the equivalent loads at eaves and apex, as shown in Fig. 278.

At each panel point at the apex these equivalent loads from the rafters are then resolved into components in the planes of the slopes, to give the panel point loads for the design of the lattice girders.

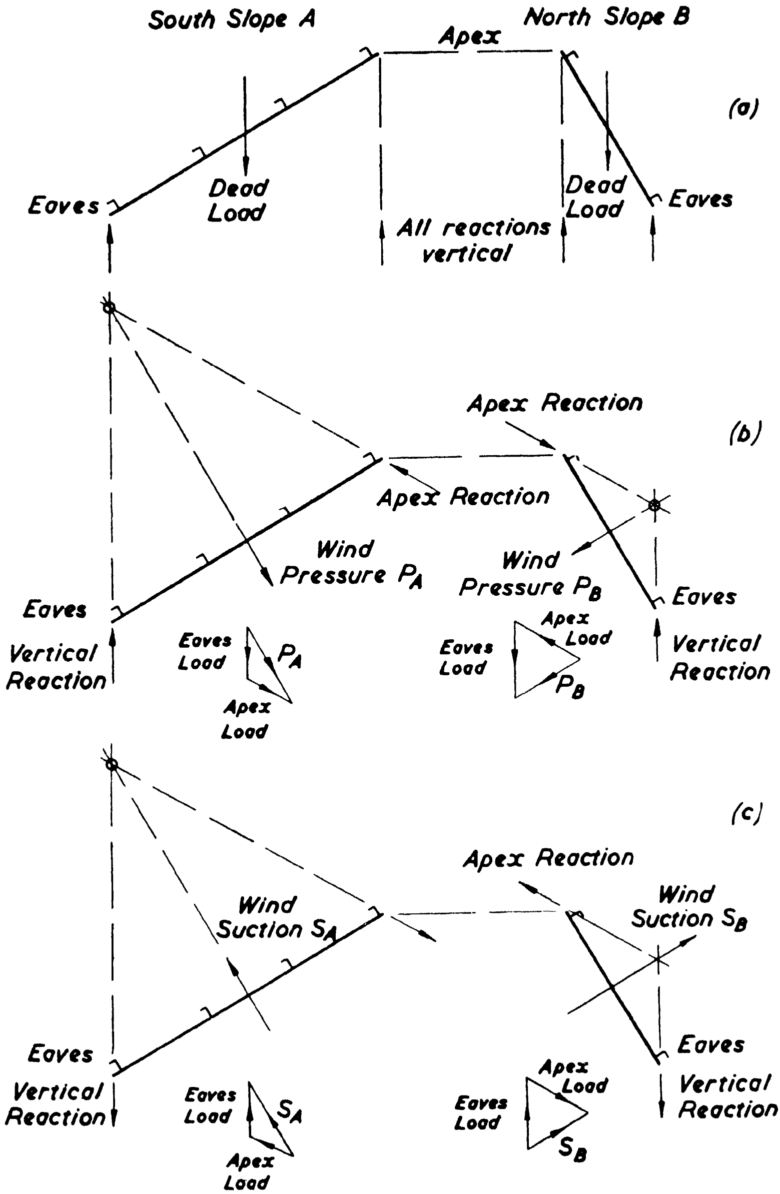
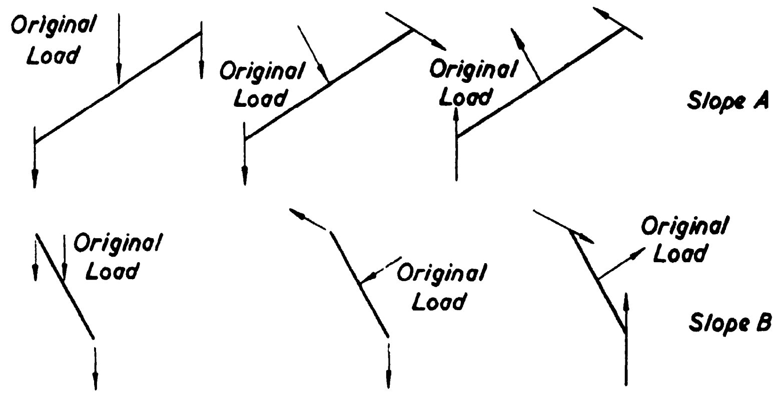


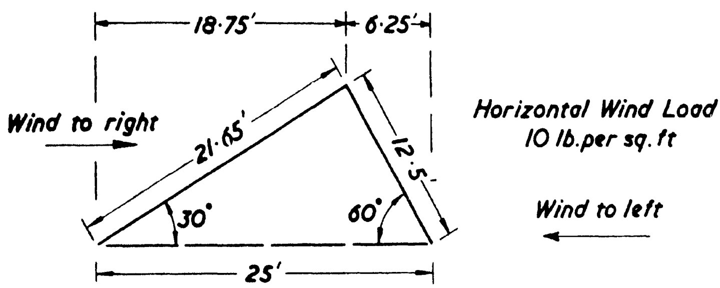
FIG. 277



EQUIVALENT EAVES AND APEX LOADS

FIG. 278

Consider, for example, the north-light roof shown in outline in Fig. 279, and assume that the rafters are at 12-ft. 6-in. centres, thereby fixing the panel lengths for both girders. With the horizontal wind pressure taken at 10 lb. per sq. ft., and assuming that the sides of the building and roof covering are such that the flow of air through them is practically negligible, the loads for design purposes will be as shown.



LOADING	SOUTH SLOPE	NORTH SLOPE	
Dead (vertical)	5 lb per sq. ft.	8 lb. per sq. ft.	Measured on slope
Wind to right	Nil	5 lb. per sq. ft. suction	
Wind to left	5 lb. per sq. ft. suction	4 lb. per sq. ft. pressure	
	Wind loads normal to surface		
Snow (vertical)	10 lb. per sq. ft	10 lb. per sq. ft.	Measured on plan

FIG. 279

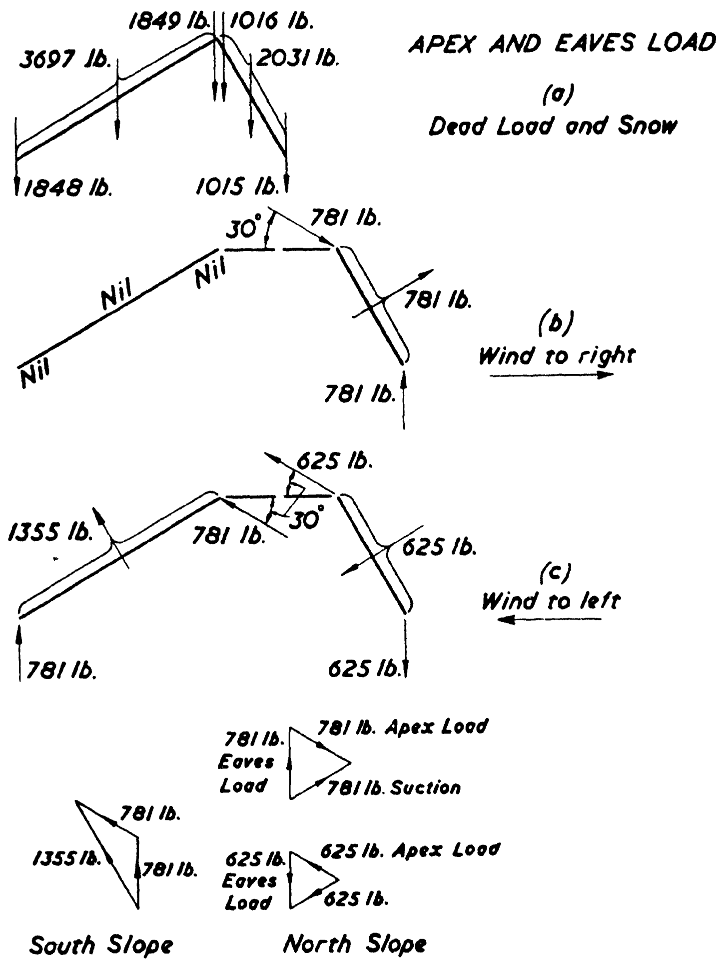


FIG. 280

The loads per panel will therefore be as shown in Fig. 280.

	South slope	*North slope*
Dead load	1353 lb.	1250 lb.
Snow load	2344 lb.	781 lb.
Wind to right	Nil	781 lb. suction
Wind to left	1355 lb. suction	625 lb.

The apex loads are also shown in this Figure and are tabulated below:

	South slope	*North slope*
Dead and snow	1849 lb.	1016 lb.
Wind to right	Nil	781 lb.
Wind to left	781 lb.	625 lb.

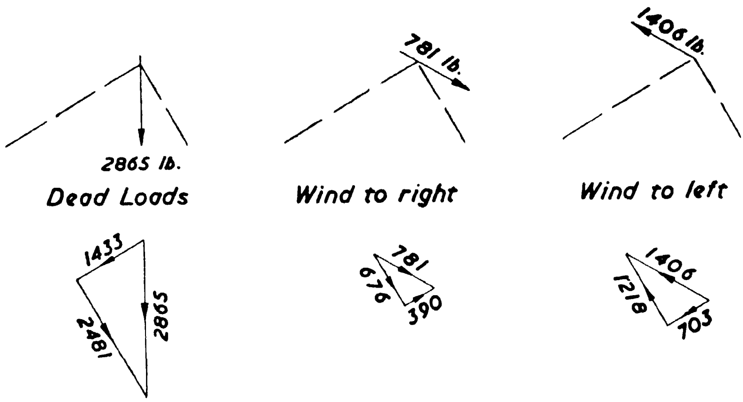


FIG. 281

These loads are then resolved into components in the respective slopes as shown in Fig. 281, from which it is obvious that the maximum panel point load for the south-slope girder is given with the wind blowing to the left and for the north-slope girder with the wind blowing to the right, and the following figures result:

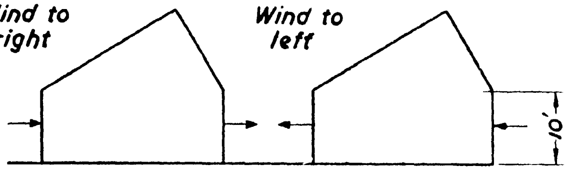
Maximum Panel Point Loads

	South slope
Dead and snow	2865 × sin 30° = 1433 lb.
Wind to left	1406 × sin 30° = 703 lb.
	2136 lb.
	North slope
Dead and snow	2865 × cos 30° = 2481 lb.
Wind to right	781 × cos 30° = 676 lb.
	3157 lb.

These are maximum panel point loads to be applied at the top chords of the lattice girders, provided that independent girders are used. The case of girders with a common top chord is considered later.

It now remains to find the forces to be applied at panel points in the bottom chords of the girders due to wind effect on the sides of the buildings. Assuming that the shed is 10 ft. high to eaves with posts at 12-ft. 6-in. centres to suit the panel length, and that there is suction and pressure as indicated in Fig. 282, then the side posts may be assumed to act as vertical beams spanning from ground to eaves level. Their horizontal reactions on each of the lattice girders will therefore be half the total load on each side, i.e. 313 lb. These horizontal reactions are then resolved into the planes of the lattice girders, resulting in panel point loads in the planes of the girders of 361 lb. in the south slope and 625 lb. in the north slope. Considering wind blowing to the

left, then for the south slope the load due to suction on the south side acts in the same direction as the wind effect at the apex, whilst wind blowing to the right increases the loads in the north slope girder.



	SOUTH SIDE	NORTH SIDE
WIND TO RIGHT	5 lb./sq. ft. pressure	5 lb./sq. ft. suction
WIND TO LEFT	5 lb./sq. ft. suction	5 lb./sq. ft. pressure

FIG. 282

The final loadings for the two lattice girders are thus as shown in Fig. 283 (a) and (b).

The vertical forces in the side posts are combined from the vertical reactions shown in Fig. 280, and the additional vertical component caused by resolving the wind loads on the sides of the building into the planes of the lattice girders, and are:

	South side posts	*North side posts*
Dead and snow	+1848 lb.	+1015 lb.
Wind to left,		
from rafter	− 781 lb.	+ 625 lb.
from side wind	− 180 lb.	+ 541 lb.
Wind to right,		
from rafter	Nil	− 781 lb.
from side wind	+ 180 lb.	− 541 lb.
Maximum value	2028 lb.	2181 lb.

If the lattice girders in the two slopes have a common top chord, some reduction can be made in the panel point loads at the apex, since the values given in Fig. 283 (a) and (b) are the maximum which can exist in either slope and are not possible at the same time; it is therefore necessary to consider the load which can exist in both slopes simultaneously. See Fig. 283 (c) and (d).

Apex loads	*South slope*	*North slope*
Dead and snow	1433 lb.	2481 lb.
Wind to left	703 lb.	−1218 lb.
	2136 lb.	1263 lb.
Dead and snow	1433 lb.	2481 lb.
Wind to right	−390 lb.	676 lb.
	1043 lb.	3157 lb.
Eaves loads		
Wind to left	361 lb.	−625 lb.
Wind to right	−361 lb.	625 lb.

It will be seen that the web members and the bottom chord members of the lattice girders in both slopes should be designed for the loads previously

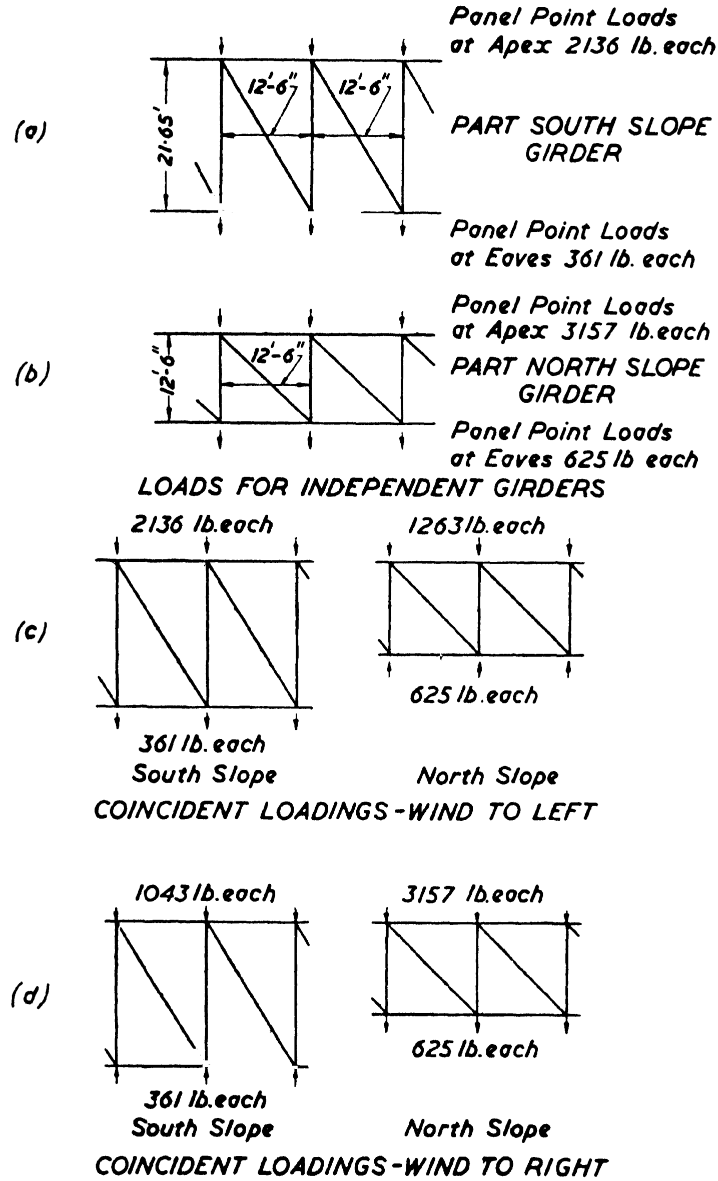


FIG. 283

given, but that the common top chord can be designed for the reduced total loads given in Fig. 283 (d).

From this example it will be apparent that the basic principle of this method of design is that advantage is taken of the line common to two inclined planes,

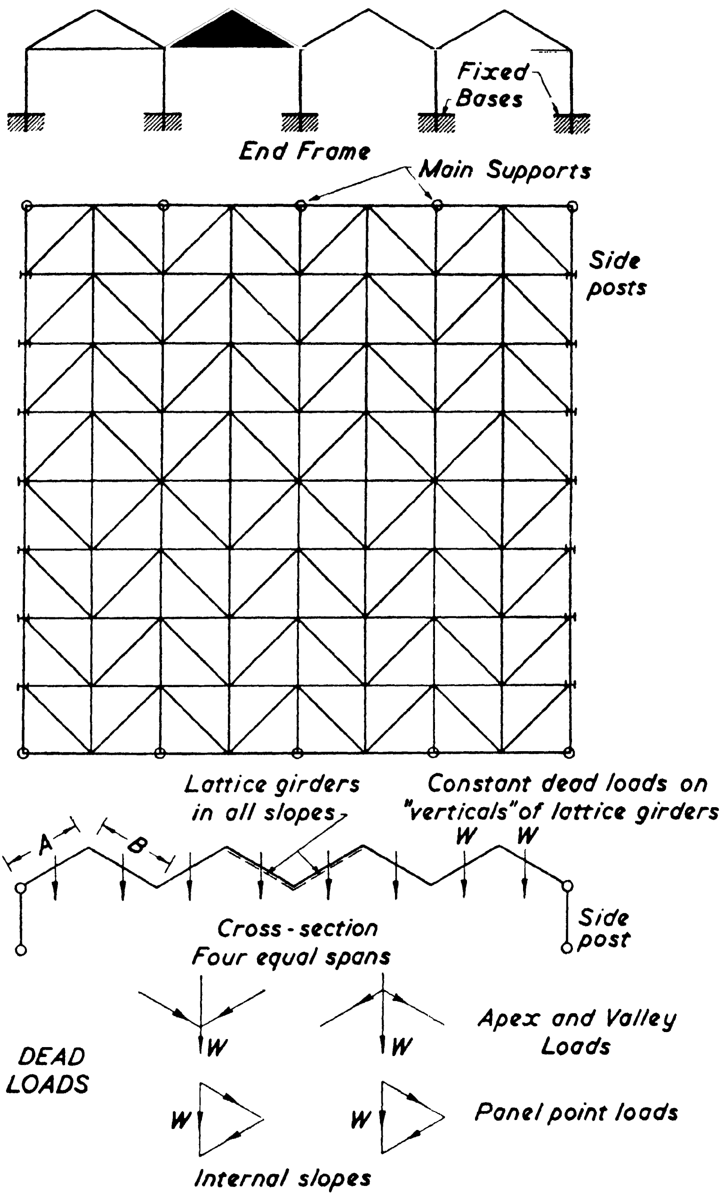


FIG. 284

since any loads acting at this line can be resolved into components acting in these planes, and any such line is regarded as the boom of two lattice girders.

The end reactions of the lattice girders are provided by triangulated or other stiff frames at the ends of the building or intermediate points. These frames transfer the loads to the foundations.

The system can easily be extended to the case of multi-bay roofs as shown in Fig. 284, which depicts, under the action of dead loads only, a four-bay roof without internal supports. It will be noticed that lattice girders extend the full depth of each slope, and in consequence, for any given overall width, an increase in the number of lattice girders will result in a reduction in the length of the 'verticals' of the girders, with a consequent reduction of the B.M.s in these members. It is thus economical in framing to utilise a fairly

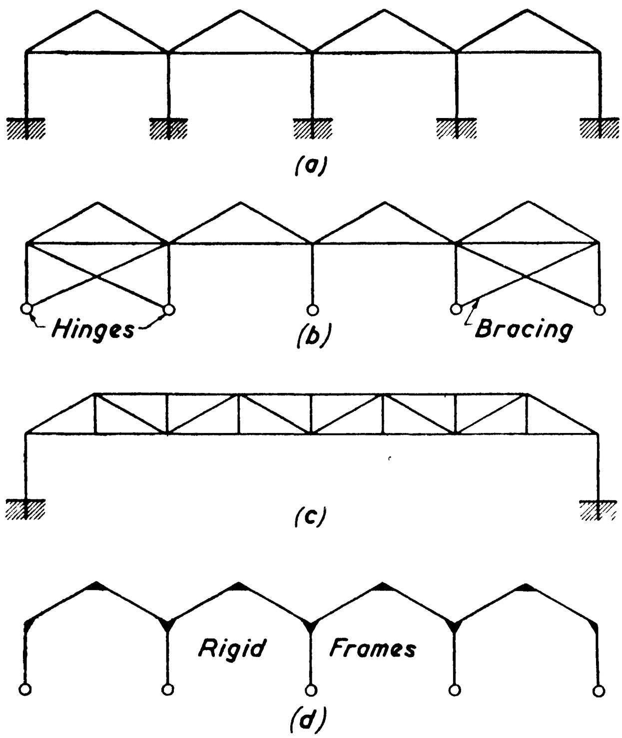


FIG. 285

large number of slopes. Under the action of dead loads only, all lattice girders, other than those in the two outer slopes, receive equal panel point loads at both top and bottom chords provided that the slopes are of the same length and the dead load per unit of area is constant. The two outer girders (*A*) are loaded at their top chords only, the reaction at the eaves end of each 'vertical' being supplied by a post. This top-chord load is the same amount as for all other top-chord loads in the structure.

The action of the roof framing under wind loads is analysed as explained in connection with the example of the north-light roof, but it must be remembered when finding the panel point loads that the reactions at the eaves are vertical, which thus affects the loads in the two slopes *A* and *B*.

The example in Fig. 285 shows several types of end frame. In diagram (a) a tie at eaves level ensures stability of the sloping lattice girders, and the horizontal components of their reactions are taken to foundation level by columns fixed at their bases.

In diagram (b) the three inner columns serve to take vertical components only, and horizontal forces are transferred through the eaves level tie to the braced frames. Alternatively, as shown in diagram (c), supports may be provided at the eaves only, the inclined lattice girders being supported by another lattice girder in a vertical plane. Rigid frames can also be utilised as sketched in diagram (d).

The examples so far discussed have utilised lattice girders of equal depth to the length of the roof slope, but this is not essential. The rafters used in the lattice girders must in all cases span from valley to apex as beams, in addition

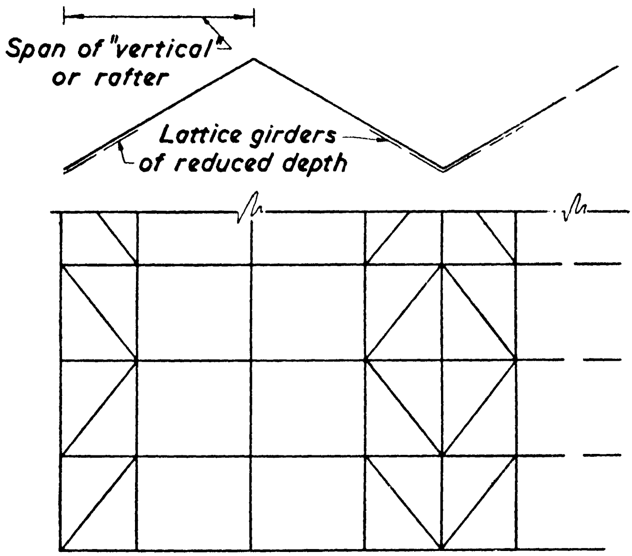


FIG. 286

to forming the struts of the lattice girders of reduced depth as shown in Fig. 286. There must always be at least one lattice girder per slope.

The construction can also be extended to mansard roofs, and Fig. 287 demonstrates this application. The loads from the roof are carried to the node points of the lattice girders by the rafters which also act as the struts in the girders. The loads at the nodes are resolved into the planes of the girders which meet at the nodes as previously explained. Horizontal lattice girders in Fig. 287 (b) and (c) will take no load other than that due to unsymmetrical loading such as wind, as the dead loads are self-cancelling.

A closer approach to the ideal of stressed-skin construction can be obtained if, in any width, the lattice girders are increased in number, their depth being dictated by the purlin spacing. In such cases it may be possible to combine the functions of purlin and lattice girder chord in a single member, and if

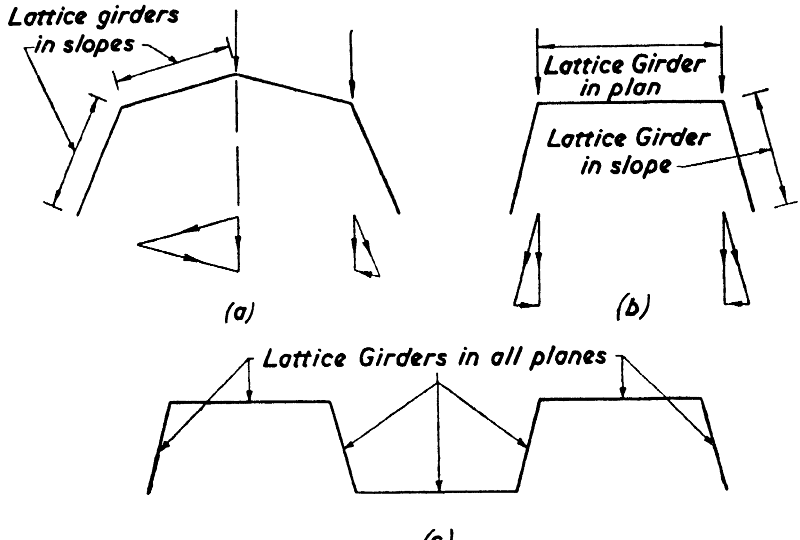


FIG. 287

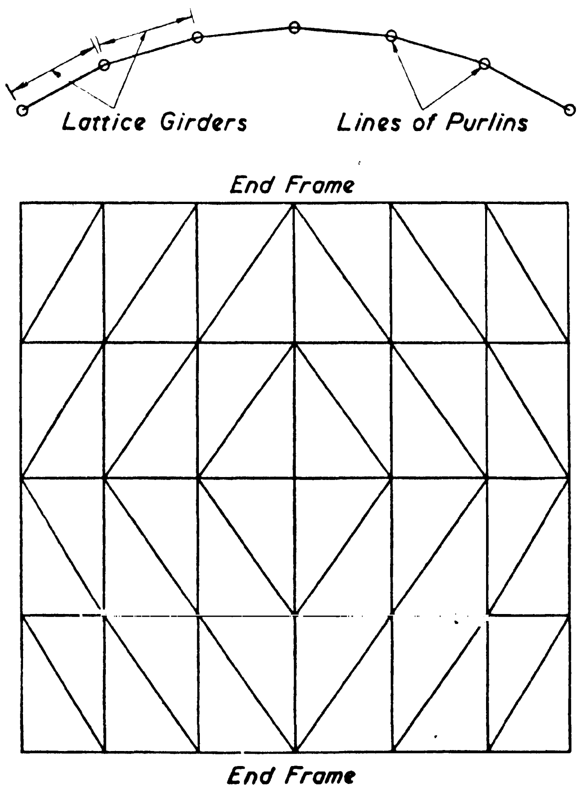


FIG. 288

the intersections of the slopes are made at purlin points as indicated in Fig. 288 there will be no B.M.s in the struts of the lattice girders.

Lattice Girders and Roof Trusses—Stress Data

In the following pages of this section stress coefficients are tabulated for the members of various standard types of lattice girders and roof trusses by which the forces in them may be readily obtained under the loading conditions indicated.

The coefficients are calculated for equal loads at all panel points other than end supports and, for the pitched trusses, they are to be multiplied by the load at one panel point to obtain the force in any member while, for the lattice girders, they are to be multiplied by the panel point load qualifier as indicated in the preamble.

Throughout the tables compression is indicated as negative (–) and tension as positive (+).

Parallel Flange Lattice Girders

In the following tables,

W = the load per panel,

d = constant depth of girder

and p = constant width of panel.

The stress coefficients are to be multiplied by qualifiers as follows:

Type of Member	*Multiply by Qualifier*
Top chords TC Bottom chords BC	$\frac{W.p}{d}$
Verticals V	W
Diagonals D	$\frac{W}{d}\sqrt{(d^2+p^2)}$

Figs. 289 and 290 show two types of girder with even and odd numbers of panels respectively. In both types there are four cases of loading, viz.:

1. Vertical struts, diagonal ties, load on compression flange.
2. Vertical struts, diagonal ties, load on tension flange.
3. Vertical ties, diagonal struts, load on compression flange.
4. Vertical ties, diagonal struts, load on tension flange.

The tables of stress coefficients are given on pages 463 to 465.

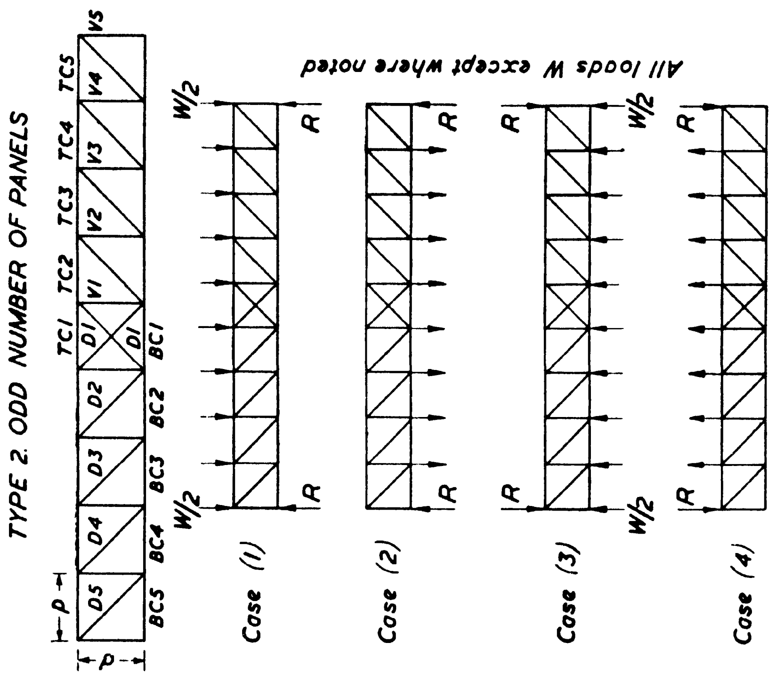


FIG. 290

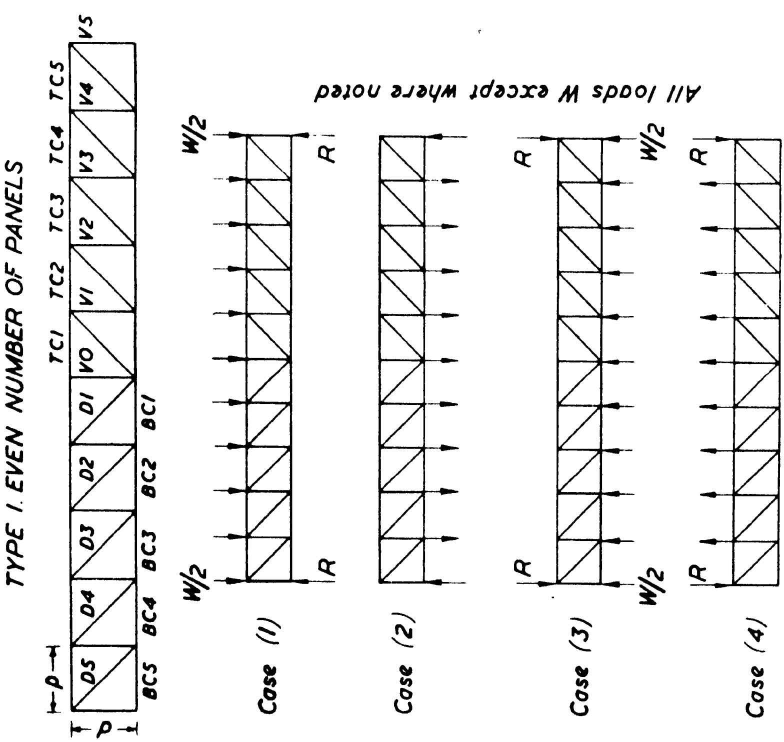


FIG. 289

STRESS COEFFICIENTS FOR LATTICE GIRDERS

TYPE 1 EVEN NUMBER OF PANELS

		Case			
		(1)	(2)	(3)	(4)
2-Panel Girder					
Chords	*TC*1	−0·5	−0·5	+0·5	+0·5
	*BC*1	nil	nil	nil	nil
Verticals	*V*0	−1·0	nil	nil	+1·0
	*V*1	−1·0	−0·5	−0·5	nil
Diagonals	*D*1	+0·5	+0 5	+0·5	+0·5
4-Panel Girder					
Chords	*TC*1	−2·0	−2·0	+2·0	+2·0
	*TC*2	−1·5	−1·5	+1·5	+1·5
	*BC*1	+1·5	+1·5	−1·5	−1·5
	*BC*2	nil	nil	nil	nil
Verticals	*V*0	−1·0	nil	nil	+1·0
	*V*1	−1·5	−0·5	+0·5	+1·5
	*V*2	−2·0	−1·5	−0·5	nil
Diagonals	*D*1	+0·5	+0·5	−0·5	−0·5
	*D*2	+1·5	+1·5	−1·5	−1·5
6-Panel Girder					
Chords	*TC*1	−4·5	−4·5	+4·5	+4·5
	*TC*2	−4·0	−4·0	+4·0	+4·0
	*TC*3	−2·5	−2·5	+2·5	+2·5
	*BC*1	+4·0	+4·0	−4·0	−4·0
	*BC*2	+2·5	+2·5	−2·5	−2·5
	*BC*3	nil	nil	nil	nil
Verticals	*V*0	−1·0	nil	nil	+1·0
	*V*1	−1·5	−0·5	+0·5	+1·5
	*V*2	−2·5	−1·5	+1·5	+2·5
	*V*3	−3·0	−2·5	−0·5	nil
Diagonals	*D*1	+0·5	+0·5	−0·5	−0·5
	*D*2	+1·5	+1·5	−1·5	−1·5
	*D*3	+2·5	+2·5	−2·5	−2·5
8-Panel Girder					
Chords	*TC*1	−8·0	−8·0	+8·0	+8·0
	*TC*2	−7·5	−7·5	+7·5	+7·5
	*TC*3	−6·0	−6·0	+6·0	+6·0
	*TC*4	−3·5	−3·5	+3·5	+3·5
	*BC*1	+7·5	+7·5	−7·5	−7·5
	*BC*2	+6·0	+6·0	−6·0	−6·0
	*BC*3	+3·5	+3·5	−3·5	−3·5
	*BC*4	nil	nil	nil	nil
Verticals	*V*0	−1·0	nil	nil	+1·0
	*V*1	−1·5	−0·5	+0·5	+1·5
	*V*2	−2·5	−1·5	+1·5	+2·5
	*V*3	−3·5	−2·5	+2·5	+3·5
	*V*4	−4·0	−3·5	−0·5	nil
Diagonals	*D*1	+0·5	+0·5	−0·5	−0·5
	*D*2	+1·5	+1·5	−1·5	−1·5
	*D*3	+2·5	+2·5	−2·5	−2·5
	*D*4	+3·5	+3·5	−3·5	−3·5

For qualifiers, see p. 461

STRESS COEFFICIENTS FOR LATTICE GIRDERS—*continued*

TYPE 1—*continued*

		Case			
10-*Panel Girder*		(1)	(2)	(3)	(4)
Chords	*TC*1	-12·5	-12·5	+12·5	+12·5
	*TC*2	-12·0	-12·0	+12·0	+12·0
	*TC*3	-10·5	-10·5	+10·5	+10·5
	*TC*4	-8·0	-8·0	+8·0	+8·0
	*TC*5	-4·5	-4·5	+4·5	+4·5
	*BC*1	+12·0	+12·0	-12·0	-12·0
	*BC*2	+10·5	+10·5	-10·5	-10·5
	*BC*3	+8·0	+8·0	-8·0	-8·0
	*BC*4	+4·5	+4·5	-4·5	-4·5
	*BC*5	nil	nil	nil	nil
Verticals	*V*0	-1·0	nil	nil	+1·0
	*V*1	-1·5	-0·5	+0·5	+1·5
	*V*2	-2·5	-1·5	+1·5	+2·5
	*V*3	-3·5	-2·5	+2·5	+3·5
	*V*4	-4·5	-3·5	+3·5	+4·5
	*V*5	-5·0	-4·5	-0·5	nil
Diagonals	*D*1	+0·5	+0·5	-0·5	-0·5
	*D*2	+1·5	+1·5	-1·5	-1·5
	*D*3	+2·5	+2·5	-2·5	-2·5
	*D*4	+3·5	+3·5	-3·5	-3·5
	*D*5	+4·5	+4·5	-4·5	-4·5

TYPE 2 ODD NUMBER OF PANELS

		Case			
3-*Panel Girder*		(1)	(2)	(3)	(4)
Chords	*TC*1	-1·0	-1·0	+1·0	+1·0
	*TC*2	-1·0	-1·0	+1·0	+1·0
	*BC*1	+1·0	+1·0	-1·0	-1·0
	*BC*2	nil	nil	nil	nil
Verticals	*V*1	-1·0	nil	nil	-1·0
	*V*2	-1·5	-1·0	-0·5	nil
Diagonals	*D*1	nil	nil	nil	nil
	*D*2	+1·0	+1·0	-1·0	-1·0
5-*Panel Girder*					
Chords	*TC*1	-3·0	-3·0	+3·0	+3·0
	*TC*2	-3·0	-3·0	+3·0	+3·0
	*TC*3	-2·0	-2·0	+2·0	+2·0
	*BC*1	+3·0	+3·0	-3·0	-3·0
	*BC*2	+2·0	+2·0	-2·0	-2·0
	*BC*3	nil	nil	nil	nil
Verticals	*V*1	-1·0	nil	nil	+1·0
	*V*2	-2·0	-1·0	+1·0	+2·0
	*V*3	-2·5	-2·0	-0·5	nil
Diagonals	*D*1	nil	nil	nil	nil
	*D*2	+1·0	+1·0	-1·0	-1·0
	*D*3	+2·0	+2·0	-2·0	-2·0

For qualifiers, see p. 461

STRESS COEFFICIENTS FOR LATTICE GIRDERS—*continued*

TYPE 2—*continued*

		Case (1)	Case (2)	Case (3)	Case (4)
7-Panel Girder					
Chords	*TC*1	−6·0	−6·0	+6·0	+6·0
	*TC*2	−6·0	−6·0	+6·0	+6·0
	*TC*3	−5·0	−5·0	+5·0	+5·0
	*TC*4	−3·0	−3·0	+3·0	+3·0
	*BC*1	+6·0	+6·0	−6·0	−6·0
	*BC*2	+5·0	+5·0	−5·0	−5·0
	*BC*3	+3·0	+3·0	−3·0	−3·0
	*BC*4	nil	nil	nil	nil
Verticals	*V*1	−1·0	nil	nil	+1·0
	*V*2	−2·0	−1·0	+1·0	+2·0
	*V*3	−3·0	−2·0	+2·0	+3·0
	*V*4	−3·5	−3·0	−0·5	nil
Diagonals	*D*1	nil	nil	nil	nil
	*D*2	+1·0	+1·0	−1·0	−1·0
	*D*3	+2·0	+2·0	−2·0	−2·0
	*D*4	+3·0	+3·0	−3·0	−3·0
9-Panel Girder					
Chords	*TC*1	−10·0	−10·0	+10·0	+10·0
	*TC*2	−10·0	−10·0	+10·0	+10·0
	*TC*3	−9·0	−9·0	+9·0	+9·0
	*TC*4	−7·0	−7·0	+7·0	+7·0
	*TC*5	−4·0	−4·0	+4·0	+4·0
	*BC*1	+10·0	+10·0	−10·0	−10·0
	*BC*2	+9·0	+9·0	−9·0	−9·0
	*BC*3	+7·0	+7·0	−7·0	−7·0
	*BC*4	+4·0	+4·0	−4·0	−4·0
	*BC*5	nil	nil	nil	nil
Verticals	*V*1	−1·0	nil	nil	+1·0
	*V*2	−2·0	−1·0	+1·0	+2·0
	*V*3	−3·0	−2·0	+2·0	+3·0
	*V*4	−4·0	−3·0	+3·0	+4·0
	*V*5	−4·5	−4·0	−0·5	nil
Diagonals	*D*1	nil	nil	nil	nil
	*D*2	+1·0	+1·0	−1·0	−1·0
	*D*3	+2·0	+2·0	−2·0	−2·0
	*D*4	+3·0	+3·0	−3·0	−3·0
	*D*5	+4·0	+4·0	−4·0	−4·0

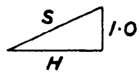
For qualifiers, see p. 461

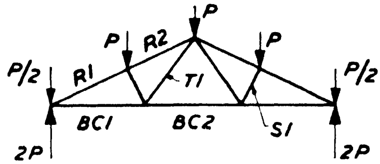
Pitched Roof Trusses

The coefficients for pitched roof trusses are given on the following six pages. It will be noted that charts are provided for span/rise ratios between 3 and 6 and that the expressions for calculation are in terms of S and H, S being the rafter length and H half the span when the rise is unity.

STRESS COEFFICIENTS FOR DEAD LOADS

Coefficients are all in terms of P.





Member	Expression for stress	Span/Rise 3	Span/Rise 4	Span/Rise 5	Span/Rise 6
		1·803 / 1 / 1·5	2·236 / 1 / 2	2·693 / 1 / 2·5	3·162 / 1 / 3
R1	1·5PS	2·71	3·35	4·04	4·74
R2	$R1 - \frac{P}{S}$	2·16	2·90	3·67	4·42
BC1	1·5PH	2·25	3·00	3·75	4·50
BC2	2/3 BC1	1·50	2·00	2·50	3·00
S1	PH/S	0·832	0·894	0·929	0·949
T1	1/3 BC1	0·75	1·00	1·25	1·50

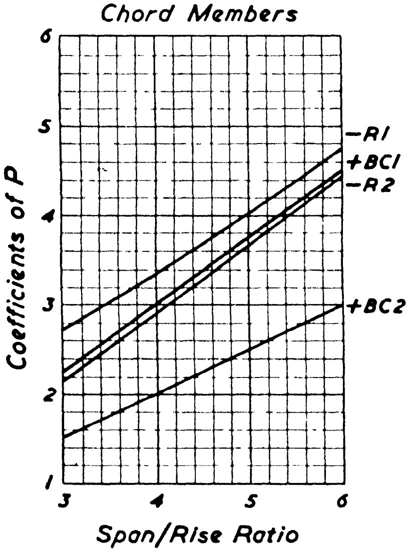
DEAD LOAD STRESSES

To find the stress in any member, multiply the coefficient by P

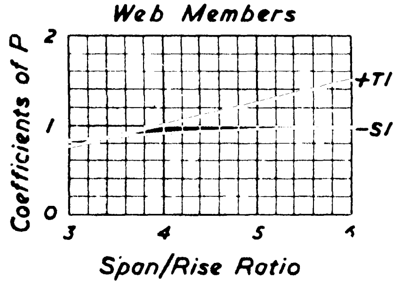
+ = TENSION

− = COMPRESSION

Chord Members

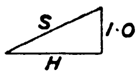


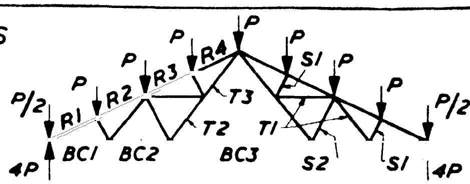
Web Members



STRESS COEFFICIENTS FOR DEAD LOADS

Coefficients are all in terms of P.





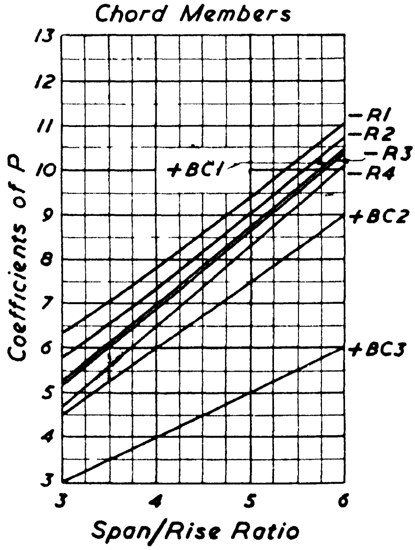
Member	Expression for stress	Span/Rise 3	4	5	6
R1	$3{\cdot}5PS$	6·31	7·83	9·43	11·07
R2	$R1 - \frac{P}{S}$	5·76	7 38	9·05	10·75
R3	$R1 - \frac{2P}{S}$	5·20	6·93	8·68	10·44
R4	$R1 - \frac{3P}{S}$	4·65	6·49	8·31	10·12
BC1	$3{\cdot}5PH$	5·25	7·00	8·75	10·50
BC2	$\frac{6}{7}BC1$	4·50	6·00	7·50	9·00
BC3	$\frac{4}{7}BC1$	3·00	4·00	5·00	6·00
S1	$\frac{PH}{S}$	0·832	0·894	0·929	0·949
S2	$\frac{2PH}{S}$	1·66	1·79	1·86	1·90
T1	$\frac{1}{7}BC1$	0·75	1·00	1·25	1·50
T2	$\frac{2}{7}BC1$	1·50	2·00	2·50	3·00
T3	$\frac{3}{7}BC1$	2·25	3·00	3·75	4·50

DEAD LOAD STRESSES

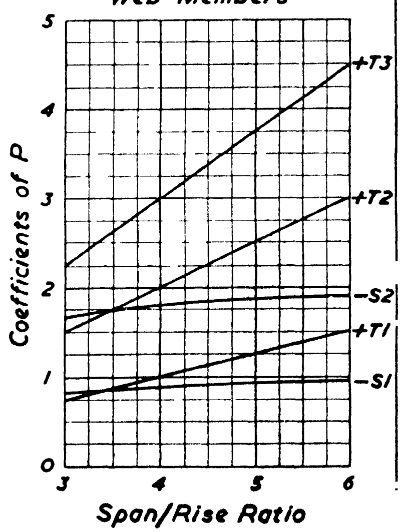
To find the stress in any member, multiply the coefficient by P

+ = TENSION — = COMPRESSION

Chord Members

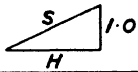


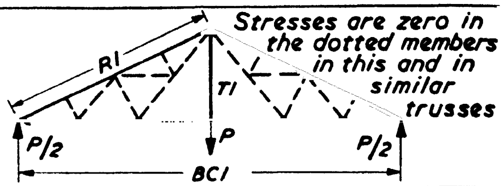
Web Members



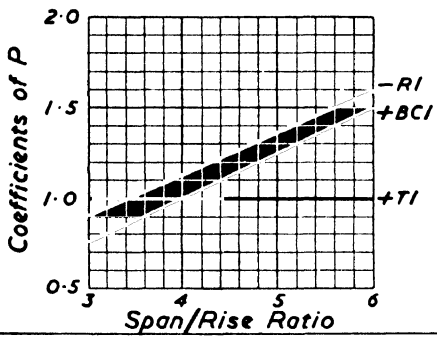
STRESS COEFFICIENTS FOR CEILING LOADS

Coefficients are all in terms of P





Member	Expression for stress	Span/Rise			
		3	4	5	6
R1	$PS/2$	0·9	1·12	1·35	1·58
BC1	$PH/2$	0·75	1·00	1·25	1·50
T1	P	1·00	1·00	1·00	1·00



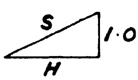
CEILING LOAD STRESSES

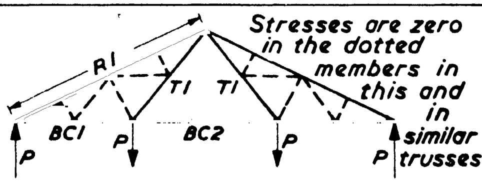
To find the stress in any member, multiply the coefficient by P

+ = TENSION
− = COMPRESSION

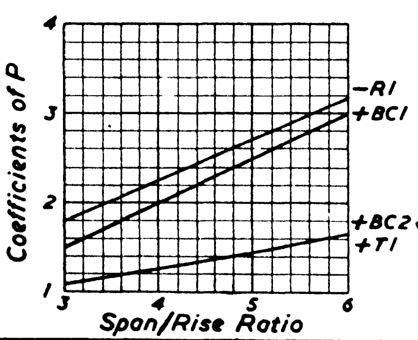
STRESS COEFFICIENTS FOR CEILING LOADS

Coefficients are all in terms of P.





Member	Expression for stress	Span/Rise			
		3	4	5	6
R1	PS	1·80	2·24	2·69	3·16
BC1	PH	1·50	2·00	2·50	3·00
BC2, T1	$P\frac{S^2}{2H}$	1·083	1·25	1·45	1·67



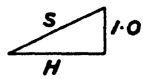
CEILING LOAD STRESSES

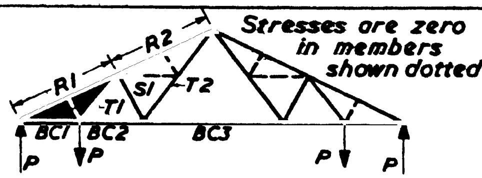
To find the stress in any member, multiply the coefficient by P

+ = TENSION
− = COMPRESSION

STRESS COEFFICIENTS FOR CEILING LOADS

Coefficients are all in terms of P.



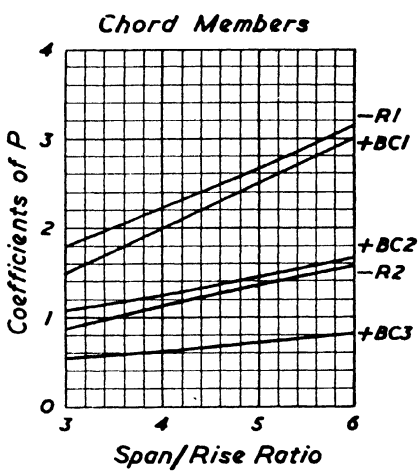


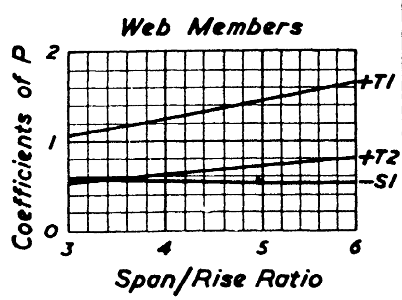
Member	Expression for stress	Span/Rise			
		3	4	5	6
R1	PS	1·80	2·24	2·69	3·16
R2	PS/2	0·90	1·12	1·35	1·58
BC1	PH	1·50	2·00	2·50	3·00
BC2	$P\frac{S^2}{2H}$	1·083	1·25	1·45	1·67
BC3	$\frac{1}{2}$ BC2	0·54	0·625	0·725	0·833
S1	$\frac{PS}{2H}$	0·60	0·56	0·54	0·53
T1	= BC2	1·083	1·25	1·45	1·67
T2	= BC3	0·54	0·625	0·725	0·833

CEILING LOAD STRESSES

To find the stress in any member, multiply the coefficient by P

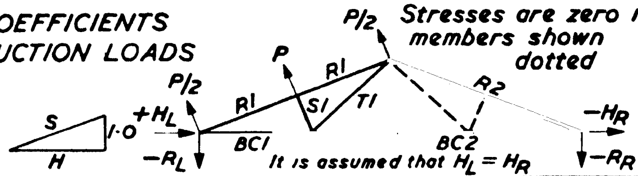
+ = TENSION — = COMPRESSION





STRESS COEFFICIENTS FOR WIND SUCTION LOADS

Coefficients are all in terms of P



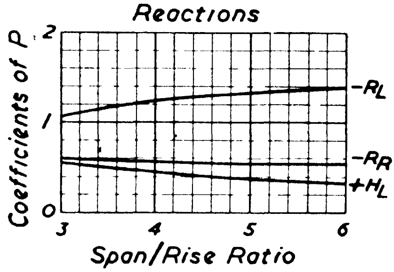
Member	Expression for stress	Span/Rise 3	4	5	6
R_L	$2\left(\frac{H}{S}-\frac{S}{4H}\right)$	1·06	1·23	1·32	1 38
R_R	$S/2H$	0·60	0·56	0·54	0·53
$H_L = H_R$	$1/S$	0·55	0·45	0·37	0·32
R1	$H-\frac{1}{2H}$	1·17	1·75	2·30	2·83
R2	$S^2/2H$	1·08	1·25	1·45	1·67
BC1	$S-\frac{1}{S}$	1·25	1·79	2·32	2·84
BC2	$\frac{S}{2}-\frac{1}{S}$	0·34	0·67	0·98	1·26
S1	1·00	1·00	1·00	1·00	1·00
T1	$S/2$	0·90	1·12	1·35	1·58

STRESSES DUE TO SUCTION

(For Wind Pressures reverse all the signs for stresses given below)

To find the stress in any member multiply the coefficient by P

Reactions

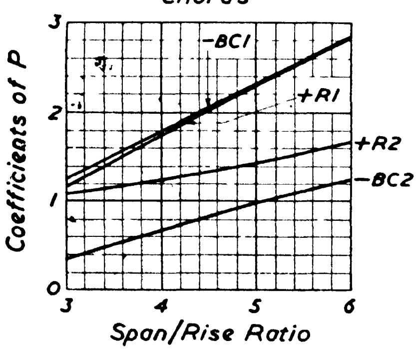


In the two lower charts

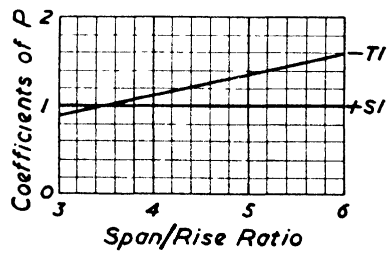
+ = TENSION

− = COMPRESSION

Chords

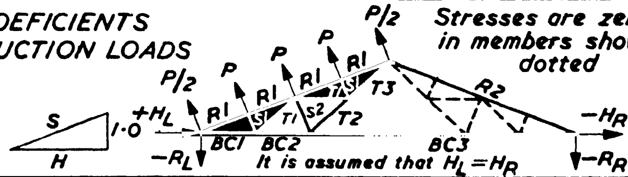


Web Members



STRESS COEFICIENTS FOR WIND SUCTION LOADS

Coefficients are all in terms of P



Stresses are zero in members shown dotted

Member	Expression for stress	Span/Rise			
		3	4	5	6
R_L	$4\left(\frac{H}{S}-\frac{S}{4H}\right)$	2·12	2·46	2·64	2·74
R_R	S/H	1·20	1 12	1·08	1·05
$+H_L=-H_R$	$2/S$	1·11	0·89	0·74	0·63
R1	$\frac{5H}{2}-\frac{1}{H}$	3·08	4·50	5·85	7·17
R2	S^2/H	2 17	2·50	2·90	3·33
BC1	$\frac{5S}{2}-\frac{2}{S}$	3·40	4 70	5·99	7 27
BC2	$2\left(S-\frac{1}{S}\right)$	2 50	3·58	4·65	5·69
BC3	$S-\frac{2}{S}$	0·69	1·34	1·95	2·53
S1	1·00	1·00	1·00	1·00	1·00
S2	2·00	2·00	2·00	2·00	2·00
T1	$S/2$	0·90	1·12	1·35	1·58
T2	S	1·80	2 24	2·69	3·16
T3	$\frac{3S}{2}$	2·70	3·35	4·04	4·74

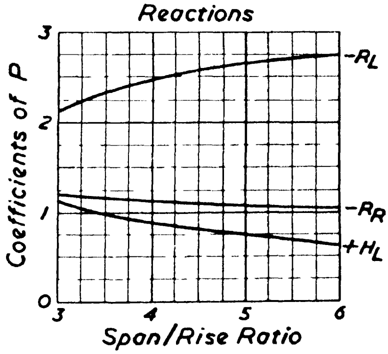
STRESSES DUE TO SUCTION

(For Wind Pressures reverse all the signs for stresses given below.)

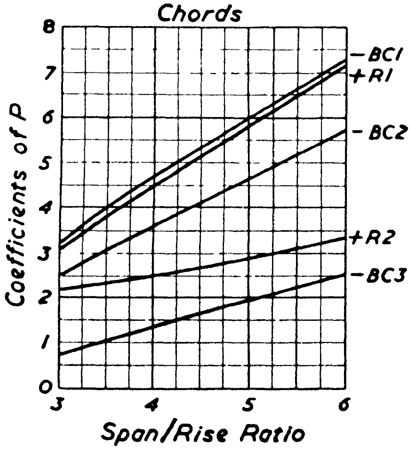
To find the stress in any member, multiply the coefficient by P

In the two lower charts
+ = TENSION
− = COMPRESSION

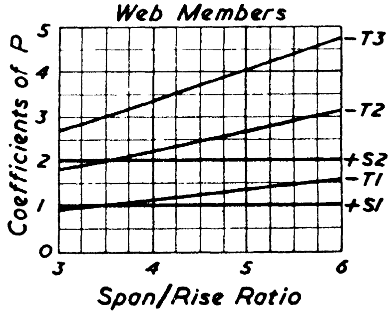
Reactions



Chords



Web Members



APPROXIMATE WEIGHTS OF WELDED ROOF TRUSSES

Roof Slope: 1 in 2·5 Centres of trusses 12′-6″

Approximate Formulae:

Weight in lb./sq. ft.

Part Glazed – $\left[1{\cdot}1 + \dfrac{\text{Plan Area}}{825}\right]$

Sheeted – $\left[1{\cdot}1 + \dfrac{\text{Plan Area}}{1000}\right]$

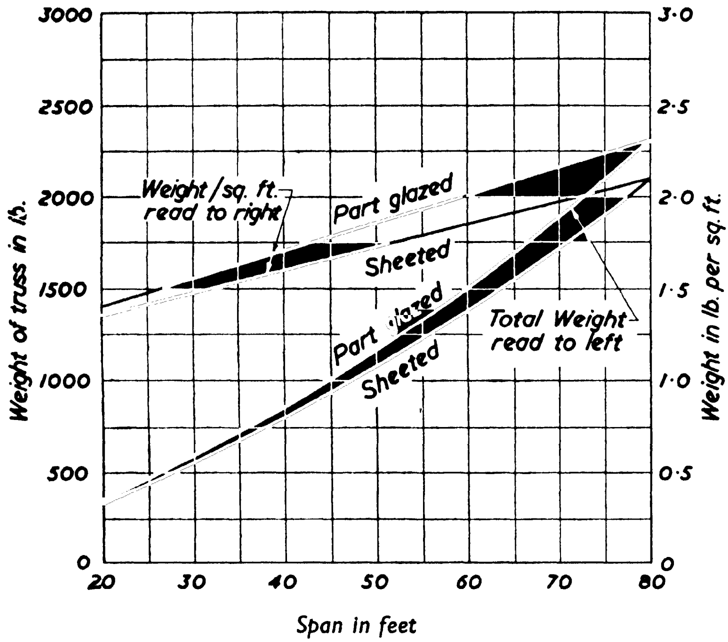
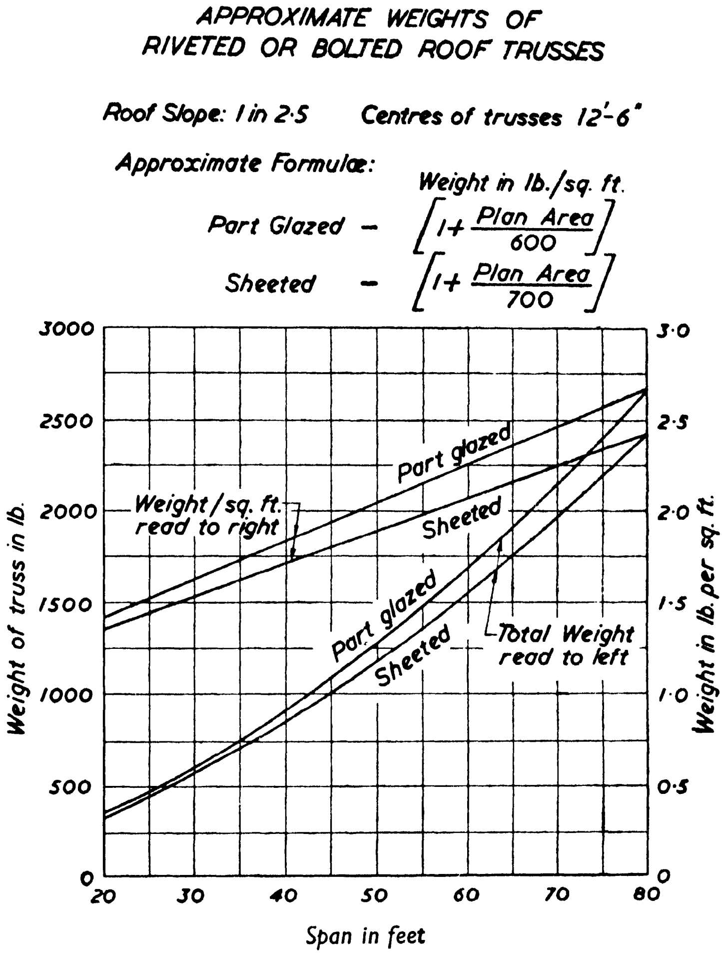


FIG. 291


APPROXIMATE WEIGHTS OF
RIVETED OR BOLTED ROOF TRUSSES
Roof Slope: 1 in 2·5
Centres of trusses 12'-6"
Approximate Formulæ:
Weight in lb./sq. ft.
Part Glazed – $\left[1+\frac{\text{Plan Area}}{600}\right]$
Sheeted – $\left[1+\frac{\text{Plan Area}}{700}\right]$
3000
2500
2000
1500
1000
500
0
3·0
2·5
2·0
1·5
1·0
0·5
0
20
30
40
50
60
70
80
Weight/sq. ft. read to right
Part glazed
Sheeted
Part glazed
Sheeted
Total Weight read to left
Weight of truss in lb.
Weight in lb. per sq. ft.
Span in feet
FIG. 292

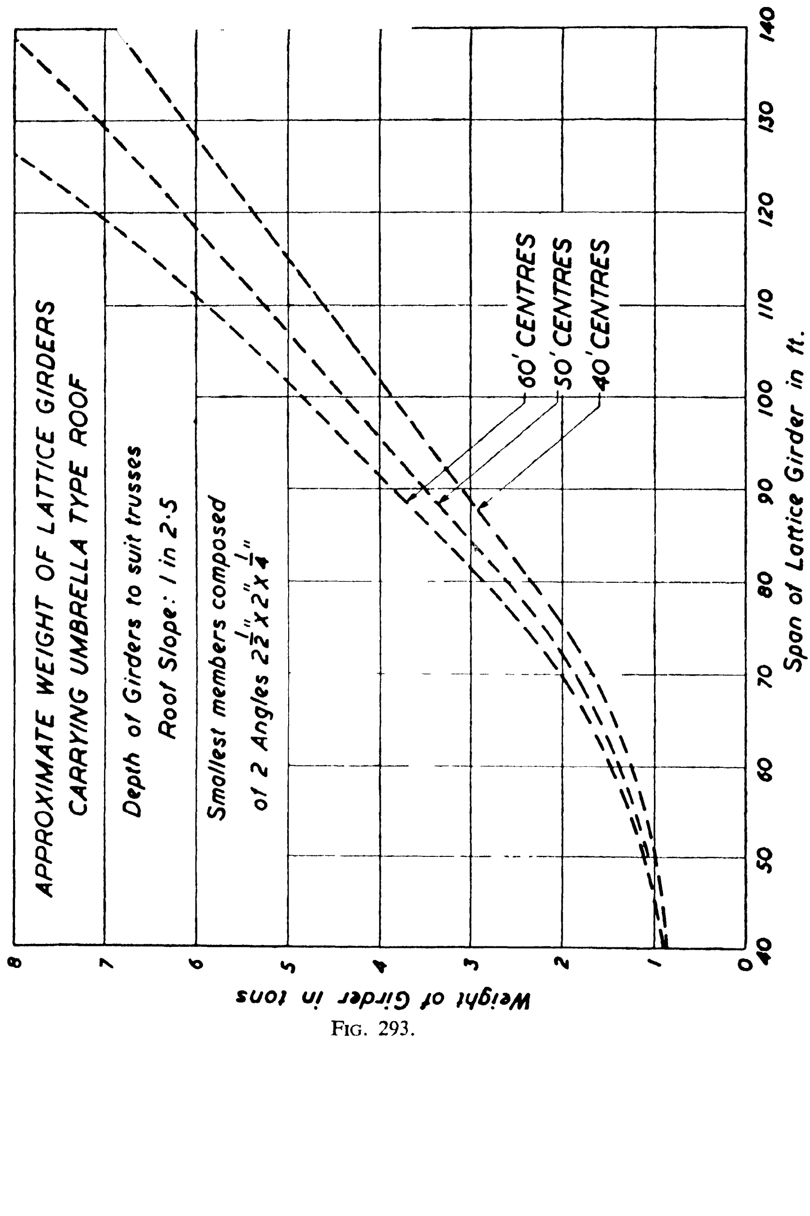

APPROXIMATE WEIGHT OF LATTICE GIRDERS
CARRYING UMBRELLA TYPE ROOF
Depth of Girders to suit trusses
Roof Slope: 1 in 2·5
Smallest members composed
of 2 Angles $2\frac{1}{2}'' \times 2'' \times \frac{1}{4}''$
60' CENTRES
50' CENTRES
40' CENTRES
40
50
60
70
80
90
100
110
120
130
140
Span of Lattice Girder in ft.
0
1
2
3
4
5
6
7
8
Weight of Girder in tons

FIG. 293.

19. NODE CONNECTIONS

THE common assumptions made in the determination of forces in lattice frames are that the external loads are applied at the node points or joints and that all the members of the frame meeting at a point are represented by lines which meet at that point. Logically, it therefore follows that the frame should be detailed in accordance with the design assumptions. This means that the

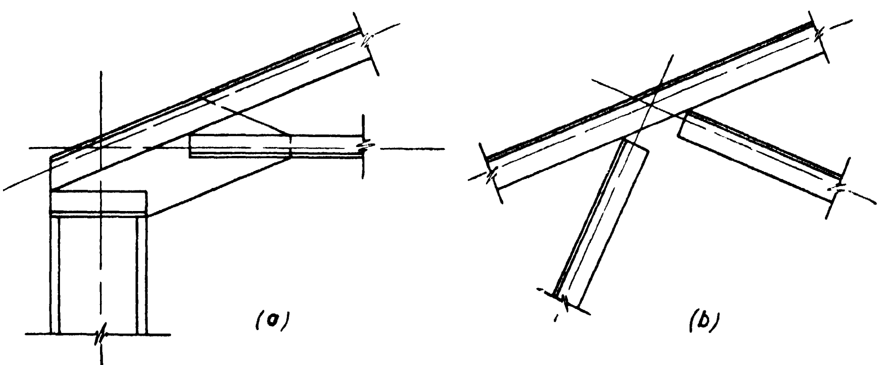


FIG. 294

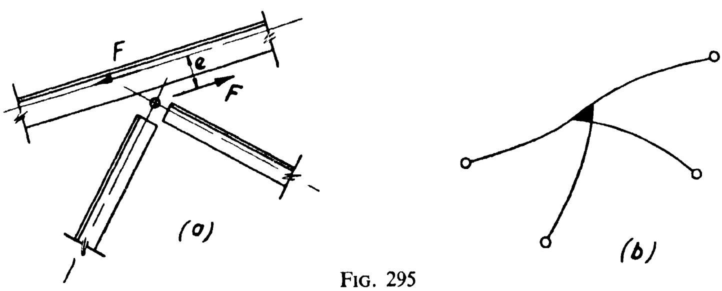


FIG. 295

members should be set out so that the lines of their C.G.s or gravity axes meet at points.

Two common cases of the correct procedure are illustrated in Fig. 294.

If the gravity axes do not intersect, then in addition to the direct forces in the members there will be a B.M. in the joint. See Fig. 295 (a).

This B.M. will be distributed into the members meeting at the joint in proportion to their stiffnesses. See Fig. 295 (b).

The types of joint shown in Fig. 294 are more economical than that shown in Fig. 295 from the standpoint of the members, but they do involve somewhat larger gusset plates.

Fig. 296 illustrates a method of reducing the size of the gusset without losing the advantage of having the gravity axes meeting at a point. The cropping of the angles will, of course, add to the cost.

While it is a comparatively simple matter to arrange that the members meeting at a joint carry direct forces only, it is not so simple to arrange that the gusset plates themselves carry direct forces only.

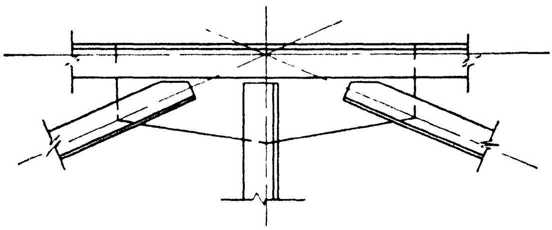


FIG. 296

In order that there shall be no B.M. in the gusset plate, the resultant force at any section must lie on the centre line of the plate at that section. It will be found that this can rarely be done except on the simplest of connections, and consequently the gusset plates should be investigated from the point of view of B.M. as well as direct load.

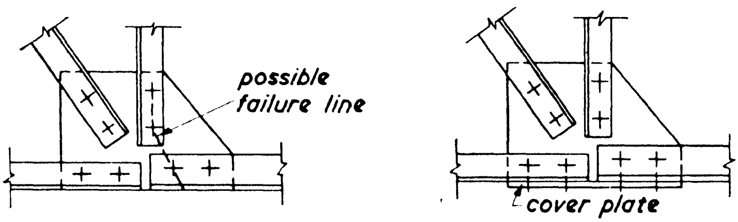


FIG. 297

Where members which are substantially in the same straight line are discontinuous at a joint, the gusset plate should not be used as both gusset and cover plate unless the calculated stresses show that it is capable of doing so. The use of cover plates in such cases is, in general, more economical than the use of a gusset in the dual capacity. Figs. 297 and 298 illustrate the two most common instances of this type of connection.

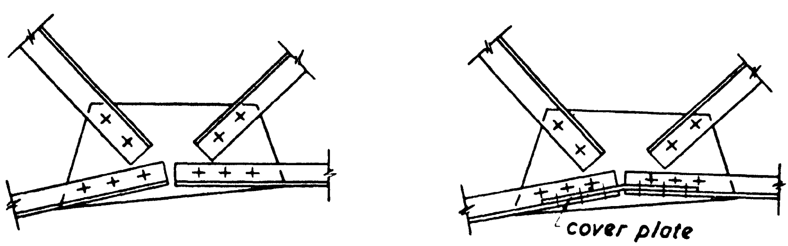


FIG. 298

20. FLAT PLATES

Contributed by C. C. Pounder, M.I.C.E., M.I.Mech.E.

FLAT plates may be divided into two general groups, viz. those which are simply supported at their perimeters—of which the bolted plate or cover is the nearest approximation in practice—and those having their edges rigidly fixed or built-in, i.e. encastré. Each group contains a variety of possible forms. Sometimes a simple plate of uniform thickness is sufficient for a particular purpose. Sometimes it is necessary to employ ribbing. In large plates exposed to high pressures a double thickness of plating may be desirable, the plates being kept the required distance apart by connecting ribs.

The fundamental differential equation for the contour of a deflected plate and the expression for the necessary boundary conditions were established well over a century ago. Strictly, the assumptions made in the theory limit the solutions to thin plates having very small deflections. The results of the theory, however, can be applied with reasonable accuracy to practical cases. Complete solutions have been obtained for: (a) circular plates, either simply supported or fixed at the perimeter, with any distribution of concentric loads; (b) elliptical plates fixed at the perimeter and exposed to either a constant or a uniformly varying pressure.

An accurate solution for the rectangular plate has not yet been determined. The difficulty arises from the absence of a complete solution to the equation which fulfils the boundary conditions. Until this difficulty is overcome rational and approximate treatment will have to serve.

(a) Plates without Stiffening Ribs

Encastré Rectangular Plate Supporting a U.D.L. Grashof's Rule

The expression which is universally quoted for the strength of rectangular encastré plates of uniform thickness, as shown in Fig. 299, is that of Dr. Grashof.

Grashof's expression is:

$$f=\frac{1}{2}\times\frac{L^4}{L^4+B^4}\times\frac{B^2}{t^2}\times p \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where f = maximum stress in lb. per sq. in.,

L = length of plate in in.,

B = breadth of plate in in.,

t = thickness of plate in in.

and p = pressure on plate in lb. per sq. in.

The corresponding formula for the maximum deflection, which occurs at the centre of the plate, is

$$d=\frac{L^4}{L^4+B^4}\times\frac{pB^4}{384EI}=\frac{L^4}{L^4+B^4}\times\frac{pB^4}{32Et^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Grashof's formulæ have been shown by experiment to be approximately correct for values of L/B exceeding 2, but when the plate is square, i.e. $L/B=1$, the stress is underestimated by 28 per cent. The reasons for the discrepancy in the case of approximately square plates may be demonstrated by reference to Fig. 299. Consider the central strip *ab* which is the most heavily stressed and will therefore govern the design of the plate. If this

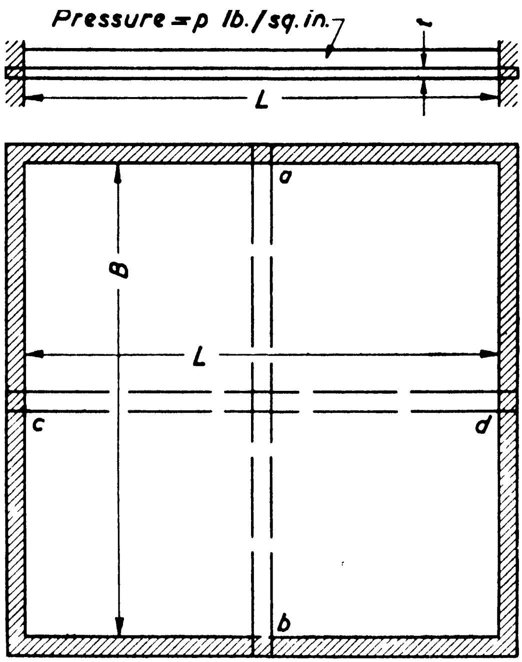


FIG. 299

narrow strip, which is of unit width, were completely isolated from the remainder of the plate, it would act as an encastré beam, the span being B and the unit pressure being p.

The maximum B.M. would be $pB^2/12$ and the maximum stress would be

$$\frac{M}{Z}=f=\frac{pB^2}{12}\times\frac{6}{t^2}$$
$$=\frac{1}{2}\times\frac{B^2}{t^2}\times p.$$

The maximum deflection at the centre of the strip would be $d=pB^4/384EI$.

Similarly, if a unit strip *cd* were isolated the maximum stress would be

$$f=\frac{1}{2}\times\frac{L^2}{t^2}\times p$$

and the maximum deflection would be $pL^4/384EI$.

However, as the plate is homogeneous the strips *ab* and *cd* cannot deflect at will. Consequently, some of the load is transferred into the plate adjoining

the strips, the amount of transfer being a function of the deflection of the plate.

Consider the intersection of the strips *ab* and *cd*. Let the unit pressure on *ab* be p_1 and that on *cd* be p_2. Then $p_1 + p_2 = p$.

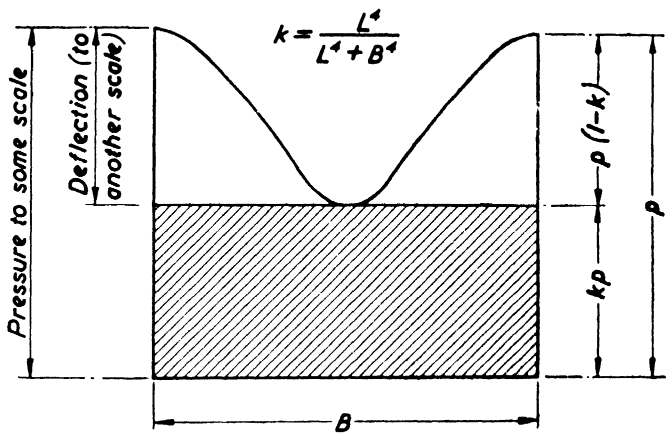
If it were assumed that the pressures p_1 *and* p_2 *were constant over ab and cd respectively,* then the maximum deflection in *ab* would be $d_1 = p_1 B^4/384EI$ and in *cd* would be $d_2 = p_2 L^4/384EI$.

But d_1 must equal d_2.

Hence $p_1 B^4 = p_2 L^4$, and $p_1/p_2 = L^4/B^4$, but $p = p_1 + p_2$.

Therefore $$\frac{p_1}{p} = \frac{p_1}{p_1 + p_2} = \frac{L^4}{L^4 + B^4}.$$

Hence, on this assumption, the value $L^4/(L^4 + B^4)$ is the fraction of the load which the strip *ab* would actually take compared with the load it would take if isolated. This gives rise to Grashof's equation (1).



GRASHOF'S RULE

FIG. 300

Unfortunately, this assumption is untrue as the pressures are not constant along the central strips *ab* and *cd*. At the ends of the strips, as there is no deflection, the pressure must be *p* but it gradually diminishes, at varying rates along each strip, until the centre of the plate is reached. The effect of these changes in pressure may be shown by Fig. 300, which is a diagram relating to an approximately square plate. The shaded portion shows the load which Grashof's formula assumes to act throughout the strip *ab*. The unshaded portion is the amount it ignores.

Obviously rational formulæ must take the unshaded portion into account. A number of investigators have evolved coefficients to modify Grashof's basic formulæ, but here it is proposed to give those evolved by C. C. Pounder, which agree very closely indeed with experimental data.

Encastré Rectangular Plate—Pounder's Formulæ

After making allowance for the unshaded portions of the diagram in Fig. 300 Pounder produced the following formulæ:

Maximum skin stress in plate,

$$f=\frac{kpB^2}{2t^2}\left[1+\frac{11}{35}(1-k)+\frac{79}{141}(1-k)^2\right] \text{ lb. per sq. in.} \quad . . . (3)$$

where $k=L^4/(L^4+B^4)$ and the remaining symbols have the significance of equation (1).

The maximum skin stress occurs at the side of the plate at the mid-point of its length.

Equation (3) can be rewritten thus:

$$f=K_R\times\frac{pB^2}{t^2} \text{ lb. per sq. in.} \quad . . . . (4)$$

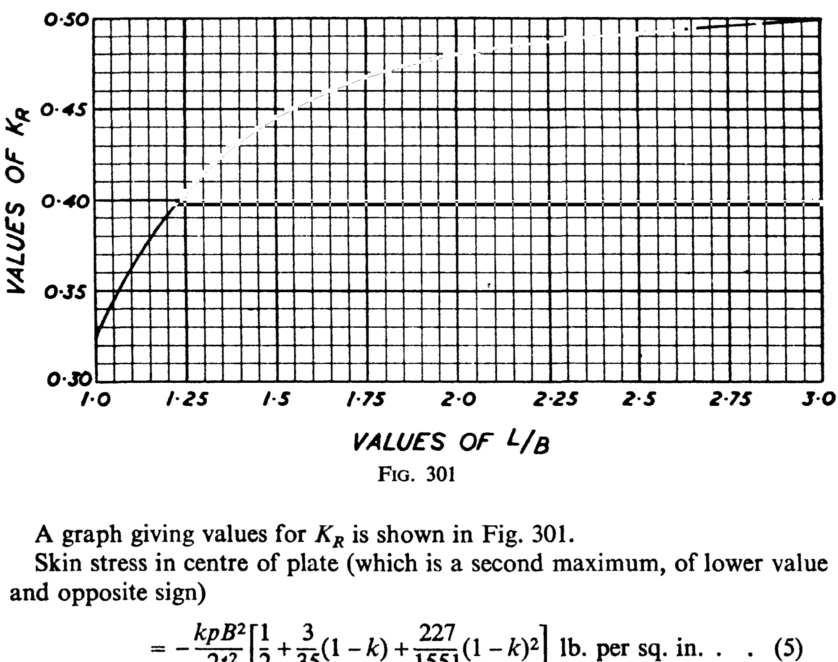


FIG. 301

A graph giving values for K_R is shown in Fig. 301.

Skin stress in centre of plate (which is a second maximum, of lower value and opposite sign)

$$=-\frac{kpB^2}{2t^2}\left[\frac{1}{2}+\frac{3}{35}(1-k)+\frac{227}{1551}(1-k)^2\right] \text{ lb. per sq. in.} \quad . . (5)$$

The skin stress attains a third maximum value in the middle of the short edge of the plate.

Skin stress at middle of short side of plate

$$=\frac{k_1pL^2}{2t^2}\left[1+\frac{11}{35}(1-k_1)+\frac{79}{141}(1-k_1)^2\right] \text{ lb. per sq. in.} \quad . . (6)$$

$$k_1=\frac{B^4}{L^4+B^4}.$$

The skin stress at the edge of the plate decreases from its maximum value at the middle of the long sides, towards the corners of the plate, and from a lower maximum in the middle of the short sides towards the corners.

The stress pattern in the region of the corners is very complex.

Pounder's formula for the maximum deflection, in the centre of the plate, is

$$d=\frac{m^2-1}{m^2}\times\frac{kpB^4}{32Et^3}\left[1+\frac{47}{210}(1-k)+\frac{200}{517}(1-k)^2\right] \text{ in.} \quad . \; . \; (7)$$

where $1/m$ = Poisson's ratio and the remaining symbols are as before; m may be taken as 3·0 or 3·5.

The Behaviour of Clamped Plates

In a deflected plate there are curvatures in all directions at every point, and the curvature in any definite direction is due primarily to the stress in that direction, but secondarily to the stresses and strains in other directions. Curvature in any definite direction does not mean that there are bending stresses only in that direction, or at all, and conversely the absence of curvature does not imply the absence of bending stresses in that direction.

In clamped plates, or in any of the approximations which occur in practice, there is a likelihood of the stress at the edge of the plate not being so high as that given by equation (3), by reason of the portion within the clamping giving relief to the plate in the region of maximum stress; in such designs the stress in the middle of the plate may be somewhat increased as a consequence. This tendency to equalisation is not a disadvantage, but rather may it be a gain, so long as the edges do not yield to such an extent as to set up a stress in the middle of the plate in excess of that which would have been realised at the edges were the plate non-yielding.

Simply Supported Rectangular Plates

Just as a simply supported beam is weaker than a built-in beam, so a simply supported plate is weaker than one which is encastré.

It should be noted that the corners of a simply supported plate tend to lift under load, and arrangements should be made to hold them down.

A formula in much use for supported plates is one attributed to Navier. It is:

$$\text{Maximum stress}=\frac{0{\cdot}92pB^2}{t^2}\times\frac{L^4}{(L^2+B^2)^2} \text{ lb. per sq. in.} \quad . \; . \; (8)$$

For a plate whose ratio of length to breadth is great, this expression gives reasonable results, but for a square plate it shows a stress less than that of Grashof's equation for an encastré plate, a result obviously not sound.

Pounder, using the form of formulæ given for encastré plates, has produced the following formula for the maximum skin stress in a plate subjected to a U.D.L.:

$$f=\frac{3}{4}\frac{kpB^2}{t^2}\left[1+\frac{14}{75}(1-k)+\frac{20}{57}(1-k)^2\right] \text{ lb. per sq. in.} \quad . \; . \; (9)$$

This can be rewritten:

$$f = K_s \times \frac{pB^2}{t^2} \text{ lb per sq. in.}$$

A graph giving values for K_s is shown in Fig. 302.

This stress occurs at the centre of the plate. The maximum deflection, also in the centre of the plate, is

$$d = \frac{m^2 - 1}{m^2} \times \frac{5kpB^4}{32Et^3} \times \left[1 + \frac{37}{175}(1-k) + \frac{79}{201}(1-k)^2\right] \text{ in.} \quad . \quad (10)$$

The limits of dimensions for rectangular plates are when the plates are square and when they are infinitely long relative to their breadth. For practical purposes the upper L/B ratio can be taken as 3, above which the values for maximum stresses and deflections do not change.

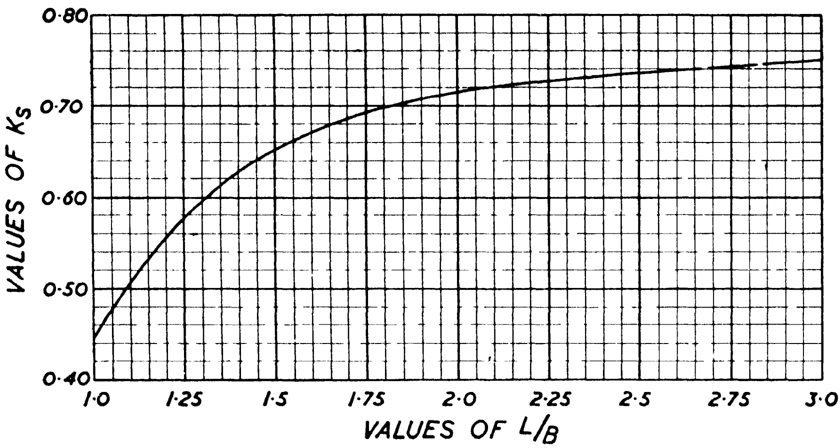


FIG. 302

Encastré Circular Plate of Uniform Thickness

The complete theory of the circular plate was given by Grashof, based upon an earlier analysis by Poisson, and experiment has shown it to be near the truth. The two forms in which the expression for the maximum stress is given are:

$$f_{max} = \frac{1}{6} \times \frac{pD^2}{t^2} \text{ lb. per sq. in.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

$$f_{max} = \frac{3}{16} \times \frac{pD^2}{t^2} \text{ lb. per sq. in.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

where p = pressure in lb. per sq. in.,
D = diameter of plate in in.
and t = thickness in in.

The first equation assumes that the maximum principal strain determines the elastic strength of the plate; the second, that the maximum principal stress is the criterion. As British engineers tend to follow the maximum principal stress theory, equation (12) is the one which should be used.

Simply Supported Circular Plate of Uniform Thickness

The remarks made for the encastré plate equally apply here. There are two expressions for the elastic strength of a circular plate simply supported at its perimeter, based respectively on the maximum principal strain and the maximum principal stress theories. They are:

$$f_{\max} = \frac{5}{24} \times \frac{pD^2}{t^2} \text{ lb. per sq. in.} \quad . \quad . \quad . \quad . \quad (13)$$

$$f_{\max} = \frac{5}{16} \times \frac{pD^2}{t^2} \text{ lb. per sq. in.} \quad . \quad . \quad . \quad . \quad . \quad (14)$$

where the symbols are as for the encastré circular plate. Equation (14) should be used.

(b) Ribbed Plates

Encastré Rectangular Plate (ribs in both directions)

A plate of constant thickness is always to be preferred to a ribbed plate. But for large sizes, and for heavy pressures, the plate of uniform thickness becomes too heavy and too costly. Ribbed plates are then used. The advantage of a ribbed construction is that, weight for weight, it is much stronger and stiffer than a simple plate, the disposition of metal giving an enhanced value both to the moment of inertia and the section modulus. Its drawback is the opportunity offered by corners of ribs for flaws to occur and for cracks to develop.

A ribbed plate bends as a whole and also bends locally between the ribs. See Fig. 303. In calculating the strength of a ribbed plate, the stress across the whole section must be added to the stress in the plate due to local bending between the ribs. That is, the results of equations (16) and (3) must be added together.

If I_e = moment of inertia of the whole plate parallel to long side L, taken at the edge of the plate, with y_p and y_r as the distances of the top of the plate face and the underside of the ribs from the neutral axis respectively, as shown in Fig. 304, then:

Maximum skin stress at middle of long side, at underside of rib

$$= f_r = y_r \times \frac{kpB^2L}{12I_e}\left[1 + \frac{11}{35}(1-k) + \frac{79}{141}(1-k)^2\right] \text{ lb. per sq. in.} \quad (15)$$

Maximum skin stress at middle of long side on top of plate

$$= f_p = y_p \times \frac{kpB^2L}{12I_e}\left[1 + \frac{11}{35}(1-k) + \frac{79}{141}(1-k)^2\right] \text{ lb. per sq. in.} \quad (16)$$

The stresses given by the above equations are only accurate for plates wherein the ribs in both the long and short sides are the same in depth, thickness and pitch—that is, the plate is ribbed into squares. It is quite common to make the ribs which are perpendicular to the long sides of greater pitch than those perpendicular to the short sides. With this arrangement the stress at the middle of the long sides is less than one would expect from the number

of ribs stiffening the long sides, while the stress at the middle of the short sides is greater than the number of ribs would lead one to believe. The point is not usually of importance.

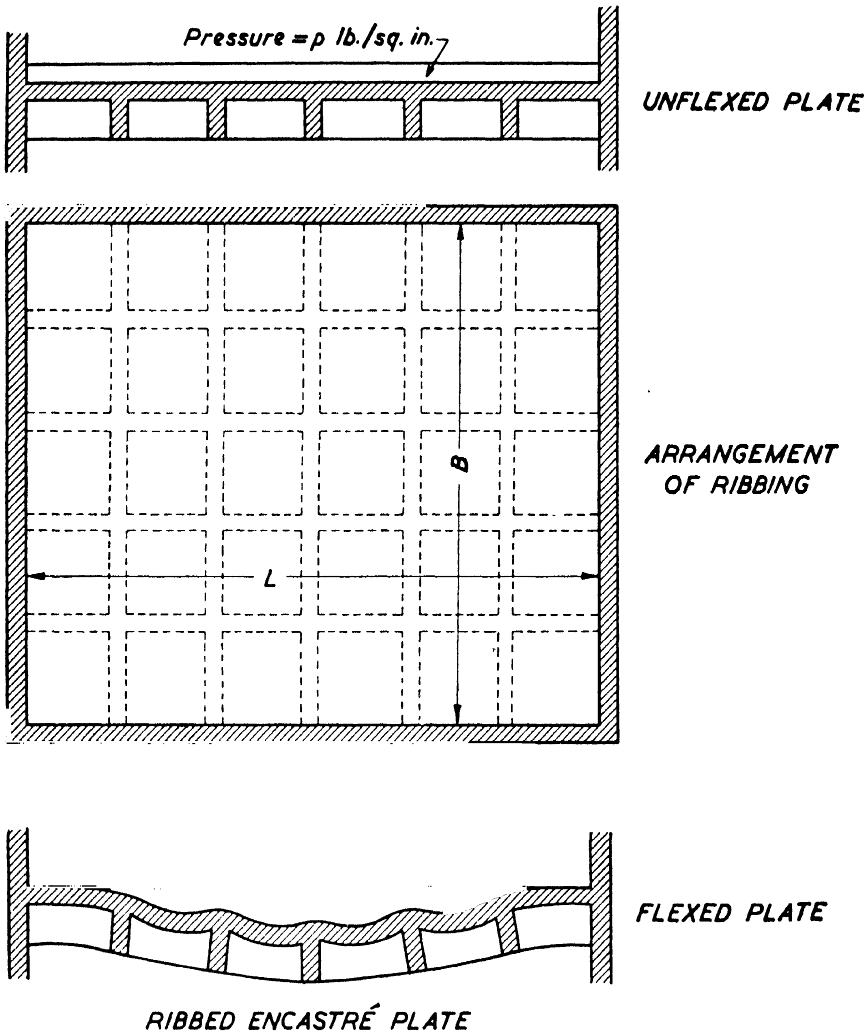


FIG. 303

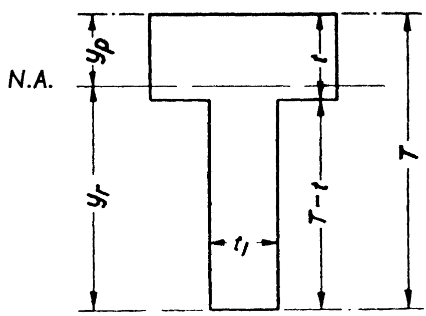
Plates of other Forms and Fixings

In their ribbed forms, the simply supported rectangular plate, the encastré circular plate and the simply supported circular plate are not amenable to satisfactory treatment. For supported plates, systems of equating the B.M. —caused by the pressure—against the moment of resistance of the plate material across the diagonal, or the diameter, can, however, be reasonably comparative.

(c) General Details

To obtain maximum strength for weight of metal used, the ribs in any plate should, where possible, be arranged to carry a compressive stress. That is, in a bolted plate, whether rectangular or circular, the ribs should be provided on the face of the plate exposed to pressure. In an encastré plate the sign of the stress changes across the plate, and the ribs should be placed on the face remote from the load to ensure a compressive stress in the ribs at the region of maximum stress.

If ribs cannot be placed on the faces named—and in many designs they cannot by reason of external circumstances—then the designer must reduce rib stresses by all legitimate means. With ribs in tension, especially under fluctuating loads, fatigue of material, initial flaws, undue stress magnitudes,



ELEMENTAL RIB

FIG. 304

etc., can combine to induce cracks which grow and deepen until the structure fails.

The system of ribbing to be used for any plate necessarily depends on the way the plate will fail if exposed to destructive loading. Thus, a rectangular bolted plate of uniform thickness may be not quite strong enough of itself, and light ribbing may be needed. As such a plate would ultimately fail across the diagonals, a suitable arrangement of ribbing would be one giving maximum strength to these places. To avoid a local aggregation of material where the ribs meet obliquely, with possibility of initial strains, a circular rib may be arranged. The circular rib breaks up the continuity of the straight ribs—but this is not normally important. Two perpendicular ribs intersecting at the centre would stiffen the plate, but not to the same degree, weight for weight, as they would not increase the modulus across the diagonal very much. A single rib across the plate would have but little value.

For rectangular plates rigidly held at their edges, the ribs should be perpendicular and not diagonal. One heavy rib across the plate in a direction parallel to the long side would be effective in reducing the breadth of the unsupported area. Two ribs intersecting at the centre are better.

For bolted and rigidly held circular plates, a single rib across a diameter is

helpful, but two arranged perpendicularly are stronger. Where heavier ribbing is needed the ribs may be radial or perpendicular.

Differences of opinion always exist as to the most suitable proportions for ribbing. If a plate requires strengthening it does not seem logical to use ribs which are much thinner than the plate to be stiffened. Broad ribs, not too deep, are advisable for general use. The pitch of the ribs must be determined by the general circumstances of the design.

Deep ribs are not always the source of strength which they are intended to be. This is only to be expected when it is recollected that in a heavy plate having deep, thin ribs, the neutral axis is a long way from the free edge of the ribs, and the stress at any plane in a section is directly proportional to its distance from the neutral axis. If thin, deep ribs cannot, for some reason, be avoided, then an ample number should be provided.

It is desirable, especially in important plates, to make calculations for a few alternative schemes of ribbing, chiefly as regards sizes, proportions and dispositions, to ensure that the maximum real strength from the ribbing is obtained.

The determination of the scantlings for any design should take into proper account the kind of load experienced under running conditions. It is no use assuming a steady load if the load is not steady, nor is it any use to allow 50 per cent for eventualities when a little investigation would show the necessity for an allowance of ten times this amount. Probably more failures take place as the result of underestimation of the magnitudes of forces and loads, and the misinterpretation of their behaviour and effect, than from any other cause.

In fabricated steel constructions, ribbing should be reduced to the limit. A welding ought not to be given a casting-like shape. In fact, it is an error of conception to design a fabricated steel plate so that it has the appearance of a complicated casting, if only because such designs are disproportionately expensive.

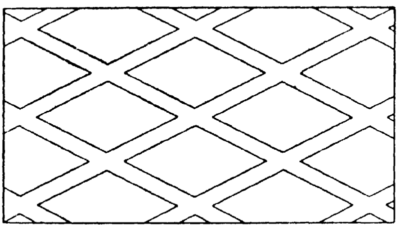
21. FLOOR PLATES

ROLLED-STEEL floor plates are obtainable in increments of $\frac{1}{16}$ in between $\frac{3}{16}$ in. and $\frac{3}{4}$ in. in the four patterns shown in the diagrams below. These plates may be cut to any sizes within the maximum lengths, widths and areas shown on the next page.

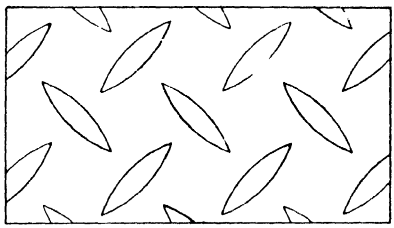
On the two following pages are safe load tables for rectangular plates either simply supported or encastré on all four sides. These have been prepared on the assumption that the maximum permissible stress is 10 tons per sq. in. It should be noted that deflections for simply supported plates are about five times as great as for encastré plates.

Pounder's formulæ 3, 7, 9 and 10 were used in the preparation of the tables, the value of *m* being taken as 3.

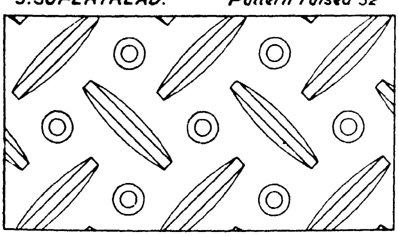
1. DIAMOND. *Pattern raised* $\frac{1''}{16}$



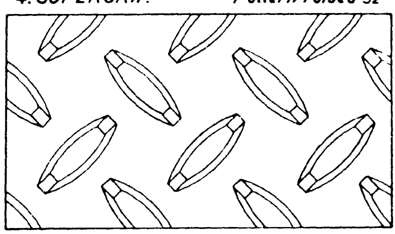
2. DURBAR. *Pattern raised* $\frac{1''}{16}$



3. SUPERTREAD. *Pattern raised* $\frac{3''}{32}$



4. SUPERGRIP. *Pattern raised* $\frac{3''}{32}$



Note: Patterns 1, 3, and 4 are made by the Consett Iron Company, Ltd., Consett, County Durham, and Patterns 1 and 2 are made by the South Durham Steel and Iron Company, Ltd., Cargo Fleet, Middlesbrough.

Weights and Dimensions of Floor Plates

Patterns 1 and 2:

Thickness on Plain	*Weight*		*Maximum Sizes—Patterns* 1 *and* 2		
	Pattern 1	*Pattern* 2	*Length*	*Width*	*Area*
in.	*lb. per sq. ft.*	*lb. per sq. ft.*	*ft.*	*in.*	*sq. ft.*
$\frac{3}{16}$	9·0	8·7	30	72	150
$\frac{1}{4}$	11·5	11·25	30	72	200
$\frac{5}{16}$	14·0	13·8	30	72	200
$\frac{3}{8}$	16·3	16·35	30	72	180
$\frac{7}{16}$	18·85	18·9	30	72	160
$\frac{1}{2}$	21·4	21·45	30	72	150
$\frac{9}{16}$	24·0	24·0	30	72	140
$\frac{5}{8}$	26·5	26·55	30	72	130
$\frac{11}{16}$	29·0	29·1	30	72	120
$\frac{3}{4}$	31·5	31·65	30	72	100

Patterns 3 and 4:

Thickness on Plain	*Weight*		*Maximum Sizes—Patterns* 3 *and* 4		
	Pattern 3	*Pattern* 4	*Length*	*Width*	*Area*
in.	*lb. per sq. ft.*	*lb. per sq. ft.*	*ft.*	*in.*	*sq. ft.*
$\frac{3}{16}$	8.42	8·71	25	60	80
$\frac{1}{4}$	10.97	11·26	30	63	100
$\frac{5}{16}$	13·52	13·81	30	66	130
$\frac{3}{8}$	16·07	16·36	30	68	130
$\frac{7}{16}$	18·62	18·91	30	68	130
$\frac{1}{2}$	21·17	21·46	30	68	130
$\frac{9}{16}$	23·72	24·01	20	66	110
$\frac{5}{8}$	26·27	26·56	20	66	110
$\frac{11}{16}$	28·82	29·11	20	66	110
$\frac{3}{4}$	31·37	31·66	20	66	110

RECTANGULAR FLOOR PLATES STRESSED TO 10 TONS PER SQ. IN.

Nett Thickness on plain in in.	*Safe Uniformly Distributed Loads in lb. per sq. ft. for plates simply-supported on four sides. The deflections of plates to the right of the zig-zag line exceed B/100* — Length L in feet: 2.5	3	3.5	4	4.5	5	5.5	6	Breadth B in ft.
3/16	284	226	201	188	181	176	174	172	2.5
		197	162	144	134	128	125	122	3
			145	121	109	101	97	93	3.5
1/4	505	403	357	334	321	313	309	306	2.5
		351	288	256	239	228	222	217	3
			258	216	194	180	172	166	3.5
				198	168	152	141	134	4
					156	135	122	114	4.5
						126	111	101	5
5/16	790	630	558	521	502	489	482	478	2.5
		549	449	400	373	357	346	339	3
			403	337	303	281	268	260	3.5
				309	263	237	221	210	4
					244	211	191	178	4.5
						198	173	157	5
3/8	1137	907	803	751	723	704	694	688	2.5
		791	647	576	537	514	499	489	3
			580	485	436	405	386	374	3.5
				444	379	342	318	302	4
					351	304	275	256	4.5
						284	249	227	5
7/16	1548	1234	1093	1022	984	959	945	937	2.5
		1076	881	784	731	700	679	665	3
			790	660	593	551	526	[illegible]	3.5
				605	515	465	432	411	4
					478	414	374	348	4.5
						387	339	308	5
1/2	2022	1612	1427	1335	1285	1252	1234	1224	2.5
		1406	1150	1023	954	914	886	869	3
			1037	862	775	720	686	664	3.5
				790	673	608	564	537	4
					624	540	489	455	4.5
						506	443	403	5
							418	370	5.5
								351	6
9/16	2558	2041	1807	1689	1626	1585	1562	1549	2.5
		1779	1456	1295	1208	1156	1122	1100	3
			1306	1092	981	911	869	841	3.5
				1000	852	769	714	679	4
					790	684	618	576	4.5
						640	560	510	5
							529	468	5.5
								444	6
5/8	3159	2519	2230	2086	2007	1957	1928	1912	2.5
		2196	1797	1599	1491	1428	1385	1358	3
			1612	1348	1211	1125	1072	1038	3.5
				1235	1052	949	882	839	4
					975	844	763	711	4.5
						790	692	630	5
							652	577	5.5
								549	6
11/16	3822	3048	2699	2524	2429	2368	2333	2314	2.5
		2657	2174	1935	1804	1727	1676	1643	3
			1951	1631	1465	1361	1298	1256	3.5
				1494	1273	1149	1067	1015	4
					1180	1021	924	860	4.5
						956	837	762	5
							790	699	5.5
								664	6
3/4	4548	3628	3212	3003	2890	2818	2777	2754	2.5
		3163	2588	2303	2147	2056	1994	1955	3
			2322	1941	1743	1620	1544	1495	3.5
				1778	1514	1367	1270	1208	4
					1405	1215	1099	1023	4.5
						1138	996	907	5
							940	831	5.5
								790	6

RECTANGULAR FLOOR PLATES STRESSED TO 10 TONS PER SQ. IN.

Nett Thickness on plain in in.	*Safe Uniformly Distributed Loads in lb. per sq. ft. for plates encastré on all four sides. No deflection exceeds B/100* — Length L in feet: 2·5	3	3·5	4	4·5	5	5·5	6	Breadth B in ft.
3/16	388	321	291	276	267	262	259	257	2·5
		269	228	207	196	189	185	182	3
			198	171	156	147	142	138	3·5
1/4	689	571	519	491	476	467	461	457	2·5
		479	406	369	349	337	329	324	3
			352	304	277	262	252	245	3·5
				269	236	217	204	196	4
					213	188	174	164	4·5
						172	155	143	5
5/16	1077	893	810	768	743	729	720	714	2·5
		748	635	576	545	526	514	506	3
			549	475	433	410	394	384	3·5
				421	369	338	319	307	4
					332	294	271	256	4·5
						269	242	223	5
3/8	1551	1286	1167	1105	1070	1050	1037	1029	2·5
		1077	914	829	785	758	740	729	3
			791	684	624	590	568	553	3·5
				606	532	487	460	441	4
					479	424	391	369	4·5
						388	348	321	5
7/16	2111	1750	1588	1504	1456	1429	1412	1400	2·5
		1466	1244	1129	1068	1031	1007	992	3
			1077	931	849	803	773	753	3·5
				824	724	663	626	601	4
					651	577	532	502	4·5
						528	474	437	5
1/2	2757	2286	2074	1965	1902	1867	1844	1828	2·5
		1914	1625	1474	1395	1347	1315	1296	3
			1407	1216	1110	1048	1009	983	3·5
				1077	946	866	818	785	4
					851	754	695	655	4·5
						689	619	571	5
							570	514	5·5
								479	6
9/16	3489	2893	2625	2487	2407	2363	2333	2314	2·5
		2423	2056	1866	1766	1705	1665	1640	3
			1780	1539	1404	1327	1277	1244	3·5
				1363	1197	1096	1035	993	4
					1077	954	879	829	4·5
						872	783	723	5
							721	651	5·5
								606	6
5/8	4308	3571	3241	3070	2972	2917	2880	2857	2·5
		2991	2538	2304	2180	2104	2055	2025	3
			2198	1900	1733	1638	1577	1536	3·5
				1683	1478	1354	1278	1226	4
					1330	1178	1086	1023	4·5
						1077	967	893	5
							890	803	5·5
								748	6
11/16	5212	4321	3921	3715	3596	3529	3486	3457	2·5
		3620	3072	2788	2638	2546	2487	2451	3
			2659	2299	2098	1982	1908	1859	3·5
				2036	1788	1638	1546	1484	4
					1609	1425	1314	1239	4·5
						1303	1170	1080	5
							1077	972	5·5
								905	6
3/4	6203	5143	4667	4421	4280	4200	4148	4114	2·5
		4308	3655	3317	3139	3030	2960	2916	3
			3165	2735	2496	2359	2271	2212	3·5
				2423	2128	1949	1840	1766	4
					1915	1696	1563	1474	4·5
						1551	1392	1286	5
							1282	1157	5·5
								1077	6

22. BUNKERS

THIS section deals with the design of a typical bunker of steel construction, the requirements being as follows:

The bunker to be of single-compartment trough type with a capacity of not less than 500 tons when filled with coal, trimmed level with the top. The weight of the coal will be taken as 56 lb. per cu. ft. The length and width of the bunker to be about 50 ft. and 25 ft. respectively, and the inside surface to be provided with an anti-attrition lining of concrete, or blue-brick tiles. Assume that the bunker will be supported by steel stanchions and that provision is to be made for one coal-charging conveyor and four coal outlets.

General Considerations

The stipulated capacity and dimensions indicate, as will be seen later, that the cross-section of the bunker will necessarily include a parallel-sided upper part and a trough bottom, the first of which provides more capacity per ton of structure and should therefore be as deep as possible. If the vertical sides of the upper part are not more than about 8 ft. deep they could be conveniently transported as shop-fabricated units, to save expensive assembly work at the site. The trough sides, however, must be steep enough to ensure effective discharge and for this reason the angle between the side and the horizontal should be not less than 50°, preferably more. With flat slopes the tendency is for discharge to occur more from the centre of the bunker, the side material being retained indefinitely, which may cause spontaneous combustion. An angle of 56° 19′ gives a very convenient rise of 1½ to 1, whilst a rise of 2 to 1 or even 2½ to 1 is often used in order to avoid dead material in the valleys of pyramidal hoppers because the material passing to the outlet is mainly that from an inverted cone.

Principal Dimensions

Fig. 305 gives the principal dimensions of the bunker, as determined from the foregoing considerations, and the capacity is very simply calculated as follows:

Part A.	1 × 51·5 ft. × 26·5 ft. × 7·5 ft. =	10,250	cu. ft.
,, B.	1 × 39·5 ft. × 18·0 ft. × 2·5 ft. =	1,780	,,
Parts C.	2 × 27·5 ft. × 12·0 ft. × 9·0 ft. =	5,940	,,
,, D.	4 × 12·0 ft. × 12·0 ft. × 6·0 ft. =	3,460	,,
		21,430	,,
Deduct lining, approx. 3500 sq. ft. × 3 in. =		875	,,
Coal at 56 lb./cu. ft. ×		20,555	= 515 tons.

Coal Pressure

In assessing the pressure of the coal on the plating throughout the depth of the bunker, it is questionable whether extreme refinement can be justified, because a number of variables enter into the complete design. The weight of the stored material may vary up to 10 per cent either way from the assumed value, depending upon the moisture present, whilst the angle of repose is rarely a precise figure and the permissible stresses and deflections in the structure vary with individual designers and purchasers.

The Rankine formula is commonly used for finding the lateral pressure from the stored material. Let w represent the weight of the material per cu. ft., h the height from the level being considered to the surface of the

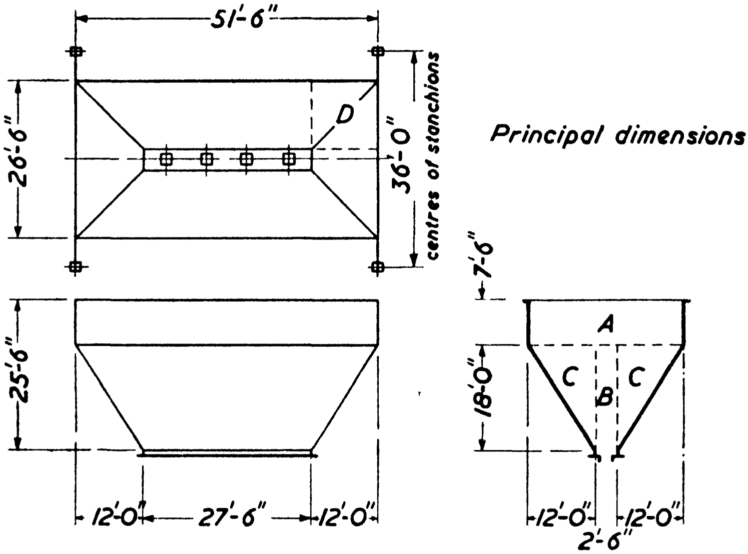


FIG. 305

material and ϕ the angle of repose of the material. Then the horizontal pressure p in lb. per sq. ft.

$$= wh\frac{1-\sin\phi}{1+\sin\phi}.$$

With coal at 56 lb./cu. ft. and $\phi = 35°$,

$$p = 56 \times h \times 0{\cdot}271,$$

which for practical purposes can be taken as

$$= 15h \text{ lb./sq. ft.,} \quad \text{or} \quad 0{\cdot}0067 \times h \text{ tons/sq. ft.}$$

For the design of the trough plate and stiffeners it is necessary to know the pressure normal to the plate, and a simple method of finding this is shown in Fig. 306.

Diagram (i) gives the proportions of the trough slope, and diagram (ii)

the base and rise dimensions for a slope length of one foot. For one foot length of the bunker the vertical load

$$V = 56 \text{ lb./cu. ft.} \times h \times 0{\cdot}555 \text{ ft.} = 31{\cdot}08h \text{ lb.}$$

and the horizontal force

$$H = 15 \text{ lb.} \times h \times 0{\cdot}832 \text{ ft.} = 12{\cdot}48h \text{ lb.}$$

Resolve these into normal and tangential forces by similar triangles, thus:

$$N = 0{\cdot}555V + 0{\cdot}832H = 17{\cdot}25h + 10{\cdot}38h = 27{\cdot}63h \text{ lb.}$$

$$S = 0{\cdot}832V - 0{\cdot}555H = 25{\cdot}86h - 6{\cdot}93h = 18{\cdot}93h \text{ lb.}$$

The dead weight of the trough and the lining must be taken into account in computing the total forces N and S.

As the force N acts on one foot slope length of the trough, it is the normal

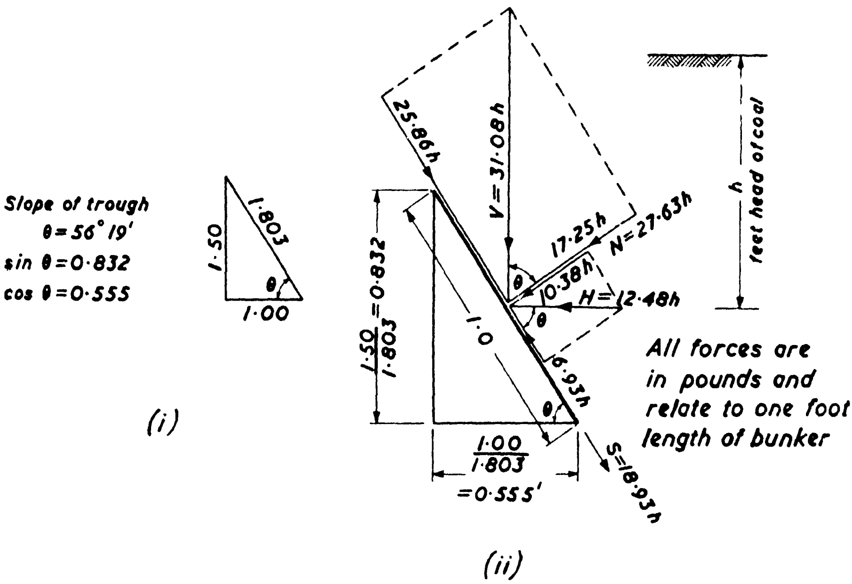


FIG. 306

pressure per sq. ft. required for the plate and stiffeners. The force S is also per sq. ft. of the sloping length, and forms part of the total tension in the trough construction. The other part is the component of the tension in the bottom plate, as shown later in Fig. 308. If the trough were of plate construction only, or if the stiffeners were arranged horizontally, the tension would have to be taken up in the plate only, whilst if the stiffeners ran from top to bottom, which is more usual for large bunkers, they would have to be designed to take all the tension.

Trough Plate

With a simple arrangement of stiffeners running from top to bottom and plain plate between, the maximum B.M. in the plate is $WL/12$ when it is fully continuous over the stiffeners. The moment is greater when the plate is not

fully continuous such as at the joining edges, and in this case $WL/10$ is usually taken. Where joints occur over every stiffener, $WL/8$ should be taken. These increased moments could be brought nearer to $WL/12$ by providing adequate cover plates and rivets, but the cost of doing so will offset the saving on plate thickness, and cover plates are not very desirable inside a bunker.

At the bottom of the bunker the trough plate joins the bottom plate and can there be deemed as supported on three sides. To allow for this, take h as the depth of the bunker less say 1 ft. 6 in., then the pressure N normal to the plate is

$$27{\cdot}63 \times 24{\cdot}0 = 663 \text{ lb./sq. ft.}$$

Add dead weight of plate and lining,

$$40{\cdot}0 \text{ lb./sq. ft.} \times \cos 56^\circ\ 19' = \underline{\ \ 22} \text{ lb./sq. ft.}$$

$$w = \underline{685} \text{ lb./sq. ft.}$$

For convenience in calculation convert this to 4·75 lb./sq. in., then for a B.M. of $WL/12$ and a permissible stress of 18,000 lb./sq. in., the maximum span L for a $\frac{3}{8}$-in. plate can be computed.

$$L^2 = \frac{0{\cdot}375^2 \times 12 \times 18{,}000}{4{\cdot}75 \times 6} = 1066,$$

hence $L = 32{\cdot}7$ in.

For $WL/10$ the limit of span would be 29·8 in. whilst for $\frac{1}{2}$-in. plate the limits would be 43·5 in. and 39·7 in. respectively. The question of which thickness to use can be left until the stiffener design is considered, because economy is a function of the two.

Trough Stiffeners

The vertical and horizontal loads imposed upon the bunker are shown in Fig. 307. They are lettered (a) to (f) and include the dead weight of the trough and the lining. The dead weight of the 8-ft. vertical side is not included because it does not affect the stiffener design. The external reactions required to maintain equilibrium are numbered (1) to (4). All values are for one foot length of the bunker and half the width, which is symmetrical about the centre.

With coal at 56 lb./cu. ft. = 0·025 tons/cu. ft., the loads are as follows:

Horizontal pressure p at hinge $B = 0{\cdot}0067 \times 7{\cdot}5$ ft. $= 0{\cdot}05$ tons/ sq. ft.,

Hence Load (a) = 7·5 ft. × 0·025 = 0·19 ton,

and Load (b) = 18 ft. × 0·05 = 0·90 ton.

At bottom of bunker,

$$p = 0{\cdot}0067 \times 25{\cdot}5 \text{ ft.} = 0{\cdot}17 \text{ ton/sq. ft.},$$

hence $$\text{Load (c)} = 18 \text{ ft.} \times \frac{0{\cdot}17 - 0{\cdot}05}{2} = 1{\cdot}08 \text{ tons.}$$

Total horizontal load = 0·19 + 0·90 + 1·08 = 2·17 tons.

(Check: $0{\cdot}0067 \times 25{\cdot}5^2/2 = 2{\cdot}18$.)

Vertical Loads:

(d) coal, 12 ft. × 7·5 ft. × 0·025 = 2.25 tons
d.w. of plate, stiffeners
and lining, 22 ft. × 56 lb. = 0·55 ton

2·80 tons

(e) coal 12 ft. × 9 ft. × 0·025 = 2·70 tons

(f) coal, 1·25 ft. × 25·5 ft. × 0·025 = 0·80 ton
bottom plate, lining, valves = 0·20 ton

1·00 ton

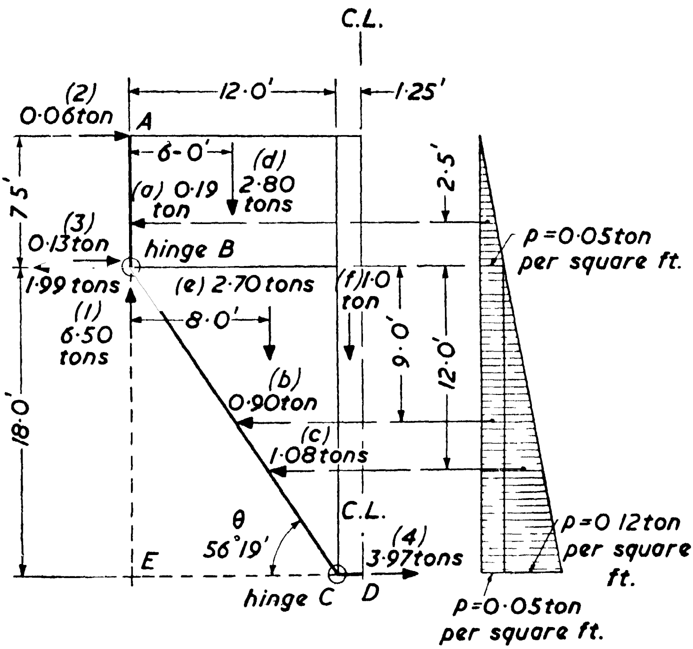


FIG. 307

For computing the reactions, Load (f) must be taken as a shear at hinge C, and this is always the case whatever the width of the bunker bottom plate.

Reaction (1) = 2·80 + 2·70 + 1·00 tons = 6·50 tons.
Reaction (2) = one-third of Load (a) = 0·06 ton.
Reaction (3) above hinge B = two-thirds of (a) = 0·13 ton.

As reaction (4) must be horizontal in order to balance the other half of the

bunker, compute for equilibrium about point *E*, then Reaction (3) from loads acting on the trough

$$= \frac{2{\cdot}8 \times 6 \text{ ft.} + 2{\cdot}7 \times 8 \text{ ft.} + 1{\cdot}0 \times 12 \text{ ft.} - 0{\cdot}9 \times 9 \text{ ft.} - 1{\cdot}08 \times 6 \text{ ft.}}{18 \text{ ft.}}$$

$$= \frac{50{\cdot}40 - 14{\cdot}58}{18} = 1{\cdot}99 \text{ tons.}$$

For equilibrium about hinge *B*,

$$\text{Reaction (4)} = \frac{50{\cdot}40 + 0{\cdot}9 \times 9 \text{ ft.} + 1{\cdot}08 \times 12 \text{ ft.}}{18 \text{ ft.}} = 3{\cdot}97 \text{ tons.}$$

(Check: $+0{\cdot}06 + 0{\cdot}13 + 3{\cdot}97 - 0{\cdot}19 - 1{\cdot}99 - 0{\cdot}90 - 1{\cdot}08 = 0$.)

The reactions at hinges *B* and *C* can be resolved into components representing direct tension and shear in the trough construction. At hinge *B* there is an upward reaction of 6·5 tons and a leftward reaction of 1·99 hence by graphics or calculation, as the slope of the trough is 1½ to 1, the tension is

$$6{\cdot}5 \times 0{\cdot}832 + 1{\cdot}99 \times 0{\cdot}555 = 6{\cdot}51 \text{ tons}$$

and the shear is

$$6{\cdot}5 \times 0{\cdot}555 - 1{\cdot}99 \times 0{\cdot}832 = 1{\cdot}95 \text{ tons.}$$

At hinge *C* the tension in the trough is

$$3{\cdot}97 \times 0{\cdot}555 + 1{\cdot}0 \times 0{\cdot}832 = 3{\cdot}04 \text{ tons}$$

and the shear is

$$3{\cdot}97 \times 0{\cdot}832 - 1{\cdot}0 \times 0{\cdot}555 = 2{\cdot}76 \text{ tons.}$$

Fig. 308 is a graphic resolution of

(i) the trough side loads
(ii) the trough bottom load
and (iii) the resultant tension and end shears,

proving the calculated values.

The end shears of 1·95 tons at hinge *B* and 2·76 tons at hinge *C* represent the total load normal to the trough and causing bending. Hence $W = 4{\cdot}71$ tons distributed as shown in Fig. 308 (iii). The exact B.M. for this trapezoidal load can be calculated, but it is approximately 0·127 *WL*, hence

$$\text{B.M.} = 0{\cdot}127 \times 4{\cdot}71 \times 21{\cdot}6 = 13{\cdot}0 \text{ tons ft. per foot length of bunker.}$$

In addition, however, there is a direct tension which at the centre of the span amounts to 6·51 tons less about 1·40 say 5·0 tons per foot length of bunker. With 12 in. × 6 in. × 44 lb. **I**-stiffeners at 2-ft. 6-in. centres the stress would be

$$f = \frac{13{\cdot}0 \times 2{\cdot}5 \text{ ft.} \times 12}{52{\cdot}79} + \frac{5{\cdot}0 \times 2{\cdot}5 \text{ ft.}}{13{\cdot}0} = 7{\cdot}40 + 0{\cdot}96 = 8{\cdot}36 \text{ tons/sq. in.}$$

With 14 in. × 6 in. × 46 lb. **I** at 3-ft. centres,

$$f = \frac{13{\cdot}0 \times 3{\cdot}0 \text{ ft.} \times 12}{63{\cdot}22} + \frac{5{\cdot}0 \times 3{\cdot}0 \text{ ft.}}{13{\cdot}59} = 7{\cdot}40 + 1{\cdot}10 = 8{\cdot}50 \text{ tons/sq. in.}$$

The comparative weight is then,

with ⅜-in. trough plate and 12-in. **I**-stiffeners at 2-ft. 6-in. centres,

$$\frac{44{\cdot}0}{2{\cdot}5} + 15{\cdot}3 = 32{\cdot}9 \text{ lb./sq. ft.}$$

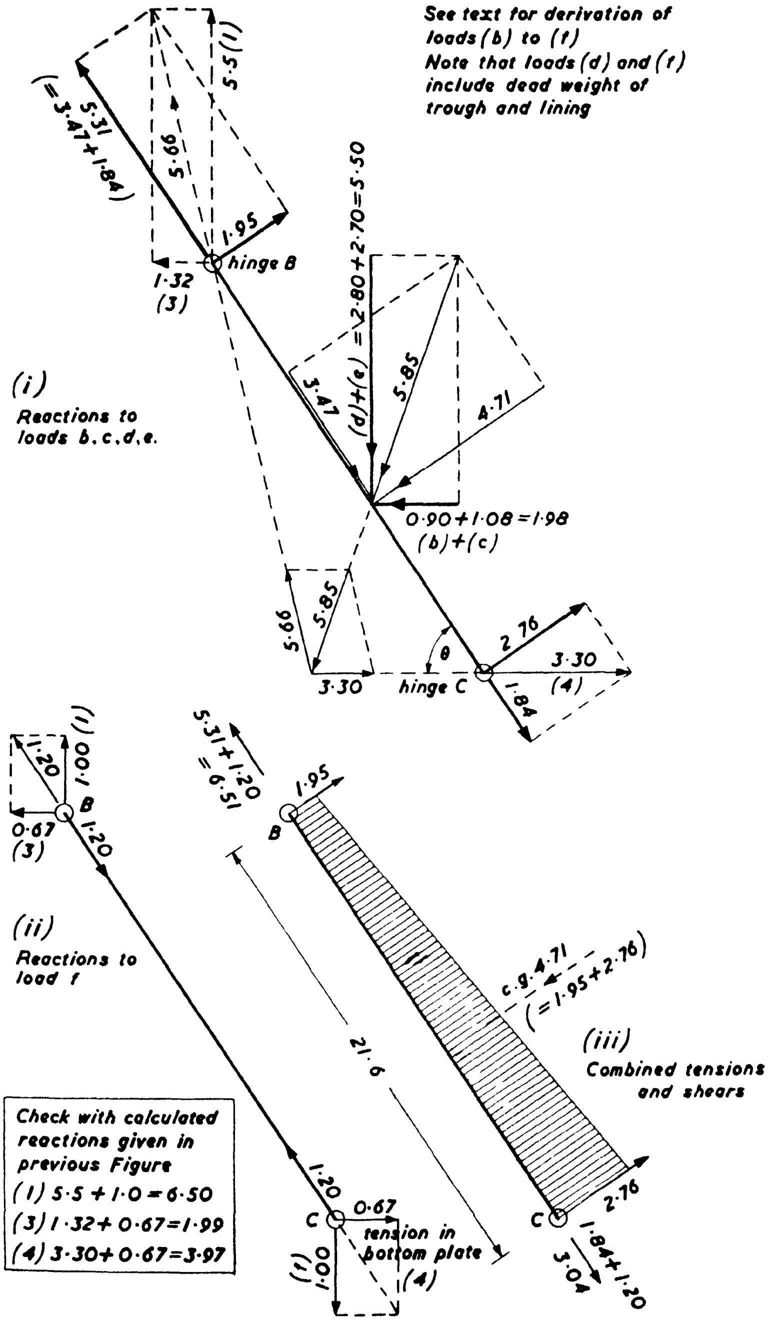

See text for derivation of loads (b) to (f)
Note that loads (d) and (f) include dead weight of trough and lining
5·5(1)
5·31 (=3·47+1·84)
5·66
1·95
hinge B
1·32 (3)
(d)+(e) = 2·80+2·70=5·50
5·85
3·47
4·71
(i)
Reactions to loads b, c, d, e.
0·90+1·08=1·98 (b)+(c)
5·66
5·85
θ
2·76
3·30
3·30
hinge C
(4)
1·84
1·00 (1)
1·20
B
0·67 (3)
1·20
(ii)
Reactions to load f
5·31+1·20 = 6·51
1·95
B
21·6
c.g. 4·71 (=1·95+2·76)
(iii)
Combined tensions and shears
Check with calculated reactions given in previous Figure
(1) 5·5 + 1·0 = 6·50
(3) 1·32+ 0·67=1·99
(4) 3·30+ 0·67=3·97
1·20
C
0·67
tension in bottom plate
(4)
1·00 (1)
C
2·76
1·84+1·20
3·04

FIG. 308

with $\frac{1}{2}$-in. plate and 14-in. I-stiffeners at 3-ft. centres,

$$\frac{46 \cdot 0}{3 \cdot 0} + 20 \cdot 4 = 35 \cdot 7 \text{ lb./sq. ft.}$$

Adopt the 12-in. I-stiffener with $\frac{3}{8}$-in. plate continuous over three spans.

Fig. 308 has been developed from the trough loads given in Fig. 307 and is useful in showing the step-by-step solution. The bending load on the stiffener can, however, be determined very simply by using the expression given under 'Coal pressure' and shown in Fig. 306, viz. the load N normal to the trough slope $= 27 \cdot 63h$ lb./sq. ft.

At the centre of the trough slope,

$$h = 7 \cdot 5 \text{ ft.} + 9 \cdot 0 \text{ ft.} = 16 \cdot 5 \text{ ft.}$$

hence

$$N = 27 \cdot 63 \times 16 \cdot 5 \text{ ft.} = 456 \text{ lb./sq. ft.}$$

to which must be added the dead weight of the plate, stiffeners and lining

$$= 56 \text{ lb./sq. ft.} \times \cos 56° \, 19' = 31 \text{ lb.},$$

giving a total of 487 lb./sq. ft. and a total load normal to the trough of

$$\frac{21 \cdot 6 \text{ ft.} \times 487}{2240} = 4 \cdot 71 \text{ tons.}$$

This also checks up with the resolution of the total vertical and horizontal loads on the slope length BC, thus:

Total normal pressure

$$= (2 \cdot 80 + 2 \cdot 70) \cos \theta + (0 \cdot 90 + 1 \cdot 08) \sin \theta$$
$$= 3 \cdot 06 + 1 \cdot 65 = 4 \cdot 71 \text{ tons.}$$

The moment can also be obtained by taking moments of each load on the projected spans, thus:

$$12(2 \cdot 8 \times 0 \cdot 125) + (2 \cdot 7 \times 0 \cdot 128) + 18(0 \cdot 9 \times 0 \cdot 125) + (1 \cdot 08 \times 0 \cdot 128) = 13 \text{ tons ft.}$$

Bottom Plate. Span 30 in.

For each foot length of the bunker the bending load is 2·0 tons and the tension 3·97, say 4·0 tons. The tension is applied by the trough stiffeners at 30-in. centres, whilst the valve opening near to may be 18 in. wide, hence the tension in the plate adjacent to the valve may approach (4·0 × 30 in.)/18 in., say 7·0 tons.

The Section Modulus of a $\frac{7}{8}$-in. plate 12 in. wide is

$$\frac{12 \times 0 \cdot 875^2}{6} = 1 \cdot 53.$$

Total stress

$$= \frac{2 \cdot 0 \times 30 \text{ in.}}{8 \times 1 \cdot 53} + \frac{7 \cdot 0}{12 \times \frac{7}{8}} = 4 \cdot 90 + 0 \cdot 70 = 5 \cdot 6 \text{ tons/sq. in.}$$

Girder Loads

The trough is of full section for a length of 27 ft. 6 in. as shown in Fig. 305, and the vertical load on each side girder is 6·50 tons per foot run for this full section. The trough slope is returned along each end of the bunker which

reduces the load per foot at the ends, due to the cut-off and because part of the load is carried by the end girders. The volume of the cut-off at each of the four corners

$$=\frac{18\text{ ft.}\times 12\text{ ft.}\times 1{\cdot}25\text{ ft.}}{2}+\frac{18\text{ ft.}\times 12\text{ ft.}\times 12\text{ ft.}}{2\times 3}=567\text{ cu. ft.}$$

which is equivalent to 14 tons of coal. Thus the total load of the corner, instead of being 12 ft. × 6.5 tons is 14 tons less, i.e. 64·0 tons and this is shared by the side and end girders flanking the corner. For simplicity the 32 tons per girder is treated as a triangular load.

Side Girder. Span 51 ft. 6 in.:

Triangular load next left end	= 32·0 tons
Distributed load over centre 27 ft. 6 in., 27·5 ft. × 6·5 tons per ft.	= 180·0 ,,
Triangular load next right end	= 32·0 ,,
Dead weight of girder, lining, conveyor, gangways, etc.	= 56·0 ,,
	300·0 ,,

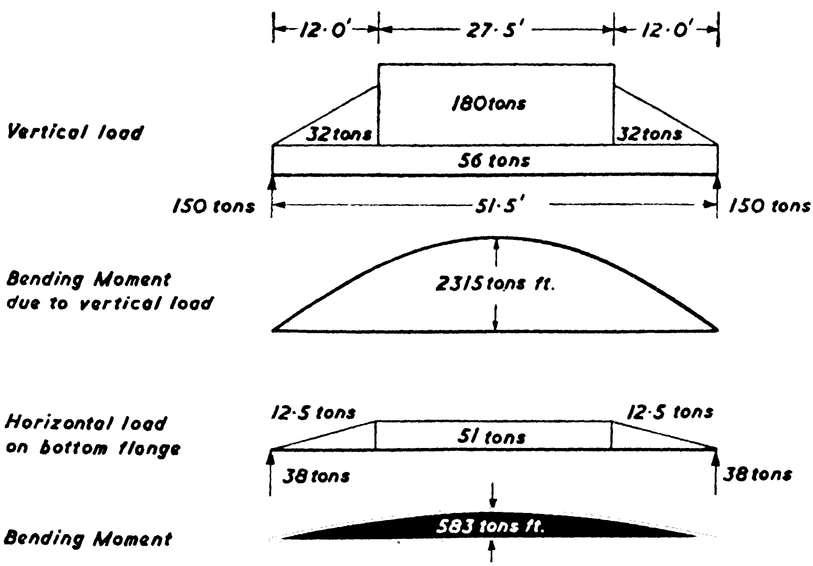


FIG. 309

B.M. at centre

$$=(150\times 25{\cdot}75\text{ ft.})-(32\times 17{\cdot}75\text{ ft.}+90\times 6{\cdot}875\text{ ft.}+28\times 12{\cdot}875\text{ ft.})$$

$$=3863-(568+619+361)=2315\text{ tons ft.,}$$

due to vertical loads.

The horizontal load at hinge *B*, when the bunker is trimmed full, is 1·99 − 0·13 tons = 1·86 tons per foot run for the centre 27 ft. 6 in. = a centrally

distributed load of 51 tons with a triangular load at each end, of say $12\frac{1}{2}$ tons. Hence the end shear = 38 tons, and

$$\text{B.M.} = (38 \times 25{\cdot}75 \text{ ft.}) - (12{\cdot}5 \times 17{\cdot}75 \text{ ft.} \times 25{\cdot}5 \times 6{\cdot}875)$$
$$= 980 - (222 + 175) = 583 \text{ tons ft.}$$

due to horizontal load.

Fig. 310 shows the form of girder proposed, and assuming for the first approximation an effective depth of 8 ft. and an effective width of 3 ft. the flange forces are as follows:

Top flange, 2315 tons ft./8 ft. = 290 tons (30·5 sq. in.)

Outrigger flange, 583 tons ft./3 ft. = 195 tons (20·5 sq. in.)

Bottom flange of vertical girder, 290 + 195 = 485 tons (51 sq. in.)

The figures given in brackets are the nett areas required for a stress of 9·5 tons per sq. in., and to decide upon the build-up the web plates and angles must first be considered.

The web of the side girder is subjected to coal pressure which at 7 ft. down = 105 lb. per sq. ft. With stiffeners at 5-ft. centres the load = 525 lb., say 0·24 ton, hence if the web is $\frac{1}{2}$ in. thick the bending stress would be

$$\frac{0{\cdot}24 \times 60 \text{ in.} \times 6}{12 \times 12 \times 0{\cdot}5^2} = 2{\cdot}4 \text{ tons/sq. in.},$$

and for $\frac{5}{8}$-in. web, 1·6 tons per sq. in., whilst the shear stresses in tons per sq. in. due to the 150 tons reaction would be $150/96 \times \frac{1}{2} = 3{\cdot}1$ for $\frac{1}{2}$-in. and 2·5 for $\frac{5}{8}$-in. web. For this girder depth and stiffener spacing, a $\frac{1}{2}$-in. web would be considered thin and $\frac{5}{8}$-in. will be provisionally adopted.

The end reaction of 150 tons induces a horizontal shear of 150/8 ft. = 19 tons per foot run, requiring four $\frac{3}{4}$-in. or three $\frac{7}{8}$-in. rivets in the vertical legs of the angles where the rivets are in double shear. In the case of the bottom flange the rivets connecting the angles and flange plates have to resist also the horizontal shear from the outrigger girder which amounts to 38/3 ft. = 13 tons/foot run. The rivets resisting the combined shears are partly in double and partly in single shear; eight $\frac{3}{4}$-in. or seven $\frac{7}{8}$-in. will be required. 6-in. × 6-in. × $\frac{5}{8}$-in. angles should suit and can be tried in the flange area computation.

The web of the outrigger girder has to resist a shear of 38 tons and could be suitably formed of $\frac{3}{8}$-in. plate.

Trial section for bottom flange of vertical girder (51 sq. in. required):

One-eighth of vertical web = 12 in. × $\frac{5}{8}$ in.	= 7·50	sq. in.
Two 6-in. × 6-in. × $\frac{5}{8}$-in. angles	= 14·22	,,
Part of outrigger web, 14-in. × $\frac{3}{8}$-in.	= 5·25	,,
Plates, two 14-in. × $1\frac{1}{4}$-in.	= 35·00	,,
	61·97	,,
Less rivet holes, one $\frac{15}{16}$-in. × $1\frac{7}{8}$-in., two $\frac{15}{16}$-in. × $3\frac{1}{2}$-in.	= 8·32	,,
	53·65	,,

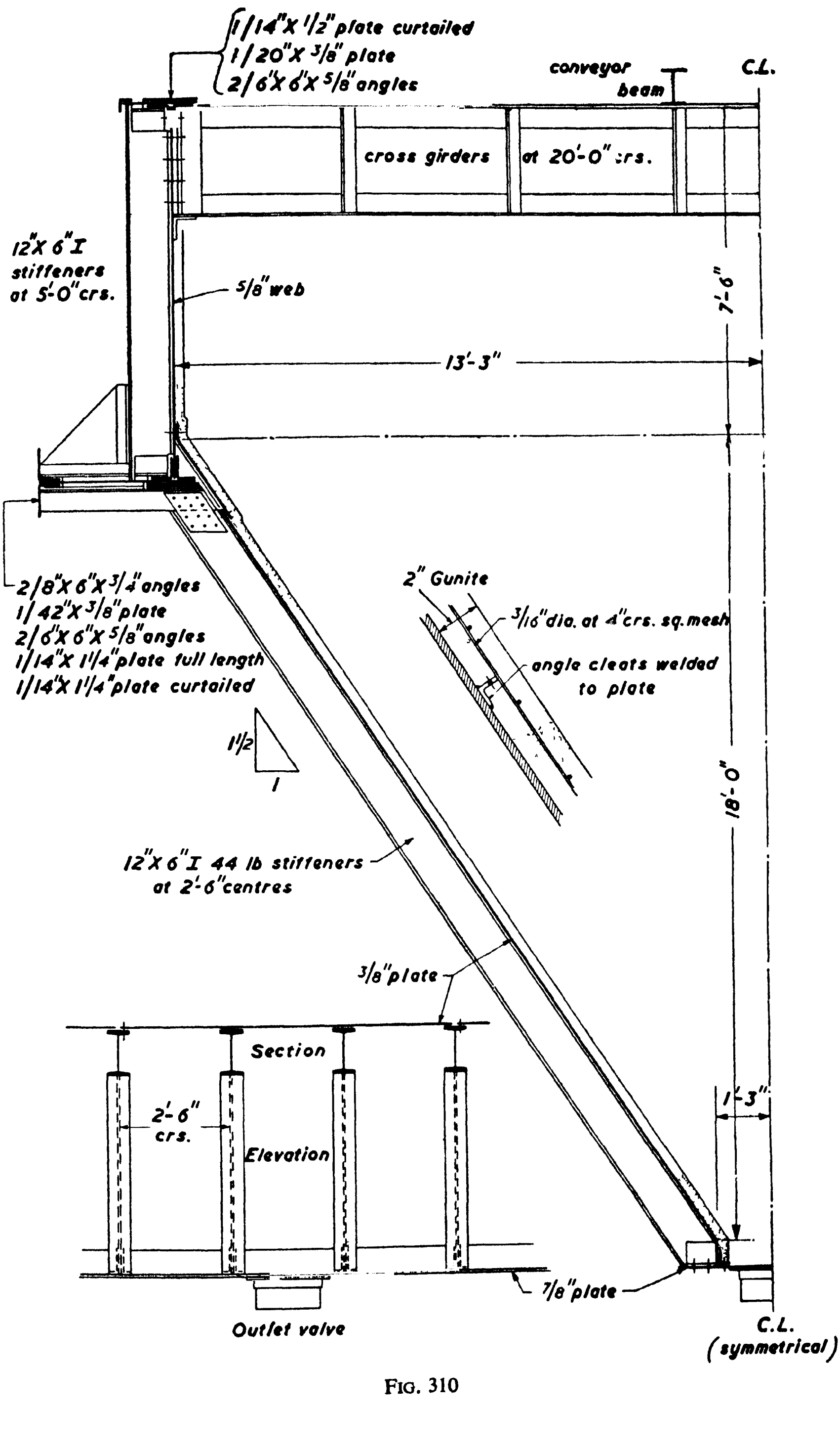

1/14"X 1/2" plate curtailed
1/20"X 3/8" plate
2/6"X 6"X 5/8" angles
conveyor beam
C.L.
cross girders at 20'-0" crs.
12"X 6"I stiffeners at 5'-0" crs.
5/8" web
7'-6"
13'-3"
2/8"X 6"X 3/4" angles
1/42"X 3/8" plate
2/6"X 6"X 5/8" angles
1/14"X 1 1/4" plate full length
1/14"X 1 1/4" plate curtailed
2" Gunite
3/16" dia. at 4" crs. sq. mesh
angle cleats welded to plate
1 1/2
1
18'-0"
12"X 6"I 44 lb stiffeners at 2'-6" centres
3/8" plate
Section
2'-6" crs.
Elevation
1'-3"
7/8" plate
Outlet valve
C.L. (symmetrical)

FIG. 310

Top flange of vertical girder (30·5 sq. in. required):

One-eighth of web	= 7·50 sq. in.	
Two 6-in. × 6-in. × $\frac{5}{8}$-in. angles	= 14·22	,,
Plates: one 20-in. × $\frac{3}{8}$-in.	= 7·50	,,
one 14-in. × $\frac{1}{2}$-in.	= 7·00	,,
	36·22	,,
Less rivet holes	= 4·57	,,
	31·65	,,

Outer flange of outrigger girder (20·5 sq. in. required):

One-eighth of web (36-in. × $\frac{3}{8}$-in.)/8	= 1·70 sq. in.	
Two 8-in. × 6-in. × $\frac{3}{4}$-in. angles	= 19·88	,,
	21·58	,,

The top flange of the vertical girder is subjected to a horizontal force, due to coal pressure, of 0·06 ton per foot run, but the cross girders carrying the

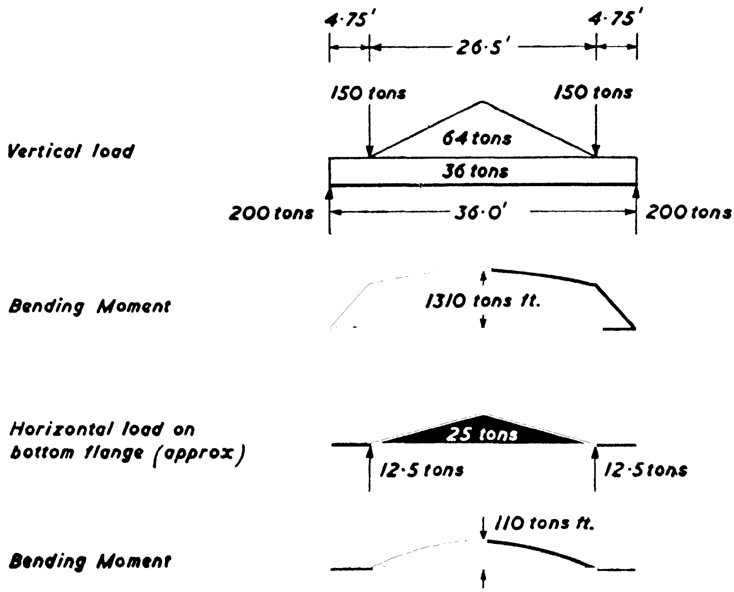


FIG. 311

charging conveyor give lateral support. Assuming these to be at 20-ft. centres, the bending load = 1·2 tons and B.M. = 1·2 × 20/12 = 2·0 tons ft., which is not significant.

For a final design, not completed here, the moment of inertia of the girder should be computed for each axis and the stresses checked. The outer flange plates can be curtailed as usual.

The web stiffeners have to resist a small B.M from coal pressure, but their

principal purpose is to prevent the web from buckling and to maintain the girder in the vertical position. Adopt 12-in. × 6-in. I-stiffeners at 5-ft. centres subject to final check for compliance with B.S. 449.

End Girders

The load and B.M. in each end girder is given in Fig. 311, and the section can be calculated in a similar manner to that given for the side girder.

Conveyor Girders

Design for bending load and direct tension from coal pressure on side girders. The stress in these girders should be kept low to minimise deflection.

Fig. 312 shows the general arrangement of the complete bunker. Note that to avoid awkward connections the bunker has been so proportioned in relation to the stiffener spacing that no stiffener occurs at the corners of the bottom plate. On each side of the valleys the stiffener lines coincide with each other, the lower ends being bevel cut to give a neat and simple connection. Down the valley the two trough plates would be connected together by means of a bent cover or by flanging one of the plates.

Corrosion and Wear

The design of a bunker and its connection details should be such that corrosion is prevented as far as possible and all surfaces exposed to attack are accessible for repainting. Details which leave inaccessible pockets should either be avoided, fully sealed or filled solid with concrete. Plate seams should preferably be vertical, not horizontal. All steel surfaces held in contact by rivets or bolts should be painted before final assembly and brought together whilst the paint is still wet, and this applies particularly to the site connections of the trough-plate edges to the stiffeners. Coal with an excess of moisture should be anticipated and the aim should be to make the trough seams fully watertight by the bedding of paint and with rivets or bolts at close pitch.

No long-term protection would be gained by painting where abrasion is severe, which occurs with hard coal and coke. Early bunkers were sometimes lined with plates of cast iron, which is particularly resistant to wear and corrosion. Renewable plates of thin steel were also used, but, whilst efficient enough in regard to wear, they tended to aggravate corrosion of the structure plate. Copper-bearing steel has been used for the structure plating, with no lining, but current good practice provides a lining of concrete or tiles which serves the dual purpose of preventing both corrosion and wear of the steelwork.

The surface of the lining should be smooth and the trough sides steep, otherwise the tendency of the bunker contents to arch over the outlet will be aggravated, especially with large or wet coal.

For the concrete lining a 4-in. square mesh fabric is wired to steel cleats or spacing blocks welded to the plating, and a 2-in. thickness of concrete applied by means of a cement-gun. The alternative is a lining of Staffordshire blue- or Accrington red-brick tiles, usually 9-in. × $4\frac{1}{2}$-in. × 2-in. thick,

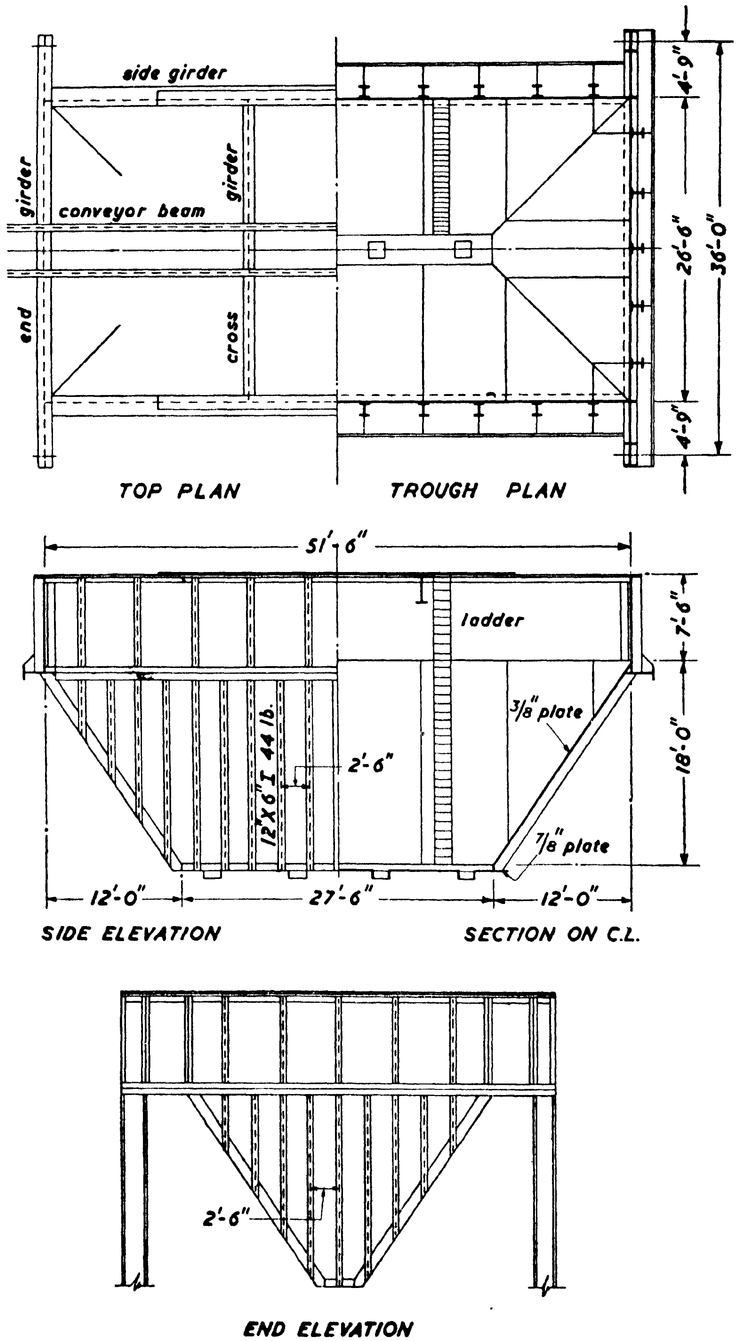

side girder
end girder
conveyor beam
cross girder
4'-9"
26'-6"
36'-0"
4'-9"
TOP PLAN
TROUGH PLAN
51'-6"
ladder
7'-6"
3/8" plate
18'-0"
12"X6" I 44 lb.
2'-6"
7/8" plate
12'-0"
27'-6"
12'-0"
SIDE ELEVATION
SECTION ON C.L.
2'-6"
END ELEVATION

FIG. 312

bedded and jointed with cement mortar. The bed should be not less than about $\frac{3}{4}$ in., and the joints kept as narrow as possible. On vertical work the tiles are sometimes provided with tongued and grooved edges and held back to the structure with dowel bolts.

Where the bunker is not lined, but severe abrasion is expected, all bolt and rivet heads should be countersunk flush with the inside surface of the plating.

The outside surfaces of steel bunkers are usually protected from corrosion by paint, which should be periodically examined for signs of corrosion, especially at seams.

Welding is being increasingly used in the construction of steel bunkers. It can effect an economy of material and give good resistance to corrosion provided that all seams liable to attack are fully sealed by continuous welds.

Suspension-type bunkers have been used in a number of recent schemes with the primary object of economising steel, although the cost per ton is higher than for trapezoidal bunkers. In the suspension type the trough is formed of plates which are bent to follow the curve of a catenary based upon the full load to be carried, and in which case the plate is subjected only to tension and stiffeners are not required.

Consideration should, however, be given to cases of part loading which cause local bending and stress concentration due to the potentially different catenaries.

The possibility of deformation, however slight, should also be taken into account when deciding whether a lining shall be used, and, if so, the type.

Tabular Data

The following table gives data on pressures and angles of repose for various kinds of contents for bunkers.

BULK STORAGE OF MATERIALS

APPROXIMATE WEIGHT AND ANGLE OF REPOSE

The values given assume that the moisture content is moderate. Excess moisture increases the weight and decreases the angle of repose.

Material	*lb. per cu. ft*	*Angle of Repose*
Sand, gravel, shingle	110	30°–35°
Clay, in lumps	70	30°
Chalk, in lumps	80	35°
Crushed brick	90	35°
Crushed stone	110	35°–40°
Slag	90	35°
Coal, in lumps	56	35°
Coke	35	30°
Clinker	65	40°
Ashes	45	40°
Haematite iron ore	230	35° (Haematite to Lead ore)
Lincolnshire iron ore	110	
Magnetite iron ore	250	
Copper, manganese and zinc ore	160–180	
Lead ore	320	
Cement	90	20°
Cement clinker	90	30°
Salt	60	30°

23. BEAMS IN TORSION

Contributed by Professor W. Fisher Cassie, Ph.D., M.S., F.R.S.E., M.I.C.E., M.I.Struct.E.

WHEN a prismatic bar is uniformly twisted through an angle θ per unit length by a uniform couple T the stiffness or resistance to twisting is given by the expression

$$\frac{T}{K}=C\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where C is the modulus of rigidity of the material and K the 'torsion constant.' K is independent of the material of which the bar is made and dependent only on the geometrical properties of its cross-section.

The principal cause of the difficulty of studying structural sections under torsion is the difference in behaviour shown by circular and non-circular sections. A plane cross-section of a cylindrical bar remains plane after the bar is twisted, but if the cross-section of the bar is not circular the cross-section warps when torsion is applied. This was pointed out by St. Venant in 1855.

Warping of this kind means that some points in the material of the bar move in a longitudinal direction relative to other points, and longitudinal stresses are developed if this warping is restrained. Shear stresses, too, vary over the perimeter of the cross-section, being lowest at outstanding points such as the edges of 'I'-beam flanges and highest at re-entrant angles. The shear stresses developed in a rod of rectangular cross-section under torsion, are, for example, lowest at the corners and highest at the centres of the longitudinal sides. As the shape of the non-circular cross-section becomes more complex, the difficulties of determining stresses also increase.

Studies of the torsion of non-circular sections can be grouped into two stages:

(a) the determination of the relationship between torque and angle of twist by the calculation of the torsion constant, K, used in equation (1); and
(b) the determination of the stresses developed due to torque applied directly to a straight beam or indirectly because of the shape of the beam (e.g. stresses in a bow girder).

Determination of the Torsion Constant

The torsion constant of a circular section is equal to its polar moment of inertia, but for even simple non-circular shapes such an easy solution is not possible. St. Venant showed how a solution of the differential equation of torsion

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=-2C\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

can be obtained for several of the simpler geometrical shapes. For rolled or extruded sections, such as are used in structures, no formal solution of the equation is possible. Approximate analytical solutions, the use of analogies, or direct experimental twisting must be employed for all complex cross-sections.

Membrane Analogy

Of all the methods which have been employed in studying the torsion of structural sections probably none is more illuminating than that employing soap films. If a soap film is drawn across a hole in a flat plate—the hole being of the same shape as the section in question—a visual conception of what happens to that section in torsion can readily be obtained. The differential equation connecting the rate of change of slope of such a film blown up by a light pressure, with the tension of the membrane and the pressure employed, happens to be of the same form as the differential equation of torsion. This results in several valid comparisons:

1. The intensity of shear stress caused by torsion at any point in the cross-section is represented by the slope of the stretched membrane: the steeper the slope the higher the shear stress.
2. The torsion constant of the section is proportional to the volume of air enclosed between the stretched membrane and the plane of the plate in which the hole is cut.

The experimental determination of torsion constants by this method is not easy but is possible, and satisfactory checks can be made by Southwell's Method of Relaxation which solves the differential equation of torsion by the use of finite differences.

Once the torsion constants of simple geometrical shapes have been determined, the torsion constant of a more complex shape can be deduced by subdividing it into several simpler forms of which the torsion constants are known. A simple summation, however, is not sufficient. If inflated soap films stretched over two rectangular openings are merged (when the two simple forms which represent the values of K are formed into 'T' shape), it can be realised that more volume is confined within the combined soap film than is accounted for by the simple summation of the volumes under the rectangles. At the point where the two films join, an extra volume is provided by the bridging over of the soap film. This has been called a 'junction effect' and when a soap film is stretched over the shape of a structural member such as an 'I'-section, the extra volume provided by the junctions between web and flanges is clearly visible as small 'mounds' on the soap film. This extra volume indicates how the junction of two simpler shapes into a more complex one assists in strengthening the section against torsion, for a larger torsion constant provides extra torsional stiffness, and the value of the torsion constant is proportional to the volume enclosed under the soap film.

Cassie and Dobie (1948) have evaluated, by the methods described, the torsion constants for all the British Standard Beams and these values are shown in the following table:

TABLE 1

TORSION CONSTANTS FOR BRITISH STANDARD BEAMS

B.S.B. No.	K (in.⁴)	B.S.B. No.	K (in.⁴)	B.S.B. No.	K (in.⁴)	B.S.B. No.	K (in.⁴)
140	8·62	130	2·00	120	4·15	110	0·94
139	4·82	129	1·69	119	2·21	109	0·55
138	8·61	128	6·43	118	1·04	108	0·19
137	3·98	127	4·21	117	0·68	107	0·66
136	7·23	126	2·30	116	3·92	106	0·18
135	6·40	125	1·25	115	0·49	105	0·07
134	2·93	124	6·05	114	1·80	104	0·16
133	6·89	123	4·23	113	1·08	103	0·03
132	4·32	122	2·40	112	0·35	102	0·13
131	2·54	121	1·00	111	0·30	101	0·03

Cullimore and Pugsley (1951) have, similarly, obtained the torsion constants for aluminium alloy sections.

Determination of Maximum Torque

As was pointed out at the beginning of the section, the equation linking torque, twist and torsion constant in a prismatic bar is

$$\frac{T}{K}=C\theta \qquad \qquad (1)$$

When the stresses caused by these conditions are considered the whole equation becomes:

$$\frac{T}{K}=C\theta=\frac{q}{R} \qquad \qquad (3)$$

where q is the shear stress at a point in the section represented by R. If the section is a circular one, R is the radius at which the shear stress is to be determined. As R increases so does q until the surface of the circular bar is reached, when both q and the radius R are a maximum.

When the section is non-circular, R cannot be a simple radius and becomes a parameter which is equal to $q/C\theta$ where q is the shear stress, C is the modulus of rigidity and θ is the rate of twist or twist per unit length at the point in question.

The value of $q/C\theta$ can be determined by study of a membrane stretched over a hole of the same shape as the section. This can be done by using a soap film and determining the shape of the stretched membrane, but it is probably more readily accomplished by Southwell's relaxation method. The slope of the membrane or stretched soap film is steepest at the re-entrant fillet in 'I' and other rolled sections, and, as has been mentioned above, this slope represents the maximum shear stress in the section. In an 'I'-beam the value of the slope, and therefore of the shear stress, depends on the diameter of the largest circle which can be inscribed at the junction of web and flange. This circle is tangential to the re-entrant fillets (Fig. 313).

By relaxation methods and by the study of stretched soap films Dobie (1952) obtained a relationship between two parameters for 'I'- beams and angles allowing the elastic shear stress to be calculated at the re-entrant

Contact angle and inscribed circle in 'I' beam

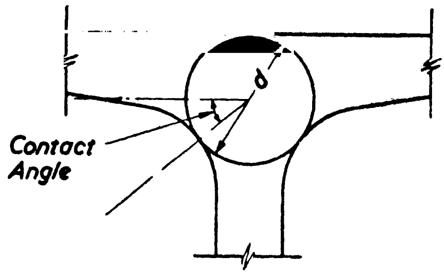


FIG. 313

fillet. Fig. 314 gives the values obtained. In the figure, r is the radius of the re-entrant fillet and d the diameter of the largest circle which can be inscribed at the junction of flange and web.

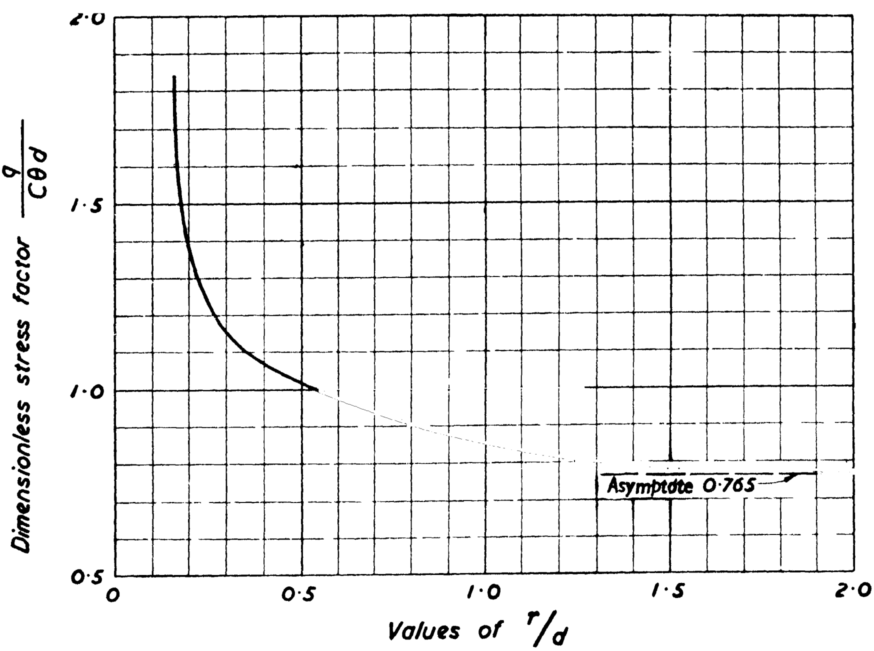


FIG. 314

By entering this diagram with the value r/d, which defines the conditions at the junction of flange and web, the investigator can determine a factor from which the maximum stress can be deduced. In this dimensionless factor only q and θ are unknown, and if one is specified the other can be determined.

This appears to be a simple solution but such simplification can be justified

only as a means of giving the reader a quick survey of the problem. Other factors must also be taken into account. Of these, which include such variables as flange fineness, probably the two most important are flange taper and contact angle.

Dobie (1952) has shown how the dimensionless stress factor from Fig. 314 must be adjusted as the taper of the flange of an 'I'-beam varies. With the usual B.S. taper of 8°, the factor is 0·94. The magnitude of the contact angle (Fig. 313) also has an effect on the stress factor. For a contact angle of 10° the 'angle factor' is 0·95, and the value of the factor increases directly with the size of the angle up to a value of 1·05 for a contact angle of 60°. The original value of $q/C\theta d$ read off from Fig. 314 is multiplied by the product of these factors, and the value of the maximum shear stress can be calculated.

The Criterion of the Safe Limit of Loading

A coarse membrane blown up through a hole of the shape of an 'I'-section cannot be used for quantitative measurement but does show how much steeper is the slope (and therefore the shear stress in torsion) at the re-entrant fillets than at any other point. If the shear stress at the re-entrant fillets is assumed to be limited to the normal working shear stress, it can be understood that the rest of the section is understressed in shear and thus ineffectively used. If, however, other points are allowed to be stressed to the working shear stress, the material at the fillets is certain to reach the yield point and a partially plastic state develops. The rate of twist along the length of the member also increases.

A decision, therefore, must be reached regarding the criterion which should be used in each design. One method is to assume that the section becomes fully plastic; the torque which produces this state is determined (the maximum possible torque), and a load factor is applied to find the working torque Another method is to assume that the material at the re-entrant fillet is just commencing to yield and use the torque so determined. A third is to limit the rate of twist of the beam. This must be done, for example, where the beam is twisted by the action of secondary beams coming in on the web, for the slope of these beams at the ends helps to determine their deflection.

The method adopted must vary with circumstances. A structure relatively insensitive to movement might be designed by allowing a certain amount of plastic deformation. In other buildings the rate of twist might be of more importance, controlling as it does the displacements of the twisted beam.

Example 1. An 8-in. × 6-in. B.S.B. (No. 114) is to be subjected to torque, but the rate of twisting must not exceed 100×10^{-6} rad. per in. $C = 11{\cdot}5 \times 10^6$ lb. per sq. in. Determine the maximum possible torque and the maximum shear stress developed.

The first step is to record the radius of the re-entrant fillet (from steel tables) and the diameter of the largest circle which can be inscribed between flange and web. This is found by means of a drawing (Fig. 313).

In this instance $r = 0{\cdot}61$ in. and $d = 1{\cdot}08$ in.

Therefore $$\frac{r}{d} = 0{\cdot}565.$$

From Fig. 314 $\frac{q}{C\theta d}=1{\cdot}0$

and thus $\frac{q}{C\theta}=1{\cdot}08.$

This must be slightly modified, first by the factor for flange taper (0·94) and secondly by the factor for contact angle. The contact angle is 50° and the factor thus 1·03 (the factor decreases from 1·05 at 60° by 0·02 for each ten degrees).

Final value of $\frac{q}{C\theta}=1{\cdot}08\times 0{\cdot}94\times 1{\cdot}03$

$=1{\cdot}04$

Therefore $q=1{\cdot}04\times C\theta$

$=1{\cdot}04\times 11{\cdot}5\times 10^6\times 100\times 10^{-6}$

$=1200$ lb. per sq. in.

Torque is obtained from the expression

$\frac{T}{K}=C\theta \quad K$ (from Table 1) $=1{\cdot}80$

$T=KC\theta$

$=1{\cdot}80\times 11{\cdot}5\times 100$

$=2070$ lb. in.

Example 2. A 15-in × 5-in. B.S.B. (No. 129) is subjected to torque. The shear stress at the re-entrant fillet may be allowed to rise to 8 tons per sq. in. Determine the maximum permitted torque.

Data: Diameter of largest inscribed circle,

$d=1{\cdot}06$ in. (scale drawing),

Contact angle $=50°$,

$K=1{\cdot}69$ (Table 1)

and $r=0{\cdot}53$ in.

Value of q:

Stress factor $\frac{q}{C\theta}=1{\cdot}02d=1{\cdot}08.$

Flange taper factor $=0{\cdot}94.$

Contact angle factor $=1{\cdot}03.$

Final value of $\frac{q}{C\theta}=1{\cdot}08\times 0{\cdot}94\times 1{\cdot}03$

$=1{\cdot}04.$

Therefore $C\theta=\frac{q}{1{\cdot}04}=\frac{8\times 2240}{1{\cdot}04}=17{,}200$

but $\frac{T}{K}=C\theta$

therefore $T=C\theta K$

$=17{,}200\times 1{\cdot}69=29{,}000$ lb. in.

As a matter of interest,

$$\theta = \frac{17{,}200}{C} = \frac{17 \cdot 2}{11 \cdot 5 \times 10^3} = 1 \cdot 50 \times 10^{-3} \text{ rad. per in.}$$

Torque and Load

The question of deciding how the torque, shape of beam, and loading of beam are related lies in the realm of structural analysis. We assume throughout these calculations that the section of the 'I'-beam is free to warp longitudinally and is not restricted by restraint as, for example, at a built-in end.

The extent to which structural connections restrict warping of a twisted beam and so induce high longitudinal stresses is not a point on which a simple answer can be given. Usually a true encastré condition can be obtained only with special designs, and the usual structural connections of beam to column and beam to beam—especially if riveted—can be considered free from restraint in the practical calculations which form the subject of this article.

Bow Girder

'I'-beams may be twisted by an eccentric load such as the end reaction of a cross girder, or may have a torque applied by being bent in a plane at right

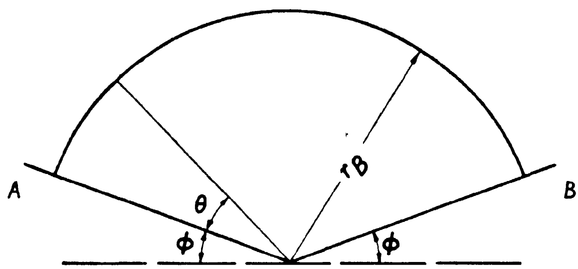


FIG. 315. Dimensions of bow-girder

angles to the direction of the loading. Such 'bow' girders bent into the form of a circular arc are frequently used in steel construction. Fig. 315 shows the symbols used here to describe the various features of this type of structure.

Commencing with a semi-circular arc we can reduce this to any subtended angle by subtracting 2ϕ. For $\phi = 45°$, for example, the subtended angle becomes 90° and the bow girder turns through a right angle—for example, round the right-angled corner of a building.

Gibson and Ritchie (1914) carried out a very full investigation of the circular-arc bow girder, and their book is still available and of considerable value to the designer of bow girders. Their section on the torsion constant has been modified by later work, but the chapters on the moments and torques sustained by bow girders is a valuable source of information. (See references at the end of the article.)

Fig. 316 is extracted from Gibson and Ritchie's work by kind permission of the publishers and shows how torque and moment at the support can be

related to the load per unit-length for a circular-arc bow girder under uniformly distributed load. This chart is correct if the ratio EI/CK is not less than about 20.

For example, if the B.S.B. in Example 2, above, were bent into a 90° bow girder of 10 ft. radius, the maximum permitted torque can be related to allowable uniformly distributed load by Fig. 316. It was found that the torque which could be permitted was 28,800 lb. in.

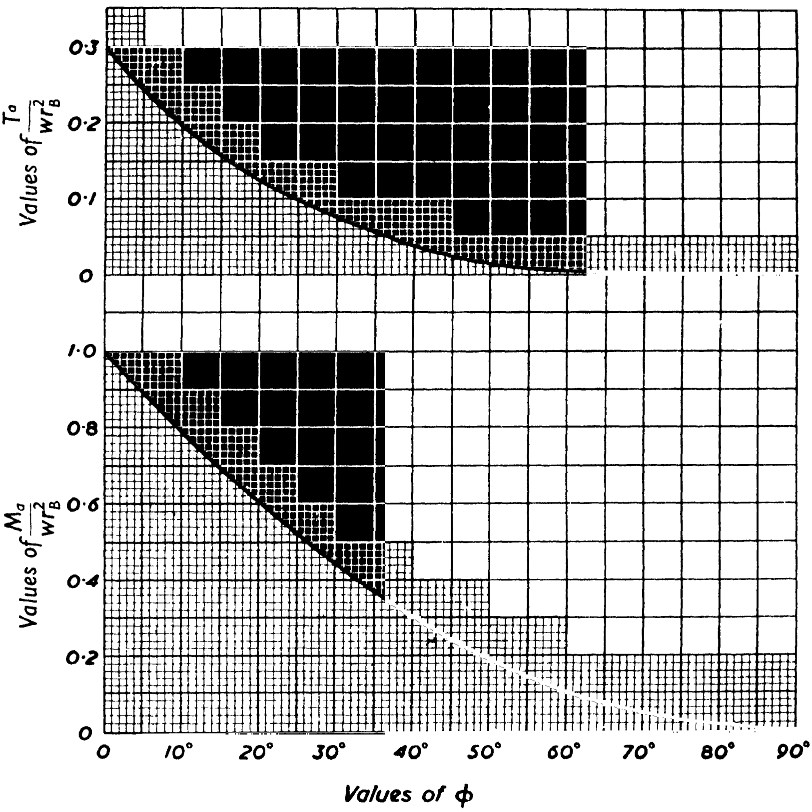


FIG. 316. Torque and Moment at support of bow-girder

The values of EI and CK must first be found before load per unit length can be determined. The values are as follows:

$E = 30 \times 10^6$ lb. per sq. in. $\qquad C = 11{\cdot}5 \times 10^6$ lb. per sq. in.

$I = 428$ in.^{4} (steel tables). $\qquad K = 1{\cdot}69$ in.^{4} (Table 1, above).

$\frac{EI}{CK} = \frac{30 \times 428}{11{\cdot}5 \times 1{\cdot}69} = 660.$ $\qquad r_B = 120$ in. $\quad \theta$ in radians.

The value of the torque at any point along the girder is given by

$$T_\theta = (T_a - R_a r_B)\cos\theta - (M_a - wr_B^2)\sin\theta + R_a r_B - wr_B^2\theta \quad . \quad . \quad (4)$$

where R_a is the reaction at A. From this equation a maximum value of the torque for any point on the beam can be determined. For a 90° bow girder the maximum torque may occur either at the support A or at some point between A and the centre. At the centre of the beam the torque, from symmetry, is zero. Between the centre and a point of contraflexure which occurs between the centre and the supported end, the torque is of the opposite sign to that of the fixing torque T_a, and reaches a maximum, T_m (Fig. 317).

From Fig. 316, for $\phi = 45°$ (90° bow girder): $T_a = 0{\cdot}025wr_B^2$, $M_a = 0{\cdot}240wr_B^2$ and R_a is half the total load, or $wr_B\pi/4$. Substituting these values in equation (4) and equating the differential coefficient $dT_\theta/d\theta$ to zero we obtain the position of the point of maximum torque, T_m. When $dT_\theta/d\theta = 0$, $\theta = 23\frac{1}{2}°$.

Substituting in equation (4) the values of sine and cosine for $23\frac{1}{2}°$ we find that $T_m = 0{\cdot}020\ wr_B^2$. Thus for a 90° bow girder the values of the torque T_m and the fixing torque T_a are nearly equal.

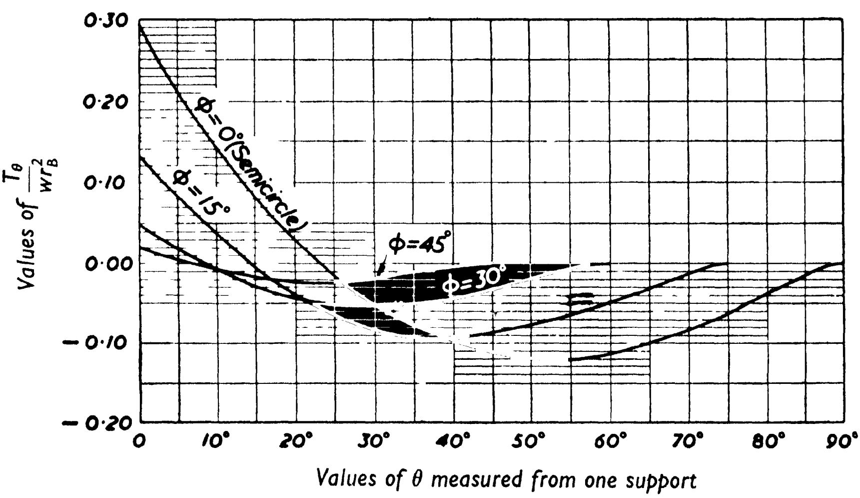


FIG. 317

From Example 2 above, the maximum torque permitted was 28,800 lb.in. Thus

$$28{,}800 = 0{\cdot}025\ wr_B^2$$

$$r_B = 120 \text{ in.}$$

$$28{,}800 = 0{\cdot}025w \times 120^2$$

$$w = \frac{28{,}800}{360} = 80 \text{ lb. per in.}$$

$$= 0{\cdot}43 \text{ ton per ft.}$$

There are implicit assumptions in such mathematical treatments never fully met in practice, but calculations based on the above derivations can be accepted as guides useful in practical design.

Gibson and Ritchie show a number of valuable curves giving the magnitudes of torque and moment at all points along circular-arc bow girders of various shapes and with various loadings and with these curves before him the designer need not carry out the differentiation mentioned above. These curves show that, as ϕ decreases and the bow girder subtends a greater angle, the fixing torque T_a increases at a greater rate than the torque T_m and the fixing torque is the controlling factor in design.

Conclusion

The treatment of an 'I'-section given in this section refers to the method devised at King's College in the University of Durham after experiments and calculations by Cassie, Dobie and Gent. It determines within practical limits of accuracy the shear stress at the most stressed point in the section—the re-entrant fillet. The problem of the design of structural sections subjected to torsion is not yet, by any means, completely solved, and other experimenters have devised quite different methods of attack from that given here. Unfortunately the final results do not always agree, as can be expected in such a complex problem.

The study of shear stress in this section represents only one part of the calculations necessary in the design of a twisted beam. The determination of bending stress is, however, a familiar procedure. The problem of deflection can also be solved, for the application of Castigliano's method gives the displacement of the beam. If the beam is in torsion because it is a bow girder, the designer is advised to study the thorough investigation made by Gibson and Ritchie (see bibliography) for in a section of this length only one aspect of the matter can be pursued with any degree of detail.

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DOBIE, W. B. 'The Torsional Strength of Structural Members', *The Structural Engineer*, Vol. 30 (1952), page 34.

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24. MISCELLANEOUS FORMS OF CONSTRUCTION

Castella Sections

These special sections incorporate a simple system of increasing the strength of a section without increasing its weight. By a combination of flame-cutting and electric welding, the ordinary standard section is converted into a much deeper member. The principle can be applied, not only to joist sections, but to channels and plates.

The section to be treated is flame-cut along the web to a pre-determined profile, as shown in Fig. 318 (a). One of the resulting pieces is then turned

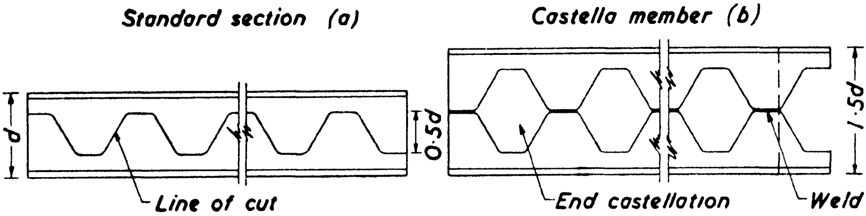


FIG. 318

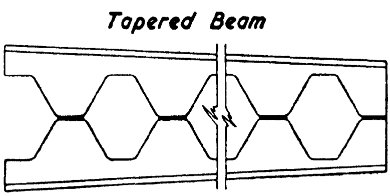


FIG. 319

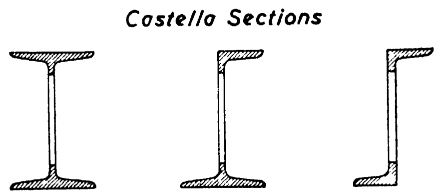


FIG 320

end for end and rejoined to the other piece, in such a way that the crests of the undulations meet, as shown in Fig. 318 (b). It will be observed that the depth of the section is increased by 50 per cent, with a result that the section modulus is also increased, thus making the members suitable for use over larger spans. The shear strength is, of course, reduced, although this only affects short spans, but this disadvantage may be rectified by welding suitably shaped plates in the end castellations of the members.

Considerable scope is offered to the designer to produce shapes and profiles which are not available in the normal range of rolled sections. For example, Fig. 319 shows how a tapered beam may be produced when the zig-zag cuts have been made asymmetrically about the centre line of the original joist. Similarly, Fig. 320 shows some of the different kinds of sections which may be produced by combining joists, channels and plates.

Castella sections were introduced originally by Mr. G. M. Boyd, but the Patent Rights have now been taken over by the Appleby-Frodingham Steel Company, a branch of the United Steel Companies Ltd. The Company will undertake supply to customers' requirements or, alternatively, will issue licences to fabricators in Great Britain to manufacture castellated members.

Castella Tables

The tables on pages 523 to 526 give the geometrical properties of Castella Joists, Channels and Zed Bars.

The thicknesses of the web and flanges are not quoted as they are identical with those of the original section. The values of the moments of inertia and moduli of section are all associated with the nett cross-sectional area of the castellated member.

Open-web Girders

A Castella section is a particular kind of girder with open web. In recent years many types of light girder with open webs have been produced which are used as purlins or floor joists. The light member which is illustrated in Fig. 321, consists of 'T'-flanges held apart by loops of round bar which

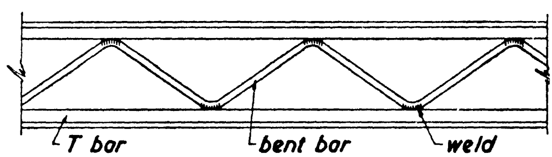


FIG. 321

are welded to the stalks. Other beams are similar, but the flanges are composed of flats. The ends of open-web girders may be closed by flats, angles or 'T'-bars, depending on the method of end support or attachment.

Composite-beam Construction

Fig. 322 shows an R.S.J. supporting a concrete slab. When the slab is loaded, both the slab and joist deflect individually, the resulting stress diagram for the joist being as shown.

Had there been some kind of bond between the slab and the joist, then together they would have acted as a 'T'-beam in which all or most of the compression would have been taken by the concrete and the tension by the joist. Even if the joist were encased in concrete, the encasement by itself could not be considered to have provided sufficient bond to ensure composite action. It is an unfortunate fact that the greatest shear stress occurs at the neutral axis of the composite section, which is always very near the top flange of the joist. Consequently, composite action can be achieved only by employing shear connectors, such as cuttings from plates, angles or channels, spirals or bent and hooked bars, which are welded to the top flange of the joist as shown in Fig. 323.

Joist sections have been mentioned because they are commonly used in light composite construction, but the profiles shown in Fig. 324 can be used and, indeed, they are more economical where the spans are great, or the superimposed load is heavy owing to the fact that the area of the top flange of the steel component is smaller than that of the bottom flange.

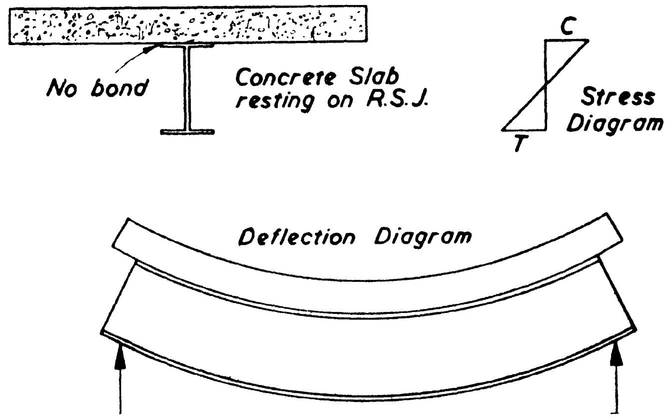


FIG. 322

Although examples of composite construction are quite common abroad where they have been used for over twenty years, they are unusual in Great Britain. Consequently, there are no regulations or codes covering their use, but it is suggested that they might be designed, where applicable, in a similar fashion to normal 'T'-beams in reinforced concrete construction. This being

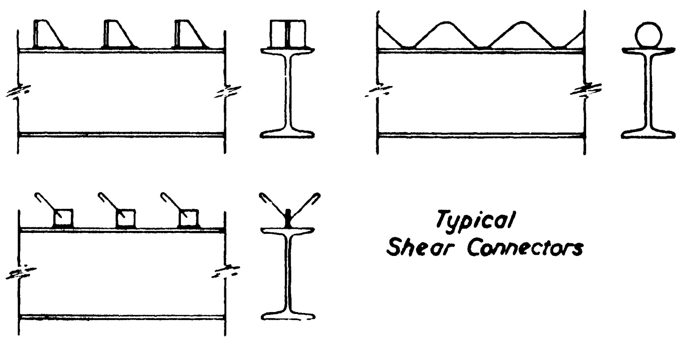


FIG. 323

so, then the limit of width of concrete slab which could be considered as forming part of the composite construction would be the least of the three following dimensions:

1. One-third of the effective span of the 'T'-beams;
2. Distance between the centres of the ribs of the 'T'-beams;
3. Twelve times the thickness of the slab plus the breadth of the rib, where the steel member is encased.

While the normal steel stresses may be employed for the steel component of the composite beam, those to be used for the concrete will depend on the mix and upon the type of structure. For example, the B.S. Code of Practice for reinforced concrete could be used for buildings, and the Memorandum on

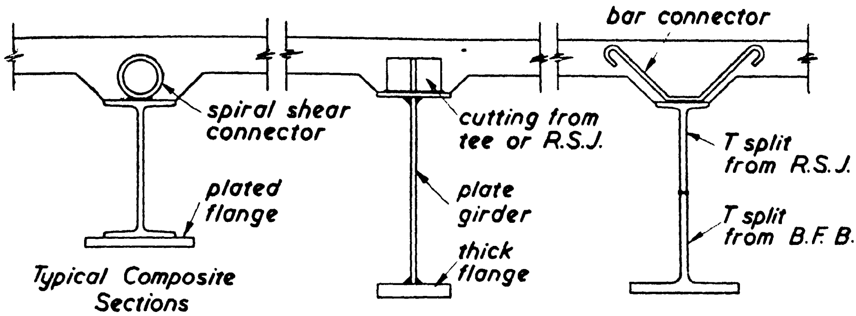


FIG. 324

Bridge Design and Construction issued by the Ministry of Transport for bridges.

In the Memorandum various values are quoted for the modular ratio m,

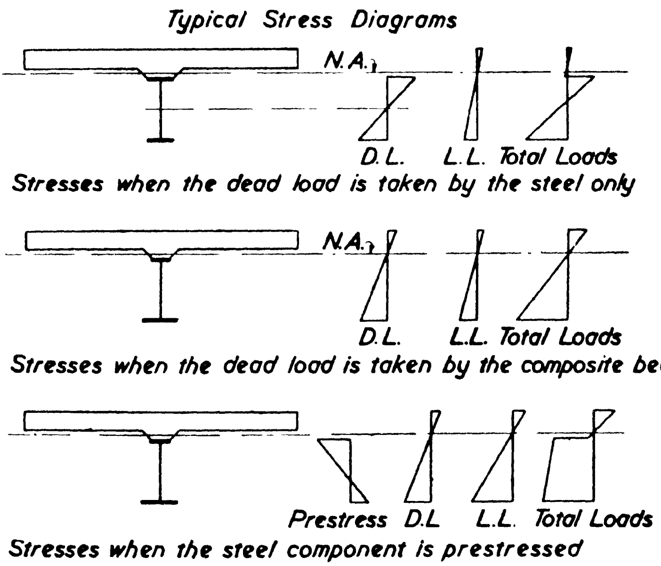


FIG. 325

depending on the quality of the concrete used, but the single value of 15 is quoted in the Code of Practice.

The strength of a composite member depends on a number of factors. If the shuttering and wet concrete are supported by the steel component alone, then the steel section takes the entire dead load, composite action being

effective only for superimposed loads. If, however, the shuttering is supported independently, then the only stress in the steel section acting alone will be that due to its own weight. From the point of view of the strength of the member, the first method is the least economical way of constructing a composite member. The second method, which is normally adopted, is better, but as an alternative it is possible to bend the steel section upwards by jacks or in other ways, so that it is prestressed, thus providing the most economical use of materials. The three methods are demonstrated in Fig. 325.

Although it is beyond the scope of this section of the book, it is also possible to employ prestressed concrete in composite construction, but at the present time this practice is not common.

Design Procedure

The design procedure is analogous to that employed for 'T'-beams in normal reinforced concrete construction.

The slab is designed like a continuous beam spanning transverse to the steel components which support it. In bridges the slab depth is at least 7 in., but in buildings it can be as little as 5 in. It is important that a sufficient depth of concrete be provided to ensure adequate anchorage of the shear connectors, and, consequently, it may be desirable to haunch the concrete over the steel components. The latter are spaced at from 5-ft. to 8-ft. centres in bridges but may be at much greater spacings in buildings.

The limits suggested for the breadth B of the 'T'-beam have already been stated. The size of the effective concrete slab having been fixed, it is normal to adjust the size and location of the steel component so that the resultant stresses in concrete and steel are approximately the maximum permissible. Subsequently, the calculations resemble those for a flitched beam, where the area of concrete may be divided by m and the resulting value considered as equivalent steel, or, alternatively, the area of steel may be multiplied by m and considered as equivalent concrete. Considering Fig. 326, and adopting the first course, when $\bar{y}$ is equal to, or greater than, d:

$$\bar{y}=\frac{(A_c y_c/m)+A_s y_s}{(A_c/m)+A_s}$$

When, however, the slab is thick and the neutral axis lies inside it, an unusual case, then

$$\bar{y}=\frac{mA_s}{B}\left[\sqrt{\left(1+\frac{2y_s \, . \, B}{mA_s}\right)}-1\right]$$

Having determined the position of the neutral axis, the gross moment of inertia I_g of the composite section may be found. When $\bar{y} \gg d$,

$$I_g=\frac{I_c}{m}+\frac{A_c}{m}(\bar{y}-y_c)^2+I_s+A_s(y_s-\bar{y})^2$$

where I_c and I_s are the moments of inertia of the concrete slab and steel component respectively about their own neutral axes.

The area of the haunch of concrete over the steel is usually small and its effect can be neglected. In this case,

$$A_c = B \times d \quad \text{and} \quad y_c = d/2$$

When $\bar{y} < d$,

$$I_g = \frac{B\bar{y}^3}{3m} + I_s + A_s(y_s - \bar{y})^2$$

The section moduli Z in terms of concrete and steel respectively are as follows:

$$Z_c = \frac{mI_g}{\bar{y}}; \quad Z_s = \frac{I_g}{(D - \bar{y})}$$

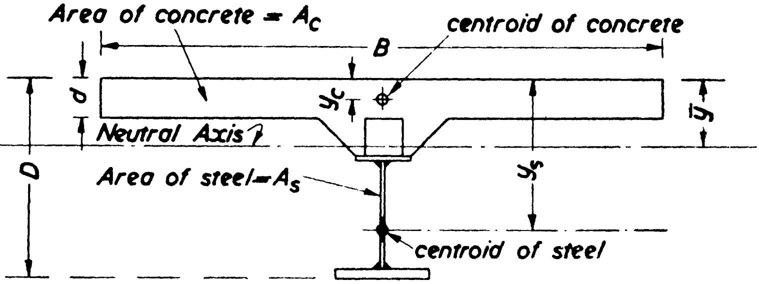


FIG. 326

The horizontal shear, s, at the top of the steel component may be found from the usual formula, slightly adjusted,

$$s = \frac{S \, . \, a \, . \, y}{mI_g}$$

where S = the vertical shear force at the section considered,

a = the area of concrete above the top flange of the steel component,

and y = the distance between the neutral axis and the centroid of the area of concrete.

For shear connectors welded at centres x, the shear force F on the connectors and the welds $= s \times x$.

Obviously the shear connectors will be fairly close together at the ends of the composite member, where the shear forces are maxima, but at the point of zero shear the spacing could be infinite. However, the maximum spacing for shear connectors is generally accepted to be four times the depth of the concrete slab.

Continuous composite beams have been constructed in various countries, notably in Canada and Switzerland. It will be appreciated that difficulties arise at the supports where the top face of the member is in tension and the bottom in compression. The difficulties may be surmounted in several ways. The concrete can be reinforced longitudinally by steel bars, or the effect of the concrete can be entirely ignored, the steel component being strengthened by the addition of extra flange plates or by increasing the depth of the

section, or combinations of these expedients may be employed. A Swiss example is shown in Fig. 327.

In this case, the overall depth D of the composite girder was 20 in. and the depth of the joist 10 in. At the support the top of the girder was reinforced with four tension bars, approximately $1\frac{3}{8}$ in. diameter, while the

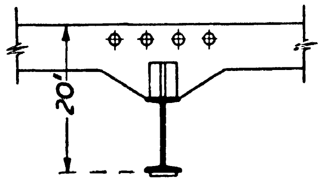


FIG. 327

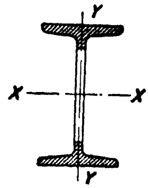
bottom flange of the joist was reinforced by welding on a plate, 4 in. $\times \frac{5}{16}$ in. in cross-section.

It will be appreciated that there is a reduction in the amount of steel used in a composite beam compared with the usual forms of construction, but that there is a considerable amount of fabrication involved. Where steel is plentiful and labour expensive, composite construction is not usually economical, but where steel has to be imported or hauled long distances to remote areas, large economies can be made.

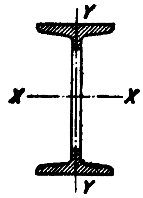
CASTELLA JOISTS

DIMENSIONS AND PROPERTIES

IN INCH UNITS



Size		Wt. per ft. lb.	Area Castellated		Thickness		Moments of Inertia		Moduli of Section		$\frac{r_{xx}}{r_{yy}}$
Original	Castellated		Gross	Nett	Web	Flange	Axis x–x	Axis y–y	Axis x–x	Axis y–y	
24 × 7½	36 × 7½	95	34·78	21·10	·57	1·011	5917·7	62·35	328·8	16·6	10·2
22 × 7	33 × 7	75	27·56	16·56	·50	·834	3910·6	40·95	237·0	11·7	10·3
20 × 7½	30 × 7½	89	32·19	20·19	·60	1·010	3909·5	62·36	260·6	16·6	8·3
20 × 6½	30 × 6½	65	23·62	14·62	·45	·820	2863·1	32·48	190·9	9·9	9·8
18 × 8	27 × 8	80	28·03	19·03	·50	·950	3035·4	69·34	224·8	17·3	6·9
18 × 7	27 × 7	75	27·04	17·14	·55	·928	2699·0	46·43	199·9	13·2	8·0
18 × 6	27 × 6	55	19·96	12·40	·42	·757	1959·1	23·58	145·1	7·8	9·6
16 × 8	24 × 8	75	25·90	18·22	·48	·938	2300·5	68·23	191·7	17·0	6·0
$16 \times 6_H$	24 × 6	62	22·61	13·81	·55	·847	1700·3	27·03	141·7	9·0	8·3
$16 \times 6_L$	24 × 6	50	17·91	11·51	·40	·726	1441·6	22·43	120·1	7·5	8·4
$15 \times 6_H$	22½ × 6	59	21·10	13·60	·50	·880	1476·7	28·14	131·3	9·4	7·6
$15 \times 6_L$	22½ × 6	45	16·09	10·39	·38	·655	1143·2	19·83	101·6	6·6	7·9
15 × 5	22½ × 5	42	15·51	9·21	·42	·647	1000·1	11·76	88·9	4·7	9·7
14 × 8	21 × 8	70	23·81	17·37	·46	·920	1664·8	66·61	158·6	16·6	5·2
$14 \times 6_H$	21 × 6	57	20·28	13·28	·50	·873	1252·7	27·87	119·2	9·3	7·0
$14 \times 6_L$	21 × 6	46	16·39	10·79	·40	·698	1027·9	21·41	97·9	7·1	7·2
14 × 5½	21 × 5½	40	14·36	9·18	·37	·627	882·5	14·76	84·0	5·3	8·0
13 × 5	19½ × 5	35	12·575	8·025	·35	·604	662·3	10·79	67·9	4·3	8·2

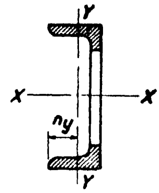


CASTELLA JOISTS

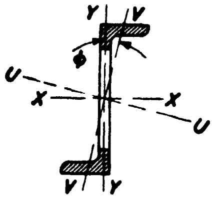
DIMENSIONS AND PROPERTIES IN INCH UNITS

Size		*Wt. per ft.*	*Area Castellated*		*Thickness*		*Moments of Inertia*		*Moduli of Section*		$\frac{r_{xx}}{r_{yy}}$
Original	*Castellated*	*lb.*	*Gross*	*Nett*	*Web*	*Flange*	*Axis x–x*	*Axis y–y*	*Axis x–x*	*Axis y–y*	
12 × 8	18 × 8	65	21·70	16·54	·43	·904	1162·8	65·14	129·2	16·3	4·3
12 × 6_H	18 × 6	54	18·89	12·89	·50	·883	893·4	28·22	99·3	9·4	5·8
12 × 6_L	18 × 6	44	15·40	10·60	·40	·717	767·4	22·09	85·3	7·3	6·1
12 × 5	18 × 5	32	11·55	7·35	·35	·550	515·5	9·67	57·3	3·8	7·6
10 × 8_H	15 × 8	70	23·60	17·60	·60	·970	834·1	71·58	111·2	17·9	3·5
10 × 8_L	15 × 8	55	18·18	14·18	·40	·783	689·0	54·71	91·9	13·6	3·6
10 × 6	15 × 6	40	13·57	9·97	·36	·709	484·8	21·74	64·7	7·2	4·9
10 × 5	15 × 5	30	10·65	7·05	·36	·552	345·4	9·71	46·1	3·9	6·2
10 × 4½	15 × 4½	25	8·85	5·85	·30	·505	284·2	6·48	37·9	2·9	6·9
9 × 7	13½ × 7	50	16·51	12·91	·40	·825	501·3	40·15	74·3	11·4	3·6
9 × 4	13½ × 4	21	7·53	4·83	·30	·457	188·2	4·14	27·9	2·0	7·0
8 × 6	12 × 6	35	11·70	8·90	·35	·648	265·7	19·52	44·3	6·5	3·8
8 × 4	12 × 4	18	6·42	4·18	·28	·398	129·4	3·50	21·6	1·7	6·3
7 × 4	10½ × 4	16	5·62	3·87	·25	·387	92·8	3·36	17·7	1·6	5·4
6 × 3	9 × 3	12	4·22	2·84	·23	·377	49·6	1·45	11·0	0·97	6·0
5 × 3	7½ × 3	11	3·81	2·71	·22	·376	31·6	1·44	8·4	0·96	4·8
4¾ × 1¾	7⅛ × 1¾	6·5	2·33	1·48	·18	·325	15·2	0·25	4·3	0·29	8·0

CASTELLA CHANNELS
DIMENSIONS AND PROPERTIES IN INCH UNITS



Size		Wt. per ft.	Area Castellated		Thickness		Moments of Inertia		Moduli of Section		$\frac{r_{xx}}{r_{yy}}$	Distance n_y
Original	Castellated	lb.	Gross	Nett	Web	Flange	Axis x–x	Axis y–y	Axis x–x	Axis y–y		
17 × 4	25½ × 4	44·34	17·12	8·96	0·48	0·68	1214·70	12·44	95·30	4·49	9·9	2·77
15 × 4	22½ × 4	36·37	13·77	7·63	0·41	0·62	812·77	10·77	73·88	3·96	8·7	2·72
12 × 4	18 × 4	31·33	11·61	6·81	0·40	0·60	468·85	9·69	52·09	3·67	6·9	2·64
12 × 3½	18 × 3½	26·37	10·04	5·48	0·38	0·50	373·26	5·80	41·47	2·41	8·0	2·40
10 × 3½	15 × 3½	24·46	8·99	5·39	0·36	0·56	257·41	5·90	34·32	2·60	6·6	2·27
10 × 3	15 × 3	19·28	7·27	4·07	0·32	0·45	192·31	3·22	25·64	1·58	7·7	2·03
9 × 3½	13½ × 3½	22·27	8·08	5·02	0·34	0·54	194·52	5·51	28·81	2·45	5·9	2·25
9 × 3	13½ × 3	17·46	6·49	3·79	0·30	0·44	147·12	3·01	21·79	1·51	7·0	2·00
8 × 3½	12 × 3½	20·21	7·22	4·66	0·32	0·52	143·27	5·07	23·88	2·30	5·3	2·21
8 × 3	12 × 3	15·96	5·81	3·57	0·28	0·44	109·70	2·87	18·28	1·47	6·2	1·95
7 × 3½	10½ × 3½	18·28	6·43	4·33	0·30	0·50	101·73	4·67	19·37	2·14	4·6	2·18
7 × 3	10½ × 3	14·22	5·09	3·27	0·26	0·42	79·52	2·60	15·14	1·36	5·5	1·91
6 × 3½	9 × 3½	16·48	5·69	4·01	0·28	0·48	69·01	4·27	15·33	1·98	4·0	1·15
6 × 3	9 × 3	16·51	6·00	3·72	0·38	0·48	62·70	2·91	13·93	1·56	4·6	1·87
6 × 3	9 × 3	12·41	4·40	2·90	0·25	0·38	50·27	2·27	11·17	1·19	4·7	1·91
5 × 2½	7½ × 2½	10·22	3·63	2·38	0·25	0·38	28·24	1·31	7·53	0·84	4·6	1·56
4 × 2	6 × 2	7·09	2·57	1·16	0·24	0·31	12·09	0·55	4·03	0·44	4·6	1·26
3 × 1½	4½ × 1½	4·60	1·65	1·05	0·20	0·28	5·07	0·20	2·25	0·22	5·0	0·91



CASTELLA ZED BARS
DIMENSIONS AND PROPERTIES
IN INCH UNITS

Size		*Wt. per ft.*	*Area Castellated*		*Thickness*		*Moments of Inertia*		*Moduli of Section*		$\frac{r_{xx}}{r_{yy}}$	*Angle* ϕ
Original	*Castellated*	*lb.*	*Gross*	*Nett*	*Web*	*Flange*	*Axis x–x*	*Axis y–y*	*Axis x–x*	*Axis y–y*		° ′
17 × 4	25½ × 4	44·34	17·12	8·96	0·48	0·68	1214·70	21·13	95·30	5·62	8·1	4 3
15 × 4	22½ × 4	36·37	13·77	7·63	0·41	0·62	812·77	19·51	73·88	5·14	6·9	4 55
12 × 4	18 × 4	31·33	11·61	6·81	0·40	0·60	468·85	18·86	52·09	4·96	5·3	6 38
12 × 3½	18 × 3½	26·37	10·04	5·48	0·38	0·50	373·26	10·27	41·47	3·10	6·4	5 13
10 × 3½	15 × 3½	24·46	8·99	5·39	0·36	0·56	257·41	11·86	34·32	3·57	4·9	7 12
10 × 3	15 × 3	19·28	7·27	4·07	0·32	0·45	192·31	5·86	25·64	2·06	6·1	5 32
9 × 3½	13½ × 3½	22·27	8·08	5·02	0·34	0·54	194·52	11·38	28·81	3·42	4·3	8 8
9 × 3	13½ × 3	17·46	6·49	3·79	0·30	0·44	147·12	5·77	21·79	2·02	5·3	6 30
8 × 3½	12 × 3½	20·21	7·22	4·66	0·32	0·52	143·27	11·05	23·88	3·31	3·8	9 17
8 × 3	12 × 3	15·96	5·81	3·57	0·28	0·44	109·70	5·79	18·28	2·03	4·6	7 42
7 × 3½	10½ × 3½	18·28	6·43	4·33	0·30	0·50	101·73	10·56	19·37	3·15	3·2	10 50
7 × 3	10½ × 3	14 22	5·09	3·27	0·26	0·42	79·52	5·60	15·14	1·95	3·9	9 14
6 × 3½	9 × 3½	16·48	5·69	4·01	0·28	0·48	69·01	10·12	15·33	3·01	2·7	12 32
6 × 3	9 × 3	16·51	6·00	3·72	0·38	0·48	62·70	6·19	13·93	2·20	3·3	10 13
6 × 3	9 × 3	12·41	4·40	2·90	0·25	0·38	50·27	4·95	11·17	1·72	3·3	11 15
5 × 2½	7½ × 2½	10·22	3·63	2·38	0·25	0·38	28·24	2·88	7·53	1·21	3·3	10 31
4 × 2	6 × 2	7·09	2·57	1·61	0·24	0·31	12·09	1·17	4·03	0·62	3·4	10 10
3 × 1½	4½ × 1½	4·60	1·65	1·05	0·20	0·28	5·07	0·45	2·25	0·32	3·5	11 3

25. THE APPLICATION OF THE PLASTIC THEORY TO THE DESIGN OF MILD-STEEL BEAMS AND RIGID FRAMES

The following section of the Manual is a reproduction in part of Publication No. 3 of the British Constructional Steelwork Association of Artillery House, Westminster, S.W.1, written by F. A. Partridge, B.Sc.(Eng.) London, A.C.G.I., M.I.C.E., M.Inst.W., to whom grateful acknowledgement is made.

B.S. 449 gives the designer the choice of three methods for the design of a steel framework, namely simple design, semi-rigid design and fully rigid design. One of the methods of producing a fully-rigid design is to use the Plastic Theory.

The purpose of this article is to explain what happens when members are bent beyond the elastic range to failure, to show how failure loads can be estimated, and so to introduce the Plastic Theory—in its application to beams and portals—as a basis for fully rigid design.

General

In accordance with the principle governing the design of all engineering structures—that there shall be a reserve of strength against failure—it is a criterion in the conventional design of steelwork in particular, that stresses developing in the several parts of a structure under working loads shall not exceed certain prescribed limits.

The limit for tension is set well within the elastic range of the steel, to give, in fact, two safeguards—the first against permanent deformation and the second against actual fracture. Thus in a tension member of mild steel, of yield point 15·25 tons per sq. in. and breaking-strength 28–33 tons per sq. in., designed to B.S. 449 at a working stress of 9 tons per sq. in., the measure of the safeguard against permanent elongation is $15{\cdot}25 \div 9 = 1{\cdot}7$ and against fracture $28 \div 9 = 3{\cdot}1$.

The limit for compression is somewhat different. For 'slender columns' the working stress is limited by B.S. 449 to one-half the average stress at which the column is assumed to fail by buckling. This average stress being lower than yield stress, there is thus only one safeguard—against ultimate failure—and its measure is 2·0.

Members subjected to bending are limited by B.S. 449, with certain provisions for support against lateral buckling, to a flange fibre stress of 10 tons per sq. in. The safeguard against permanent deformation is therefore represented by $15{\cdot}25 \div 10 = 1{\cdot}525$. Whether the structure be a simple beam or a complex rigid frame, this safeguard is applied against the occurrence of permanent strain at one isolated point in its length. The margin of strength against ultimate collapse is, however, unknown in conventional design. Its determination lies beyond the scope of the theory of elasticity on which

conventional design is based. There is no analogy with the tension member; the ratio of collapse load to working load is *not* $28 \div 10 = 2{\cdot}8$.

The behaviour of mild-steel beams stressed beyond the elastic limit has received close attention in recent years, and a 'simple plastic theory of bending' has been developed in explanation. With this theory it has become possible to estimate with fair accuracy the load at which a conventionally designed beam will actually fail, and hence the hitherto unknown ratio of collapse load to working load. This ratio—which may also be regarded as the quantity by which the working load must be multiplied to produce the condition of collapse—will, for convenience, be referred to in this article as the 'load factor'.

The more recent work of Professor Baker* of Cambridge University is, however, of much greater importance. He has seen in the theory not only a means of determining the ultimate strength of all beams and rigid frames, but also the possibility of evolving a completely new method for designing them. The work he has already carried out on beams and portals shows that in structures designed to fixed limits of stress, the load factors vary considerably from structure to structure. They depend on the type and disposition of the loading, on the proportions of the frame and on its degree of redundancy. With 'constant load factor' as the new design basis, it is the maximum stresses developing under working load that will vary from structure to structure. These working stresses may be higher—sometimes much higher—than designers have been accustomed to use in the past. Deflections, too, may be greater, but not necessarily excessive. The economies to be made, on the other hand, will be considerable.

Plastic Theory and Load Factors

The theory derives from the study of the stress-strain curve for annealed mild steel, as produced during a tension test in a spring-loaded testing machine, as in Fig. 328.

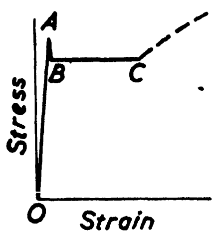


FIG. 328

In this curve A represents the upper yield point. When this stress has been reached and straining is continued, there is an immediate drop in stress to B, the lower yield point. Further straining continues to C without increase in load. At this point plastic flow under constant stress ceases and the load must be increased before further strain can be produced. From O to A the strain is elastic; BC is the range of plastic strain; at C strain-hardening commences. The considerable magnitude of the plastic strain as compared with the elastic strain is *the* characteristic property of mild steel on which the plastic theory is based.

For the purpose of examining the behaviour of structural members subjected to bending beyond the elastic range, upper yield and strain hardening

* BAKER, J. F. 'A Review of Recent Investigations into the Behaviour of Steel Frames in the Plastic Range', *Journal, Institution of Civil Engineers*, Vol. 31 (1948–9), page 188.
BAKER, J. F. 'The Design of Steel Frames', *Structural Engineer*, Vol. 27 (1949–50), page 397.

are ignored and it is assumed that the plastic range is entered when lower yield stress is reached. It is assumed also that the stress-strain relationship for compression is identical with that for tension, and that plane sections remain plane in the plastic condition.

The working stress across the section of a simple beam subject to load W has a straight-line distribution as in diagram (a), Fig. 329, where xw represents the stress in the extreme outer fibres.

Under increasing load the extreme fibres will eventually reach yield stress xy as in diagram (b) and will offer no further increase in resistance. Beyond this point increase of load will induce resistance from more and more inner fibres, each in turn reaching the yield stress, diagram (c), until ultimately, under a load C yield stress spreads, in effect, to the centre of the section, diagram (d). The section is then 'fully plastic'. In this condition it has reached its effective maximum capacity for resistance; the beam can carry no further increase of load. Any attempt to increase the load will merely

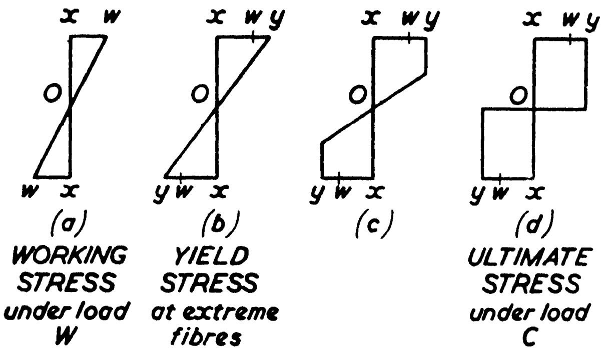


FIG. 329

result in increased deflection; the beam will behave as if 'hinged' at the plastic section; a condition of collapse has been reached. The ratio, C/W, of the 'collapse load' to the 'working load' is the load factor.

The load factor for a simple beam can be calculated directly from the shape of the section and the cross-sectional stress distributions at working load and collapse. This is most easily demonstrated by considering the beam to be of rectangular cross-section of breadth B in. and depth D in., as shown in Fig. 330. For a working stress of 10 tons per sq. in. and a yield stress of 15·25 tons per sq. in.:

Moment of Resistance under *Working Load*

$$=B\times\frac{D}{2}\times\frac{10}{2}\times\frac{2}{3}D=\frac{10\times BD^2}{6}$$

Moment of Resistance at *Collapse*

$$=B\times\frac{D}{2}\times 15{\cdot}25\times\frac{D}{2}=15{\cdot}25\times\frac{BD^2}{4}.$$

Hence the load factor $=\dfrac{15{\cdot}25}{10}\times\dfrac{BD^2/4}{BD^2/6}$

$$=1{\cdot}525\times 1{\cdot}5=2{\cdot}2875.$$

It will be observed that the shape of the section has an important influence on the load factor and that, just as the moment of resistance at working stress is the product of working stress and modulus Z of the section, so the moment of resistance at collapse is the product of yield stress and the first moment of area of the section about its centre. This first moment of area S is called the 'plastic modulus' and the ratio S/Z the 'shape factor'. As will be seen,

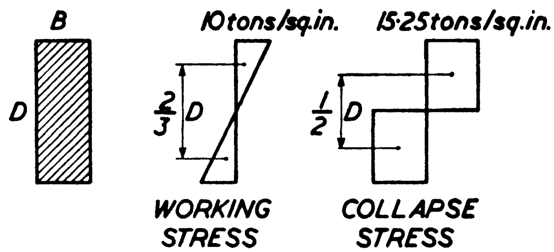


FIG. 330

the shape factor for a rectangular section is $(BD^2/4) \div (BD^2/6) = 1{\cdot}5$. For joists—and indeed for built-up 'I'-sections of similar proportions—it varies slightly around an average value of 1·15. Thus the load factor for a simple joist beam on the same assumptions of working stress and yield stress as above would be $1{\cdot}525 \times 1{\cdot}15 = 1{\cdot}75$. This low value may surprise those who imagined it to be in the region of 2·8.

The distribution of stress across the section of the beam at collapse has

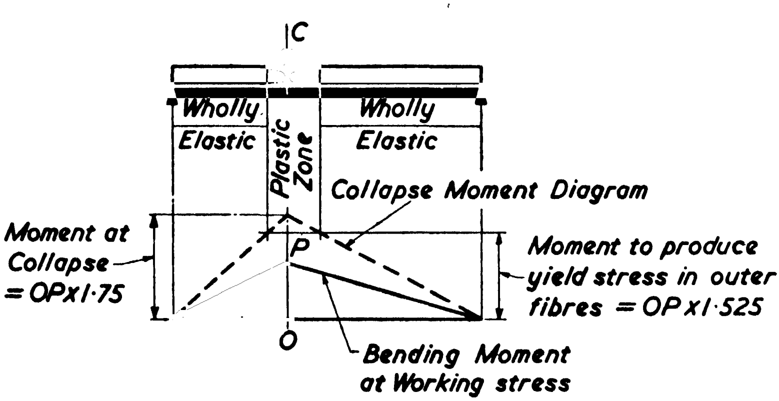


FIG. 331

been explained; it is interesting to note the corresponding distribution of elastic and plastic stress *along* the beam, as shown in Fig. 331. At collapse the section immediately beneath the point load is fully plastic and becomes virtually a hinge as far as an attempt to increase the load is concerned. The case illustrated is a simple beam under a single concentrated load; for other conditions of loading the plastic hinge develops at the point of maximum elastic moment.

Redundancy, such as fixity at one end of the beam, introduces a new phenomenon, and a further influence on load factor. Such a case is considered below in which a point load, W tons, is carried, for convenience of example, at the centre of the span L in., as shown in Fig. 332.

The joist would, by conventional methods, be designed on the end moment of $\frac{3}{16}$ WL. Let it be assumed that the joist section selected is exactly suited

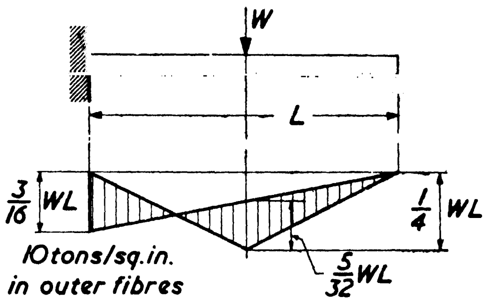


FIG. 332

to requirements at a working stress of 10 tons per sq. in. in the extreme flange fibres. In order to determine the load factor, consider the development of stress in the joist as the load is increased from its working value of W to its value C at collapse.

Stresses and strains are wholly elastic until yield stress develops in the extreme fibres of the joist at the fixed end. The B.M. there, is then

$$\frac{15 \cdot 25}{10} \times \frac{3}{16} WL, \quad \text{and at the centre} \quad \frac{15 \cdot 25}{10} \times \frac{5}{32} WL,$$

as shown in Fig. 333.

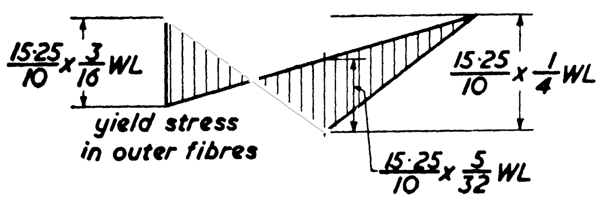


FIG. 333

With further increase in load, plasticity develops at the fixed end and is complete when the moment there is equal to

$$\frac{15 \cdot 25}{10} \times 1 \cdot 15 \times \frac{3}{16} WL, \quad \text{i.e.} \quad 1 \cdot 75 \times \frac{3}{16} WL.$$

The moment at the centre increases also, though not exactly at the same rate. Further increase of load has no effect on the moment at the fixed end, where a plastic hinge has formed, but increases the moment at the centre until the section there becomes fully plastic, as shown in Fig. 334.

The full carrying-capacity of the beam has been reached; failure is imminent because of the development of two plastic hinges. If the collapse load is C tons, then from the geometry of the collapse moment diagram:

$$\frac{CL}{4}=\frac{3}{2}\times(1\cdot75\times\frac{3}{16}WL).$$

Hence the load factor

$$=\frac{C}{W}=4\times\frac{3}{2}\times1\cdot75\times\frac{3}{16}=1\cdot97.$$

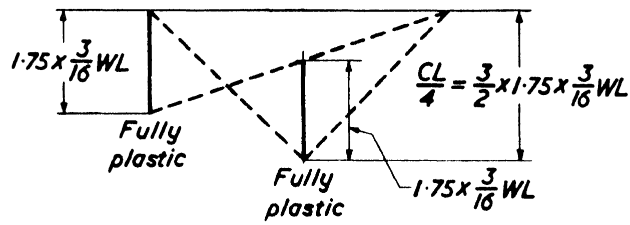


FIG. 334

If the loading is uniformly distributed instead of concentrated, the B.M. under working load of w tons per in. and the collapse load of c tons per in. would be as shown in Fig. 335.

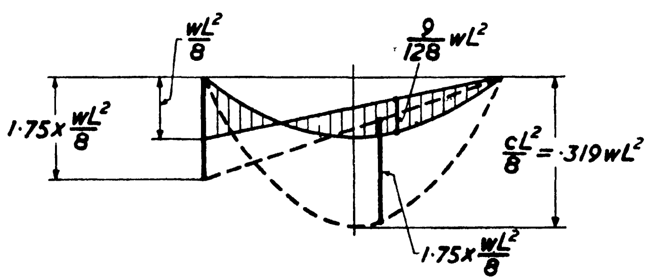


FIG. 335

From the geometry of the B.M. curves:

$$\frac{cL^2}{8}=0\cdot319wL^2.$$

Hence load factor

$$=\frac{c}{w}=8\times0\cdot319=2\cdot55.$$

Applying the same processes to the simpler case of the beam fixed at *both* ends, the load factors:

for a single central load $=1\cdot75$

and for a uniformly distributed load $=2\cdot34$

The following table summarises these load factors for the single span beams:

	Point Load at Centre	*Uniformly Distributed Load*
Simply supported	1·75	1·75
Fixed one end	1·97	2·55
Fixed both ends	1·75	2·34

This variation in the load factor is the result of designing on a limited fibre stress, and the question arises—'If a load factor of 1·75 is acceptable in some cases, why not in all?'

Design of Beams

It has been shown how the plastic theory of simple bending can be used to calculate the load factor of a single span beam designed to a specified working stress. It will have become obvious that the theory may be used conversely as the basis for the design of a beam to any preferred load factor. The method involves three simple steps:

1. The working load is first multiplied by the load factor, to give the collapse load.
2. The B.M. diagram for this collapse load is constructed as for a freely supported beam without end restraint.
3. A line representing the collapse restraining moment, and becoming the base line of the complete collapse moment diagram, is drawn across the curve in such a way as to make the moments at the plastic hinge points equal.

By way of illustration, in the case of the beam fixed at one end and carrying a central load as shown in Fig. 336:

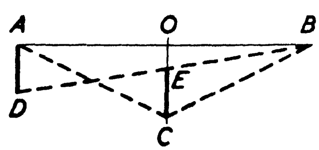


FIG. 336

ACB is the simple span collapse moment diagram.

AD is the collapse restraining moment at the fixed end,

DB being drawn to make $AD = EC$.

In this case it is obvious from the geometry of the diagram that $AD = EC = \frac{2}{3}OC$.

The case for uniformly distributed load gives rise to a simple problem in the geometry of the parabola, as shown in Fig. 337.

ACB is the simple span collapse moment diagram.

AD is the collapse restraining moment at the fixed end,

DB being drawn to make $AD = FG$ (max.).

This may be done with sufficient accuracy by trial and error. The exact solution, obtained analytically, gives:

$$AD = FG = 0{\cdot}6863 \times OC, \quad \text{and} \quad AQ = 0{\cdot}5858 \times AB.$$

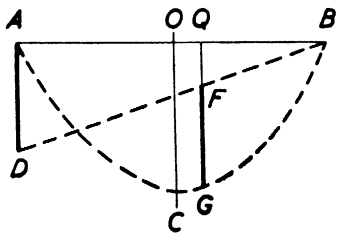


FIG. 337

Beams built in at both ends are even more simply dealt with, as shown in Fig. 338. DH is drawn parallel to AB to make $AD = EC = BH$, i.e. so that $EC = \frac{1}{2}OC$.

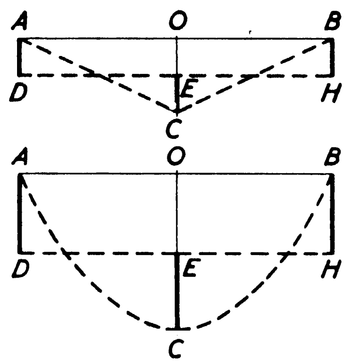


FIG. 338

In all these cases the modulus of section required for joist sections is

$$AD \div (1{\cdot}15 \times 15{\cdot}25),$$

AD being expressed in tons ins.

For the purpose of explaining the method of design, the cases considered have been standard cases of simple symmetrical loading. Where loading is complex and unsymmetrical the considerable advantage of the method becomes apparent. All that is necessary after the construction of the simple-span collapse moment diagram, is the drawing of a straight line across it—through the simple support end in the case of the beam fixed at one end, and

parallel with the base line for the beam fixed at both ends, as shown in Fig. 339, in such a way that (in both cases) $AD = EC$.

In designing a beam by this method, the ideas to be kept clearly in mind are:

That under the collapse load a condition of imminent collapse exists in the beam; that this condition results from the development of plastic hinges in the beam—one in the simple span, two in the beam fixed at one end and

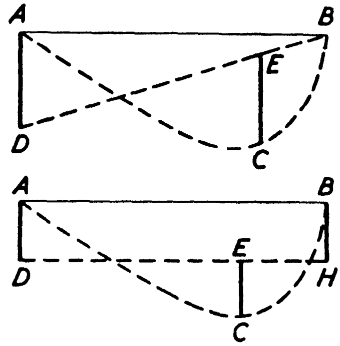


FIG. 339

three in the beam fixed at both ends; that the moments at adjacent plastic hinges are of opposite sign; that at a plastic hinge the full plastic moment of resistance of the beam is developed; and that the required moment of resistance, determined by the conditions of the problem, can be found by a simple graphical construction applied to the collapse moment diagram drawn for the simple span condition.

Application to Continuous Beams

The method has a ready application to the case of continuous beams. The infinite number of cases resulting from three variables—load, span length and number of spans—and the wide choice of beam sections appropriate to

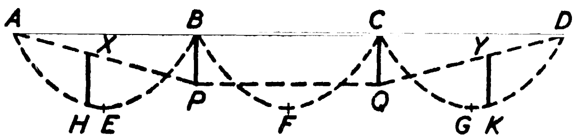


FIG. 340

each case makes it impossible to deal adequately with the subject within the limits of this article, but by way of illustration three solutions are given below for the case of three equal spans with static uniformly distributed load.

In the continuous beam *ABCD*:

AHB, *BFC* and *CGD* are the parabolic B.M. diagrams for the uniformly distributed collapse load applied to *AB*, *BC* and *CD* as simple spans.

For a beam of uniform section throughout, *APQD* is drawn to make $HX = BP = CQ = YK$, as shown in Fig. 340.

There may be some economy in making the centre span of lighter section, in which case $APQD$ is drawn to make $BP = FZ = CQ$ to determine this sec-

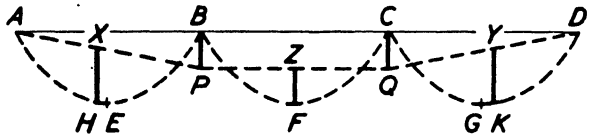


FIG. 341

tion. Then HX ($= YK$) determines the section of the outer spans, as shown in Fig. 341.

Alternatively the centre section could be used throughout with the outer

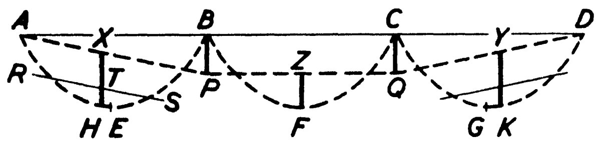


FIG. 342

spans reinforced. RTS is drawn parallel to AP, making $XT = BP$. Then TH represents the plastic moment of resistance required from the flange reinforcing plates covering the portion RS, as shown in Fig. 342.

Application to Portals

The development of the theory of plasticity as applied to portal frames stressed beyond the elastic range has been the comparatively recent work undertaken for the British Welding Research Association by Professor J. F. Baker and his research team at Cambridge University. By the autumn of 1945 they had progressed sufficiently far to be able to invite structural engineers to collaborate in an attempt to establish an entirely new practical method of design. This invitation was accepted by the British Constructional Steelwork Association, who arranged for a small panel of engineers and designers to spend frequent periods at Cambridge during the following year, to work in close contact with the team of scientists. It was found that the graphical method of designing beams as already described could very easily be applied to the solution of the more complicated problem of the portal frame; and the application of this convenient method proved most useful when investigating the modes of failure of portal frames, and estimating the material savings to be made by the adoption of the criterion of constant load factor. The work is described in detail in the *Plastic Design of Single Bay Portal Frames*, published by the British Welding Research Association in August 1947.

Hinged-base Portal

The best way, perhaps, of explaining the method is to apply it to the simple hinged-base portal, rectangular in shape, composed of members of uniform

section, and subjected to single vertical and horizontal loads, as shown in Fig. 343.

Ignoring, for the moment, axial load and lateral buckling, such a portal would be designed in the conventional way by determining the greatest B.M. occurring in the frame under working loads and providing an appropriate section to carry it at prescribed working stresses. The greatest B.M. would occur either at C or at O, depending on the relative magnitudes of the loads V and H and the dimensions of the frame L and h.

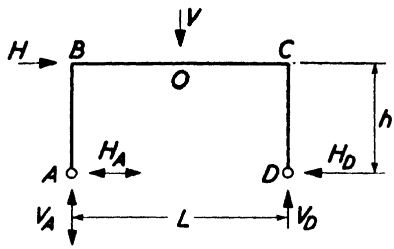


FIG. 343

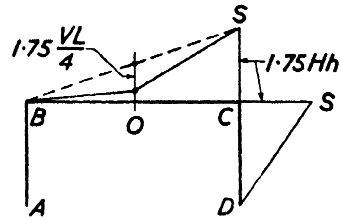


FIG. 344

Now, under collapse loads, plastic hinges must develop at *two* points, for with one plastic hinge only, the frame would have the stability of a three-pin portal. These two points must be either C and O, or C and B, depending on the relative values of $VL/4$ and Hh.

First, then, the working loads V and H are multiplied by the load factor, say 1·75, and with the redundant horizontal reaction H_A at A reduced to zero, the 'free' collapse moment diagram is drawn, as shown in Fig. 344.

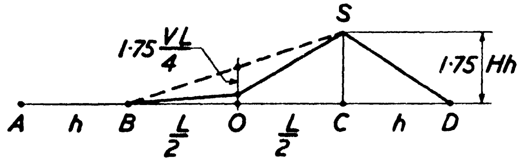


FIG. 345

The variation and continuity of the B.M. around the frame is much more clearly seen if the diagram is drawn with AB and DC swung up into line with BC, as shown in Fig. 345.

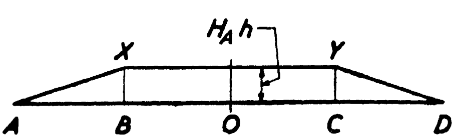


FIG. 346

The effect of restoring reaction H_A is to produce a moment throughout the frame of the form shown in Fig. 346.

Now whereas in elastic stress design the horizontal reaction at *A* is the force that would reduce the horizontal deflection of *A* to zero, in plastic collapse design it is the force that will make the collapse moment at *C* equal to the

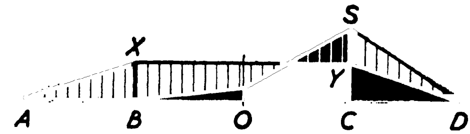


FIG. 347

collapse moment at *O*, or at *B*. It is apparent in this case that *XY* should be drawn midway between *C* and *S*, and the complete B.M. diagram for collapse must be as shown in Fig. 347, giving plastic hinges at *C* and *B*, and a required plastic moment of resistance equal to *BX* and *SY*.

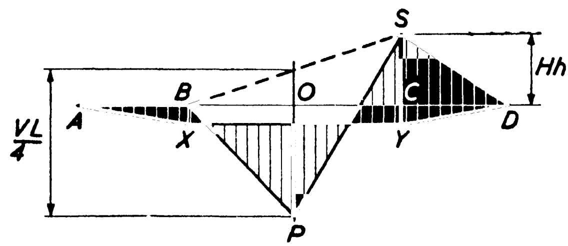


FIG. 348

For larger values of $VL/4$ the solution would be as shown in Fig. 348, with *XY* drawn midway between *S* and *P*, and with plastic hinges at *C* and *O*.

Fixed-base Portal

In the case of the fixed-base portal frame, plastic hinges must form, at collapse, at *A*, *B*, *C* and *D*; or at *A*, *O*, *C* and *D*; or at *B*, *O* and *C*. *Where* they form will again depend entirely on the magnitude of the loads and the dimensions of the frame, or more specifically, as before, on the relative values of $VL/4$ and *Hh*.

In the first two cases, in which plastic hinges form at the bases, the B.M. throughout the frame due to these equal collapse restraining moments *m*, is of the form shown in Fig. 349.

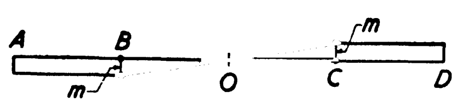


FIG. 349

The appropriate superimposition of this diagram, combined with the diagram of moment due to the horizontal reaction H_A (Fig. 350) on the free collapse moment diagram will give the solution. The method of procedure,

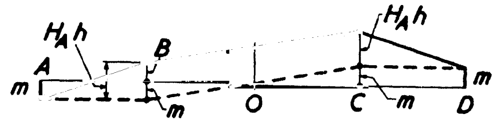


FIG. 350

then, is to make a trial, graphically, as shown in Fig. 351, and from it form the equations for hinge moments at A, B, C and D, which give the exact solution:

$$m = (H_A h - m) = (CS - H_A h - m) = m.$$

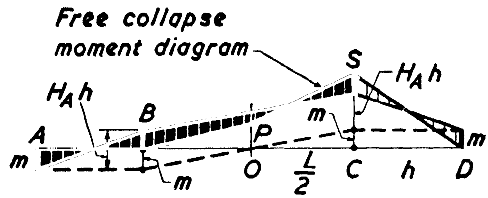


FIG. 351

If, in the free moment diagram, CS, for example = 100 tons ft., and $h =$ 12 ft.:

then the solution gives $m = 25$ tons ft.

and the modulus of section required

$$= \frac{25 \times 12}{15 \cdot 25 \times 1 \cdot 15} = 17 \cdot 1.$$

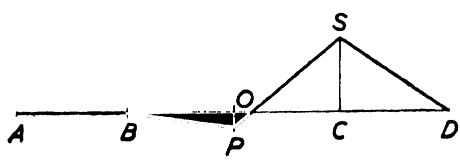


FIG. 352

If, however, the centre load is of such magnitude as to give a free collapse moment diagram of the form shown in Fig. 352, it will be obvious from the previous case that the fourth plastic hinge will occur at O, and not at B. Applying trial moment curves, as before, for base restraint and horizontal reaction, and equating moments at A, O, C and D as shown in Fig. 353.

$$m = (H_A h + OP) = (CS - H_A h - m) = m.$$

Assuming, for example, that $OP = 20$ tons ft. and $CS = 100$ tons ft.; and $h = 12$ ft. as before, the solution gives:

$$m = 40 \text{ tons ft.} \quad \text{and} \quad 12\ H_A = 20 \text{ tons ft.}$$

and the correct collapse moment diagram would be as shown in Fig. 354. If the central load is greater still it is possible for the plastic hinge to develop

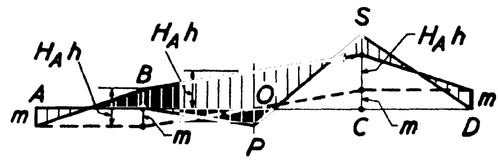


FIG. 353

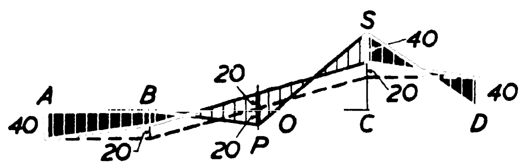


FIG. 354

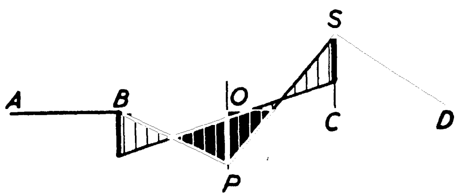


FIG. 355

at *B* instead of *A*, in which case the portion *AC* fails as a beam fixed at both ends, giving a solution as shown in Fig. 355.

Experimental Confirmation

In order to verify the theory in its application to the design of rectangular portals as just described, full-scale tests have been carried out at the British Welding Research Association's Station at Abington, near Cambridge. Three portals, one with hinged bases and two with fixed bases, of 16 ft. span and 8 ft. height, constructed from 8-in. × 4-in. rolled-steel joist section, were loaded gradually to collapse. Behaviour throughout the tests was almost exactly as the theory had predicted.

A more comprehensive series of tests was carried out on miniature portal frames of 4 in. span and 2 in. height. In these portals the members were rectangular in cross-section, and although 'collapse' for this section is not so clearly defined as with an 'I'-section, the results gave added proof of the soundness of the plastic theory in regard to the formation of plastic hinges and the estimation of collapse loads.

Ridged Portals

The method of design is applicable to all forms of hinged and fixed portals met with in practice. A portal of the ridged type, as shown in Fig. 356,

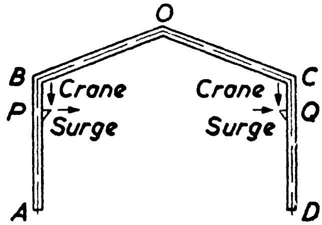


FIG. 356

subjected to dead load, wind, crane load and surge would have a 'free' collapse moment diagram, as shown in Fig. 357.

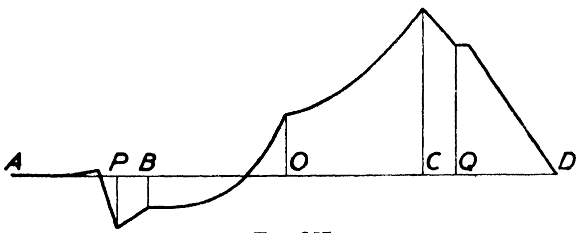


FIG. 357

If the bases are hinged, the solution would be as shown in Fig. 358, the plastic hinges forming at *P* (not *B*) and *C*. If the bases are fixed, then the

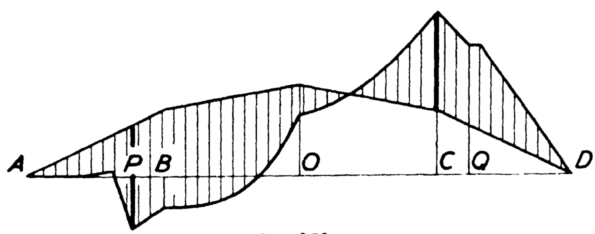


FIG. 358

solution would be as shown in Fig. 359, with plastic hinges at *A*, *R* (not *B*), *C* and *D*.

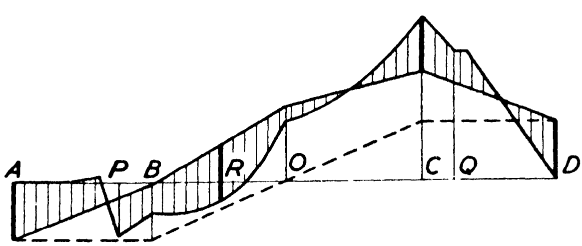


FIG. 359

In the foregoing treatment of rectangular and ridged portals, the frame members have been considered as being of the same uniform section throughout. As a general rule, in the simple form of portal, uniform section will give the cheapest solution, but in other arrangements it may be economical to employ, for example, one section for the columns and another for the beam or rafter. The number of combinations of sections suitable for any particular case is theoretically infinite, but a solution is reached by selecting the one section and determining the other, remembering that the selected section can carry a B.M. no greater than 15·25 *S*, the product of yield stress and plastic modulus of the section.

There is one important difference to be noted between the stress method and the collapse method of design. In the stress method, stresses due to the various kinds of loading, e.g. dead load, live load, wind, crane and crane surge, etc., are estimated separately, and added finally in accordance with the conditions in which they can act in combination. This is not possible in collapse design; the 'worst condition' of loading must be predetermined by inspection, and the problem solved for the loads applied in combination. It may be necessary to examine an alternative 'worst condition'.

In the stress design 25 per cent higher working stresses are permitted by B.S. 449 for those conditions in which the effect of wind is included. In collapse design similar relief is afforded by reducing the load factor. The comparable reduction would be from 1·75 to 1·40.

Combined Bending and Axial Load

So far direct axial load has been neglected; more often than not it is of little consequence, but where it has to be considered, a correction must be made to the solutions already outlined.

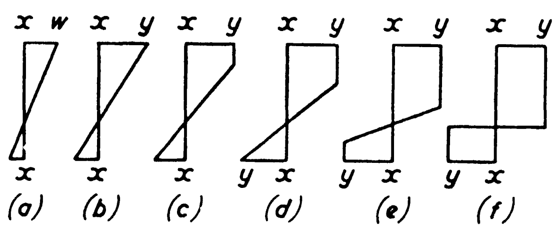


FIG. 360

The working stress across the section of a member subject to B.M. and axial load has a straight-line distribution as in diagram (a), Fig. 360, where xw represents the stress in the outer flange fibres on, say, the compression side.

Under increase of both moment and load, stress develops in the following way (Fig. 360):

(b) Yield stress xy reached on the compression side.
(c) Plasticity developing on the compression side, tension side still elastic.
(d) Yield stress xy reached on the tension side.
(e) Plasticity developing on both sides.
(f) Section fully plastic.

The distribution of stress at full plasticity, as indicated in diagram (f), Fig. 360, can be derived in the following way, with a clearer understanding of the relationship between B.M. and axial load, as shown in Fig. 361.

The first and last diagrams represent the stress across a fully plastic symmetrical section (a) under B.M. only and (d) under direct load only. It follows that (b) must represent full plasticity under a large moment combined with a small direct load and (c) a small B.M. with a large direct load.

Bearing in mind that the direct load is central, and that the rectangle on the left-hand side, representing tension, must be balanced by an equal rectangle of compression on the right-hand side, it will become obvious, on reflection, that the shaded area must represent the direct load, and that the full plastic moment of resistance of the section is reduced by an amount

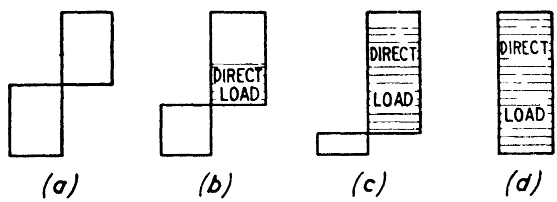


FIG. 361

equal to the plastic moment of resistance of that central portion of the section considered as stressed to full plasticity by the direct load.

If, for example, the axial load in an 'I'-beam under working conditions is 25 tons, i.e. the axial collapse load $= 25 \times 1{\cdot}75 = 43{\cdot}75$ tons, then the area of web required to support it

$$= 43{\cdot}75 \div 15{\cdot}25 = 2{\cdot}87 \text{ sq. in.}$$

Assuming first (and later confirming) a web thickness of $\frac{1}{2}$ in., then the depth of web required

$$= 2{\cdot}87 \div 0{\cdot}5 = 5{\cdot}74 \text{ in.}$$

The plastic moment of resistance of this central portion of the web, $\frac{1}{2}$ in. thick and 5·74 in. deep

$$= \frac{0{\cdot}5 \times 5{\cdot}74^2}{4} \times 15{\cdot}25 = 63 \text{ tons in.}$$

This means that the moment of resistance of an 'I'-beam, with a $\frac{1}{2}$-in. web, supporting an axial load at collapse of 43·75 tons, is reduced by 63 tons in. In other words, the available resistance to bending is reduced to $(15{\cdot}25\, S - 63)$ tons in.

In the solution of frames by the collapse design method, therefore, the quantities in the equations representing plastic hinge moments must be increased for the effect of any axial load by amounts calculated in this way.

Lateral Instability

The foregoing conceptions of the ultimate failure of beams and portals by reason of the formation of plastic hinges, is based on the assumption that

adequate support is provided throughout the frame to prevent lateral instability. The assumption may be justified in those cases in which wall and roof coverings are connected to the frames either directly or through purlins or girts; but cases arise in which adequate support of this kind is normally lacking and, unless suitable provision is made, failure may occur by lateral instability—by twisting and bending of a frame member about the minor axis—before the full plastic moment of resistance of the member about its major axis can be developed.

The lateral instability of slender beams has been the subject of a considerable amount of research at Cambridge. Although the investigation is by no means complete it can already be concluded that if lateral support is provided at distances not greater than would be permitted by conventional stress design for the development of flange stresses of 10 tons per sq. in., i.e. at distances which would give an l/r value not greater than 100, then the loss of ultimate carrying capacity due to lateral instability in the plastic range is almost negligible.

The vertical member of the portal being subject to axial compression together with bending about the major axis presents a problem of some complexity. It has, however, been so far ascertained that where the axial load is relatively small—as in the verticals of a shed-type portal frame—it can for all practical purposes be neglected. Each vertical can then be considered as a beam and, provided the l/r is not greater than 100 and provided there is adequate end restraint against bending about the minor axis, the principles of plastic collapse design can be applied.

Shear

In normal cases shear will have little effect on design. Professor Baker has found from tests on small-scale 'I'-beams that shear stresses up to 4 tons per sq. in. produce no observable effect, and that even with stresses considerably greater only a slight reduction in plastic moment of resistance can be detected. The subject is still under investigation.

Further details of the Plastic Theory are contained in Publication No. 5, issued by the British Constructional Steelwork Association, "The Collapse Method of Design, being the application of the Plastic Theory to the design of mild steel beams and rigid frames."

26. DESIGN OF STANCHIONS

Working Stress

A stanchion or column carrying axial load depends for its carrying capacity on its resistance to buckling. This resistance to buckling is, to some extent, measured by the slenderness ratio

$$\frac{\text{effective length}}{\text{least radius of gyration}}$$

i.e. l/r of the stanchion.

If mild steel stanchions are designed in accordance with B.S. 449, Table 7 of that document sets out the permissible working stresses for varying values of l/r. The London Building (Constructional) By-Laws 1952, Table XX, give the same values as B.S. 449.

The formula from which the values of the stresses are derived is an empirical formula and is based on the Perry-Robertson formula.

The actual expression is:

$$K_2F_a = \frac{f_y+(\eta+1)f_e}{2} - \sqrt{\left\{\left[\frac{f_y+(\eta+1)f_e}{2}\right]^2 - f_y . f_e\right\}}$$

where F_a = the permissible average stress in tons per sq. in.,

K_2 = load factor, taken as 2 in the expression,

$f_y = 15{\cdot}25$ tons per sq. in.,

f_e = the Eulerian value $= \dfrac{13{,}000\pi^2}{(l/r)^2}$,

and $\eta = 0{\cdot}003\ l/r$.

The expression is used only for values of l/r greater than 80. For values of l/r less than 80, the value of F_a is obtained from a linear variation between $F_a = 5{\cdot}12$ tons per sq. in. for $l/r = 80$ and $F_a = 9{\cdot}00$ tons per sq. in. for $l/r = 0$.

The above expression is the result of a theoretical formula which has been modified in the light of experimental results.

The ideal column is that which is initially straight and which is loaded concentrically.

The behaviour of the ideal long column is represented, very closely, by the Eulerian formula.

However, in practice, there is always an initial curvature, i.e. a deviation from the ideal straightness, and in addition, it is not possible even under the careful conditions of a testing laboratory, to ensure that the load is concentrically applied.

It has been found that the initially curved and eccentrically loaded column behaves in a very different manner from the ideal column.

Tests on columns have given diverse results depending on the amounts of initial curvature and eccentricity of loading and many attempts have been made to find an empirical formula to fit the test results. In these formulæ there have been widely differing estimates of the imperfections of practical columns, resulting in corresponding differences in the formulæ.

There is, in addition, a further weakness in columns due to cold straightening, initial stresses set up during manufacture, local defects, etc., which have greater influence in compression members than in tension members or beams, making it wise to adopt a lower working stress for the former than would be suitable for the latter.

For some time past it has been accepted that the Perry formula, with constants suggested by Robertson, and consequently often referred to as the Perry-Robertson formula, most nearly represents the behaviour of the practical column.

Briefly, this formula assumes that a column has an initial curvature resulting in an initial eccentricity of loading $=0{\cdot}003l/r$ and a load factor to allow for other imperfections. This load factor was originally suggested at 2·36 by Robertson, but in the B.S. 449 formula it has been reduced to 2·0.

The effective length of a stanchion is governed by the conditions of restraint at the ends. For stanchions designed in accordance with B.S. 449, the coefficients given in Table 15 for various end conditions may be used.

Figs. 9 to 23 in B.S. 449 give some indications of the interpretation of the effective length coefficients given in Table 15.

It should be noted that, according to B.S. 449, a slab base at the bottom of a stanchion constitutes only a positional restraint.

In dealing with stanchions subjected to both compression and bending, Clause 22 of B.S. 449 requires that the expression $f_a/F_a + f_{bc}/F_{bc}$ shall not exceed unity.

At first sight, this expression looks complicated by reason of the fact that the value of F_{bc} is itself governed by the formula given in Clause 19c.

The following reasoning, however, will reduce the expression to simple terms.

The value of F_{bc} is either 10 tons per sq. in. or some lesser value given by the formula

$$F_{bc}=\frac{1000}{l/r}\times K_1.$$

K_1 is a factor which varies between 1·0 and 1·5 and depends on the ratio r_{xx}/r_{yy}. It should be noted that K_1 is only greater than 1·0 in the case of those sections, which include rolled steel joists, that are symmetrical about both principal axes.

Accordingly, if F_{bc} is not to be less than 10 tons per sq. in., l must not be greater than $100 \times r_{yy} \times K_1$.

If the properties of plain and plated joists are studied, the following points will be apparent:

1. Considering all B.S. sections, the value of r_{yy} and the ratio r_{xx}/r_{yy} for varying flange widths of joist can be summed up in the following table:

Width of Joist (*in.*)	*Maximum Value of* r_{xx}/r_{yy}	*Minimum Value of* r_{yy} (*in.*)
4½	4·35	0·94
5	6·01	1·05
6	5·96	1·30
7½ or 8	6·35	1·50

2. Considering only the heavy B.S. sections which are normally used for stanchions, the corresponding table is:

Width of Joist (*in.*)	*Maximum Value of* r_{xx}/r_{yy}	*Minimum Value of* r_{yy} (*in.*)
4½	2·54	0·96
5	3·86	1·05
6	3·80	1·30
8	3·77	1·72

3. Considering single joists plated, the corresponding table is:

Section	*Maximum value of* r_{xx}/r_{yy}	*Minimum value of* r_{yy}
4½-in. flange with 6-in. plates	2·06 approx.	1·35 in. approx.
5-in. flange with 6-in. plates	3·50 ,,	1·25 ,,
6-in. flange with 8-in. plates	3·17 ,,	1·55 ,,
8-in. flange with 10-in. plates	2·86 ,,	2·21 ,,

From these figures it will be seen that, in order that F_{bc} shall not be less than 10 tons per sq. in., the effective length of a plain heavy beam used as a stanchion should not exceed $100 \times 1{\cdot}05 \times 1{\cdot}23 = 134$ in. or 11 ft. 2 in., while that for a plated section should not exceed $100 \times 1{\cdot}25 \times 1{\cdot}375 = 172$ in. or 14 ft. 4 in.

When these effective lengths are used in conjunction with the effective length coefficients, the following table giving maximum storey heights, to keep F_{bc} at 10 tons per sq. in., is derived.

Type of Section	*Effective Length Coefficient*		
	1·0	0·85	0·70
Plain heavy B.S. section	11 ft. 2 in.	13 ft. 2 in.	16 ft. 0 in.
Plated section	14 ft. 4 in.	16 ft. 10 in.	20 ft. 6 in.

It will now be clear that for multi-storey building work, in the lower lengths of most continuous stanchions and the intermediate lengths of almost all stanchions, the value of F_{bc} may be taken as 10 tons per sq. in. Consequently, in such cases, the expression $f_a/F_a + f_{bc}/F_{bc}$ may be re-written $f_a/F_a + f_{bc}/10$.

Stanchions for single-storey sheds, and other similar slender stanchions, would, of course, have to be treated on their merits.

The modified expression given above is used in the following examples. Example 1 deals with an internal stanchion in a six-storey building designed as uncased, but, as most stanchions must be cased to comply with fire-resistance regulations, an allowance has been made for the weight of such casing.

Example 2 deals with a cased stanchion carrying the same loading as in Example 1, designed in accordance with Clause 18(b) of B.S. 449, by which the stiffening effect of the casing allows a greater r to be used.

Example 4 deals with a corner stanchion in the same building designed as uncased, while Example 5 deals with the cased alternative.

It will be noticed that, in Examples 2 and 5, two calculations have been made for the value of l/r and the corresponding permissible stresses. This is because, even if the stanchion is cased, the allowable stress must not exceed $1\frac{1}{2}$ times that permitted on the uncased section.

Therefore, if the stanchion is designed as cased, the permissible stress is either that permitted for a cased stanchion or 150 per cent of that permitted for an uncased stanchion, whichever is the lesser.

Eccentricity of Loading

All loads applied by beams, except in the case of beams which are centrally disposed on the cap plate at the top of a stanchion, cause bending stresses in the stanchion by the fact that they are applied eccentrically. The nature of the connection, beam to stanchion, will determine the eccentricity of the load.

If stanchions are designed in accordance with B.S. 449, Table 16 of Clause 37 sets down the eccentricities that may be assumed with various types of connection.

It should be noted that a stiffened bracket implies one that has the horizontal portion of the bracket stiffened against bending. It should also be noted that, notwithstanding the type of connection, a minimum eccentricity of 2 in. from the centroid of the section is to be assumed except in the cases of certain types of cap connections.

In the examples, unless the eccentricities are known and given, the following assumptions, which may be regarded as usual practice, are made for design purposes.

Axis XX – 1 in. from flange of section.

Axis YY – 2 in. from centroid of section.

In continuous stanchions, the B.M.s caused by eccentric loading at any one level must affect the lengths of the stanchions both above and below that level.

The B.M. should be divided between the lengths above and below the level under consideration in proportion to the stiffnesses of those lengths, but unless the stiffnesses vary by more than 50 per cent it is usual to divide the moment equally between the two lengths.

Clause 37(b) of B.S. 449 sets down the assumptions usually made in this connection, and those assumptions have been used in the Examples.

Effective Length of Bottom Lengths

It will be noted that, in Examples 1, 2, 4 and 5 (see pages 560–7) the bottom lengths have been assumed to have slab bases. In such cases the

effective length coefficients are, as noted previously, greater than for the corresponding intermediate lengths and are 0·85 for Examples 1 and 2, and 1·0 for Examples 4 and 5.

If the stanchions of Examples 1 and 4 had been provided with gusseted or substantial bases, the effective length coefficients could have been taken as 0·7 and 0·85 respectively and Examples 3 and 6 have been given to illustrate this point.

In Example 3 it will be seen that the smallest possible reduction in section, i.e. from plates 14-in. × 1-in. to 14-in. × $\frac{7}{8}$-in., results in the section being slightly overstressed, even with the smaller l/r.

Example 6 shows that the similar smallest possible reduction in section would provide a section adequately strong.

However, as will be seen later, page 553, it would not be economic to use these smaller sections, even if the slight over-stressing in Example 3 were allowed.

It is apparent, therefore, from Example 3, that in the range of values of l/r of about 40 to 50 little is gained by reducing the effective length coefficient, while from Example 6 it is seen that in the range between 80 and 90 the gain is somewhat greater.

Effect of Casing

A comparison between Examples 1 and 2 shows that a slight saving in the steel of the stanchion shaft is effected by designing the stanchion cased as against uncased. However, if the effect of the casing is taken into account, it is necessary to fulfil certain requirements in relation to the reinforcement of the casing.

It is frequently necessary to case the members of a steel-framed building to meet statutory requirements for fire-resistance, and this casing must have a certain minimum reinforcement, which, however, is less than that specified for a stanchion which is designed as a cased stanchion.

Therefore, in order to make a comparison as to the saving, if any, achieved by one method over the other, it is necessary to assess the following alternatives:

On the one hand, in the stanchion designed as uncased, the reinforcement necessary for fire-resistance purposes; and on the other, in the stanchion designed as cased, the saving in weight of the stanchion section together with the increased reinforcement required.

1. *Uncased Stanchion*

The minimum reinforcement specified by both the London Building (Constructional) By-Laws 1952 and the Model Byelaws, series IV, 1953, is a mesh weighing 1 lb. per sq. yd.

In the case of Example 1, if the length 3rd floor to 2nd floor is considered as giving the average section, the overall concrete size, assuming 2-in. cover, will be 15 in. by 16 in., and the length of mesh required to wrap this section will be $2(13+14)=54$ in., if the mesh is placed centrally in the 2-in. casing.

Therefore, in a 1-ft. length of column the area of mesh will be $1 \times 54/12 =$ 4·5 sq. ft., which at 1 lb. per sq. yard will weigh 0·5 lb.

2. *Cased Stanchion*

The minimum reinforcement specified by both the London By-Laws and B.S. 449 consists of $\frac{3}{16}$-in. wire at a maximum pitch of 6 in. In addition, although not specified, it will be necessary to provide four vertical rods, minimum diameter $\frac{1}{4}$ in., to secure the binders in order that they are maintained in position.

In Example 2, again considering length 3rd floor to 2nd floor, the overall concrete size will be 14 in. $\times$ 16 in., and the length of the binders will be $2(12+14)=52$ in., to which should be added at least 3 in. for tieing round the verticals.

Therefore, in a 1-ft. length of column, the weight of reinforcement is

$$2 \times \tfrac{55}{12} \times 0{\cdot}094 = 0{\cdot}86$$

$$4 \times 1 \text{ ft.} \times 0{\cdot}167 = 0{\cdot}67$$

$$1{\cdot}53 \text{ lb.}$$

These figures show that the reinforcement required if the stanchion is designed as cased weighs 1 lb. more per foot length of stanchion than that required merely to meet the fire-resistance regulations.

Against this increased weight, however, there is a saving, in the particular example, of 5 lb. per foot length of stanchion in the main section.

Moreover it is very probable that the cost per ton of the structural steel will be greater than that of the reinforcement.

In addition to the savings mentioned above, there is that due to the use of a smaller overall section, in Example 2, than that required in Example 1. This results in a saving in the volume of the concrete casing.

It will be seen by comparing Examples 4 and 5 that the saving in the cased stanchion is more than that between Examples 1 and 2.

As a general rule, it may be taken that the more lightly loaded a stanchion is, the greater will be the saving in the use of cased stanchions over uncased stanchions. This will be exemplified by the next two Examples, Nos. 7 and 8, which deal with a lightly loaded stanchion in a four-storey school building.

Economy in Design

Since the strengh of a column depends on the value of l/r, then it follows that, for an axial load, a column which has the same r in all directions will occupy the least floor space of any number of equally strong columns.

The only sections that would fulfil this requirement are circular ones, either solid or hollow. Both these sections, however, have the defect that connections to them are far from simple.

The next most efficient section is a hollow square, which can be made from either two equal angles or two channels welded toe to toe to which connections are more easily made than to circular columns.

The plain R.S.J. section is inefficient by comparison with the two types previously mentioned, since the ratio r_{max}/r_{min} is much greater than 1, being

about 3 for heavy beam sections, such as 10 in. × 8 in. or 12 in. × 8 in, and even greater for light beam sections such as 12 in. × 5 in., 13 in × 5 in., 16 in. × 6 in., etc.

The use of plates on single R.S.J. stalks has the effect of reducing, to some extent, this inequality.

The optimum width of plate for any one depth of section can be shown approximately as follows:

Consider Fig. 362. Neglecting the effect of the R.S.J., for the plates to have equal stiffness about both major axes, I_{XX} must equal I_{YY}.

Now
$$I_{XX} \text{ of one plate} = \frac{Bt^3}{12} + Btx^2$$

$$I_{YY} \text{ of one plate} = \frac{tB^3}{12}$$

Equating,
$$\frac{Bt^3}{12} + Btx^2 = \frac{tB^3}{12}$$

and
$$\frac{t^2}{12} + x^2 = \frac{B^2}{12}$$

Now $t^2/12$ is small in comparison with the other two terms and may be neglected, whence $x^2 = B^2/12$ or $B = 3{\cdot}464x$, and since D is approximately $2x$, then $B = 1{\cdot}732D$

So that for a depth of joists of 8 in., the optimum width of plate is about 14 in., and for a depth of joist of 7 in., the optimum width of plate is 12 in.

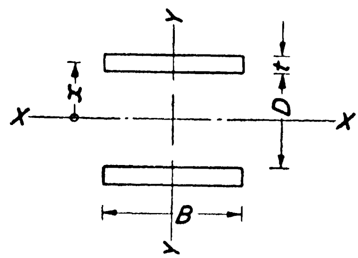


FIG. 362

Now 14-in. wide plates should not be used with flanges less than 8 in. wide, nor should 12-in. wide plates be used with flanges less than 6 in. wide and since in the range of B.S. beams these flange widths are only given by beams of 10-in. and 8-in. minimum depths respectively, it follows that all plated B.S. beams will be stronger on the XX axis than on the YY axis.

This is not necessarily true for the stockier beams in the Broad Flange range, where by using 6 in. × 6 in., 7 in. × 7 in. or 8 in. × 8 in. with the appropriate optimum plate widths, it is possible to obtain sections of equal strength in both directions.

The general deduction from the foregoing remarks is that, ignoring other considerations such as floor space, the most economical plated sections are obtained by using the widest possible plates for any given flange width of beam, e.g. 10-in. plates on 5-in. flanges, 12-in. plates on 6-in. flanges and 14-in. plates on 8-in. flanges.

Twin joist stanchions without plates can be arranged quite efficiently by adjusting the distance between the shafts so that $I_{XX} = I_{YY}$. See Fig. 363.

If A is the area of each joist,

I_{max} is the I_{XX} of each joist

and I_{min} is the I_{YY} of each joist,

then I_{XX} of the compound section $= 2I_{max}$

and I_{YY} of the compound section $= 2(I_{min} + Ax^2)$

so that, for equality $\quad 2I_{max} = 2(I_{min} + Ax^2)$

whence
$$x^2 = \frac{I_{max}}{A} - \frac{I_{min}}{A}$$
$$= r_{xx}{}^2 - r_{yy}{}^2$$

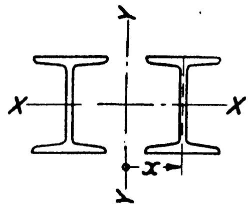


FIG. 363

In the range of heavy beams

$r_{xx} = 0{\cdot}425$ to $0{\cdot}41$ of the depth of the joist,

and $\quad r_{yy} = 0{\cdot}18$ to $0{\cdot}10$ of the depth of the joist,

from which it will be found that $2x$ varies between 0·76 and 0·8 of the depth of the joists.

In the case of twin channels, back to back, it will be found that I_{XX} and I_{YY} of the compound section will be equal when the distance, back to back, varies between 0·5 and 0·63 of the depth of the channel.

In the case of twin channels, toe to toe, the corresponding distance, back to back, varies between 0·85 and 1·05 of the depth of the channel.

In the case of twin joist stanchions with flange plates, it is not possible to

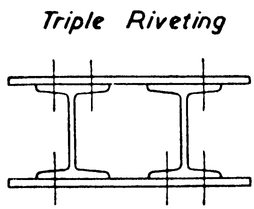


FIG. 364

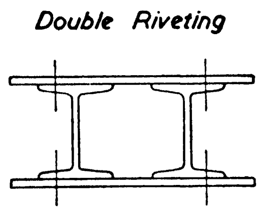


FIG. 365

achieve equal values of I_{XX} and I_{YY} unless the plates are wide compared with the depth of the joists. For example, if the joists have flanges 8 in. wide and are spaced at 10-in. centres it will be found that approximate equality is achieved with a depth of joist of 12 in. and with plates 24-in. wide. Any greater depth of joist than 12 in. retaining the same flange plate width will give a section having I_{XX} greater than I_{YY}. Again if the joists have flanges 6 in. wide and are spaced at 8-in. centres, approximate equality is achieved with a depth of joist of 12 in. and with plates 22 in. wide. Both these sections give plates which are wider than is common practice with twin joist sections. The above remarks refer to sections with triple riveting on the flanges as shown in Fig. 364.

If double riveting only is used, as shown in Fig. 365 the plates should be less wide than the values given above, and the joist depth will have to be smaller for I_{XX} to equal I_{YY}. The depth in this case will be about 10 in. against the 12 in. with triple riveting.

Practical Considerations

The remarks in the previous pages may be taken as being somewhat theoretical and will have to be modified to suit practical conditions.

Fabrication should be as simple as possible, and due regard must be paid to erection problems.

For example, the web of an 8-in. × 6-in. R.S.J. stanchion will just accommodate a 6-in. wide beam flange if the beam is centrally disposed about the *XX* axis of the stanchion. If the beam is supported by a seating cleat and secured by a top cleat, bolts can be easily inserted to effect the connection. Site riveting, however, for the same connection would be extremely difficult. If the beam width had been greater than 6 in, the flanges of the beam would have to be side notched to frame into the web of the stanchion.

Such notching, in any particular case, could be avoided by increasing the depth of the stanchion section.

For the greatest possible economy in weight, the section of a continuous stanchion should be varied in each length as the load is increased. This, however, is not always possible, particularly in lightly loaded stanchions, since, in that case, the variation could only be achieved by using a different joist section in each length. Stanchion shafts should be fabricated in units of the greatest lengths possible, consistent with the greatest economic length rolled, ease of transportation, fabrication, and erection. This usually results in the unit lengths being of two- or three-storey heights, depending on the individual storey height. This will be evident by reference to Examples 1, 2, 4, 5, 7 and 8 in which the symbol **X** marks the splices in the stanchions. The root joist section can be changed, if so desired, at these points.

Splices should be placed at some convenient distance above the appropriate floor level, so that beam cleats do not foul the splice plates.

Wherever the root joist section changes in depth at a splice a horizontal division plate should be provided between the two stanchion lengths.

In heavily loaded stanchions, where it is necessary to use plated sections, it will be possible to vary the plates in each storey height and so approach the most economic section. The minimum thickness of flange plate used is $\frac{3}{8}$ in. and the maximum total thickness of plate on one flange is normally limited to 2 in.

Referring back to Example 3 on page 549, it will be seen that, if slight overstressing is permitted, the flange plates in the bottom length could be 14 in. × $\frac{7}{8}$ in. Since, however, by reference to Example 1, the plates required in the length from 2nd to 1st floor are 14 in. × $\frac{5}{8}$ in., it would not be possible to add two 14-in. × $\frac{1}{4}$-in. plates in the bottom length. It would be possible to run the 14-in × $\frac{7}{8}$-in. plates throughout the two lengths, but the resulting section would be less economical than that provided in Example 1. Had it been possible to reduce the plates in the bottom length, to 14 in. × $\frac{3}{4}$ in., it would then be more economical to run these plates throughout the two lengths than to use the section provided in Example 1.

In the case of wall stanchions with three-way connections, Fig. 366, it is possible to balance the B.M.s about the *XX* axis by selecting the stanchion centre line in the position shown, so that the wall beams have an eccentricity

of e_x. This balancing cannot be achieved at all floor levels if the proportions of wall and floor loads vary from floor to floor, but a mean value could be selected. Again, in such a case, some thought must be given to the connections of the wall beams before the stanchion centre line is selected. The simplest connections are obtained if the wall beams are central on the stanchion, i.e. no attempt is made at balancing the B.M.s, and once again a happy medium should be sought.

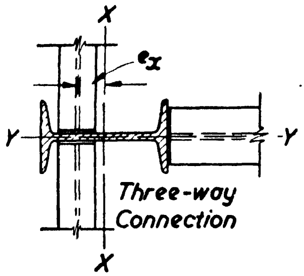


FIG. 366

This argument will also apply to corner stanchions. In such cases, as will be seen from the sketch in Example 4, the flange of the joist should be as wide as possible, consistent with economy, so as to reduce the number of rivets in connections for beam 12 at all levels.

Stanchion Bases

The base for the stanchion in Example 1 will now be designed, first as a slab base and then as a gusseted base. The allowable pressure on the concrete base is assumed at 40 tons per sq. ft. in each case.

A. *Slab Base*

Load on base = 280 tons.

Dimensions of stanchion 14 in. × 12 in.

Area of base required $= \frac{280}{40} = 7{\cdot}00$ sq. ft., say 2ft. 8 in. × 2 ft. 8 in.

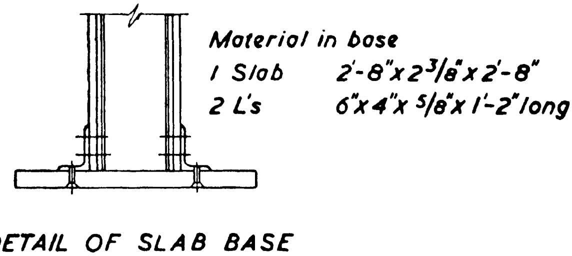


FIG. 367

The formula for the thickness of the slab base is:

$$t = \sqrt{\left[\frac{3w}{F_b} \times \left(A^2 - \frac{B^2}{4}\right)\right]}$$

$$w = \frac{280}{32 \times 32} = 0{\cdot}273 \text{ tons per sq. in.}$$

$$A = \frac{32 - 12}{2} = \frac{20}{2} = 10{\cdot}0 \text{ in.}$$

$$B = \frac{32 - 14}{2} = \frac{18}{2} = 9{\cdot}0 \text{ in.}$$

Therefore, $t = \sqrt{\left[\frac{3 \times 0{\cdot}273}{12}(100 - 20{\cdot}25)\right]} = \sqrt{\frac{3 \times 0{\cdot}273 \times 79{\cdot}25}{12}}$

$= \sqrt{5{\cdot}45} = 2{\cdot}33$ in.

Therefore, slab base size is 2 ft. 8 in × $2\frac{3}{8}$ in. × 2 ft. 8 in.

It should be noted that the calculated thickness is that after machining. See Fig. 367 for the details of the slab base.

B. *Gusseted Base*

Area of base required = 7·00 sq. ft., as in A.

Assume $\frac{5}{8}$-in. thick gusset plates and 9-in. × 4-in. × $\frac{5}{8}$-in. gusset angles.

Dimension of base plate in direction parallel to web of stanchion

$$= 12 \text{ in.} + (2 \times 0{\cdot}625 \text{ in.}) + (2 \times 4 \text{ in.})$$

$$= 12 \text{ in.} + 1{\cdot}25 \text{ in.} + 8 \text{ in.} = 21\tfrac{1}{4} \text{ in., say } 22 \text{ in.,}$$

to allow for overhang of plate over angles.

Length of plate required in the other direction

$$= \frac{7{\cdot}00 \times 12}{22}$$

$$= 3{\cdot}82, \text{ say } 3 \text{ ft. } 10 \text{ in.}$$

It should be noted that it is permitted, by B.S. 449, to take 60 per cent of the stanchion load on the base fastenings provided that the ends of the stanchion, after all gussets, cleats, etc., have been attached, are fabricated flush for bearing.

$$60 \text{ per cent of } 280 = \frac{60 \times 280}{100}$$

$$= 168 \text{ tons.}$$

These rivets are in single shear, and using $\frac{7}{8}$-in. diameter rivets in $\frac{15}{16}$-in. holes (single shear value on gross area = 4·14 tons) the number of rivets required = 168/4·14 = 41 rivets. With 14-in. wide plates on 8-in. wide stanchion flanges, it is possible to use 4 rows of rivets on each flange. If 4 rows with 5 rivets in each row are used, the remaining rivet can be catered for by the web cleats.

If the ends of the stanchion are not fabricated flush it is necessary to take all the load on the fastenings. In that event the number of $\frac{7}{8}$-in. diameter rivets required is 280/4·14 = 68 rivets, i.e. 4 rows of 8 rivets in each flange and 4 rivets in the web cleats.

It remains to determine the thickness of the base plate. There are two critical sections which require investigation.

(1) The cantilevered portion of the base plate outside the gussets. See Fig. 368.

The load on the shaded strip = (3·75/22·0)280 = 47·8 tons, and the B.M. on the strip = 47·8 × 3·75/2 = 89·4 tons in.

The M.R. of the strip $= fbt^2/6$, where $f = 10$ tons per sq. in. and $b = 3$ft. 10 in. or 46 in.

Therefore $$\frac{10 \times 46 \times t^2}{6} = 89{\cdot}4,$$

whence $$t = \sqrt{\frac{89{\cdot}4 \times 6}{10 \times 46}} = \sqrt{1{\cdot}17} = 1{\cdot}08 \text{ in.}$$

(2) The portion of the base plate between the gussets.

An approximate calculation can be made by considering the base plate as a simply supported beam with a cantilever at each end. See Fig. 369.

Consider the supports to be at the plane of junction of gusset plate and gusset angle.

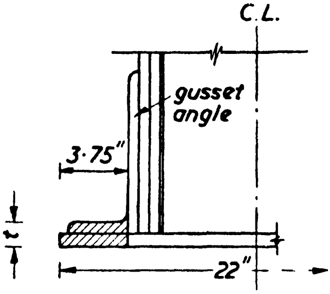


FIG. 368

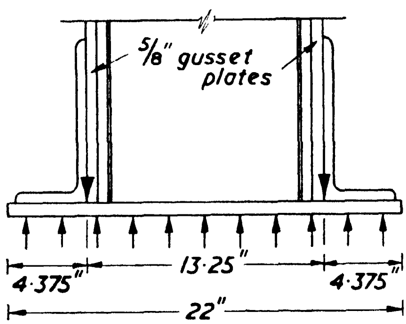


FIG. 369

Since the base is 3 ft. 10 in. or 46 in. long, the load on a strip 1 in. long $= 280/46 = 6{\cdot}08$ tons.

Hence,

the load, per 1-in. strip, on each cantilever $= \dfrac{4{\cdot}375}{22{\cdot}0} \times 6{\cdot}08 = 1{\cdot}21$ tons,

and the load on the span $= \dfrac{13{\cdot}25}{22{\cdot}0} \times 6{\cdot}08 = 3{\cdot}66$ tons.

$$\text{Cantilever B.M.} = \frac{1{\cdot}21 \times 4{\cdot}375}{2} = 2{\cdot}65 \text{ tons in.}$$

$$\text{Span B.M.} = \frac{3{\cdot}66 \times 13{\cdot}25}{8} - 2{\cdot}65 = 6{\cdot}06 - 2{\cdot}65 = 3{\cdot}41 \text{ tons ins.}$$

$$Z \text{ of strip} = \frac{1 \times t^2}{6}$$

and, using a stress of 10 tons per sq. in.,

$$\text{M.R. of strip} = \frac{10t^2}{6}$$

whence $$t = \sqrt{\frac{3{\cdot}41 \times 6}{10}} = \sqrt{2{\cdot}05} = 1{\cdot}43 \text{ in.}$$

The above calculation represents a very severe condition and assumes that the base plate is supported only at the gussets and bends in one plane, whereas

in fact the centre portion bends in two planes at right angles, and in addition receives some support from the web of the stanchion as well as from the web angles. Taking these facts into consideration, a reasonable result will be given if the thickness is made about one-half of that given by the last calculation, i.e. $\frac{3}{4}$ in.

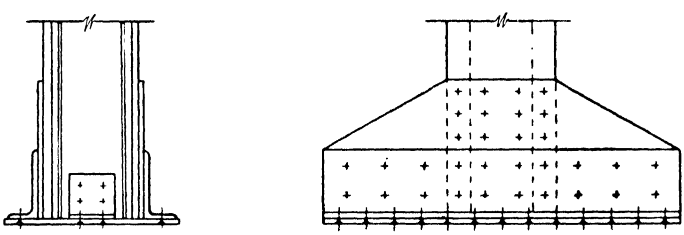
It should be noted that it is usual to make the base plate at least as thick as the gussets or gusset angles.

It will be obvious that the base plate and the gusset angle cannot be considered as acting together unless steps are taken to ensure that they do so. In other words, they must be riveted together with sufficient rivets to take the horizontal shear.

It is not easy to estimate the horizontal shear, but on the assumption that the load is uniformly distributed over the base area, the load on the cantilever

Material in base

1 Base plate	22" x 3/4" x 3'-10" long
2 L's	9" x 4" x 5/8" x 3'-10" long
2 Plates	18" x 5/8" x 3'-10" long shaped
2 L's	6" x 4" x 5/8" x 6" long



13 - 7/8" rivets at 3 1/2" pitch. csk. underside

DETAIL OF GUSSETED BASE

FIG. 370

strip is 47·8 tons. The length of the strip is 46 in., and consequently the vertical shear per inch run $= 47{\cdot}8/46{\cdot}0 = 1{\cdot}04$ tons.

If the strip is considered as a rectangular beam of depth $1\frac{3}{8}$ in., the average vertical shear stress $= 1{\cdot}04/1{\cdot}375 = 0{\cdot}76$ tons per sq. in., and the maximum horizontal shear stress in a rectangular section is $1\frac{1}{2}$ times the average shear stress and occurs at the half depth of the section.

In the example, the maximum horizontal shear stress is $1{\cdot}5 \times 0{\cdot}76 = 1{\cdot}14$ tons per sq. in., and since the base plate and the gusset angle are very nearly of equal thicknesses, it may be assumed that this shear stress occurs at the plane of junction.

Therefore the maximum spacing of $\frac{7}{8}$-in. rivets to take the horizontal shear $= 4{\cdot}14/1{\cdot}14 = 3{\cdot}62$ in. See Fig. 370 for details of the gusseted base. Note that it is common practice to use as many rivets connecting the gusset plates to the gusset angles as there are connecting the stanchion shaft to the gusset plates.

If no rivets were used to connect the two members together, the B.M. would have to be shared between the plate and the angle in proportion to their inertias, and they would be consequently thicker than if considered acting as a whole.

As a guide for obtaining limiting values for the projection and thickness for varying pressures on the concrete foundation block, the following expression can be used:

$$\frac{wa^2}{144\times 2}=\frac{fbt^2}{6}$$

where w = permissible pressure in tons per sq. ft.,

a = length of projection,

f = allowable stress in bending = 10 tons per sq. in.,

b = breadth of section taken as 1 in.

and t = thickness of section, i.e. plate plus angle.

From this expression it will be found that $a^2/t^2 = 480/w$ and the following table gives limiting values of a/t for varying values of w.

Tons per sq. ft. w	a/t
12	6·32
20	4·90
24	4·47
30	4·00
36	3·65
40	3·46

If the thicknesses of the base plate and the gusset angle are assumed equal, and working to the maximum values of a/t given in the preceding table, another table may now be drawn up as follows:

Tons per sq. ft. w	*Horizontal Shear in tons per inch run*	*Max. Pitch p of Rivets*	
		¾-in. Diam.	*⅞-in. Diam.*
12	0·79	3·94	5·25
20	1·02	3·05	4·05
24	1·12	2·77	3·69
30	1·25	2·48	3·31
36	1·37	Impracticable	3·02
40	1·44	,,	2·85

The pitches of rivets have been calculated for a single shear stress of 6 tons per sq. in. on the gross areas, which gives values of 3·11 tons and 4·14 tons on ¾-in. and ⅞-in rivets respectively.

EXAMPLE 3

UNCASED INTERNAL STANCHION WITH SUBSTANTIAL BASE

Sketch	Calculations				Total tons	Reactions		Portion and length	Section	Stan. ref.
	l/r	F_a tons per sq. in.	f_a and f_{bc} tons per sq. in.	$\frac{f_a}{F_a}+\frac{f_{bc}}{F_{bc}}$		Amount tons	Beam ref.			
	$\frac{0{\cdot}7\times192}{3{\cdot}34}=41$	7·01	$f_a=\frac{279{\cdot}4}{40{\cdot}68}=6{\cdot}87$ $f_{BXX}=\frac{2{\cdot}2\times6{\cdot}88}{2\times172{\cdot}7}=0{\cdot}05$ $f_{BYY}=\frac{12{\cdot}5\times2{\cdot}0}{2\times65{\cdot}0}=0{\cdot}19$ 0·24	$\frac{6{\cdot}87}{7{\cdot}01}=0{\cdot}979$ $\frac{0{\cdot}24}{10{\cdot}0}=0{\cdot}024$ 1·003	279·4				$11\frac{3}{4}''\times14''$ $10''\times8''\times55$ lb. I + 2 PLS. $14''\times\frac{7}{8}''$	12

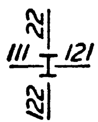
EXAMPLE 6

UNCASED EXTERNAL CORNER STANCHION WITH SUBSTANTIAL BASE

	$\frac{0{\cdot}85\times192}{2{\cdot}09}=78$	5·22	$f_a=\frac{103{\cdot}6}{21\ 5}=4{\cdot}82$ $f_{BXX}=\frac{10{\cdot}0\times5{\cdot}62-9{\cdot}6\times4{\cdot}0}{2\times69{\cdot}0}=0{\cdot}13$ $f_{BYY}=\frac{10{\cdot}0\times4{\cdot}0-9{\cdot}6\times2{\cdot}0}{2\times20{\cdot}9}=0{\cdot}50$ 0·63	$\frac{4{\cdot}82}{5{\cdot}22}=0{\cdot}923$ $\frac{0{\cdot}63}{10{\cdot}0}=0{\cdot}063$ 0·986	103 6				$9\frac{1}{4}''\times9''$ $8''\times6''\times35$ lb. I + 2 PLS. $9''\times\frac{5}{8}''$	1

EXAMPLE 1

UNCASED INTERNAL STANCHION

Sketch	Calculations: l/r	Calculations: F_a tons per sq. in.	Calculations: f_a and f_{bc} tons per sq. in.	Calculations: $\frac{f_a}{F_a}+\frac{f_{bc}}{F_{bc}}$	Total tons	Reactions: Amount tons	Reactions: Beam ref.	Portion and length	Section.	Stan. ref.
					31·7	9·0 9·0 9·5 3·2 1·0 31·7	22 122 111 121 O.W. & C.	ROOF 12′ 0″	AS BELOW	12
ALL BEAMS CENTRAL	$\frac{0{\cdot}7\times168}{1{\cdot}84}$ $=64$	5·89	$f_a=\frac{79{\cdot}4}{16{\cdot}18}=4{\cdot}91$ $f_{B_{XX}}=\frac{2{\cdot}4\times6{\cdot}0}{2\times57{\cdot}74}=0{\cdot}13$ $f_{B_{YY}}=\frac{10{\cdot}5\times2{\cdot}0}{2\times13{\cdot}69}=0{\cdot}77$ 0·90	$\frac{4{\cdot}91}{5{\cdot}89}=0{\cdot}833$ $\frac{0{\cdot}90}{10{\cdot}0}=0{\cdot}09$ 0·923	47·7 79·4	14·0 11·6 15·7 5·2 1·2 47·7	22 122 111 121 O.W. & C.	5TH FLR. 14′ 0″	10″ × 8″ × 55 lb. I	
					48·0 127·4	14·0 11·6 15·7 5·2 1·5 48·0	22 122 111 121 O.W. & C.	4TH FLR 14′ 0″	AS BELOW	

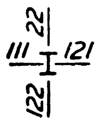
$\frac{0\cdot7 \times 168}{2\cdot66}$ $=45$	6·82	$f_a=\frac{175\cdot4}{28\cdot18}=6\cdot22$ $f_{B_{XX}}=\frac{2\cdot4 \times 6\cdot5}{2 \times 113}=0\cdot07$ $f_{B_{YY}}=\frac{10\cdot5 \times 2\cdot0}{2 \times 33\cdot1}=0\cdot32$ 0·39	$\frac{6\cdot22}{6\cdot82}=0\cdot912$ $\frac{0\cdot39}{10\cdot0}=0\cdot039$ 0·951	48·0 175·4	14·0 11·6 15·7 5·2 1·5 48·0	22 122 111 121 O.W. & C.	3RD FLR. 14′ 0″	11″ × 12″ 10″ × 8″ × 55 lb. I + 2 PLS. 12″ × ½″
$\frac{0\cdot7 \times 168}{3\cdot18}$ $=37$	7·20	$f_a=\frac{223\cdot7}{33\cdot68}=6\cdot65$ ECCY. LESS THAN LENGTH 3RD–2ND	$\frac{6\cdot65}{7\cdot20}=0\cdot923$	48·3 223·7	14·0 11·6 15·7 5·2 1·8 48·3	22 122 111 121 O.W. & C.	2ND FLR. 14′ 0″	11¼″ × 14″ 10″ × 8″ × 55″ lb.I + 2 PLS. 14″ × ⅝″
$\frac{0\cdot85 \times 192}{3\cdot40}$ $=48$	6·67	$f_a=\frac{279\cdot4}{44\cdot18}=6\cdot32$ $f_{B_{XX}}=\frac{2\cdot2 \times 7\cdot0}{2 \times 189\cdot7}=0\cdot04$ $f_{B_{YY}}=\frac{12\cdot5 \times 2\cdot0}{2 \times 73\cdot2}=0\cdot17$ 0·21	$\frac{6\cdot32}{6\cdot67}=0\cdot948$ $\frac{0\cdot21}{10\cdot0}=0\cdot021$ 0·969	55·7 279·4	16·5 14·3 17·7 5·2 2·0 55·7	22 122 111 121 O.W. & C.	1ST FLR. 16′ 0″	12″ × 14″ 10″ × 8″ × 55 lb. I + 2 PLS. 14″ × 1″

X

SLAB BASE

EXAMPLE 2

CASED INTERNAL STANCHION

Sketch	Calculations: l/r	Calculations: F_a tons per sq. in.	Calculations: f_a and f_{bc} tons per sq. in.	Calculations: $\frac{f_a}{F_a}+\frac{f_{bc}}{F_{bc}}$	Total tons	Reactions: Amount tons	Reactions: Beam ref.	Portion and length	Section	Stan. ref.
					THESE COLUMNS ALL REPEAT AS EXAMPLE 1				AS BELOW	12
	$\frac{0.7\times168}{1.65}$ $=72$ $\frac{0.7\times168}{2.2}$ $=54$	5·50 2·75 8·25 6·38	$f_a=\frac{79.4}{14.71}=5.40$ $f_{BXX}=\frac{2.4\times5.5}{2\times46.25}=0.15$ $f_{BYY}=\frac{10.5\times2.0}{2\times11.48}=0.92$ 1·07	$\frac{5.40}{6.38}=0.845$ $\frac{1.07}{10.0}=0.107$ 0·952	79·4				9″ × 7″ × 50 lb. I	X
									AS BELOW	

$\frac{0.7\times168}{2.63}$ $=45$ $\frac{0.7\times168}{3.2}$ $=37$	6·82 3·41 10·23 7·20	$f_a=\frac{175.4}{26.71}=6.57$ $f_{B_{XX}}=\frac{2.4\times6.0}{2\times95.8}=0.08$ $f_{B_{YY}}=\frac{10.5\times2.0}{2\times30.7}=0.34$ 0·42	$\frac{6.57}{7.20}=0.912$ $\frac{0.42}{10.0}=0.042$ 0·954	175·4				10″ × 12″ 9″ × 7″ × 50 lb. I + 2 Pls. 12″ × ½″ ✕
$\frac{0.7\times168}{3.16}$ $=38$ $\frac{0.7\times168}{3.6}$ $=33$	7·16 3·58 10·74 7·40	$f_a=\frac{223.7}{32.21}=6.93$ Eccy. less than Length 3rd–2nd	$\frac{6.93}{7.40}=0.936$	223·7				10¼″ × 14″ 9″ × 7″ × 50 lb. I + 2 Pls. 14″ × ⅝
$\frac{0.85\times192}{3.37}$ $=49$ $\frac{0.85\times192}{3.6}$ $=46$	6·62 3·31 9·93 6·77	$f_a=\frac{279.4}{42.71}=6.55$ $f_{B_{XX}}=\frac{2.2\times6.5}{2\times165.5}=0.05$ $f_{B_{YY}}=\frac{12.5\times2.0}{2\times71.1}=0.18$ 0·23	$\frac{6.55}{6.77}=0.968$ $\frac{0.23}{10.0}=0.023$ 0·991	279·4				11″ × 14″ 9″ × 7″ × 50 lb. I + 2 Pls. 14″ × 1″

Slab Base

EXAMPLE 4

UNCASED EXTERNAL CORNER STANCHION

Sketch	Calculations: l/r	Calculations: F_a tons per sq. in.	Calculations: f_a and f_{bc} tons per sq. in.	Calculations: $\frac{f_a}{F_a}+\frac{f_{bc}}{F_{bc}}$	Total tons	Reactions: Amount tons	Reactions: Beam ref.	Portion and length	Section	Stan. ref.
						4·1 4·3 0·6	11 12 O.W. & C.	ROOF 12′ 0″	AS BELOW	1
					9·0	9·0				
	$\frac{0·85\times168}{1·38}$ $=104$	3·95	$f_a=\frac{27·2}{10·3}=2·64$ $f_{B_{XX}}=\frac{8·9\times5·0-8·5\times4·0}{2\times28·76}=0·19$ $f_{B_{YY}}=\frac{8·9\times4·0-8·5\times2·0}{2\times6·51}=1·43$	$\frac{2·64}{3·95}=0·668$ $\frac{1·62}{10·0}=0·162$ 0·830	18·2	8·5 8·9 0·8	11 12 O.W. & C.	5TH FLR. 14′ 0″	8″ × 6″ × 35 lb. I	X
			1·62		27·2	18·2				
					18·4	8·5 8·9 1·0	11 12 O.W. & C.	4TH FLR. 14′ 0″	AS BELOW	
					45·6	18·4				

$\frac{0{\cdot}85\times168}{1{\cdot}75}$ $=82$	5·02	$f_a=\frac{64{\cdot}0}{16{\cdot}3}=3{\cdot}93$ $f_{B_{XX}}=\frac{8{\cdot}9\times5{\cdot}38-8{\cdot}5\times4{\cdot}0}{2\times49{\cdot}4}=0{\cdot}14$ $f_{B_{YY}}=\frac{8{\cdot}9\times4{\cdot}0-8{\cdot}5\times2{\cdot}0}{2\times12{\cdot}5}=0{\cdot}75$	$\frac{3{\cdot}93}{5{\cdot}02}=0{\cdot}783$ $\frac{0{\cdot}89}{10{\cdot}0}=0{\cdot}089$ 0·872	18·4	8·5 8·9 1·0	11 12 O.W. & C.	3RD FLR. 14′ 0″	8¾″ × 8″ 8″ × 6″ × 35 lb. I + 2 PLS. 8″ × ⅜″
		0·89		64·0	18·4			
$\frac{0{\cdot}85\times168}{1{\cdot}93}$ $=74$	5·41	$f_a=\frac{82{\cdot}6}{17{\cdot}0}=4{\cdot}86$ $f_{B_{XX}}=\frac{0{\cdot}14\times49{\cdot}4}{52{\cdot}4}=0{\cdot}13$ $f_{B_{YY}}=\frac{0{\cdot}75\times12{\cdot}5}{14{\cdot}2}=0{\cdot}66$	$\frac{4{\cdot}86}{5{\cdot}41}=0{\cdot}898$ $\frac{0{\cdot}79}{10{\cdot}0}=0{\cdot}079$ 0·977	18·6	8·5 8·9 1·2	11 12 O.W. & C.	2ND FLR. 14′ 0″	8¾″ × 9″ 8″ × 6″ × 35 lb. I + 2 PLS. 9″ × ⅜″
		0·79		82·6	18·6			
$\frac{1{\cdot}0\times192}{2{\cdot}14}$ $=90$	4·62	$f_a=\frac{103{\cdot}6}{23{\cdot}8}=4{\cdot}35$ $f_{B_{XX}}=\frac{10{\cdot}0\times5{\cdot}75-9{\cdot}6\times4{\cdot}0}{2\times77{\cdot}8}=0{\cdot}12$ $f_{B_{YY}}=\frac{10{\cdot}0\times4{\cdot}0-9{\cdot}6\times2{\cdot}0}{2\times24{\cdot}2}=0{\cdot}43$	$\frac{4{\cdot}35}{4{\cdot}62}=0{\cdot}940$ $\frac{0{\cdot}55}{10{\cdot}0}=0{\cdot}055$ 0·995	21·0	9·6 10·0 1·4	11 12 O.W. & C.	1ST FLR.	9½″ × 9″ 8″ × 6″ × 35 lb. I + 2 PLS 9″ × ¾″
		0·55		103·6	21·0		16′ 0″	

X

SLAB BASE

Example 5

Cased External Corner Stanchion

Sketch	Calculations: l/r	Calculations: F_a tons per sq. in.	Calculations: f_a and f_{bc} tons per sq. in.	Calculations: $\frac{f_a}{F_a}+\frac{f_{bc}}{F_{bc}}$	Total tons	Reactions: Amount tons	Reactions: Beam ref.	Portion and length	Section	Stan. ref.
					These Columns all repeat as Example 4				As Below	1
	$\frac{0.85\times168}{1.11}$ $=129$ $\frac{0.85\times168}{1.8}$ $=80$	2·93 1·46 4·39 5·12	$f_a=\frac{27.2}{8.28}=3.28$ $f_{BXX}=\frac{0.19\times28.76}{22.42}=0.24$ $f_{BYY}=\frac{1.43\times6.51}{4.08}=2.28$ 2·52	$\frac{3.28}{4.39}=0.747$ $\frac{2.25}{10.0}=0.252$ 0·999	27·2				8″ × 5″ × 28 lb. I	X
	$\frac{0.85\times168}{1.38}$ $=104$ $\frac{0.85\times168}{2.0}$ $=72$	3·95 1·47 5·42 5·50	$f_a=\frac{45.6}{10.3}=4.43$ $f_{BXX}+f_{BYY}$ as Ex 4 Length 5th Flr to 4th Flr $=1.62$	$\frac{4.43}{5.42}=0.817$ $\frac{1.62}{10.0}=0.162$ 0·979	45·6				8″ × 6″ × 35 lb. I.	

				64·0				As Below	X
$\frac{0·85 \times 168}{1·75}$ =82 $\frac{0·85 \times 168}{2·4}$ =60	5·02 2·51 —— 7·53 —— 6·09	$f_a = \frac{82·6}{16·3} = 5·07$ $f_{B_{XX}} + f_{B_{YY}}$ As Ex. 4 Length 3rd Flr to 2nd Flr =0·89	$\frac{5·07}{6·09} = 0·833$ $\frac{0·89}{10·0} = 0·089$ —— 0·922	82·6				$8\frac{3}{4}'' \times 8''$ $8'' \times 6'' \times 35''$ lb. I. + 2 Pls $8'' \times \frac{3}{8}''$	
$\frac{1·0 \times 192}{1·92}$ =100 $\frac{1·0 \times 192}{2·4}$ =80	4·13 2·06 —— 6·19 —— 5·12	$f_a = \frac{103·6}{22·3} = 4·65$ $f_{B_{XX}} = \frac{10·0 \times 5·75 - 9·6 \times 4·0}{2 \times 72·0} = 0·14$ $f_{B_{YY}} = \frac{10·0 \times 4·0 - 9·6 \times 2·0}{2 \times 20·5} = 0·51$ —— 0·65	$\frac{4·65}{5·12} = 0·908$ $\frac{0·65}{10·0} = 0·065$ —— 0·973	103·6				$9\frac{1}{2}'' \times 8''$ $8'' \times 6'' \times 35''$ lb. I. + 2 Pls $8'' \times \frac{3}{4}''$	
								Slab Base	

TABLES REPRODUCED FROM B.S.449:1948

TABLE 7

Permissible working stresses in tons/sq. in. of gross section for axial loads

l/r	F_a tons/ sq. in.	l/r	F_a tons/ sq. in.	l/r	F_a tons/ sq. in.	l/r	F_a tons/ sq. in.
0	9·00	70	5·60	140	2·57	210	1·27
10	8·51	80	5·12	150	2·30	220	1·17
20	8·03	90	4·62	160	2·06	230	1·08
30	7·54	100	4·13	170	1·86	240	0·99
40	7·06	110	3·67	180	1·68	250	0·92
50	6·57	120	3·26	190	1·52	300	0·65
60	6·09	130	2·89	200	1·39	350	0·49

Intermediate values may be determined by interpolation.

TABLE 8

Permissible working stresses in tons/sq. in. of gross section for discontinuous angle struts. (Double-bolted, double-riveted, or welded at ends.)

L/r	F_e2 tons/ sq. in.	L/r	F_e2 tons/ sq. in.	L/r	F_e2 tons/ sq. in.	L/r	F_e2 tons/ sq. in.	L/r	F_e2 tons/ sq. in.	L/r	F_e2 tons/ sq. in.
0	6·00	50	4·55	100	3·12	150	1·95	200	1·25	250	0·86
10	5·71	60	4·26	110	2·85	160	1·77	210	1·16	300	0·62
20	5·42	70	3·97	120	2·60	170	1·61	220	1·08	350	0·47
30	5·13	80	3·68	130	2·37	180	1·47	230	1·00		
40	4·84	90	3·40	140	2·15	190	1·35	240	0·93		

Intermediate values may be determined by interpolation.

TABLE 9

Permissible working stresses in tons/sq. in. of gross section for discontinuous angle struts. (Single-bolted or single-riveted at ends.)

L/r	F_e1 tons/ sq. in.	L/r	F_e1 tons/ sq. in.	L/r	F_e1 tons/ sq. in.	L/r	F_e1 tons/ sq. in.	L/r	F_e1 tons/ sq. in.	L/r	F_e1 tons/ sq. in.
0	3·00	50	2·42	100	1·85	150	1·32	200	0·96	250	0·71
10	2·88	60	2·31	110	1·73	160	1·24	210	0·90	300	0·53
20	2·77	70	2·19	120	1·62	170	1·16	220	0·84	350	0·43
30	2 65	80	2·08	130	1·51	180	1·09	230	0·79		
40	2 54	90	1·96	140	1·41	190	1 02	240	0·75		

Intermediate values may be determined by interpolation.

27. DESIGN OF ANGLE STRUTS

As in the case of stanchions, the carrying capacity of angle struts carrying axial loads depends to some extent on the slenderness ratio of the strut. According to B.S. 449 angle struts may be divided into five groups. Four groups are specifically mentioned and these are given below:

1. Single-angle discontinuous struts with single-bolted or riveted connections at each end.
2. Single-angle discontinuous struts connected at each end by not less than two bolts or rivets in line.
3. Double-angle discontinuous struts, back to back, connected to one side only of a gusset. The end connections are not considered to affect the permissible stresses on this type of strut even if they are only single-bolted or riveted.
4. Double-angle discontinuous struts, back to back, connected on both sides of a gusset by not less than two bolts or rivets in line.

The permissible stresses for the above groups are set down in Tables 7 to 9 thus:

Group 1:	Table 9	Reproduced on page 570.
Groups 2 and 3:	Table 8	
Group 4:	Table 7.	

It may be inferred that the types of angle struts not specifically mentioned fall into the fifth group. This would include single- and double-angle continuous struts, the permissible stresses on which are those given in Table 7.

A comparison of the three Tables 7, 8 and 9 will show that although the maximum permissible stresses are 9·0, 6·0 and 3·0 tons per sq. in. respectively, the subsequent values of permissible stress for given values of l/r do not maintain the same ratio in the three tables. In fact, as the value of l/r increases, the values of permissible stress in the three tables draw closer together.

The formula from which the values given in Table 7 are derived has already been discussed in the section dealing with the design of stanchions.

The values given in Tables 8 and 9 have been derived from the same basic source, but, in addition, an allowance has been made for combined bending and axial compression. Appendix D of B.S. 449 sets out the basis of derivation, i.e. in the case of Table 8 eccentricity of loading in the plane of the gusset has been neglected, being considered to be controlled by the double fastenings, while in the case of Table 9 full eccentricity in this plane has been taken, because of the single fastening.

Effective Length

It should be noted that, in Table 7, the slenderness ratio of the strut is given by l/r where l is the *effective* length of the strut. This effective length depends, as has been noted previously, on the types of restraints at the ends of the struts.

In Tables 8 and 9, however, the slenderness ratio is given by l/r, where l is $0{\cdot}8\times$ the length of the strut between centres of fastenings at each end.

It may be taken that for struts in Group 4, page 571, while the permissible stress is given by Table 7, the effective length should be the length between centres of fastenings at each end. For continuous angle struts, the effective length will depend on the types of end restraints. This latter point will be illustrated in later examples.

Eccentricity of Loading

In discontinuous struts the eccentricity of the end connections with respect to the centroid of the strut may be ignored and the strut designed as an axially loaded member.

In single-angle continuous struts, such as the rafters in roof trusses, account should be taken of such eccentricity. This point will be dealt with later in this section.

Examples.

Examples 1 and 2 deal with a discontinuous strut having an actual overall length of 6 ft. 9 in. and carrying a load of 4·0 tons.

It will be designed as a single-angle discontinuous strut, first with single-riveted connections and secondly with double-riveted connections.

Example 1. Single-angle discontinuous strut with single-riveted connections.

Overall length of strut = 6 ft. 9 in.

Allowing $1\frac{1}{2}$ in. end distance, $L = 6$ ft. 9 in. − 3in. = 6ft. 6 in. = 78 in.

Try $3\frac{1}{2}$ in. × $3\frac{1}{2}$ in. × $\frac{5}{16}$ in. angle.

$$\text{Area} = 2{\cdot}09 \text{ sq. in.}$$

$$r_{vv} = 0{\cdot}68 \text{ in.}$$

$L/r = \dfrac{78\times 0{\cdot}8}{0{\cdot}68} = 92.$ F_e1(Table 9) = 1·92 tons per sq. in.

Actual stress = 4·0/2·09 = 1·92 tons per sq. in.

The section is adequate.

Example 2. Single-angle discontinuous strut with double-riveted connections.

Overall length of strut = 6 ft. 9 in.

$L = 6$ ft. 9 in. − 6 in. = 6 ft. 3 in. = 75 in.

Try 3 in. × 3 in. × $\frac{1}{4}$ in angle.

$$\text{Area} = 1{\cdot}44 \text{ sq. in.}$$

$$r_{vv} = 0{\cdot}59 \text{ in.}$$

$L/r = \dfrac{75\times 0{\cdot}8}{0{\cdot}59} = 102.$ F_e2 (Table 8) = 3·06 tons per sq. in.

Actual stress = 4·0/1·44 = 2·78 tons per sq. in.

The section is adequate.

It will be noted that the section required in Example 1 weighs 7·11 lb. per ft. run, while that required in Example 2 weighs 4·89 lb. per ft. run, representing a saving of 2·22 lb. per ft. run or 31 per cent, a saving gained by the addition of one more rivet at each end of the strut.

Examples 3, 4 and 5 deal with a discontinuous strut having an actual overall length of 9 ft. 3 in. and carrying a load of 7·4 tons. It will be designed first as a single-angle discontinuous strut with double-riveted connections; secondly as a double-angle discontinuous strut, back to back, connected to one side only of a gusset; and thirdly as a double-angle discontinuous strut, back to back, connected to both sides of a gusset with double-riveting.

Example 3. Single-angle discontinuous strut with double-riveted connections.

Overall length of strut = 9 ft. 3 in.

L = 9 ft. 3 in. − 6 in. = 8 ft. 9 in. = 105 in.

Try 4 in. × 4 in. × $\frac{3}{8}$ in. angle.

Area = 2·86 sq. in.

r_{vv} = 0·78 in.

$L/r = \dfrac{105 \times 0{\cdot}8}{0{\cdot}78} = 108.$ F_e2 (Table 8) = 2·82 tons per sq. in.

Actual stress = 7·4/2·86 = 2·59 tons per sq. in.

The section is adequate.

Example 4. Double-angle discontinuous strut, back to back, connected to one side of a gusset, single-riveted.

Overall length of strut = 9 ft. 3 in.

L = 9 ft. 3 in. − 3 in. = 9ft. 0 in = 108 in.

Try two angles 3 in. × $2\frac{1}{2}$ in. × $\frac{1}{4}$ in., $\frac{5}{16}$ in. back to back.

Area = 2·63 sq. in.

r_{xx} = 0·93 in.

$L/r = \dfrac{108 \times 0{\cdot}8}{0{\cdot}93} = 93.$ F_e2 (Table 8) = 3·31 tons per sq. in.

Actual stress = 7·4/2·63 = 2·82 tons per sq. in.

The section is adequate.

Example 5. Double-angle discontinuous strut, back to back, connected to both sides of a gusset with not less than two rivets in line.

Overall length of strut = 9 ft. 3 in.

L = 9 ft. 3 in. − 6 in. = 9 ft. 0 in. = 105 in.

Try two angles 3 in. × 2 in. × $\frac{1}{4}$ in., $\frac{5}{16}$ in. back to back.

Area = 2·38 sq. in.

r_{xx} = 0·85 ins.

$L/r = \dfrac{105}{0{\cdot}85} = 124.$ F_a (Table 7) = 3·11 tons per sq. in.

Actual stress = 7·4/2·38 = 3·11 tons per sq. in.

The section is adequate.

The weights of the sections in Examples 3, 4 and 5 are 9·73, 8·94 and 8·08 lb. per foot run respectively. Against these weights, to the last two must be added something for intermediate fastenings.

The next two examples, Nos. 6 and 7, deal with a continuous angle strut, i.e. the main rafter of a roof truss.

The loads and B.M.s are as shown in Fig. 371. It should be noted that the continuous member is assumed to have fixed ends at the eaves and apex.

It will be designed first as a single-angle strut and then as a double-angle strut.

Example 6. Single-angle continuous strut.

An inspection of the B.M. diagram in conjunction with the direct loads in the members and the positions of the purlins, will show that the design will be governed by the conditions at one of the following points:

(a) The support B.
(b) The support C.
(c) The purlin point between these two supports.

A single angle subjected to bending, if freely supported, will not bend about the horizontal axis xx but about some other axis, as shown in Example 2 in the section on Bending and Axial Stresses.

It will have been noted, from that section, that the determination of the axis of bending is not an easy matter.

If the rafter of a roof truss is a single angle, it will tend to bend about some other axis than the horizontal axis xx, but it will be appreciated that, due to the effect of the gussets at the panel points and also due to intermediate connections at the purlin points, the determination of the axis of bending, which is not necessarily the same at both the points mentioned, is an even more complicated problem than in the case of the simply supported angle beam.

It is necessary, moreover, to take into account the effect of the gusset plate in applying the direct load eccentrically to the member.

A practical design method should take these facts into consideration and should be, at the same time, convenient to use in the design office.

The following calculations are based on such a method, which has been found, from experience, to give reasonably satisfactory results.

In ascertaining the relevant properties of the section chosen it is necessary to evaluate certain bending stress coefficients, as follows:

1. Those due to eccentrically applied direct loads.
2. Those due to transverse bending.

Fig. 372 shows the method of determining the first of these coefficients.

$$\text{Coefficient for point } B = \frac{OP \times \sin\theta \times BV}{I_{vv}} - \frac{OP \times \cos\theta \times BU}{I_{uu}}.$$

$$\text{Coefficient for point } C = \frac{OP \times \cos\theta \times CU}{I_{uu}} - \frac{OP \times \sin\theta \times CV}{I_{vv}}.$$

Maximum compressive bending stress = Load × max. coefficient.

Note.—P is the point of application of the load assumed to be at the centre of gusset thickness and on the centre line of the connecting rivets or bolts.

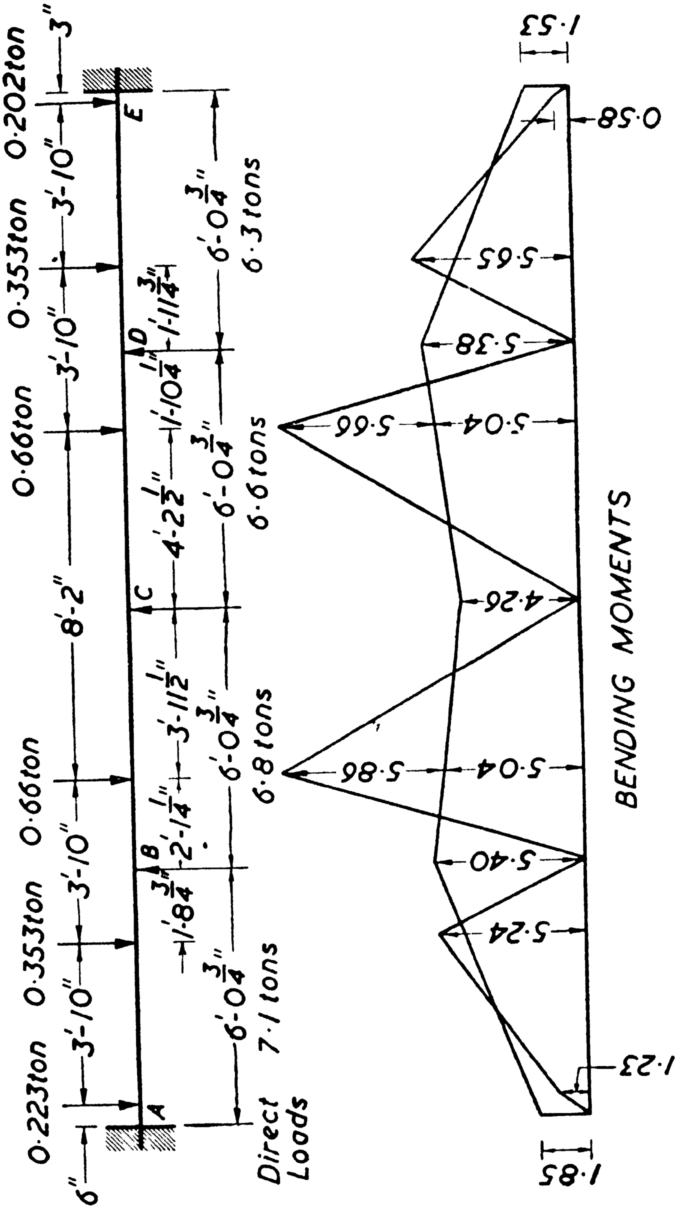

0·202ton
0·353ton
0·66ton
0·66ton
0·353ton
0·223ton
3″
3′-10″
3′-10″
8′-2″
3′-10″
3′-10″
6″
A
B
C
D
E
6′-0¾″
6′-0¾″
6′-0¾″
6′-0¾″
6·3 tons
6·6 tons
6·8 tons
7·1 tons
Direct Loads
1·53
0·58
5·65
5·38
5·66
5·04
4·26
5·86
5·04
5·40
5·24
1·23
1·85
BENDING MOMENTS

FIG. 371

In this example one angle, 4 in. × 3½ in. × ⅜ in., will be investigated. The properties of the angle are:

$$\text{Area} = 2{\cdot}67 \text{ sq. ins.}$$
$$r_{xx} = 1{\cdot}24 \text{ in.}$$
$$r_{yy} = 1{\cdot}04 \text{ in.}$$
$$r_{vv} = 0{\cdot}72 \text{ in.}$$

For the purpose of calculating the first coefficient, the dimension x will be taken as 2¼ in. and the gusset thickness as ⅜ in., whence from Fig. 373,

$$OP = 1{\cdot}547 \text{ in.}$$
$$\theta = 9° \ 54'$$
$$CU = 2{\cdot}813 \text{ in.}$$
$$CV = 0{\cdot}934 \text{ in.}$$
$$BU = 0{\cdot}387 \text{ in.}$$
$$BV = 1{\cdot}466 \text{ in.}$$

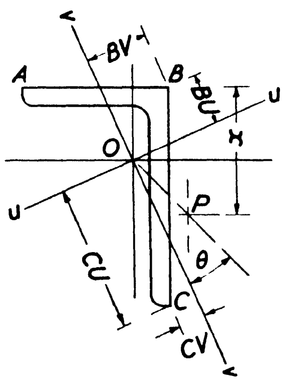


FIG. 372

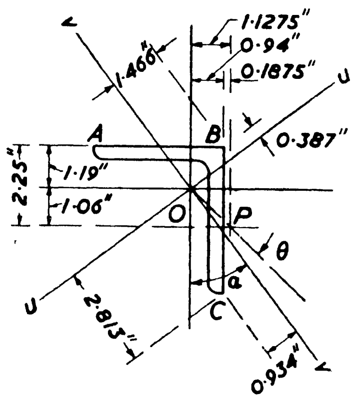


FIG. 373

The coefficient for point B

$$= \frac{OP \times \sin\theta \times BV}{I_{vv}} - \frac{OP \times \cos\theta \times BU}{I_{uu}}$$

$$= \frac{1{\cdot}547 \times 0{\cdot}1719 \times 1{\cdot}466}{1{\cdot}39} - \frac{1{\cdot}547 \times 0{\cdot}9851 \times 0{\cdot}387}{5{\cdot}61}$$

$$= 0{\cdot}28 - 0{\cdot}105 = 0{\cdot}175.$$

The coefficient for point C

$$= \frac{OP \times \cos\theta \times CU}{I_{uu}} - \frac{OP \times \sin\theta \times CV}{I_{vv}}$$

$$= \frac{1{\cdot}547 \times 0{\cdot}9851 \times 2{\cdot}813}{5{\cdot}61} - \frac{1{\cdot}547 \times 0{\cdot}1719 \times 0{\cdot}934}{1{\cdot}39}$$

$$= 0{\cdot}764 - 0{\cdot}179 = 0{\cdot}585.$$

Therefore the maximum coefficient is given at the point C. It will generally be found that such is the case, although it must not necessarily be taken that it is always so.

Fig. 374 shows the method of determining the second of these coefficients.

Coefficient for point B

$$= +\frac{BV \times \sin\alpha}{I_{vv}} + \frac{BU \times \cos\alpha.}{I_{uu}}$$

Coefficient for point A

$$= -\frac{AV \times \sin\alpha}{I_{vv}} + \frac{AU \times \cos\alpha}{I_{uu}}.$$

Coefficient for point C

$$= -\frac{CV \times \sin\alpha}{I_{vv}} - \frac{CU \times \cos\alpha}{I_{uu}}.$$

(+indicates compression; − indicates tension.)

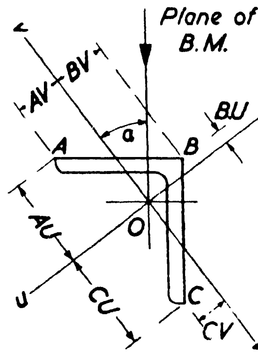


FIG. 374

The angle $\alpha = 36° 54'$ and the values of CU, CV, BU and BV are as given previously. $AV = 1{\cdot}334$ in., and $AU = 2{\cdot}487$ in.

The coefficients will be found to be as follows:

Coefficient for point $B = +0{\cdot}687$

„ „ „ $C = -0{\cdot}804$

„ „ „ $A = -0{\cdot}220$.

(a) *Support* B

(i) Axis xx:

Effective length $= 0{\cdot}7 \times$ panel length

$= 0{\cdot}7 \times 72{\cdot}75 = 50{\cdot}9$ in.

$l/r_{xx} = 50{\cdot}9/1{\cdot}24 = 41$.

(ii) Axis yy:

Effective length $= 1{\cdot}0 \times$ distance between purlins

$= 1{\cdot}0 \times 46 = 46{\cdot}0$ in.

$l/r_{yy} = 46{\cdot}0/1{\cdot}04 = 44{\cdot}2$

(iii) Axis vv:

Effective length $= 1{\cdot}0 \times$ maximum distance between support and adjacent purlin

$= 1{\cdot}0 \times 25{\cdot}25 = 25{\cdot}25$ in.

$l/r_{vv} = 25{\cdot}25/0{\cdot}72 = 35{\cdot}1$.

Therefore greatest value of $l/r = 44{\cdot}2$.

$F_a = 6{\cdot}84$ tons per sq. in.

$f_a = 7{\cdot}1/2{\cdot}67 = 2{\cdot}66$ tons per sq. in.

Bending moment at support B $= 5{\cdot}40$ tons in. negative.

At this point, it will be the point C which has the maximum compressive bending stress.

$$f_{bc} = (7{\cdot}1 - 6{\cdot}8) \times \text{coeff. from Fig. 373} + 5{\cdot}40 \times \text{coeff. from Fig. 374}$$
$$= 0{\cdot}3 \times 0{\cdot}585 + 5{\cdot}40 \times 0{\cdot}804$$
$$= 0{\cdot}18 + 4{\cdot}35 = 4{\cdot}53 \text{ tons per sq. in.}$$

Since $l/r_{yy} = 44{\cdot}2$, i.e. less than 100, it is obvious from clause 19(c) that F_{bc} will be given by the maximum allowable value of 10 tons per sq. in.

$$\frac{f_a}{F_A} + \frac{f_{bc}}{F_{bc}} = \frac{2{\cdot}66}{6{\cdot}84} + \frac{4{\cdot}53}{10{\cdot}0} = 0{\cdot}389 + 0{\cdot}453 = 0{\cdot}842.$$

Therefore the section is adequate at support B.

(b) *Support* C

(i) Axis xx:

$$l/r_{xx} \text{ as for support } B = 41.$$

(ii) Axis yy:

$$\text{Effective length} = 1{\cdot}0 \times 98 = 98{\cdot}0 \text{ in.}$$
$$l/r_{yy} = 98{\cdot}0/1{\cdot}04 = 94{\cdot}0.$$

(iii) Axis vv:

$$l/r_{vv} = 50{\cdot}5/0{\cdot}72 = 70{\cdot}0.$$

Therefore greatest value of $l/r = 94$.

$$F_a = 4{\cdot}42 \text{ tons per sq. in.}$$
$$f_a = 6{\cdot}8/2{\cdot}67 = 2{\cdot}55 \text{ tons per sq. in.}$$

Bending moment at support C = 4·26 tons in. negative.

At this point, it will again be the point C which has the maximum compressive stress.

$$f_{bc} = (6{\cdot}8 - 6{\cdot}6) \times 0{\cdot}585 + 4{\cdot}26 \times 0{\cdot}804$$
$$= 0{\cdot}12 + 3{\cdot}43 = 3{\cdot}55 \text{ tons per sq. in.}$$

Once again, by reference to clause 19(c) it will be obvious that $F_{bc} = 10$ tons per sq. in.

$$\frac{f_a}{F_A} + \frac{f_{bc}}{F_{bc}} = \frac{2{\cdot}55}{4{\cdot}42} + \frac{3{\cdot}55}{10} = 0{\cdot}577 + 0{\cdot}355 = 0{\cdot}932.$$

Therefore the section is adequate at support C.

(*c*) *At the purlin point between B and C*

(i) Axis xx:

$$l/r_{xx} \text{ as for support B} = 41.$$

(ii) Axis yy:

$$l/r_{yy} \text{ as for support C} = 94.$$

(iii) Axis vv:

$$l/r_{vv} = 47{\cdot}5/0{\cdot}72 = 66.$$

Therefore the greatest value of $l/r = 94$.

$$F_a = 4{\cdot}42 \text{ tons per sq. in.}$$
$$f_a = 6{\cdot}8/2{\cdot}67 = 2{\cdot}55 \text{ tons per sq. in.}$$

Bending moment at purlin point = 5·86 tons in. positive.

At this point, it will be the point B which has the maximum compressive stress, but it is reasonable to assume that, since the purlin point is approximately half-way between the supports B and C, the bending stress due to the eccentricity of the end connections is negligible.

$$f_{bc} = 5{\cdot}86 \times 0{\cdot}687 = 4{\cdot}03 \text{ tons per sq. in.}$$

$$\frac{f_a}{F_A} + \frac{f_{bc}}{F_{bc}} = \frac{2{\cdot}55}{4{\cdot}42} + \frac{4{\cdot}03}{10} = 0{\cdot}577 + 0{\cdot}403 = 0{\cdot}980.$$

Therefore the section is adequate at the purlin point.

Accordingly, the section selected is adequate.

Example 7. Double-angle continuous strut.

In this case, there will be bending only about the axis xx, i.e. in a vertical plane.

Moreover it will be obvious, by consideration of Example 6, that only support C need be investigated.

Try two angles 3 in. × $2\frac{1}{2}$ in. × $\frac{1}{4}$ in., $\frac{5}{16}$ in. back to back.

$$\text{Area} = 2{\cdot}63 \text{ sq. in.}$$

$$r_{xx} = 0{\cdot}93 \text{ in.}$$

$$r_{yy} = 1{\cdot}09 \text{ in.}$$

$$\text{Min. } Z_{xx} = 1{\cdot}08 \text{ in.}^3.$$

Support C

(i) Axis xx:

$$l/r_{xx} = 0{\cdot}7 \times 72{\cdot}75/0{\cdot}93 = 55.$$

(ii) Axis yy:

$$l/r_{yy} = 98{\cdot}0/1{\cdot}09 = 90.$$

Therefore the greatest value of $l/r = 90$.

$$F_a = 4{\cdot}62 \text{ tons per sq. in.}$$

$$f_a = 6{\cdot}8/2{\cdot}63 = 2{\cdot}59 \text{ tons per sq. in.}$$

Bending moment at C = 4·26 tons in. negative.

$$f_{bc} = 4{\cdot}26/1{\cdot}08 = 3{\cdot}95 \text{ tons per sq. in.}$$

$$\frac{f_a}{F_a} + \frac{f_{bc}}{F_{bc}} = \frac{2{\cdot}59}{4{\cdot}62} + \frac{3{\cdot}95}{10{\cdot}00} = 0{\cdot}561 + 0{\cdot}395 = 0{\cdot}956.$$

Therefore the section is adequate.

The comparative weights of the sections in Examples 6 and 7 are 9·09 and 8·94 lb. per ft. run respectively, showing a slight saving by the use of a double angle.

There is probably not a great deal to choose between the sections, for the greater r_{yy} of the double angle is offset by the smaller bending resistance, and the ultimate criterion of economy will depend on the relation of the direct stress to the bending on the section.

The single angle has the advantage that there is less fabrication than with the double angle because the latter requires washer-riveting at intervals. It

also has the advantage from the point of view of maintenance, since it is virtually impossible to paint the inner vertical surfaces of a double angle.

Against this, the double angle has two principal advantages: first, since the rivets at the joints will be in double shear or bearing, fewer will be needed than in the single angle; secondly, the purlin cleats, which should be connected to the rafter by a minimum of two rivets, will be of smaller section, since in the double angle the two rivets can be placed one in each angle, whereas in the single angle they must be placed in line down the rafter back.

28. WIND ON MULTI-STOREY BUILDINGS

WIND forces induce stresses in all buildings to some degree, depending on the type of building.

In the case of multi-storey framed buildings this effect is much smaller than in the case of the single-storey shed type of structure, and Clause 12, B.S. 449, states that providing the overall dimensions of the building are such that the height of the building is not more than 3 times the effective width, and providing also that the framework is stiffened by walls, or by floors and walls, no calculations for the effects of wind are required, except in regard to the foundations and wall panels.

It should be noted that floors, of themselves, are not considered capable of providing any bracing resistance against wind forces.

By Clause 25, the normal permissible stresses in the members may be exceeded by 25 per cent in cases where such increases are due solely to the stresses induced by wind.

This higher working stress is allowed because of the transient nature of the load and also because the structure is of a sufficiently elastic nature to allow it to absorb such transient loads without permanent defect.

The increase is not permissible on foundations because the soil is not capable of acting in the same elastic manner as the frame itself, and consequently the overturning effect on the foundation must be taken into consideration. Moreover, there are not the same safety factors in safe soil pressures as there are in steel-frame stresses.

Many cases occur of buildings in which, although the height/width ratio of 3 is not exceeded, it is not practicable to build the cross walls which according to B.S. 449 can be considered as providing the requisite lateral stiffness against the wind forces. A warehouse building, in which large open spaces are required, is a case in point.

This lateral bracing may be provided in one of two ways. First by some system of vertical diagonal bracing, or secondly by making the beam-to-stanchion connections capable of transmitting the forces and moments imposed by the transverse loads. The former might not be practicable in a building where large open spaces are required. In the second case some form of analysis must be applied so that the forces and moments in the members of the framework can be determined.

There are several approximate methods by which the forces can be assessed, and of these methods the three in most common use will be dealt with in the following pages.

These methods are applicable only to multi-storey buildings designed on the basis of simple design as set out in B.S. 449, and must not be applied to a rectangular frame designed on any other basis.

It is not claimed that the methods are theoretically correct, but they give reasonable results and are simple in application.

Before any analysis can be made it is necessary to make certain basic assumptions. These assumptions are:

1. That the wind loads are applied at the panel points, i.e. the joints between beams and columns.
2. That the total horizontal shear at any level is resisted by the columns at the points of contraflexure immediately below that level. The actual distribution of the horizontal shear between the columns varies with the method used.
3. That a point of contraflexure exists at the mid-height of each column This assumption is probably not correct, and it is likely that in practice the point of contraflexure will be somewhere in the middle third of the column height, depending on the relative stiffnesses of beams and columns. Any error resulting from this assumption will not, however, be of practical significance.

These assumptions are illustrated in Fig. 375 (a), which for the sake of simplicity is shown as referring to a single-bay frame.

Referring to Fig. 375 (b), the upper portion of the diagram shows the forces on the portion of the frame above the points of contraflexure A and D, while the lower portion shows the forces induced in the portion $ABCDEF$ of the frame. The vertical forces are found by taking moments about the appropriate points of contraflexure. The shear force in the beam BE is the difference between the vertical forces in the columns, i.e.

$$\frac{W_1H_1}{L}+\frac{(W_1+W_2)H_2}{2L}-\frac{W_1H_1}{2L}=\frac{W_1H_1+(W_1+W_2)H_2}{2L}.$$

The resulting moments in the frame are shown in Fig. 375 (c).

The moment in the upper portion of the column at B due to the force at A

$$=\frac{W_1}{2}\times\frac{H_1}{2}=\frac{W_1H_1}{4},$$

while that in the lower portion of the column at B due to the force at C is

$$\frac{W_1+W_2}{2}\times\frac{H_2}{2}=\frac{(W_1+W_2)H_2}{4}.$$

The moment in the beam at B = the shear force in the beam $\times L/2$

$$=\frac{W_1H_1+(W_1+W_2)H_2}{2L}\times\frac{L}{2}=\frac{W_1H_1+(W_1+W_2)H_2}{4},$$

from which it will be seen that the moment in the beam at B is equal to the sum of the moments in the column, both upper and lower, at B. This is true for every joint in the frame and irrespective of the method used, i.e. the algebraic sum of the moments at any joint must be zero.

The three methods will now be examined in detail and, in order to simplify the working, an example with actual figures will be used.

The building is 10 storeys high, each storey being of 12 ft. height. The

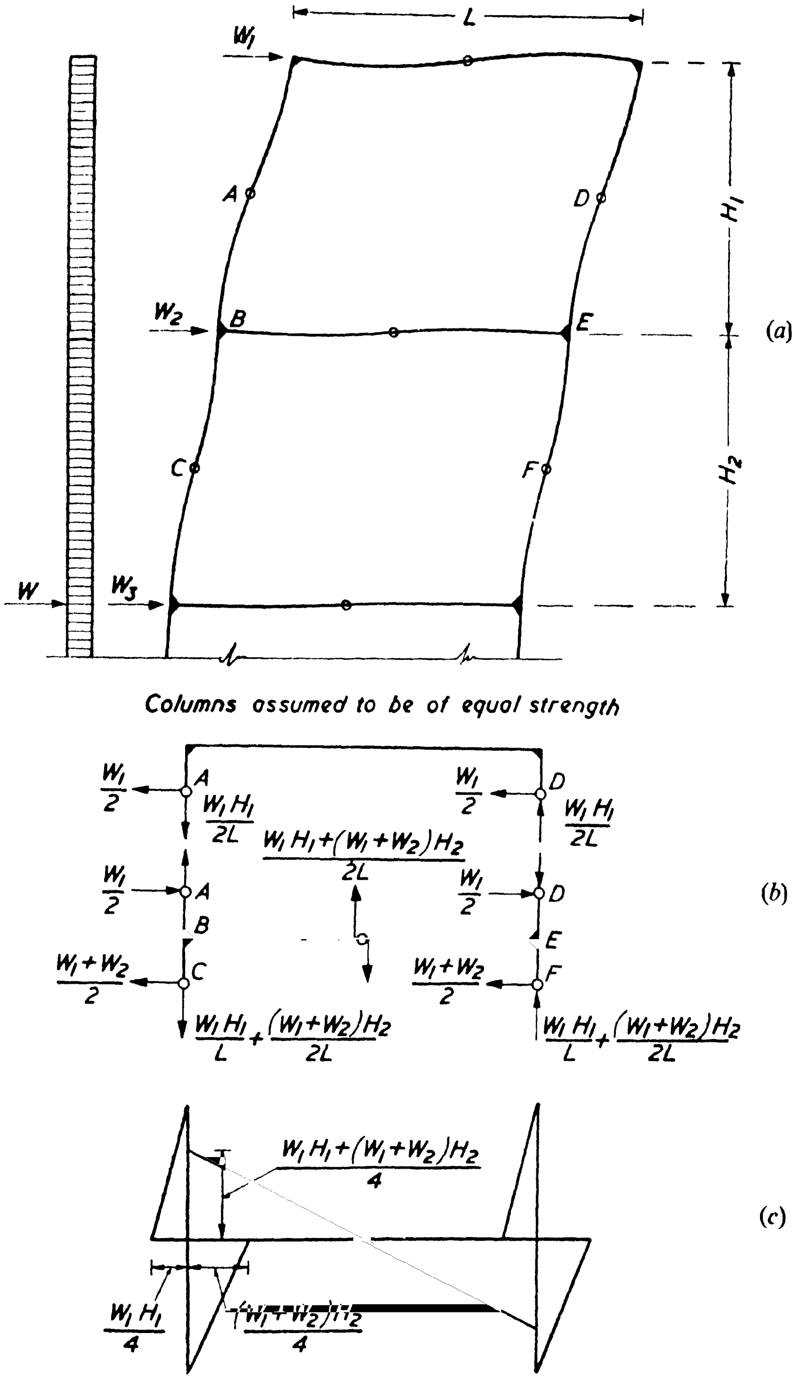

L
W_1
W_2
W_3
W
A
B
C
D
E
F
H_1
H_2
(a)
Columns assumed to be of equal strength
$\frac{W_1}{2}$
$\frac{W_1 H_1}{2L}$
$\frac{W_1 H_1 + (W_1 + W_2) H_2}{2L}$
$\frac{W_1 + W_2}{2}$
$\frac{W_1 H_1}{L} + \frac{(W_1 + W_2) H_2}{2L}$
(b)
$\frac{W_1 H_1 + (W_1 + W_2) H_2}{4}$
$\frac{W_1 H_1}{4}$
(c)

FIG. 375

width of the building is 60 ft. divided into three equal bays each of 20 ft. The spacing of the main frames is 15 ft. The unit wind pressure is 10 lb. per sq. ft. derived from Tables 3 and 4 in B.S. 449 as follows: The building is in central England in a position such that natural protection is afforded and consequently the approximate wind velocity is 50 miles per hour. The height above ground assumed to be sheltered by permanent obstacles is 40 ft., resulting in an effective height of $120-40=80$ ft. From Table 4, the unit wind pressure is 10·0 lb. per sq. ft.

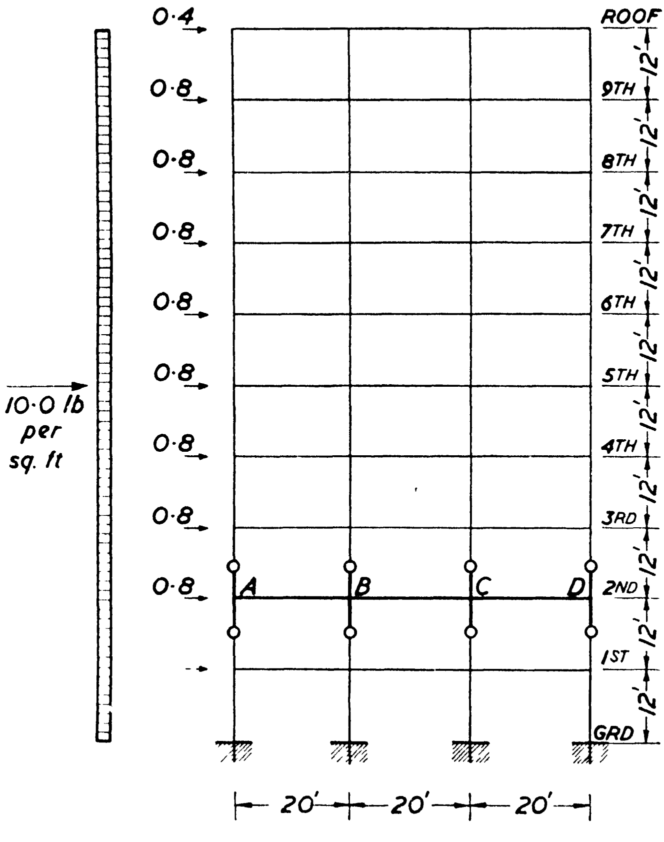


FIG. 376

The loads and moments at the 2nd-floor level will be investigated.

Fig. 376 shows a cross-section through the building together with the wind loads at the panel points.

$$\text{The wind load per panel} = \frac{10{\cdot}0 \times 12 \times 15}{2240} = 0{\cdot}8 \text{ ton.}$$

For the purposes of the calculation in all three methods the columns will be assumed to have equal areas and equal stiffnesses.

Method 1

This method is sometimes called the *Continuous Portal* method.

The assumptions made are as follows:

(a) There is a point of contraflexure at the mid-height of each column.
(b) The axial *stress* in the columns is proportional to the distances of the columns from the C.G. of the columns as a whole.
(c) The total horizontal shear is divided between the columns in proportion to their stiffnesses in the direction of the wind force.

The total horizontal force above the point of contraflexure above the 2nd floor $= 6{\cdot}0$ tons. Therefore each column takes

$$\frac{6{\cdot}0}{4} = 1{\cdot}5 \text{ tons.}$$

The total horizontal force above the point of contraflexure below the 2nd floor $= 6{\cdot}8$ tons. Therefore each column takes

$$\frac{6{\cdot}8}{4} = 1{\cdot}7 \text{ tons.}$$

From assumption (b) and because the columns have equal areas, the axial *loads* in the columns are proportional to the distances from the C.G. of the columns as a whole. Therefore, the axial load in the outer columns $= 3 \times$ the axial load in the inner columns.

Let P_1 be the axial load in the inner columns above 2nd-floor level and P_2 be the axial load in the inner columns below 2nd-floor level. Then by taking moments at the point of contraflexure above 2nd-floor level,

$$P_1 \times 20 + 3P_1 \times 60 = 0{\cdot}8(6 + 18 + 30 + 42 + 54 + 66 + 78) + 0{\cdot}4 \times 90$$

whence $P_1 = 1{\cdot}356$ tons, and the axial load in the outer columns at this level $= 3 \times 1{\cdot}356 = 4{\cdot}068$ tons.

Similarly by taking moments at the point of contraflexure below 2nd floor: $P_2 = 1{\cdot}74$ tons and the axial load in the outer columns at this level $= 3 \times 1{\cdot}74 = 5{\cdot}22$ tons.

The forces acting on the portion of the frame under review are shown in Fig. 377 (a).

The shear force in beam AB equals the difference between the axial forces in the columns above and below the 2nd-floor level and at $A = 5{\cdot}22 - 4{\cdot}068 = 1{\cdot}152$ tons, while that in beam $BC = 1{\cdot}152 + (1{\cdot}74 - 1{\cdot}356) = 1{\cdot}152 + 0{\cdot}384 = 1{\cdot}536$ tons.

The B.M.s in beams and columns are shown in Fig. 377 (b).

The B.M. at the floor level due to the force in the upper portion of the column is the same at all points and equals $1{\cdot}5 \times 6{\cdot}0 = 9{\cdot}0$ tons ft., while that due to the force in the lower portion of the column $= 1{\cdot}7 \times 6{\cdot}0 = 10{\cdot}2$ tons ft.

The B.M. in beam AB at A is therefore equal to $9{\cdot}0 + 10{\cdot}2 = 19{\cdot}2$ tons ft.

The B.M. in beam AB at B is found by taking moments about B and equals $19{\cdot}2 - 1{\cdot}152 \times 20 = 19{\cdot}2 - 23{\cdot}04 = -3{\cdot}84$ tons ft.

The B.M. in beam BC at B is found from the fact that the algebraic sum of the B.M.s at B must be zero. Therefore B.M. in beam BC at $B = 19{\cdot}2 - 3{\cdot}84 = 15{\cdot}36$ tons ft.

Since the frame is symmetrical, the point of contraflexure in beam BC must be at the midpoint of the span. Accordingly the B.M. at the ends of the beam can be found as follows:

$$\text{B.M.} = \text{S.F. in beam} \times \text{half span} = 1{\cdot}536 \times 10{\cdot}0 = 15{\cdot}36 \text{ tons ft.}$$

which is the same result as before and is a check on the working.

The following points should be noted in connection with Method 1:

(a) If the columns are unequally spaced, or have different areas, the C.G. of the columns must be found.

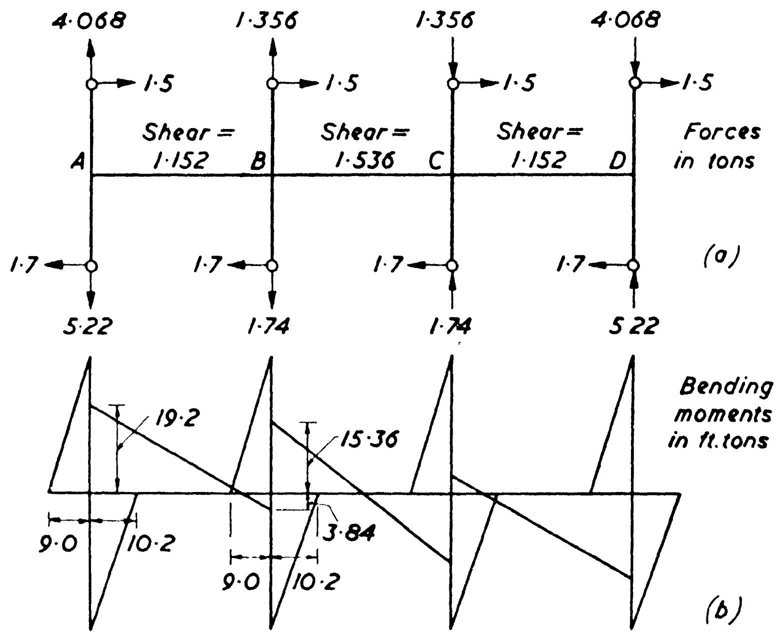


FIG. 377

(b) It will be seen, from Fig. 377 (b), that the points of contraflexure in the outer beams are $\frac{1}{6}$ of the span from the internal columns. If the frame had four equal bays, it would have been found that these points of contraflexure occurred at the inner ends of the beams, while for any number of equal bays greater than four, there will be no point of contraflexure in the outer beams. It follows that this method cannot be applied to a frame with more than four equal bays. In such cases, an approximate solution can be found by assuming that each external column takes half as much horizontal shear as an internal column, i.e., that each has half the stiffness of an internal column.

Method 2

This method is sometimes called the *Cantilever* method. The assumptions made are as follows:

(a) As (a) in Method 1.
(b) As (b) in Method 1.
(c) There is a point of contraflexure at the mid-span of each beam.

It follows from assumptions (a) and (b) that the vertical loads in the columns, and the S.F.s in the beams, are the same as in Method 1. It also follows, from assumption (c), that the B.M. at the ends of the beams AB and $CD = 1{\cdot}152 \times 10 = 11{\cdot}52$ tons ft., while the B.M. at the ends of beam $BC = 1{\cdot}536 \times 10 = 15{\cdot}36$ tons ft.

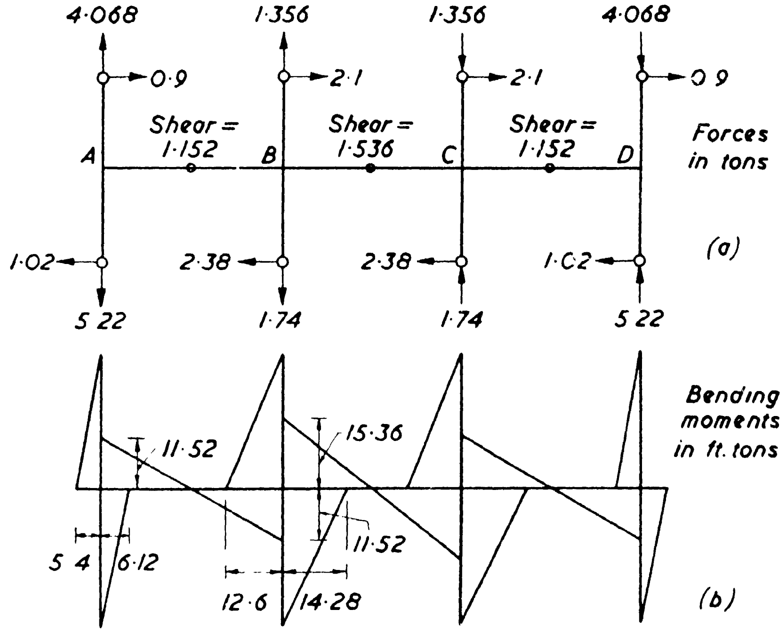


FIG. 378

The horizontal shears in the columns are found as follows: The outer column must carry a certain fixed proportion of the total horizontal shear at any level. If this proportion is x, then the horizontal shear in the outer column above the 2nd floor $= x \times 6{\cdot}0$, while that below the 2nd floor $= x \times 6{\cdot}8$. The B.M.s in the column at A due to these shears are $6{\cdot}0x \times 6{\cdot}0$ and $6{\cdot}8x \times 6{\cdot}0$ respectively and they must add up to the B.M. in beam AB at A.

Therefore $$6{\cdot}0x \times 6{\cdot}0 + 6{\cdot}8x \times 6{\cdot}0 = 11{\cdot}52$$

whence $$x = 0{\cdot}15$$

so that the horizontal shear in the outer columns above 2nd floor

$$= 0{\cdot}15 \times 6{\cdot}0 = 0{\cdot}9 \text{ tons,}$$

and the horizontal shear in the outer columns below 2nd floor

$$=0{\cdot}15\times 6{\cdot}8=1{\cdot}02 \text{ tons.}$$

Consequently the horizontal shear in the inner columns above 2nd floor

$$=\frac{6{\cdot}0}{2}-0{\cdot}9=2{\cdot}1 \text{ tons,}$$

and the horizontal shear in the inner columns below 2nd floor

$$=\frac{6{\cdot}8}{2}-1{\cdot}02=2{\cdot}38 \text{ tons.}$$

The loads on the frame are shown in Fig. 378 (a), while the B.M.s are shown in Fig. 378 (b).

The following points should be noted in connection with Method 2:

(a) If the columns are unequally spaced, or have different areas, the C.G. of the columns must be found.

(b) The method can be applied to any number of bays and it will be seen, from Fig. 378 (a), that the outer columns take just under half the horizontal shear taken by the internal columns. This method, then, is similar to the approximate solution suggested by Method 1.

Method 3

This method is sometimes called the *Portal* method. The assumptions made are as follows:

(a) As (a) in Method 1.

(b) There is a point of contraflexure at the mid-span of each beam.

(c) Each bay acts as a simple portal, and the total horizontal load is divided between the bays in proportion to the span of the bays. The result of this assumption is that the internal columns take no vertical load. With equal bays, as in the example under review, the horizontal load is divided equally between the bays. Accordingly, each internal column, since it forms part of two bays, takes twice as much horizontal load as each outer column. Again, because each internal column forms part of two portals, the uplift due to one portal is balanced by the downward load due to the other portal, resulting in no net vertical load in the internal columns.

The horizontal shears in the columns are therefore as follows:

(a) Above 2nd floor: Internal columns, 2·0 tons.
External columns, 1·0 ton.

(b) Below 2nd floor: Internal columns, 2·267 tons.
External columns, 1·133 tons.

Since only the external columns take the vertical load, the vertical load in these columns above 2nd floor is found by dividing the overturning moment at the level of the point of contraflexure above the 2nd floor by 60 ft., i.e. by the distance between the outer columns.

This load

$$= \frac{0{\cdot}8(6+18+30+42+54+66+78)+0{\cdot}4\times 90}{60}$$

$=4{\cdot}52$ tons.

Similarly the vertical load in the outer columns below 2nd floor $=5{\cdot}80$ tons.

The forces acting on the frame are shown in Fig. 379 (a).

Since there is no vertical force in any of the internal columns the shear in all the beams is constant and equals $5{\cdot}80-4{\cdot}52=1{\cdot}28$ tons.

The B.M.s in the beams and columns are shown in Fig. 379 (b).

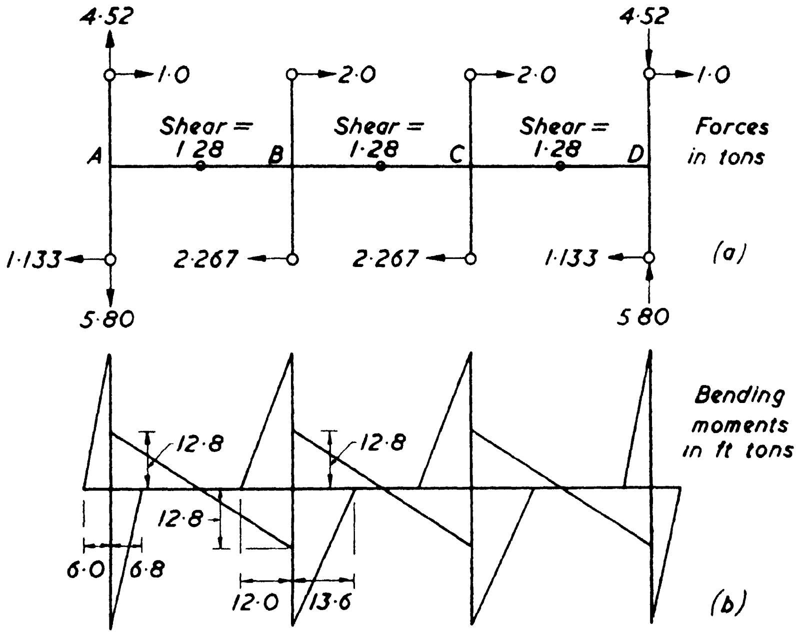


FIG. 379

The following points should be noted in connection with Method 3:

(a) Since the shear in the beams is constant, and there is a point of contraflexure at the mid-span of each beam, the B.M. at the ends of the beams is constant at any one level. This applies, of course, only if the bays are equal.

(b) There is vertical load only in the outer columns, and that load has no relation to the areas of the columns.

Summing up, it will be seen, from Figs. 377, 378 and 379, that there is no vast disparity between the results obtained from any of the three methods examined and from the point of view of both the designer and the detailer, Method 3 is the most satisfactory.

The process of the design of the beams at the 2nd-floor level and their end connections will now be studied.

It will be assumed that the overall weight of the floor inclusive of both dead and superimposed loads is 180 lb. per sq. ft. and the results from Method 3 will be taken as giving the wind B.M.s.

$$\text{Floor load on any beam, assumed U.D.,} = \frac{20 \times 15 \times 180}{2240}$$

$$= 24{\cdot}1 \text{ tons.}$$

Allowance for own weight and casing,

$$\text{say,} = \quad 1{\cdot}9 \text{ tons}$$

$$26{\cdot}0 \text{ tons}$$

$$\text{B.M.} = \frac{26{\cdot}0 \times 20{\cdot}0}{8} = 65 \text{ tons ft.}$$

The combined B.M. diagram for dead and superimposed loads and for wind loads is shown in Fig. 380.

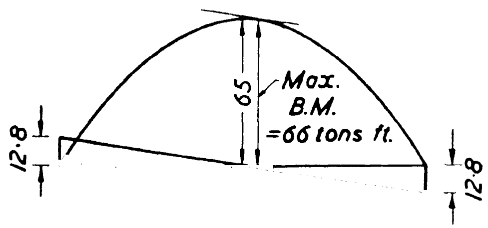


FIG. 380

The maximum B.M. under dead and superimposed loads is 65·0 tons. ft., while that due to all loads, including wind, is 66·0 tons ft. In connection with the former, the permissible stress is 10 tons per sq. in., while in connection with the latter the permissible stress could be increased to 12·5 tons per sq. in.

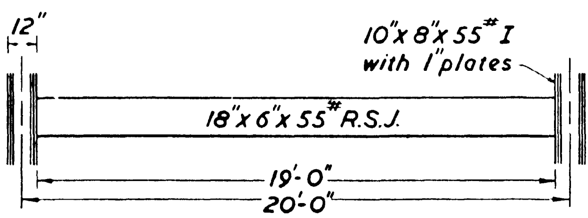


FIG. 381

It is obvious that the former governs and the wind effect may be neglected, except for the end connections. Therefore

$$Z \text{ required} = \frac{65{\cdot}0 \times 12}{10} = 78{\cdot}0 \text{ in.}^3$$

An 18-in. × 6-in. × 55-lb. 'I' will be suitable.

If the columns are assumed to be 10-in. × 8-in. × 55-lb. 'I's' with 1-in thick plates in each flange at the level concerned then the clear span between faces of columns = 20 ft. 0 in. − 2 × 6 in. = 19 ft. 0 in. See Fig. 381.

The wind B.M. at the ends of the beam, viz. 12·8 tons ft. has been calculated on the distance between centre lines and at the face of the column this reduces to

$$12{\cdot}8 \times \frac{9{\cdot}5}{10} = 12{\cdot}2 \text{ tons ft.}$$

This B.M. must be resisted by the connection of the beam to the columns.

The connections must be examined at two critical points:

1. The attachment of the connections to the beam.
2. The attachment of the connections to the column.

The simplest form of connection is shown in Fig. 382.

The connection may be made either with rivets or turned bolts. The former have been more common in the past.

At the attachment of the connection to the beam the rivets are in single shear and the lever arm of the resistance moment is the overall depth of the beam. In some cases of large B.M.s, this lever arm can be increased as will be seen later.

In the present example the overall depth of the beam is 18 in. or 1·5 ft., and therefore the force in each arm of the resistance moment = 12·2/1·5 = 8·14 tons.

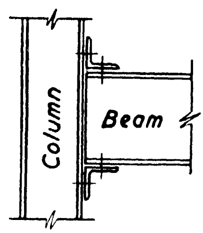


FIG. 382

Using ¾-in. rivets each having a single shear value, on the gross area, of 3·11 tons + 25 per cent = 3·11 + 0·78 = 3·89 tons, three rivets should be provided top and bottom. This number is impractical, however, and therefore four rivets will be provided, which means that the horizontal legs of the cleats should be at least 6 in. long.

At the attachment of the connection to the column, one half of the connection will be in tension and the other in compression. On the tension side the rivets or bolts alone must provide the force in the arm of the couple.

If rivets are used, the gross area may be taken, and the allowable stress is 5·0 or 4·0 tons per sq. in. depending on whether the rivets are shop or field driven. If bolts are used, the allowable stress is 6·0 tons per sq. in. on the area at the bottom of the thread. (Note.—6·0 tons per sq. in. is the allowable stress on bolts from ¾-in. to 1½-in. in diameter.) The stresses quoted in this paragraph may be increased by 25 per cent providing the excess is solely due to stresses induced by wind.

The compression force is provided by direct compression between the cleats and the column face.

It will be assumed that the centre of rotation of the connection is at the centre of the compression group of rivets.

When the B.M. is such that the upper cleat is in tension, the rivets have only tension to resist, and in the example the cleat shown in Fig. 383 will be adequate.

The rivets will be assumed to be field driven and of ⅞ in. nominal diameter,

in a flange 12 in. wide. The corresponding value of each rivet in tension, using a stress of 4 tons per sq. in. + 25 per cent on the gross area, = 2·76 tons.

$$\text{R.M. of rivets} = 2 \times 2{\cdot}76 \times \frac{25{\cdot}88}{12} + 2 \times 2{\cdot}76 \times \frac{23{\cdot}62}{25{\cdot}88} \times \frac{23{\cdot}62}{12}$$

$$= 11{\cdot}9 + 9{\cdot}9 = 21{\cdot}8 \text{ tons ft.}$$

The dimensions given are for a 6-in. cleat.

When the lower cleat is in tension, the rivets connecting the cleat to the column must be capable of taking not only the tension due to the wind B.M. but also the reaction from the beam due to dead and superimposed loads. In the example this load in the beam is 26·0 tons, and therefore the reaction is 13·0 tons.

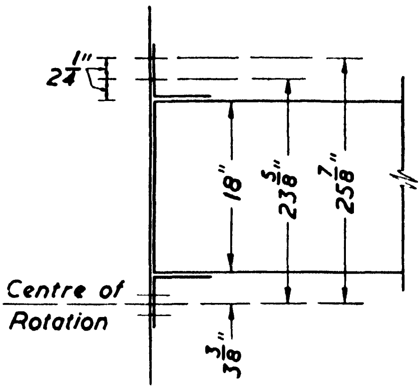


FIG. 383

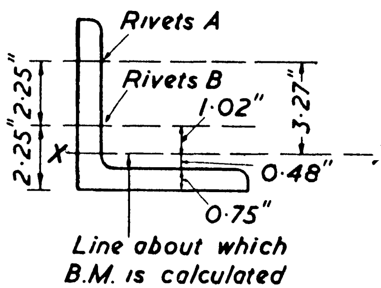


FIG. 384

The same cleat as at the top will be tried.

$$\text{Shear load per rivet} = \frac{13{\cdot}0}{4} = 3{\cdot}25 \text{ tons.}$$

$$\text{Maximum tension load per rivet} = \frac{12{\cdot}2}{21{\cdot}8} \times 2{\cdot}76 = 1{\cdot}55 \text{ tons.}$$

These two forces act at 90° to each other, and therefore the resultant force

$$= \sqrt{(3{\cdot}25^2 + 1{\cdot}55^2)} = \sqrt{(10{\cdot}56 + 2{\cdot}41)} = \sqrt{12{\cdot}97}$$

$$= 3{\cdot}60 \text{ tons.}$$

Under this composite loading it may be assumed that the allowable stress is 6·0 tons per sq. in. + 25 per cent because the excess load is due to wind and consequently the allowable load on the rivet = 5·17 tons. The connection therefore is adequate.

6-in. × 6-in. angle cleats will be satisfactory for the connection from the standpoint of rivet strength. The cleats should be stiff enough to transmit the B.M.s without undue distortion. It will be assumed that the cleats are 6-in. × 6-in. × ¾-in. thick, and the B.M. on the angle will be calculated at the start of the root curve of the angle. See Fig. 384.

The load on the rivets $A = 1{\cdot}55$ tons each, and the load on the rivets B, by proportion, $= 1{\cdot}55 \times 23{\cdot}62/25{\cdot}88 = 1{\cdot}42$ tons.

$$\text{B.M. about line } XX = 2 \times 1{\cdot}55 \times 3{\cdot}27 + 2 \times 1{\cdot}42 \times 1{\cdot}02$$
$$= 10{\cdot}1 + 2{\cdot}9 = 13{\cdot}0 \text{ tons in.}$$

$$Z \text{ required} = \frac{13{\cdot}0}{12{\cdot}5} = 1{\cdot}04 \text{ ins.}^3$$

$$Z \text{ per inch width of cleat} = \frac{1{\cdot}0 \times 0{\cdot}75^2}{6} = \frac{3}{32} \text{ in.}^3.$$

$$\text{Width required} = \frac{1{\cdot}04 \times 32}{3} = 11{\cdot}1 \text{ in.}$$

Therefore a cleat 12 in. wide is adequate.

The calculation for these angle cleats demonstrates that, of themselves, they

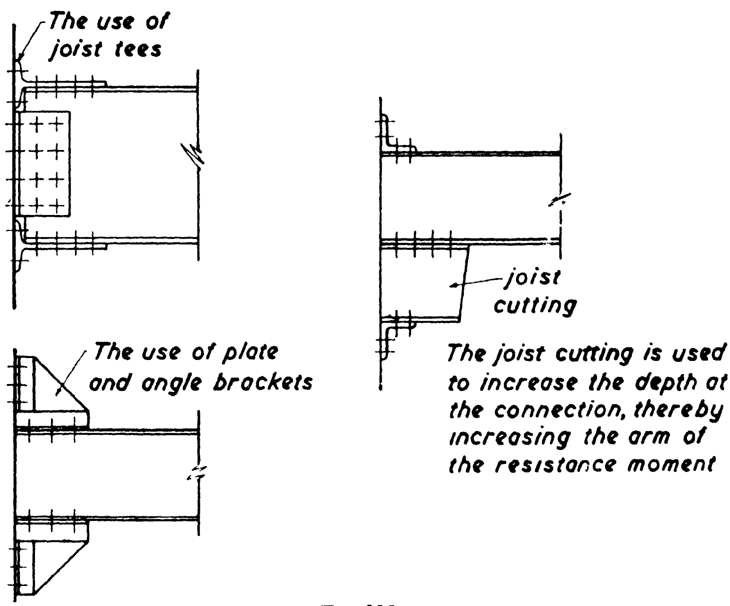


FIG. 385

must be of considerable thickness to withstand even a comparatively small B.M., and they are therefore uneconomic.

An angle of lesser thickness could be used, even if the configuration of riveting shown is retained, provided that it is stiffened against bending. Alternatively, if the stanchion is wide enough to allow four vertical lines of rivets, then the rivets could be placed in one horizontal line, thereby reducing the B.M. on the cleat and enabling a thinner angle to be used.

A small additional resistance can be obtained by the use of web cleats, but by reason of the smallness of the arm of the resistance moment in such cleats, they are, by comparison with flange cleats, uneconomic.

Fig. 385 shows some typical wind connections.

29. TENSION MEMBERS

THE requirements laid down in B.S. 449:1948 for the detail design of tension members relate to angle sections only.

Clause 39 states that the effective cross-sectional area shall be calculated as follows:

(A) Equal angles and unequal angles connected by the longer leg.

(i) Single angle
    Net area of connected leg plus half area of outstanding leg.

(ii) Double angles connected to both sides of a gusset and with stitch riveting or welding at not more than 3-ft. 6-in. centres
    Gross area less connection holes.

(iii) Double angles connected to one side of a gusset with stitch riveting or welding as in (ii) above.
    Net area of connected legs plus three-quarters of the area of the outstanding legs.

(B) Unequal angles connected by the shorter leg.

(i) Single angle
    Net area of connected leg plus half the gross area of the same leg.

(ii) Double angles connected to both sides of a gusset and with stitch riveting as above
    Gross area less connection holes.

(iii) Double angles connected to one side of a gusset and with stitch riveting as above.
    Net area of connected legs plus three-quarters of the gross area of the same legs.

The clause also states that if these effective areas are used the reductions make allowance for the effect of eccentricity of the connections.

It therefore follows that all other tension members require to be designed to meet the stresses caused by the direct tension in conjunction with the bending stresses, if any, induced by the end connections. It would, however, seem logical, for the design of channel and 'T'-sections to use effective areas calculated in a manner similar to angles.

30. FOUNDATIONS

The total load on the contact surface between the underside of the concrete block and the soil is that due to the imposed load upon the stanchion, plus the superimposed load at ground level on an area equal to that of the concrete block and plus also the weight of the concrete and the earth immediately above it. The pressure on the soil should not exceed the safe bearing capacity at this depth below the free surface.

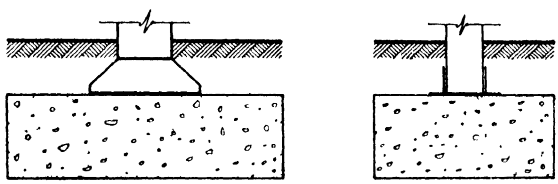


FIG. 386

In the remarks which follow:

P = vertical load on stanchion base.

W = weight of base, earth above it and superload if any.

R = total vertical load = $P + W$.

H = horizontal force at stanchion base.

M = moment at stanchion base.

Case 1

Central load:

Total load on soil = R

$$\text{Soil pressure} = \frac{R}{BL}.$$

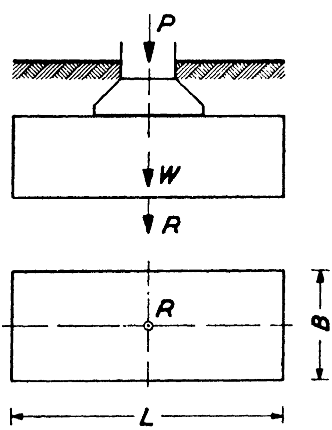


FIG. 387

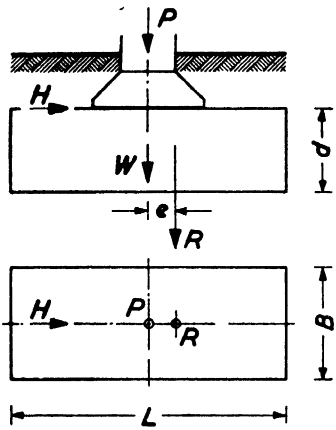


FIG. 388

Case 2

Central load with horizontal force at base of stanchion:

Total load on soil $= R$.

Eccentricity induced at underside of block due to horizontal force H

$$= \frac{Hd}{R} = e.$$

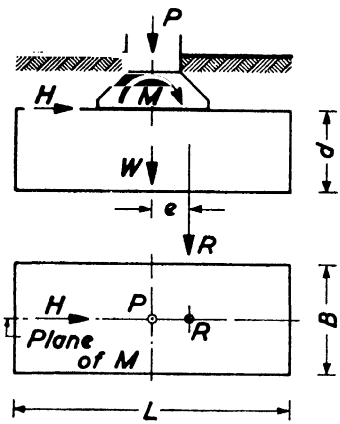


FIG. 389

Case 3

Central load with horizontal force and moment at stanchion base:

Total load on soil $= R$.

Eccentricity induced at underside of block dùe to H and M

$$= \frac{Hd + M}{R} = e.$$

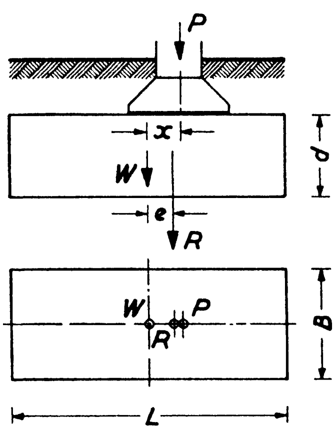


FIG. 390

Case 1a

Load eccentric on one axis:

Total load on soil $= R$.

Eccentricity at underside of block:

$$= \frac{Px}{R} = e.$$

Case 2a

Load eccentric on one axis with horizontal force at stanchion base:

Total load on soil $= R$.

Eccentricity at underside of block:

$$= \frac{Px + Hd}{R} = e.$$

For load P to left of centre line:

$$e = \frac{-Px + Hd}{R}.$$

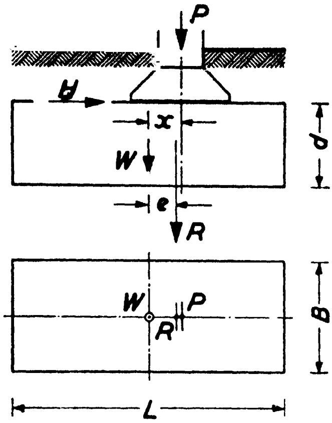


FIG. 391

Case 3a

Load eccentric on one axis with horizontal load and moment at stanchion base:

Total load on soil $= R$.

Eccentricity at underside of block

$$= \frac{Px + Hd + M}{R} = e.$$

$\left(\text{In general terms } e = \frac{\pm Px \pm Hd \pm M.}{R}\right)$

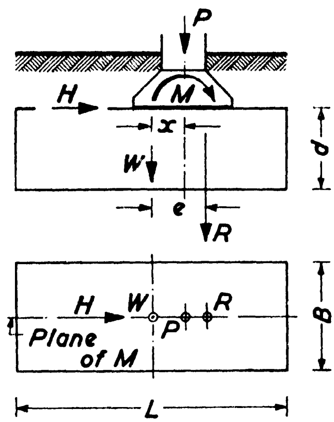


FIG. 392

All the foregoing with the exception of Case 1 result in non-uniform loading on the soil and for the whole length L to be under pressure the eccentricity e must not exceed $L/6$.

With this proviso the stresses at the edges of the base are given by the expression:

$$p = \frac{R}{BL} \pm \frac{Re}{BL^2/6}$$

$$= \frac{R}{BL}\left(1 \pm \frac{6e}{L}\right).$$

The limiting values when $e = L/6$ are $p_1 =$ zero and $p_2 = 2R/BL$ (see Fig. 393).

It is necessary to proceed on other lines should the value of e exceed $L/6$, since the preceding formula is based upon the ordinary bending theory which postulates the existence of tension. This cannot occur between the soil and the underside of the block. In such a case, let y be the distance from the line of action of R to the nearest edge of the base, i.e.

$$y = \frac{L}{2} - e.$$

Since the stress distribution will be linear and the centroid of the diagram must be on the same vertical line as R it follows that the loaded length will be

$$3y = 3\left(\frac{L}{2} - e\right). \qquad \text{(See Fig. 394)}$$

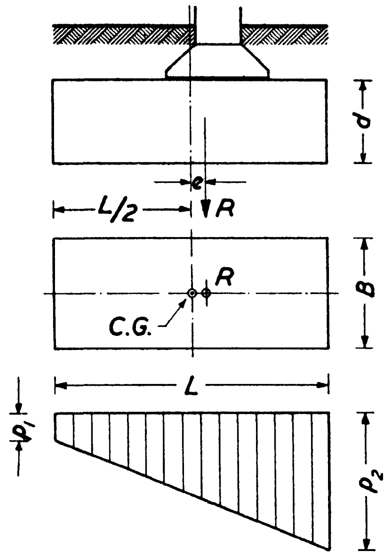


FIG. 393

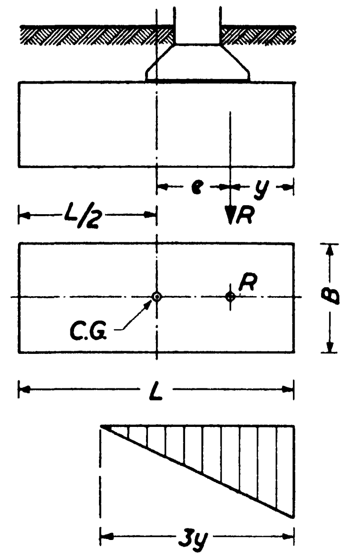


FIG. 394

Consequently the maximum edge pressure

$$= p$$

$$= \frac{2R}{3 \,.\, B \,.\, y}.$$

Loading Eccentric about both Axes

It will be seen by analogy with the foregoing that if e_L and e_B be the eccentricities of R with regard to L and B respectively then the stresses at the *four corners* of the base, using the ordinary bending theory, are given by the expression

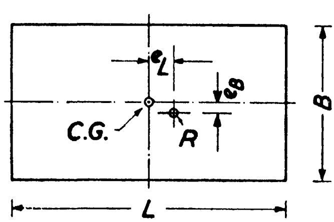


FIG. 395

$$p = \frac{R}{BL} \pm \frac{Re_L}{BL^2/6} \pm \frac{R \,.\, e_B}{B^2L/6}$$

$$= \frac{R}{BL}\left(1 \pm \frac{6e_L}{L} \pm \frac{6e_B}{B}\right).$$

For the whole area of the base to be under pressure, i.e. for there to be no tension, R must act within the 'kern' of the section, shown hatched in Fig. 396 The limiting values of stresses for the configuration shown are minimum stress = zero and maximum stress $= 2R/BL$.

Concrete bases, unreinforced, are normally not investigated for resistance to moment and shear, and the load from the stanchion base is assumed to disperse at an angle of 45° from which the thickness of the base is obtained as shown in Fig. 397.

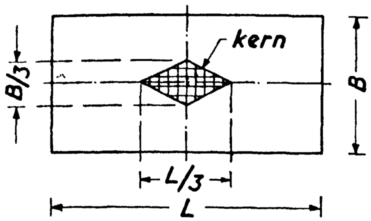


FIG. 396

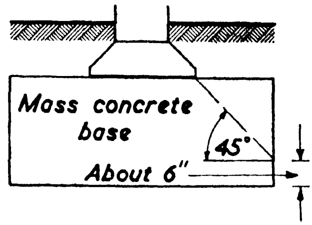


FIG. 397

For the design of reinforced concrete bases the reader is referred to any standard text-book on that subject.

Safe bearing pressures between the stanchion base and concrete block should not exceed the values permitted by the local building regulations.

Grillage Bases

Where the stanchion load is so great that the conventional slab or gusseted base becomes too large, grillages may be employed.

If the load be assumed uniformly distributed both above and below the grillage beam, then the shear diagram takes the form shown in Fig. 398, the maximum shear being

$$W\left(\frac{L-l}{2L}\right)=\frac{Wa}{L}$$

where a = overhang of grillage beam.

The B.M. at this point is $Wa^2/2L$, and the maximum B.M. occurs at the centre and equals

$$\frac{W}{8}(L-l).$$

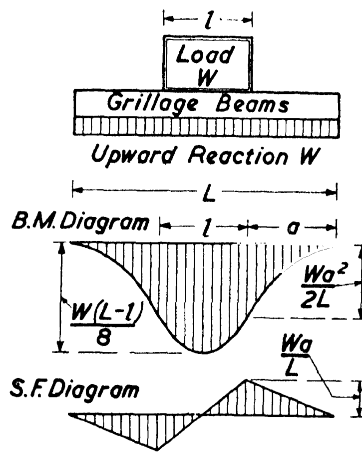


FIG. 398

If, however, the load from the stanchion is carried by a gusseted base then the assumption made above is not valid and the loads should be taken as coming down the gusset plates as shown in Fig. 399. The maximum shear force is Wa/L, as before, although the diagram is of different form. The maximum B.M now occurs at the point of maximum shear and is $Wa^2/2L$ whilst the B.M. at the centre line is reduced to

$$\frac{W}{4}\left(\frac{L}{2}-l\right).$$

The requirements of B.S. 449 are that the normal working stresses may be increased by one-third where the grillage beams are completely covered in concrete of mix not weaker than 1 : 2 : 4 provided the minimum distance between flanges is 3 in. (to allow for the satisfactory consolidation of the concrete) and that the minimum cover at any point is 4 in.

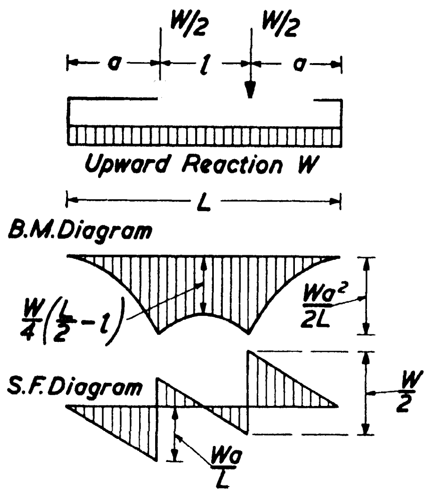


FIG. 399

In addition to checking that permissible bending and shear stresses are not exceeded it will be necessary to observe the requirements for combined bending and shear as outlined in Clause 22 of B.S. 449. It is necessary also to investigate the strength of the webs of the joists against vertical buckling due to the downward load. The length over which the load may be considered to act is calculated in accordance with Clauses 18 and 47.

Slab Bases

Solid steel slab bases have to a great extent superseded grillages since the fabrication required is reduced to a minimum. If the load W on the steel slab is assumed uniformly distributed over an area $l \times d$ and the load under the slab also uniformly distributed over the concrete base, then

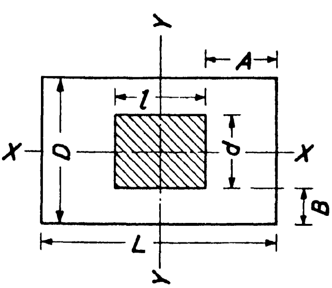


FIG. 400

$$\text{B.M. in plane } XX = M_{XX} = \frac{W}{8}(L - l)$$

$$\text{and B.M. in plane } YY = M_{YY} = \frac{W}{8}(D - d).$$

Assuming Poisson's Ratio as $\frac{1}{4}$ it follows that the effective moments are:

$$\text{For breadth } D, \quad M_{XX} - \frac{M_Y}{4}$$

$$\text{and} \quad \text{for length } L, \quad M_{YY} - \frac{M_X}{4}.$$

It is, however, generally assumed that these calculated moments are excessive since they make no allowance for the 'dishing' action of the slab and maximum moments are assumed to occur at the edges of the upper rectangle $d \times l$.

It is then more convenient to express the load on the underside in terms of tons per square inch—say w. Then if the overhang be A in., the B.M. per

inch run will be $wA^2/2$ in. tons and the moment in the direction at right angles will be $wB^2/2$ in. tons. Allowance is normally made for the effect of two-dimensional stresses as outlined previously and therefore the effective moments for design purposes are:

$$\text{For breadth } D \ldots M = \frac{w}{2}\left(A^2 - \frac{B^2}{4}\right) \text{ in. tons}$$

$$\text{for length } L \ldots M = \frac{w}{2}\left(B^2 - \frac{A^2}{4}\right) \text{ in. tons.}$$

If the base is square, i.e. $L = D$, and A is the *maximum* projection of the slab beyond the stanchion, the above reduces to

$$t = \sqrt{\left[\frac{3w}{F_b}\left(A^2 - \frac{B^2}{4}\right)\right]},$$

wherein t = slab thickness in inches,

F_b = permissible bending stress = 12 tons per sq. in.

This is the formula given in B.S. 449, Clause 50.

Alternatively, for a square base:

$$t = \frac{1}{2L}\sqrt{\left[W\left(A^2 - \frac{B^2}{4}\right)\right]},$$

where W is the total load and L is the length of one side of the base.

Where the slab is not uniformly loaded on its underside the formula given above does not apply and the example which follows illustrates the methods which can be used.

Example 1. Eccentrically Loaded Foundation.

Fig. 401 shows the foot of a heavy column for which a slab base and concrete foundation will be designed.

Data:

Direct central load = 152 tons.

M_{XX} in plane XX = 48·6 tons ft.

M_{YY} in plane YY = 6·5 tons ft.

Permissible pressures:

on soil = 4 tons per sq. ft.

on concrete = 24 tons per sq. ft.

Assume that the concrete base is 10 ft. long and 5 ft. wide, as shown in Fig. 402.

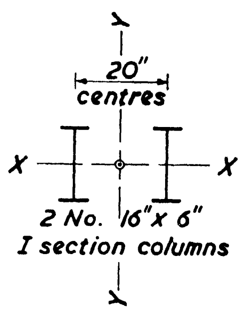


FIG. 401

Then the individual pressures on the soil are as follows:

Due to direct load = 152/50 = +3·04 tons per sq. ft.

$$\text{due to } M_{XX} = \pm\frac{48{\cdot}6 \times 6}{5 \times 10^2} = \pm 0{\cdot}58 \text{ tons per sq. ft.}$$

$$\text{due to } M_{YY} = \pm\frac{6{\cdot}5 \times 6}{10 \times 5^2} = \pm 0{\cdot}16 \text{ tons per sq. ft.}$$

When added algebraically, the resulting pressures are as shown in Fig. 402.

It will be noted that the whole base is in compression.

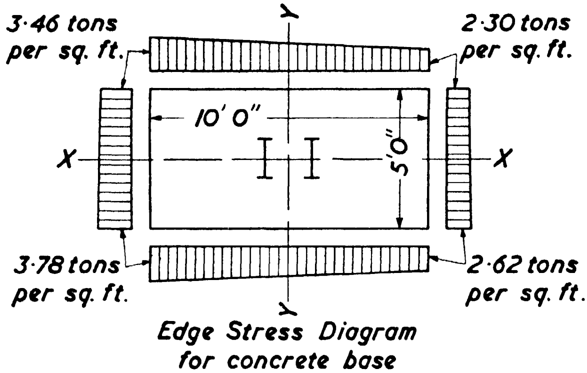


FIG. 402

Assume that the steel slab is 4 ft. long and 2 ft. 6 in. wide. Then the individual pressures on the concrete, shown in Fig. 403, are as follows:

Due to direct load $= 152/10 = +15{\cdot}2$ tons per sq. ft.

due to M_{XX} $= \pm \dfrac{48{\cdot}6 \times 6}{2{\cdot}5 \times 4^2} = \pm\ 7{\cdot}29$ tons per sq. ft.

due to M_{YY} $= \pm \dfrac{6{\cdot}5 \times 6}{4 \times 2{\cdot}5^2} = \pm\ 1{\cdot}56$ tons per sq. ft.

When added algebraically, the resulting pressures are as shown in Fig. 403.

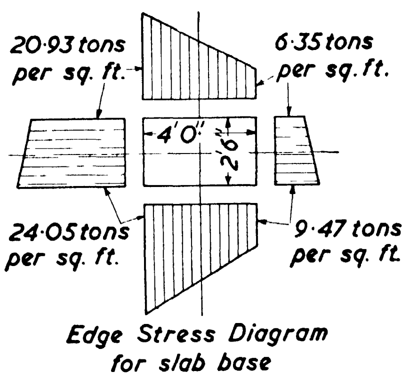


FIG. 403

Consider the loads in the stanchions, as shown in Fig. 404.

The direct load in each stanchion $= 76$ tons.

The loads in the webs due to $M_{XX} = \pm \dfrac{48{\cdot}6 \times 12}{20} = \pm 29{\cdot}16$ tons.

The loads in the flanges due to $M_{YY} = \pm \dfrac{6{\cdot}5 \times 12}{2 \times 15} = \pm\ 2{\cdot}6$ tons.

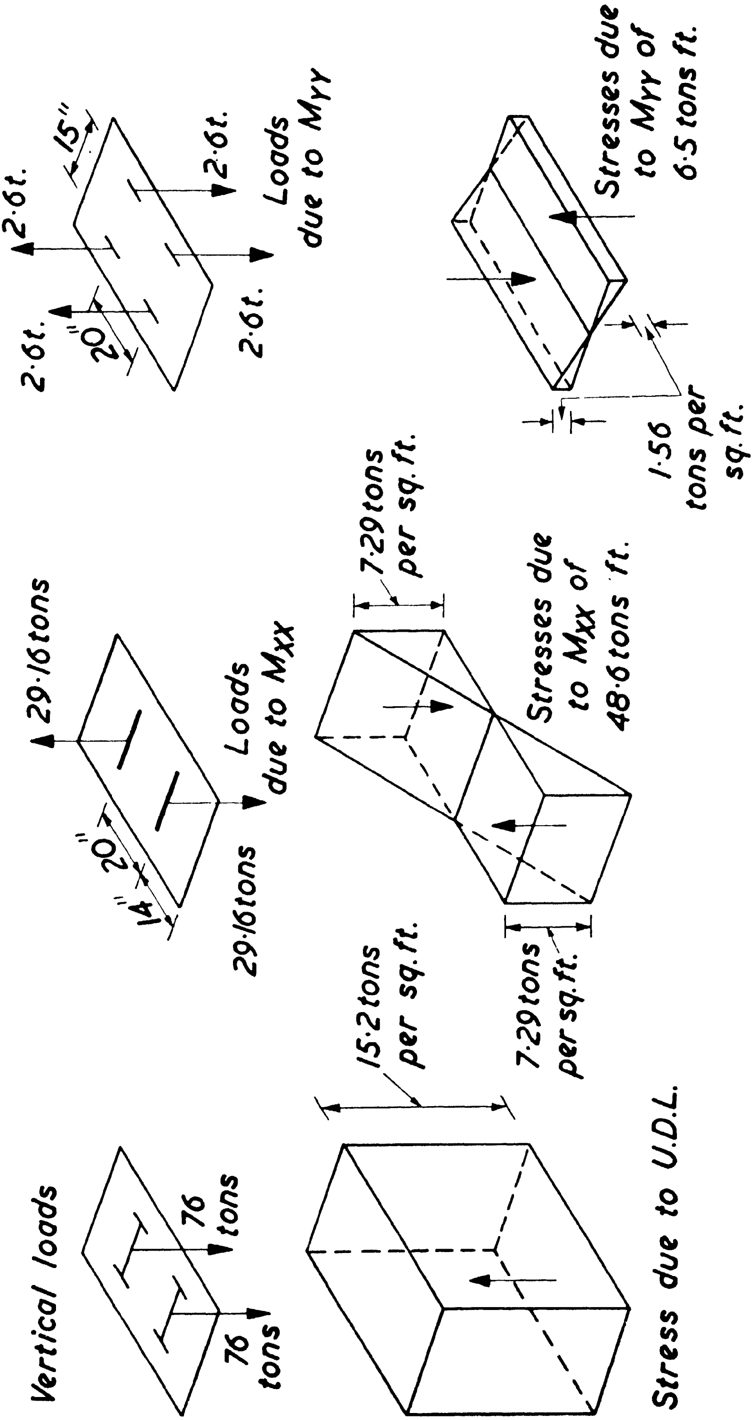

Vertical loads
76 tons
76 tons
15·2 tons per sq. ft.
Stress due to U.D.L.
29·16 tons
29·16 tons
14"
20"
Loads due to M_{XX}
7·29 tons per sq. ft.
7·29 tons per sq. ft.
Stresses due to M_{XX} of 48·6 tons ft.
2·6t.
2·6t.
2·6t.
2·6t.
15"
20"
Loads due to M_{YY}
Stresses due to M_{YY} of 6·5 tons ft.
1·56 tons per sq. ft.

FIG. 404

Now design for moments in the most heavily loaded quarter of the slab.

Check the moments about the lines XX, YY, AA (on the line of the stanchion web) and BB (at the centres of the stanchion flanges) shown in Fig. 405.

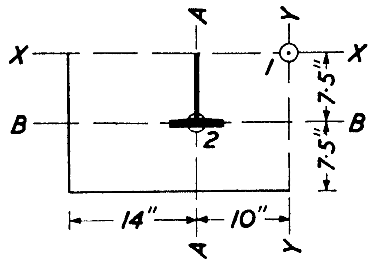


FIG. 405

Loads on quarter slab:

	Upwards	Downwards
Due to direct load 152/4	= 38 tons	38 tons.
due to M_{XX} $\frac{7{\cdot}29 \times 2 \times 1{\cdot}25}{2}$	= 9·11 tons	$\frac{29{\cdot}16}{2}$ = 14·58 tons.
due to M_{YY} $\frac{1{\cdot}56 \times 2 \times 1{\cdot}25}{2}$	= 1·95 tons	2·6 tons.

Moments about XX

Due to upward loads:

$$\left.\begin{array}{lll} \text{Direct load } 38 \text{ tons} \times & 7{\cdot}5 \text{ in.} & = +285 \\ M_{XX} & 9{\cdot}11 \times 7{\cdot}5 & = +\ 68 \\ M_{YY} & 1{\cdot}95 \times 10 & = +\ 20 \end{array}\right\} +373$$

$$\text{Or } \frac{6{\cdot}5 \times 12}{4}$$

Due to downward loads:

$$\left.\begin{array}{lll} \text{Direct load} - 38 \text{ tons} \times 4 \text{ in.} & & = -152 \\ M_{XX} & -\ 14{\cdot}58 \times 4 & = -\ 58 \\ M_{YY} & -\ 2{\cdot}6 \times 7{\cdot}5 & = -\ 20 \end{array}\right\} -230$$

$+373 - 230 = +143$ tons in.

Moments about YY

Due to upward loads:

$$\left.\begin{array}{lll} \text{Direct load} & 38 \text{ tons} \times 12 \text{ in.} & = +456 \\ M_{XX} & 9{\cdot}11 \times 16 & = +146 \\ \text{Or} & \frac{48{\cdot}6 \times 12}{4} & \\ M_{YY} & 1{\cdot}95 \times 12 & = +\ 23 \end{array}\right\} +625$$

Due to downward loads:

$$\left.\begin{array}{lll} \text{Direct load} - 38 \text{ tons} \times 10 \text{ in.} & & = -380 \\ M_{XX} & -\ 14{\cdot}58 \times 10 & = -146 \\ M_{YY} & -\ 2{\cdot}6 \times 10 & = -\ 26 \end{array}\right\} -552$$

$+625 - 552 = +73$ tons in.

Moments about AA

Due to upward loads:

$$\left.\begin{array}{lll} \text{Direct load } (15{\cdot}2 \times 1{\cdot}17 \times 1{\cdot}25) \text{ tons} \times 7 \text{ in.} & & = +155 \\ \text{Pressure at } AA \text{ due to } M_{XX} & & \\ \quad = \dfrac{7{\cdot}29 \times 10}{24} = 3{\cdot}04 \text{ tons per sq. ft.} & & \\ M_{XX}(3{\cdot}04 \times 1{\cdot}17 \times 1{\cdot}25) \text{ tons} \times 7 \text{ in.} & & = +\ 31 \\ \qquad \dfrac{4{\cdot}25}{2} \times 1{\cdot}17 \times 1{\cdot}25 \times 9{\cdot}33 & & = +\ 29 \\ M_{YY} \qquad \dfrac{1{\cdot}56}{2} \times 1{\cdot}17 \times 1{\cdot}25 \times\ 7 & & = +\ \ 8 \\ \text{Due to downward loads} & & = \text{Nil} \end{array}\right\} 223 \text{ tons in.}$$

Moments about BB

Due to upward loads:

$$\left.\begin{array}{lll} \text{Direct load} & 19 \text{ tons} \times 3{\cdot}75 \text{ in.} & = +71 \\ M_{XX} & 4{\cdot}55 \times 3{\cdot}75 & = +17 \\ \text{Pressure at } BB \text{ due to } M_{YY} & & \\ \quad = 1{\cdot}56 \times \dfrac{7{\cdot}5}{15} = 0{\cdot}78 \text{ tons per sq. ft.} & & \\ M_{YY}\ (0{\cdot}78 \times 2{\cdot}0 \times 0{\cdot}625) \text{ tons} \times 3{\cdot}75 \text{ in.} & & = +4 \\ \qquad \dfrac{0{\cdot}78}{2} \times 2{\cdot}0 \times 0{\cdot}625 \times 5 & & = +3 \\ \text{Due to downward loads} & & = \text{Nil} \end{array}\right\} +95 \text{ tons in.}$$

Taking Poisson's Ratio as 1/4, then

Moment at 1 for 1 ft. 3 in. width of slab $= +73 - 143/4 = +37$ tons in.

for 2 ft. 0 in. width of slab $= 143 - 73/4 = +125$ tons in.

Moment at 2 for 1 ft. 3 in. width of slab $= 223 - 95/4 = +199$ tons in.

for 2 ft. 0 in. width of slab $= 95 - 223/4 = +39$ tons in.

Hence design for 199 in. tons on 1 ft. 3 in. width.

Therefore thickness of slab at 12 tons per sq. in.

$$= \sqrt{\frac{199 \times 6}{12 \times 15}} = \sqrt{6{\cdot}63} = 2{\cdot}59 \text{ in.}$$

Use slab 4 ft. × 2 ft. 6 in. × $2\frac{3}{4}$ in. thick.

Assuming the base to be of mass concrete, then with an angle of dispersion at 45°, the minimum thickness of concrete equals $5 - 2 = 3$ ft.

Use base 10 ft. × 5 ft. × 3 ft. 6 in. thick.

This slab-base example has been dealt with at some length in order to demonstrate the basic principles which can be applied to design when the stanchion has more than one shaft and the load is not uniformly distributed.

Alternative Grillage Base

As before, the permissible concrete pressure is 24 tons per sq. ft.

Assume three joists of 6 in. flange width and 6 ft. long, say 8-in. × 6-in. × 35-lb. joists, as shown in Fig. 406.

Then each joist takes 152/3 = 50·67 tons from the central load

48·6/3 = 16·2 ft. tons from M_{XX}.

In addition, the two outer joists only share M_{YY} and the additional load = ±6·5/1·58 = ±4·12.

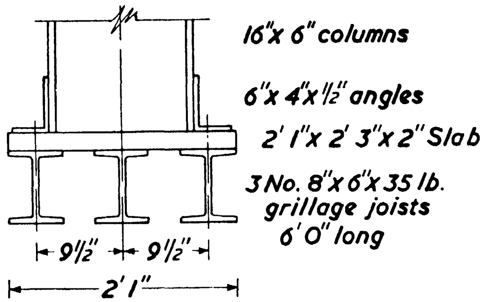


FIG. 406

Thus the loading conditions for the most heavily loaded joist are

Direct load = 50·67 + 4·12 = 54·8 tons

Moment = 16·2 ft. tons.

The bearing area is 6 in. wide by 6 ft. long and the pressures on the concrete are

Direct 54·8/3 = +18·3 tons per sq. ft.

$$M_{XX} \quad \pm\frac{16\cdot 2 \times 6}{\frac{1}{2} \times 6^2} = \pm\ 5\cdot 4 \text{ tons per sq. ft.}$$

Maximum pressure 23·7 tons per sq. ft., minimum pressure 12·9 tons per sq. ft.

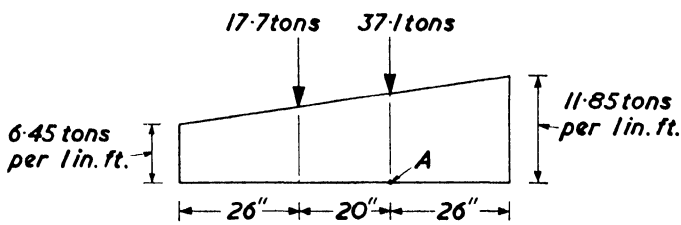


FIG. 407

The downward loads shown in Fig. 407, will be

$$27\cdot 4 \pm \frac{16\cdot 2 \times 12}{20} = 27\cdot 4 \pm 9\cdot 7 \text{ tons}$$

$$= 37\cdot 1 \text{ and } 17\cdot 7 \text{ tons.}$$

$$\text{Intensity of loading at } A = 11{\cdot}85 - \frac{26 \times 5{\cdot}4}{72} = 11{\cdot}85 - 1{\cdot}95$$
$$= 9{\cdot}9 \text{ tons per ft. run.}$$

Maximum B.M. and S.F. occur at this point

$$\text{S.F. at } A = 9{\cdot}9 \times 2{\cdot}17 + \frac{1{\cdot}95}{2} \times 2{\cdot}17$$
$$= 21{\cdot}45 + 2{\cdot}11 = 23{\cdot}56 \text{ tons.}$$
$$\text{B.M. at } A = (21{\cdot}45 \text{ tons} \times 13 \text{ in.}) + (2{\cdot}11 \text{ tons} \times 52/3 \text{ in.})$$
$$= 279 + 36$$
$$= 315 \text{ tons in.}$$

Z required at 13·33 tons per sq. in., which is the stress allowed by B.S. 449, $= 24$ in.^{3}.

Therefore 8-in. × 6-in. × 35-lb. 'I'-beams are amply strong in bending and shear. Web crippling is not a criterion in this case since the stanchion load is spread by the slab.

Slab (see Fig. 408).

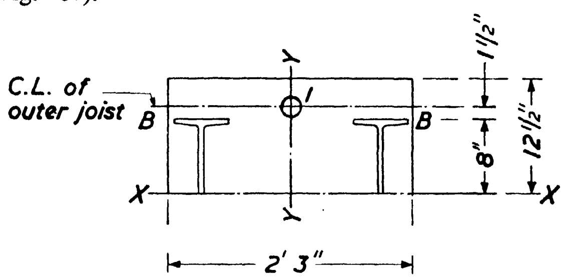


FIG. 408

The moment on line BB due to the uniform load on the outer beam

$$= 54{\cdot}8 \text{ tons} \times 1\tfrac{1}{2} \text{ in.}$$
$$= 82 \text{ tons in.}$$
$$\text{The moment about } YY = 16{\cdot}2 \text{ tons ft.}$$
$$= 194 \text{ tons in.}$$

Then taking Poisson's Ratio as 1/4 we have for the moment at point 1 for a 2-ft. 3-in. width of slab

$$M = 194 - 82/4 = 174 \text{ in. tons.}$$

Therefore thickness at 12 tons per sq. in.

$$= \sqrt{\frac{174 \times 6}{12 \times 27}} = \sqrt{3{\cdot}22} = 1{\cdot}8 \text{ in.}$$

Use 2-in. slab.

Example of Design of Holding-down Bolts

Design of a base plate for a compound stanchion of the type used in a workshop building.

The centres of the two legs of the stanchion together with the worst conditions of loading are shown in Fig. 409.

In general, it will be found that a base plate of a length about 2½ to 3 times the centres of the stanchions will be suitable.

In this example, a length of base plate of 48 in. will be used, with the holding-down bolts placed at 6 in. from the edge of the base plate.

The allowable pressure on the concrete below the base plate is 30 tons per sq. ft.

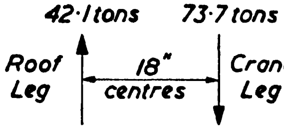


FIG. 409

Fig. 410 gives the dimensions of the base plate and the relative positions of stanchion legs and H.D. bolts.

It may be assumed that the B.M. caused by the stanchion loading is resisted by the two forces T and C, T being the tension in the H.D. bolts and C the compression between the base plates and the concrete base.

The analogy of the resisting forces to those in a R.C. beam subjected to bending will be obvious.

The solution is approximate, because the forces on the base plate do not constitute a pure couple.

Adopting the R.C. beam analogy, the allowable maximum stresses in steel and concrete are as follows:

Steel stress $= f_s$ (¾-in. diam. bolts and over in tension, B.S. 449 : 1948, Clause 23)

$= 6{\cdot}0$ tons per sq. in. $+ 25\% = 7{\cdot}5$ tons per sq. in.

Concrete stress $= f_c = 30$ tons per sq. ft. $+ 25\% = 37{\cdot}5$ tons per sq. ft.

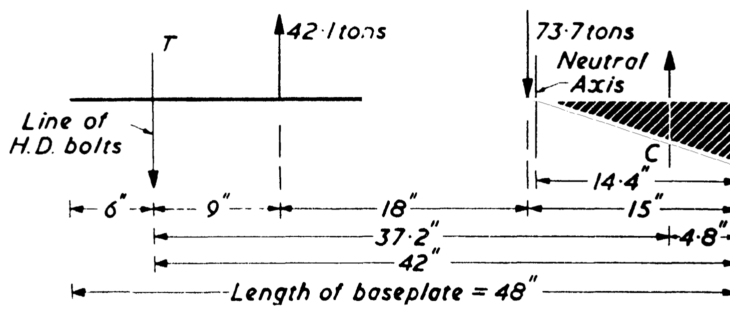


FIG. 410

The two allowable stresses may be increased by 25 per cent because the large proportion of the stanchion loads is due to wind forces.

$$f_s = 7{\cdot}5 \text{ tons per sq. in.} = 16{,}800 \text{ lb. per sq. in.}$$

$$f_c = 37{\cdot}5 \text{ tons per sq. ft.} = 582 \text{ lb. per sq. in.}$$

Assuming that the two maximum stresses occur together and that the modular ratio, m, $= 15$, then the depth to the neutral axis from the com-

pression edge $= mf_c d/(mf_c + f_s)$ where d is the effective depth to the centres of the tension steel, 42 in. in the example.

$$\text{Depth to N.A.} = \frac{15 \times 582 \times 42}{15 \times 582 + 16{,}800} = 14{\cdot}4 \text{ in.}$$

The compressive stress has a triangular distribution, as shown in Fig. 410 and the centre of compression is at $14{\cdot}4/3 = 4{\cdot}8$ in. from the compression edge.

Taking moments about T, the line of the force in the H.D. bolts,

$$C = \frac{73{\cdot}7 \times 27 - 42{\cdot}1 \times 9}{37{\cdot}2} = \frac{1991 - 379}{37{\cdot}2}$$

$$= 43{\cdot}3 \text{ tons.}$$

and

$$T = 42{\cdot}1 + 43{\cdot}3 - 73{\cdot}7$$

$$= 11{\cdot}7 \text{ tons.}$$

The maximum allowable concrete pressure is 37·5 tons per sq. ft. and, if the width of the base plate is b ft., then

$$b = \frac{43{\cdot}3 \times 12 \times 2}{14{\cdot}4 \times 37{\cdot}5} = 1{\cdot}93 \text{ ft.}$$

$$= \text{say, } 24 \text{ in.}$$

The total tension to be supplied to the H.D. bolts = 11·7 tons.

Area of steel, A_s, required = 11·7/7·5 = 1·56 sq. in.

The area of the bottom of the thread of a ⅞-in. diameter bolt = 0·422 sq. in. and four such bolts would give a total area of 1·688 sq. in., which is satisfactory.

The following table gives the safe load values of H.D. bolts between ¾ in. and 2 in. in diameter.

Diameter in inches	¾	⅞	1	1⅛	1¼	1⅜	1½	1¾	2
Area at bottom of thread in sq. in.	0·304	0·422	0·554	0·697	0·894	1·060	1·300	1·753	2·311
Value in tons of one bolt at 6 tons per sq. in.	1·82	2·53	3·32	4·18	5·36	6·36	7·80	10·52	13·87
Value in tons of one bolt at (6+25%) tons per sq. in.	2·28	3·17	4·16	5·23	6·71	7·95	9·75	13·15	17·33

31. GIRDERS

THE following types of girders are described and illustrated in this section:

1. Riveted Compound Girders.
2. Welded Compound Girders.
3. Welded Beams consisting of Three Plates.
4. Riveted Plate Girders.
5. Welded Plate Girders.

The last-named is reproduced from Publication No. 6 published by the British Constructional Steelwork Association of Artillery House, Westminster, S.W.1, to whom grateful acknowledgement is made.

Design of Riveted Compound Girders

When the B.M. to be resisted is greater than the resistance moment of a single section the bending capacity of the beam may be increased by the addition of flange plates, subject to an investigation of the strength of the web in shear and buckling.

In computing the moment of inertia and section modulus of the compound section allowance must be made for the reduced area in the tension flange due to the rivet holes, but in practice allowance is made for identical holing in both flanges in order to avoid the laborious calculations due to the shift of the neutral axis from the centre line of the gross section. The properties of compound girders listed in handbooks are generally based upon a deduction of one hole per flange which implies that the rivets in each flange must be staggered.

Theoretically the flange plates may stop at any point where the reduced section has sufficient bending resistance to meet the reduced B.M., but in practice they are extended beyond these points by a length sufficient to accommodate three rivets in line longitudinally, which may, however, be at a reduced pitch.

The theoretical points of cut-off can be found quite simply by calculation or graphically by superimposing the B.M. diagram upon a diagram representing the variations in section modulus of the section. The rivets securing the flange plates to the joist must be adequate to transmit the horizontal shear which develops the stress in the plates. If the slope of the B.M. curve is constant and a uniform rivet pitch is used throughout, then it is only necessary to investigate the rivet pitch for the section having the greatest area of flange plates; in other cases it is usually necessary to investigate the rivet pitch at the end of each flange plate.

For example, a uniformly distributed load of 100 tons carried on a span of 36 ft. with unrestricted depth and full lateral support, at a working stress of 10 tons per sq. in. and with an allowance of 3·4 tons for the weight of the

girder necessitates a section modulus of 558·4 in.^{3}. This requirement can be met by a number of sections, but that with the dead weight stated will be a 24-in. × 7½-in. × 95-lb. B.S.B. with 12-in. × 1½-in. plates on each flange. If the plating is composed of ¾-in. plates, then the section modulus of a 24-in. ×

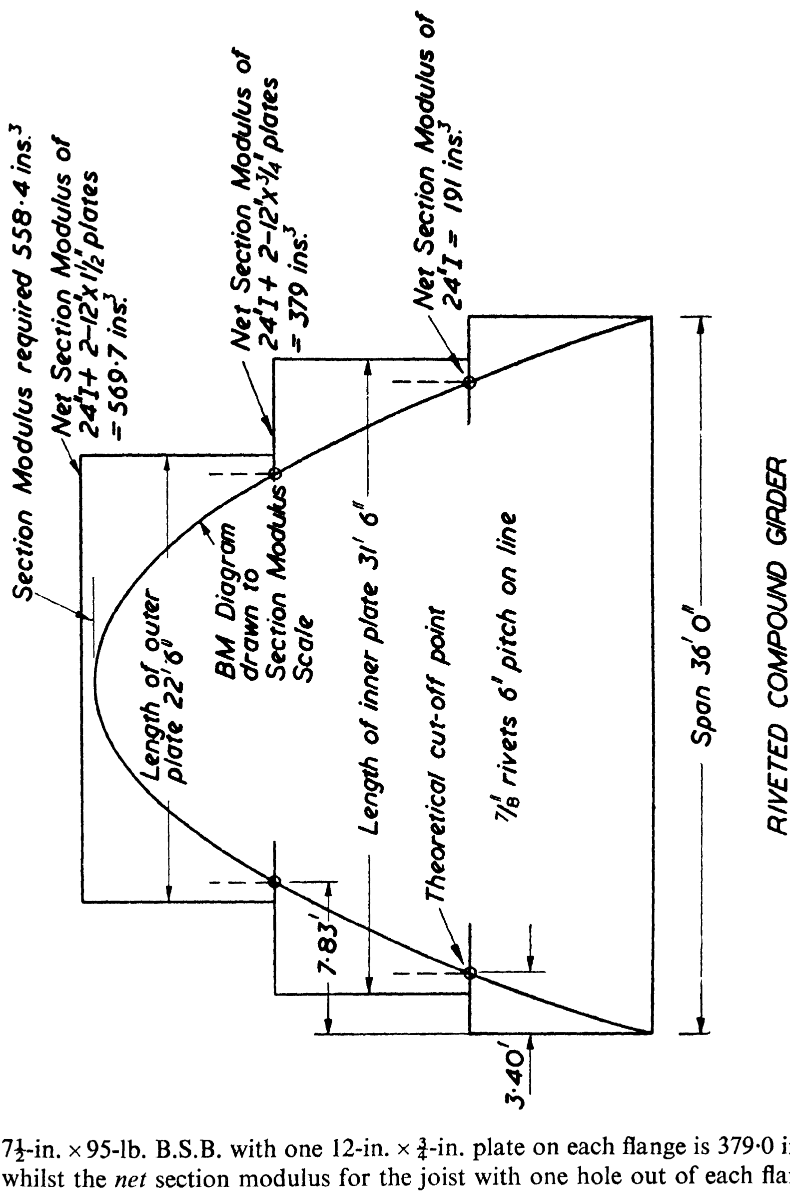


FIG. 411

7½-in. × 95-lb. B.S.B. with one 12-in. × ¾-in. plate on each flange is 379·0 in.^{3}, whilst the *net* section modulus for the joist with one hole out of each flange is 191 in.^{3}. The theoretical plate cut-off points measured from the support can be found graphically as shown in Fig. 411 or may be calculated as follows:

The B.M. at any point x ft. from either end of the girder is

$$M = 51\cdot7x - \left(\frac{103\cdot4}{36} \times \frac{x^2}{2}\right) \text{ tons ft.}$$

$$= 51\cdot7 \times 12\left(x - \frac{x^2}{36}\right) \text{ tons in.}$$

The theoretical cut-off points occur where M equals the value of fZ. When $f = 10$ tons per sq. in.,

$$Z = \frac{51\cdot7 \times 12}{10}\left(x - \frac{x^2}{36}\right)$$

or $$x^2 - 36x + \frac{30}{51\cdot7}Z = 0.$$

When $Z = 191$ in.^{3}, $x = 3\cdot40$ ft.

and when $Z = 379$ in.^{3}, $x = 7\cdot83$ ft.

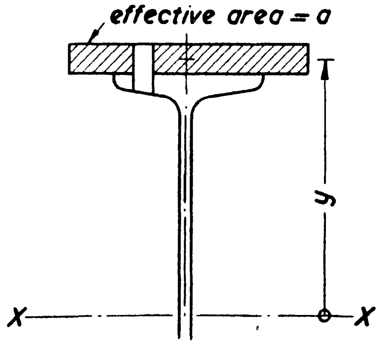


FIG. 412

These values for x are shown in Fig. 411 indicating the theoretical cut-off points for the inner and outer plates respectively.

The vertical shears at the theoretical points of cut-off are respectively 41·93 tons and 29·3 tons and it is from these values that the horizontal shear should be determined. The horizontal shear per lineal inch is given by

$$s = \frac{S \,.\, a \,.\, y}{I} \quad \text{(see Fig. 412),}$$

where s = horizontal shear in tons per lineal inch,

S = vertical shear at section in tons,

a = area of material to be connected,

y = distance from centroid of a to centroid of section

and I = moment of inertia of compound section.

The area to be taken should be that used in computing the moment of inertia and hence must be reduced by the cross-section of one rivet hole, taken as $\frac{15}{16}$ in. wide, i.e. for a $\frac{7}{8}$ in. dia. rivet. Hence at the point of cut-off of the inner plate

$$s = \frac{41\cdot93 \times 11\cdot0625 \times 0\cdot75 \times 12\cdot375}{4832}$$

$$= 0\cdot891 \text{ tons per lin. in.}$$

At the point of cut-off of the outer plate, since the rivets must now serve both plates,

$$s = \frac{29{\cdot}3 \times 11{\cdot}0625 \times 1{\cdot}5 \times 12{\cdot}75}{7691}$$

$$= 0{\cdot}806 \text{ tons per lin. in.}$$

According to B.S. 449 the shear strength of a rivet may be taken on the diameter of the hole, which at 6 tons per sq. in. gives a single shear value of 4·14 tons for a $\frac{7}{8}$ in. dia. rivet, and hence the maximum allowable pitch equals 4·14/0·891 or 4·64 in. Using two rows of rivets with staggered pitch, this is equivalent to a straight line pitch of 9·28 in., but this must be reduced to 6 in. on line, the maximum rivet pitch allowed by B.S. 449 for parts in compression.

The shear stress at the ends of the girder calculated on the gross area of the web equals

$$\frac{51{\cdot}7}{24 \times 0{\cdot}57} \quad \text{or} \quad 3{\cdot}78 \text{ tons per sq. in.}$$

which is well within allowable limits.

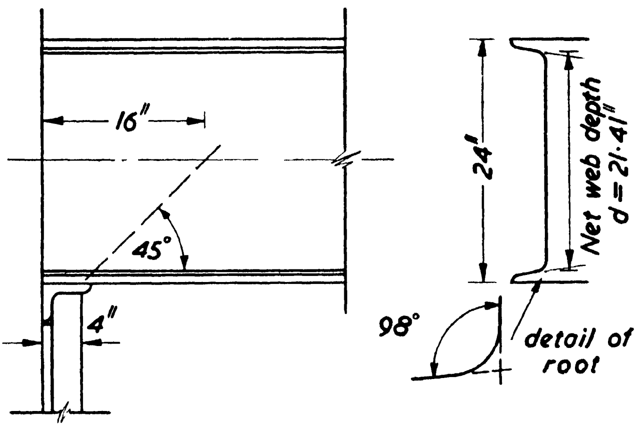


FIG. 413

Now web-buckling must be investigated in the end section (see Clause 47, B.S. 449 : 1948) and assuming that the length of stiffened seating is 4 in., the effective length of web assumed to be acting as a column will be 16 in. as shown in Fig. 413. The height of the column is to be the clear depth of web ignoring root fillets, as shown in Fig. 413, the dimension being 21·41 in. The slenderness ratio is specified as $d\sqrt{3}/t$ and is therefore

$$\frac{21{\cdot}41 \times 1{\cdot}732}{0{\cdot}57} = 65.$$

The appropriate compressive stress given in Table 7 of B.S. 449 is 5·85 tons per sq. in. giving a safe column load $5{\cdot}85 \times 16 \times 0{\cdot}57 = 53{\cdot}2$ tons. This value exceeds the reaction and therefore the web need not be stiffened.

It should be noted, however, that if the beam rested upon an unstiffened

bracket the web would require to be stiffened to meet the specification requirements.

Rivet Shear Factors

The horizontal shear per lineal inch between the plates and the parent section in a compound girder is determined from the expression:

$$s=\frac{S\,.\,a\,.\,y}{I}=\frac{S}{I/(a\,.\,y)} \qquad \text{(see p. 612)}$$

$$=\frac{S}{\text{a constant}}$$

where the constant is a shear factor depending only on the geometrical properties of the cross-section of the girder.

The table on the next page gives the shear factors for a number of single-joist compound girders. To demonstrate its application consider the theoretical plate cut-off points for the compound girder shown in Fig. 411, where the shears are 41·93 and 29·3 tons respectively.

For a 24-in. × $7\frac{1}{2}$-in. joist with 12-in. × $\frac{3}{4}$-in. plates the shear factor is 47·06, and with 12-in. × $1\frac{1}{2}$-in. plates the factor is 36·35.

In the first case,

$$s=\frac{S}{\text{Shear factor}}=\frac{41{\cdot}93}{47{\cdot}06}=0{\cdot}891 \text{ tons per lin. in.}$$

In the second case,

$$s=\frac{29{\cdot}3}{36{\cdot}35}=0{\cdot}806 \text{ ton per lin. in.}$$

The results are identical with those obtained previously.

Design of Welded Compound Girders

The design procedure for compound girders composed of flange plates welded to joists is very similar to that for riveted compound girders, except that the added plating is normally in one thickness on each flange, since each added plate requires two runs of fillet weld, the cost of which will normally outweigh any savings on material due to the curtailment of the outer plates as shown in Fig. 414. Reduction can, however, be made in the width or thickness of the flange plate where the B.M. permits, but this demands full-strength butt joints in the plates, which, again, are likely to be more costly than using the thicker plate throughout.

As an example, consider a beam to carry a uniformly distributed load of 100 tons over a span of 36 ft. assuming full lateral support. If an allowance of 3·2 tons is made for the weight of the girder the section modulus required at a working stress of 10 tons per sq. in. will be 557·3 ins.^{3} A suitable section would thus be a 24-in. × $7\frac{1}{2}$-in. × 95-lb. joist with 12-in. × $1\frac{3}{8}$-in. plates on each flange with section modulus 587·4 ins.^{3}. The next smaller size, with 12-in. × $1\frac{1}{4}$-in. plates, would be stressed to 10·1 tons per sq. in. and would not meet the specification requirements.

RIVET SHEAR FACTORS
SINGLE JOIST COMPOUND GIRDERS

Joist Size (in.)	*Hole Diam. (in.)*	*Plate Widths (in.)*	*Plate Thicknesses in Inches*							
			½	⅝	¾	⅞	1	1¼	1½	2
24×7½	15/16	8	77·44	66·76	59·69	54·68	50·95	45·82	42·48	38·52
		9	70·87	61·54	54·02	50·98	47·73	43·27	40·37	36·97
		10	65·76	57·46	51·98	48·10	45·23	41·28	38·74	35·77
		12	58·30	51·53	47·06	43·91	41·58	38·39	36·35	33·83
22×7	15/16	8	60·39	52·77	47·74	44·18	41·54	37·94	35·62	32·93
		9	55·68	49·03	44·64	41·54	39·24	36·11	34·12	31·83
		10	52·03	46·12	42·22	39·48	37·45	34·70	32·95	30·97
		12	46·68	41·87	38·71	36·48	34·85	32·63	31·25	27·44
20×7½	15/16	8	62·13	53·73	48·18	44·25	41·33	37·34	34·76	31·75
		9	56·97	49·63	44·78	41·35	38·81	35·35	33·11	30·56
		10	52·94	46·43	42·13	39·09	36·85	33·79	31·85	29·62
		12	47·08	41·76	38·27	35·80	33·99	31·53	29·98	28·25
20×6½	15/16	8	50·56	44·53	40·55	37·74	35·68	32·87	31·09	29·06
		9	46·82	41·56	38·10	35·66	33·86	31·43	29·88	28·19
		10	43·92	39·25	36·19	34·03	32·45	30·31	28·98	27·52
18×8	15/16	9	49·79	43·50	39·34	36·41	34·25	31·30	29·43	27·30
		10	46·34	40·75	37·07	34·48	32·57	29·98	28·34	26·50
		12	41·37	36·75	33·76	31·66	30·12	28·04	26·75	25·34
18×7	15/16	8	49·92	43·60	39·42	36·48	34·31	31·35	29·47	27·33
		9	46·02	40·50	36·86	34·30	32·41	29·85	28·24	26·43
		10	42·99	38·09	34·86	32·60	30·94	28·69	27·28	25·73
		12	38·56	34·57	31·95	30·12	28·78	26 99	25·88	24·71
18×6	13/16	8	41·14	36·62	33·65	31·60	30·04	27·98	26·69	—
		9	38·38	34·42	31·83	30·02	28·69	26·92	25·82	—
		10	36·22	32·70	30·40	28·81	27·63	26·08	25·14	—
16×8	15/16	9	42·89	37·58	34·09	31·63	29·82	27·37	25·83	24·12
		10	39·97	35·27	32·17	30·00	28·41	26·26	24·92	23·45
		12	35·74	31·90	29·39	27·63	26·35	24·64	23·59	22·48
16×6 H	13/16	8	38·34	33·97	31·10	29·09	27·61	25·63	24·40	—
		9	35·67	31·85	29·35	27·60	26·32	24·61	23·57	—
		10	33·59	30·19	27·99	26·43	25·30	23·81	22·91	—
16×6 L	13/16	8	35·09	31·39	28·97	27·27	26·04	24·39	23·38	—
		9	32·82	29·58	27·48	26·00	24·94	23·52	22·67	—
		10	31·06	28·17	26·30	25·01	24·07	22·84	22·11	—
15×6	13/16	8	31·26	28·13	26·10	24·68	23·65	22·29	21·48	—
		9	29·33	26·61	24·84	23·61	22·72	21·56	20·88	—
		10	27·84	25·41	23·84	22·77	21·99	20·98	20·41	—
14×8	15/16	9	36·20	31·85	28·98	26·98	25·51	23·55	22·32	21·00
		10	33·81	29·95	27·41	25·65	24·35	22·62	21·60	20·46
		12	30·32	27·18	25·14	23·70	22·67	21·30	20·49	19·67
14×6 H	13/16	8	32·67	29·04	26·66	25·01	23·80	22·19	21·21	—
		9	30·45	27·28	25·21	23·77	22·73	21·35	20·52	—
		10	28·70	25·90	24·07	22·81	21·88	20·69	19·98	—
14×6 L	13/16	8	29·62	26·61	24·66	23·31	22·32	21·03	20·26	—
		9	27·77	25·16	23·46	22·28	21·43	20·33	19·68	—
		10	26·33	24·01	22·51	21·48	20·74	19·78	19·23	—

The theoretical cut-off points for the plates will be 3·84 ft. from the supports, as shown in Fig. 415, and the shear at this point is 40·6 tons. Then the horizontal shear per lineal inch $= S \cdot a \cdot y/I = 1{\cdot}08$ tons.

This is to be taken by two fillets and hence $\frac{3}{16}$-in. continuous fillets would suffice, but this would contravene the requirements of B.S. 1856 which states that $\frac{5}{16}$-in. welds must be used for material over $1\frac{1}{4}$-in. in thickness. This size can be used continuous throughout or intermittent welds may be used. The ratio of effective length of intermittent weld to total length is found by dividing the actual shear per lineal inch by the weld strength per lineal inch, that is $0{\cdot}54/1{\cdot}42 = 0{\cdot}38$. Welds 6 in. in effective length and 9 in. clear spacing will give a ratio of 0·4 and can be adopted. It will be obvious, by reference to the riveted girder on p. 613, that the web of the joist is adequate for shear and web buckling provided that a 4-in. stiffened bearing is used.

The actual length of a weld is the effective length plus the weld size added to each free end and thus the actual space between welds is reduced as shown in Fig. 415.

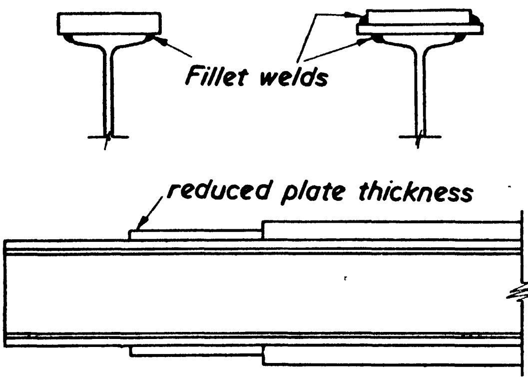


FIG. 414

Design of Welded Beams Consisting of Three Plates

The essential feature of such girders, which are really small plate girders, is that for distributed loads they should be designed to dispense with stiffeners and with flange curtailment, thereby reducing fabrication costs to a minimum.

This directs attention first to the web which must have a thickness not less than $d/75$ (where d = depth between flanges) and which must also be of sufficient area to meet the allowable shear stress, but web stiffeners will be necessary in any case at the ends to meet the requirements for plate girders in B.S. 449.

If, as in the previous example, a uniformly distributed load of 100 tons is considered on a span of 36 ft. with an allowance for the girder of 3 tons, the section modulus required, assuming full lateral support and a working stress of 10 tons per sq. in., is 556·2 ins.³. A girder with web 24-in. $\times \frac{7}{16}$ in. will meet shear and stiffness requirements, and with flanges 12 in. $\times 1\frac{7}{8}$ in. will give a section modulus of 579·9 ins.³. With 12-in. $\times 1\frac{3}{4}$-in. plates the girder would be stressed to about 10·22 tons per sq. in. Since there is no

flange curtailment, the horizontal shear per lineal inch at the ends of the girder

$$= \frac{S.a.y}{I} = \frac{51{\cdot}5 \times 12 \times 1{\cdot}875 \times 12{\cdot}94}{579{\cdot}9 \times 13{\cdot}875}$$

$$= 1{\cdot}86 \text{ tons.}$$

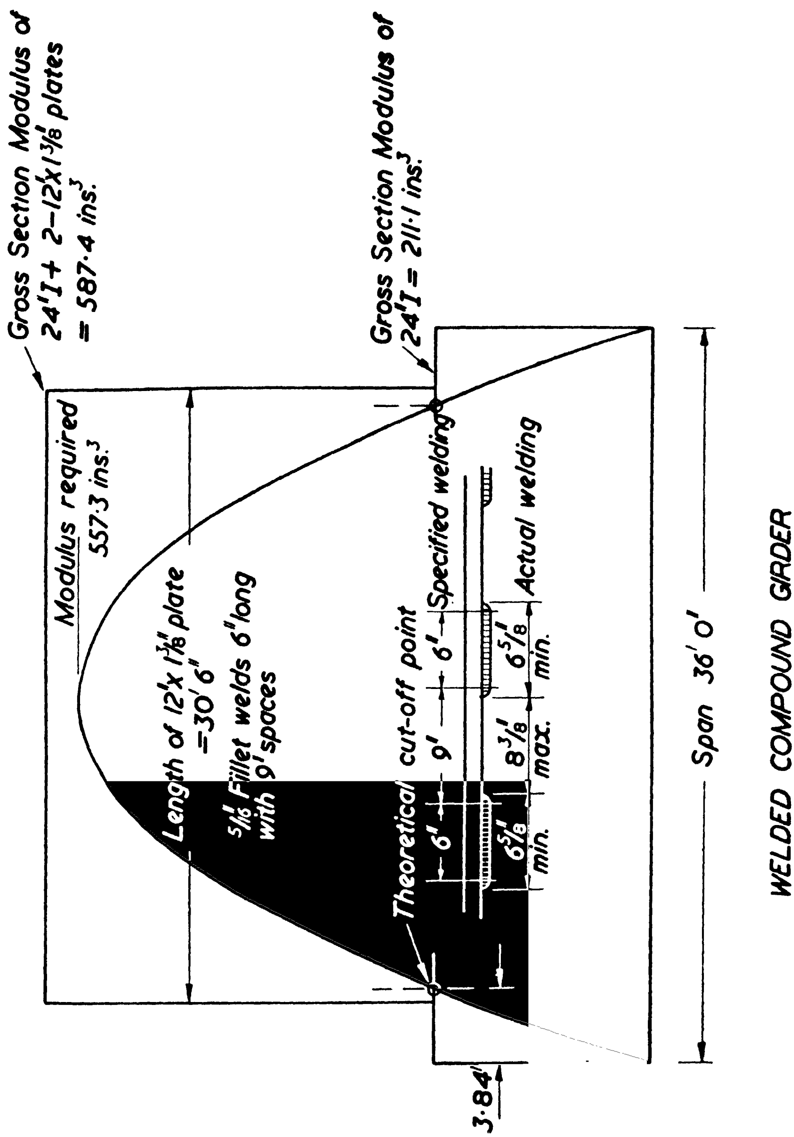


FIG. 415

This horizontal shear exists at both the top and bottom flanges, but for the top flange there is, in addition, the vertical shear due to the superimposed load, since this cannot be assumed to be transmitted by direct bearing from the flange to the web. The vertical shear equals $100/(36 \times 12) = 0{\cdot}231$ ton

per lin. in. which must be added vectorially to the horizontal shear, the resultant being 1·874 tons per lin. in., a figure only a little greater than that for the bottom flange. This is to be taken by two fillets and hence $\frac{1}{4}$-in. continuous fillets at 1·14 tons per lin. in. would suffice, but $\frac{5}{16}$-in. fillets will be required since the flanges exceed $1\frac{1}{4}$ in. in thickness. The ratio of effective length to space for intermittent welding will be $0{\cdot}887/1{\cdot}42 = 0{\cdot}625$, and, hence, $\frac{5}{16}$-in. welds, 7 in. long, with 3-in. spaces will be suitable for both flanges.

It should be noted that B.S. 449 does not require the computation of the principal stress in the welds.

The end stiffeners or end plates must carry the reaction of 51·5 tons and should be designed in a similar manner to those for the plate girder described elsewhere in this section.

Riveted Plate Girders

Detail requirements to be observed in the design of both riveted and welded plate girders are given in Clauses 47 and 48 of B.S. 449 : 1948.

The specification states that plate girders shall be designed on the basis of the moment of inertia of their cross-section, except that where the flanges are equal in gross area an approximate method may be used, based upon the area of the flanges.

The web plate is the primary element in the girder and is normally the same thickness throughout, as, if altered, filler plates will be required under the flange angles for the length of web reduced in thickness. Subject to conditions relating to transport and erection the web should be in as few pieces as possible. The maximum sizes of plates normally obtainable are 40 ft. in length, 9 ft. in width for all thicknesses from $\frac{1}{2}$ in. to $1\frac{1}{2}$ in., but there is in addition an overriding maximum area in one piece ranging from 360 sq. ft. for plates up to $\frac{3}{4}$ in. thickness falling to 180 sq. ft. at $1\frac{1}{2}$ in. thickness.

Flange angles and flange plates are riveted to the web plate. Angles are not usually readily obtainable in lengths greater than 60 ft. except by special arrangement. Flange plates up to 1 in. thickness can be obtained up to about 85–95 ft. in length in all suitable widths; the lengths fall to about half these values for plates 2 in. in thickness.

Flanges may be composed of angles only for light work, angles and flange plates for normal work and also angles, flange plates and flitch plates for heavy work as shown in Fig. 416.

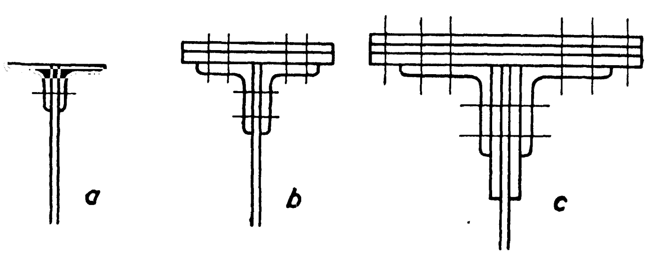


FIG. 416

The depth of plate girders, when not fixed by considerations of headroom, normally varies from about one-eighth of the span for short girders to about one-twelfth of the span for long girders. The girder of least total weight will not necessarily be that with the least total cost since this depends upon elements other than weight such as transport and erection costs, so that it is generally economical to use a depth rather less than that giving minimum weight. It has been shown that the least weight depth is given approximately by

$$d = 1{\cdot}1\sqrt{(M/f\,.\,t)},$$

where M = maximum moment on girder,

f = allowable bending stress on flange

and t = web thickness.

This formula assumes that the bending resistance of the web is taken into account, that the flange plates are curtailed and that the weight of stiffeners is 60 per cent of the web weight. A reduction in the depth of 20 per cent from that calculated by the formula gives an increase in weight of about 2 per cent, and the normal depths in practice are usually considerably below those given by the formula. To apply the preceding principles a thickness of web must first be assumed and the appropriate depth found; the resulting proportions must then be checked against the specification requirements for the details of the design.

Moment of Resistance, Approximate Method

This should only be applied to girders with flanges of the type shown in Fig. 416 (b) by reason of the assumptions made which are set out in detail below. Consider the girder shown in Fig. 417. The moment of inertia of the section equals

$$I = I_F + I_W,$$

where I_F = moment of inertia of two flanges about the neutral axis

and I_W = moment of inertia of web about the neutral axis,

let A_F = the net area of one flange (angles and flange plates), both flanges being identical,

A_W = the gross area of the web,

D = depth of girder overall,

d = depth back to back of angles

and h = effective depth or distance between centroids of flange areas.

Then $I_F = 2A_F(h/2)^2$, neglecting the moment of inertia of the flanges about their own centroids, and $I_W = td^3/12$. In the type of flange used, the centroid is not far removed from the back of the angles, hence h is approximately equal to d. It is also assumed that the flange stress f is reached at the centroid of the flange and thus, by analogy with the basic flexure formula, $M = f\,.\,I/y$, then

$$M = f\frac{[2\,.\,A_F(d/2)^2 + td^3/12]}{d/2}$$

$$= f\,.\,d\left(A_F + \frac{A_W}{6}\right).$$

If it now be assumed that at the critical section there are holes in the web of diameter d at $4d$ pitch, then the net area of the web will be three-quarters of the gross area and $A_W/6$ will be reduced to $A_W/8$ and the formula becomes, as stated in B.S. 449,

$$M = f \, . \, d\left(A_F + \frac{A_W}{8}\right).$$

The quantity $A_W/8$ is often called the web equivalent and the total flange area is therefore

$$A_P + A_A + \frac{A_W}{8},$$

where A_P = net area of flange plates,

A_A = net area of flange angles,

and A_W = gross area of web.

Rivet pitch, Approximate Method

The rivets connecting the flange angles to the web plate must transfer from the web the horizontal shear necessary to induce the bending stresses in the

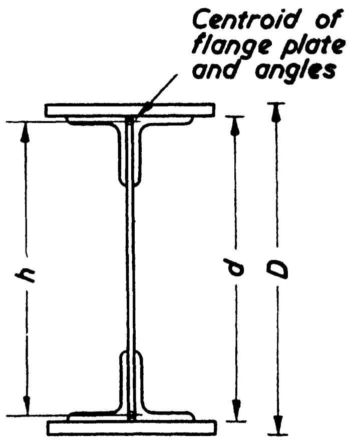


FIG. 417

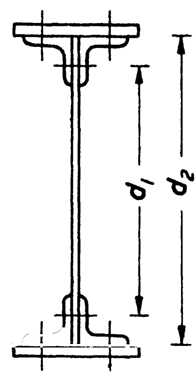


FIG. 418

flange angles and plates, and, in addition, if there is a vertical load on the flange plates must transfer this also.

Consider first the case of the flange with no vertical load. Then, for the rivets connecting the angles to the web, using the basic formula for intensity of horizontal shear stress at any point the pitch of rivets $= p_1 = r_1 d_1 / K_1 S$ (see Fig. 418), where

r_1 = safe load on rivet (double shear or bearing, whichever is least),

d_1 = depth between rivet lines,

$$K_1 = \frac{(A_P + A_A)}{(A_P + A_A + A_W/8)}$$

and S = vertical shear at section.

Similarly, if $K_2 = \dfrac{A_P}{(A_P + A_A + A_W/8)}$

r_2 = safe load on rivet in single shear

and d_2 = distance between flange plates,

then the pitch of flange-plate rivets,

$$p_2 = \frac{r_2 d_2}{K_2 S}.$$

This is the staggered pitch, and the pitch on line will be twice this amount.

If there is a vertical load on the flange plate, such as a floor or a wall resulting in a shear load of w tons per inch run, this must, in the case of rivets connecting angles and web, be combined vectorially with the horizontal shear and

$$p_1 = \frac{r_1}{\sqrt{[(K_1 S)^2/d_1 + w^2]}}.$$

Moment of Resistance of Girders with Sloping Flanges

If M is the total moment on the girder then it follows that the total flange force is M/d, where d = effective depth or distance back to back of angles. It therefore follows from what has been said earlier that the moment taken

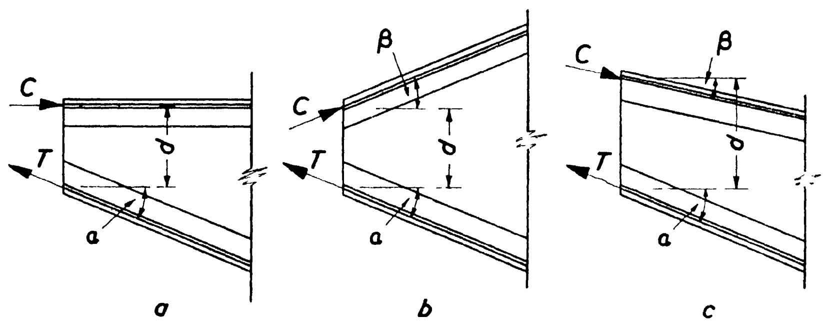


FIG. 419

by the flange angles and plates = $K_1 M$ whilst that taken by the web will be $M(1 - K_1)$. Thus for the girders shown in Fig. 419:

(a) $$C = \frac{K_1 M}{d} \quad \text{and} \quad T = \frac{K_1 M}{d \cos \alpha},$$

(b) and (c) $$C = \frac{K_1 M}{d \cos \beta} \quad \text{and} \quad T = \frac{K_1 M}{d \cos \alpha}.$$

Inclined flanges affect the shear to be taken by the web. If the sections indicated on Fig. 419 are to the left of the point of maximum moment, assuming the girder to be simply supported, S = vertical shear at section and S_W = shear to be taken by the web, then for the various cases shown:

(a) $$S_W = S - T \sin \alpha = S - \frac{K_1 M}{d} \tan \alpha$$

since the inclined tension absorbs shear.

(b) $$S_W = S - \frac{K_1 M}{d}(\tan\alpha + \tan\beta)$$

as both flanges absorb shear.

(c) $$S_W = S - \frac{K_1 M}{d}(\tan\alpha - \tan\beta).$$

In this case one flange absorbs shear and the other adds to the shear force.

It must be noted, however, that these computations refer only to the shear load in the web and not to the vertical reactions which support the girder.

The formulæ previously given for rivet pitch require modification to allow for the increased flange loads and the fact that the pitch may be on a sloping line.

Design of a Riveted Plate Girder

The girder designed in this example is 33 ft. 6 in. span, simply supported at the ends, and carries the loading shown in Fig. 420, which includes the

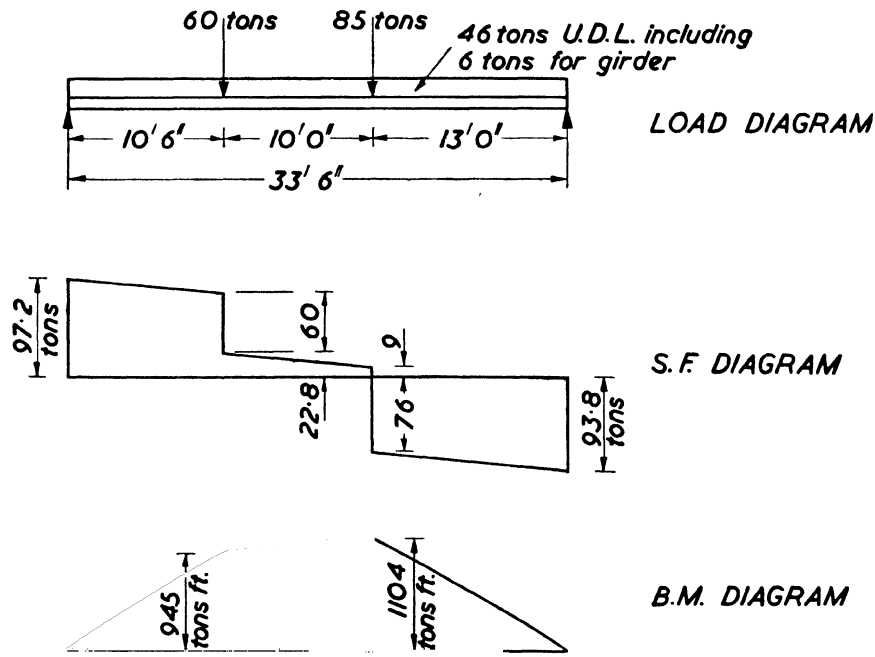


FIG. 420

weight of the girder, assumed as 6 tons, which must be checked after it has been designed. The figure also shows the shear force and bending moment diagrams. It is also assumed that effective lateral support is provided at each reaction and point load.

The allowable bending stress is therefore 9·5 tons per sq. in. by B.S. 449, Clause 19 (b) (ii). The shear stress in the web is governed by the requirements of Clauses 20 (b) and (c), which necessitate a knowledge of the stiffener spacing but with a maximum possible shear stress of 6·5 tons per sq. in. Assuming that a web thickness of $\frac{1}{2}$ in. will be satisfactory, then the least weight depth, as given on page 618,

$$=1\cdot1\sqrt{\frac{M}{f.t.}}=1\cdot1\sqrt{\frac{1104\times12}{9\cdot5\times\frac{1}{2}}}$$

$$=58 \text{ in. approximately.}$$

This can be considerably reduced without sensible increase in weight, and a depth of web of 48 in. will be used, it being assumed that headroom considerations do not dictate a lesser depth. The depth d back-to-back of angles will be made $48\frac{1}{2}$ in. in order to clear the outside edges of the web plate.

Designing upon the Approximate Method

The flange area required

$$A=\frac{M}{f\times d}=\frac{1104\times12}{9\cdot5\times48\cdot5}=28\cdot8 \text{ sq. in.}$$

This can be made up as follows:

Web equivalent $=\dfrac{48 \text{ in.}\times\frac{1}{2}\text{ in.}}{8}$	= 3·0 sq. in.
2 angles, 6 in. × 4 in. × $\frac{5}{8}$ in.	= 11·72
Plates, 16 in. × $1\frac{1}{4}$ in.	= 20·0
	34·72
Less rivet holes, $2\times\frac{15}{16}$ in. × 1·88 in. + $\frac{15}{16}$ in. × 1·75 in.	= 5·15
	29·57 sq. in.

The slight excess area will compensate for the fact that in calculating the flange areas the depth has been taken as that over the angles, whereas Clause 48 (a) stipulates that the effective depth shall be that between the centroids of the flanges.

It should be noted that B.S. 449 states that flange angles should form as large a part of the flange area as is practicable, and in the section given they form rather more than 37 per cent of the attached material. The specification also states that the number of flange plates shall be reduced to a practical minimum, and thus the $1\frac{1}{4}$ in. thickness will be made up from $\frac{5}{8}$-in. plates, one running full-length and the other curtailed; the net area of each plate being 8·83 sq. in.

The net areas of the flange components are thus:

Web equivalent	2·53 sq. in.
Flange angles	9·38 „
Inner plate	8·83 „
Outer plate	8·83 „

These are used for the flange-plate curtailment diagram given in Fig. 421 giving theoretical points of cut-off of the outer plate at 8·65 ft. and 9·0 ft. from the left and right ends respectively, the shears at these points being 85·3 and 81·5 tons.

The left-hand end of the girder is more highly stressed in shear and will be used to decide the pitch of rivets necessary as the pitch so obtained will

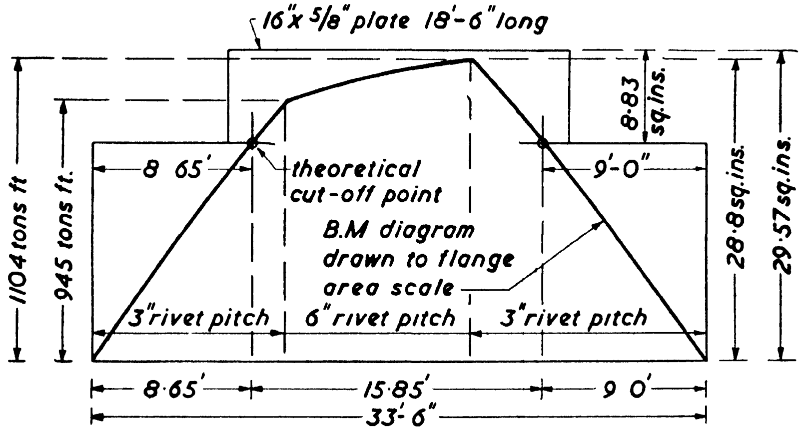


FIG. 421

be adequate for the other end. Dealing with the rivets connecting flange angles to web plate, the following figures result:

L.H. end.

Shear = 97·2 tons,

Web equivalent	2·53	
Flange angles	9·38	$K_1 = \frac{18 \cdot 21}{20 \cdot 74} = 0 \cdot 88$
One flange plate	8·83	
	20·74	

The least strength of a $\frac{7}{8}$-in. diameter rivet, assumed as $\frac{15}{16}$ in. diameter in accordance with Clause 38 (b) of B.S. 449, will be given in bearing on the $\frac{1}{2}$-in. web, at a stress of 12 tons per sq. in. plus 25 per cent (see Clause 23) and is thus 7·03 tons. The horizontal shear load per inch, the distance between rivet lines being 44 in.,

$$= \frac{97 \cdot 2 \times 0 \cdot 88}{44} = 1 \cdot 95 \text{ tons.}$$

The vertical shear load per inch run, due to the uniformly distributed load of 46 tons, = 0·115 ton. The vertical effect of the vertical load is negligible, and we therefore have a resultant load of 1·95 tons.

At point of cut-off of outer plate:

Shear = 85·3 tons,

Web equivalent	2·53
Flange angles	9·38
Two flange plates	17·66
	29·57

$$K_1 = \frac{27{\cdot}04}{29{\cdot}57} = 0{\cdot}91$$

$$\text{Horizontal shear load per inch} = \frac{85{\cdot}3 \times 0{\cdot}91}{44} = 1{\cdot}77 \text{ tons,}$$

which is less than the corresponding figure for the left-hand end.

The maximum rivet pitch is therefore 7·03/1·95 = 3·6 in., and therefore a 3-in. pitch will be used at the ends, increased to 6-in. pitch between the two point loads.

Similarly, the calculations for the rivets connecting flange plates to flange angles follow, the safe load in the rivet being that in single shear, which is 4·14 tons, and the distance between shearing surfaces 48·5 in. under the action of horizontal shear only.

L.H. end.

Shear = 97·2 tons,

$$K_2 = \frac{8{\cdot}83}{20{\cdot}74} = 0{\cdot}43.$$

$$\text{Horizontal shear load per inch} = \frac{97{\cdot}2 \times 0{\cdot}43}{48{\cdot}5}$$

$$= 0{\cdot}86 \text{ ton.}$$

At point of cut-off of outer plate:

Shear = 85·3 tons,

$$K_2 = \frac{17{\cdot}66}{29{\cdot}57} = 0{\cdot}6.$$

$$\text{Horizontal shear load per inch} = \frac{85{\cdot}3 \times 0{\cdot}6}{48{\cdot}5}$$

$$= 1{\cdot}05 \text{ tons.}$$

There are two rows of rivets connecting angles and flange plates and hence the straight line pitch

$$= \frac{2 \times 4{\cdot}14}{1{\cdot}05} = 7{\cdot}9 \text{ in.}$$

A pitch of 6 in. on line will be used to facilitate marking out, since 3 in. pitch has been used for the web rivets.

A web thickness of ½ in. has been assumed, and it is necessary to check that this complies with the requirements of B.S. 449.

The ratio d/t of the web $= 40\tfrac{1}{2}/\tfrac{1}{2} = 81$, where d = clear depth of web between flange angles, and since this is greater than 75, web stiffeners are required in accordance with Clause 20 (b). The same clause limits the spacing to $1\tfrac{1}{2}D = 72$ in., where D = depth of girder, and the spacing proposed is shown in Fig. 422.

The permissible average shear stress in the web at any section is also given by Clause 20 (b) when the bending stress in the web does not exceed 2 tons per sq. in., and by the same clause in conjunction with Clause 22 (c) when the bending stress exceeds 2 tons per sq. in.

The critical panel will be that marked in Fig. 422, and although the stresses are well within specification requirements, as will be shown, the relevant calculations will be given as an example of those which may on occasion be required.

The average shear stress on the web immediately to the left of the 60-ton point load

$$=\frac{82\cdot8}{48\times\frac{1}{2}}=3\cdot45 \text{ tons per sq. in.}$$

The maximum fibre stress in the flange plates at this point

$$=\frac{945\times12}{29\cdot57\times48\cdot5}=7\cdot92 \text{ tons per sq. in.,}$$

and therefore the bending stress in the web at this point

$$=\frac{7\cdot92\times24\cdot25}{25\cdot5}$$

$$=7\cdot54 \text{ tons per sq. in.}$$

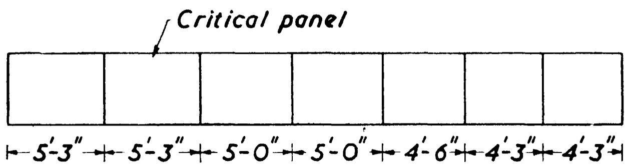


FIG. 422

The stiffeners will be 5-in. × 3-in. angles with the 5-in. leg outstanding, and the unsupported length of web in the panel is 57 in. $=a$, whilst the clear depth of web between flange angles is $40\frac{1}{2}$ in. $=b$.

Hence, in this panel, the permissible shear stress is, by Clause 20 (b) (iii),

$$F_s'=\left(\frac{225}{b/t}\right)^2\left[1+\frac{3}{4}\left(\frac{b}{a}\right)^2\right]=\left(\frac{225}{81}\right)^2\left[1+\frac{3}{4}\left(\frac{40\cdot5}{57}\right)^2\right]$$

$$=10\cdot63 \text{ tons per sq. in.}$$

It should be noted, however, that the allowable stress calculated above is only applicable when the coexistent bending stress in the web does not exceed 2 tons per sq. in., which is not so in this girder, and accordingly the 10·63 tons per sq. in. must be reduced in accordance with Clause 22.

The reduction factor

$$=1-\frac{K_3\,.\,f_b{}^2}{100} \quad \text{wherein } K_3=\left(\frac{b}{t}\div170\right)^5$$

$$=1-\frac{0\cdot0246\times7\cdot54^2}{100}$$

$$=0\cdot986.$$

Thus the maximum average shear stress must not exceed $10{\cdot}63 \times 0{\cdot}986 = 10{\cdot}48$ tons per sq. in., and hence the requirements of Clause 20 (b) (i) still govern, and the maximum allowable shear stress is 6·5 tons per sq. in.; the web is therefore adequate. The intermediate stiffeners have no applied end load, and their size is governed by Clause 47 (a); 5-in. × 3-in. × $\frac{3}{8}$-in. angles will meet the requirement that the outstanding leg shall be not less than 2 in. plus overall length of girder divided by 30, and that the thickness of the outstanding leg shall be not less than $\frac{1}{16}$ of the outstand.

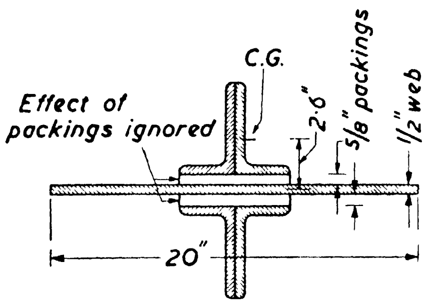


FIG. 423

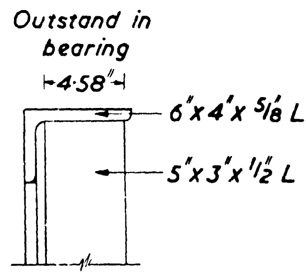


FIG. 424

The stiffeners under the applied loads will be designed to suit the larger of the two loads, that is, 85 tons. The area for design as a column includes 20 times the web thickness, = 10 in., on each side of the centre line of the stiffeners. The minimum area required for bearing will be $85/12 = 7{\cdot}08$ sq. in., and since bearing is not allowed on the root fillet of the flange angles, the length of bearing between the flange angles and the outstanding leg of the 5-in. angle will be 4·58 in. as shown in Fig. 424, and the minimum thickness, assuming four angles,

$$= \frac{7{\cdot}08}{4{\cdot}58 \times 4} = 0{\cdot}386 \text{ in.,}$$

so that 5-in. × 3-in. × $\frac{1}{2}$-in. angles will be used.

The safe load as a column must now be checked, taking into account the length of web referred to above; it must be noted that packings between stiffeners and the web plate should not be taken as forming part of the column unless their width is greater than the combined widths of the angles and they are separately riveted to the web plate.

Thus I about centre line of web

$$= \frac{20 \times \frac{1}{2}^3}{12} + 4(9{\cdot}33 + 3{\cdot}75 \times 2{\cdot}6^2)$$

$$= 139 \text{ in.}^4.$$

$$\text{Area} = 20 \times \tfrac{1}{2} + 4 \times 3{\cdot}75$$

$$= 25 \text{ sq. in.}$$

r about centre line of web

$$= \sqrt{\frac{139}{25}} = 2{\cdot}35 \text{ in.}$$

Clause 47 also states that the effective length shall be taken as three-quarters of the length of the stiffeners,

$$= \tfrac{3}{4} \times (48{\cdot}5 - 1{\cdot}25)$$
$$= 35{\cdot}4 \text{ in.}$$
$$\frac{l}{r} = \frac{35{\cdot}4}{2{\cdot}35} = 15.$$

The permissible stress by Table 7 is 8·27 tons per sq. in. and the permissible load as a column is therefore 206 tons.

The detail of the stiffeners at the end will be dependent upon the support detail, but assuming this to be a 12-in. × 8-in. joist column the arrangement shown in Fig. 425 will be investigated.

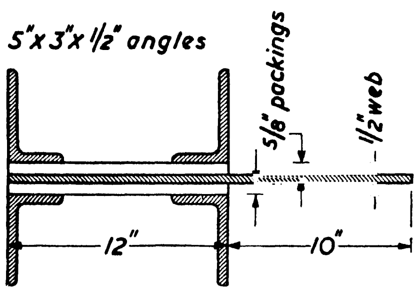


FIG. 425

The area required for bearing will be 97·2/12 = 8·1 sq. in., and consequently the minimum thickness of angle

$$= \frac{8{\cdot}1}{4{\cdot}58 \times 4} = 0{\cdot}44 \text{ in.,}$$

and thus 5-in. × 3-in. × $\frac{1}{2}$-in. angles will be satisfactory from this standpoint.

Clause 47 states that the section of the column shall consist of a pair of stiffeners, one on each side of the web, with such portion of the web as is available up to 20 times the web thickness on each side of the centre line of the pair of stiffeners. Assuming that the load is shared equally by the two pairs of angles, it follows that the outer pair must be taken in conjunction with a 6-in. length of web, and it will thus be necessary to investigate the outer pair only.

Thus I about centre line of web

$$= \frac{6 \times \frac{1}{2}^3}{12} + 2(9{\cdot}33 + 3{\cdot}75 \times 2{\cdot}6^2)$$
$$= 69{\cdot}4 \text{ in.}^4.$$
$$\text{Area} = 6 \times \tfrac{1}{2} + 2 \times 3{\cdot}75$$
$$= 10{\cdot}5 \text{ sq. in.}$$

r about centre line of web

$$= \sqrt{\frac{69{\cdot}4}{10{\cdot}5}} = 2{\cdot}57 \text{ in.}$$
$$\frac{l}{r} = \frac{35{\cdot}4}{2{\cdot}57} = 13{\cdot}7.$$

The permissible load equals $8{\cdot}33 \times 10{\cdot}5 = 87{\cdot}5$ tons, compared with 47·6 tons actual load and the detail is satisfactory.

The complete girder is shown in Fig. 426.

Designing upon the Exact Method

The design of the girder in this method is based upon the moment of inertia, and the following are the calculations where they differ from the preceding figures:

Moment of Inertia about centroid, see Fig. 427, at maximum section.

Web, 48 in. $\times \frac{1}{2}$ in.	$= 4{,}608{\cdot}0$	p. 650
Flange angles, four 6 in. $\times$ 4 in. $\times \frac{5}{8}$ in.		
Self inertia, $4 \times 7{\cdot}37$	$= 29{\cdot}5$	p. 661
Ay^2, where $y = 23{\cdot}23$ in., $= 23{\cdot}44 \times 539{\cdot}6$	$= 12{,}648{\cdot}2$	p. 661 and 654
Flange plates, two 16 in. $\times 1\frac{1}{4}$ in.		
Self inertia, $2 \times 2{\cdot}6$	$= 5{\cdot}2$	p 653
Ay^2, where $y = 24{\cdot}875$ in., $= 2 \times 16 \times 1{\cdot}25 \times 619$	$= 24{,}760{\cdot}0$	p. 654
	Gross $I = 42{,}051$ in.^{4}	

Deduct rivet holes

Ay^2 only, self inertia negligible

$y = 22{\cdot}0,\ 2 \times \frac{15}{16} \times 1\frac{3}{4} \times 484$	$= 1589$		p. 654
$y = 24{\cdot}6,\ 4 \times \frac{15}{16} \times 1\frac{7}{8} \times 605$	$= 4254$		p. 654
		5,843	

Net $I = 36{,}208$ in.^{4}

$$Z = \frac{36{,}208}{25{\cdot}5} = 1420 \text{ in.}^3.$$

$$Z \text{ required} = 1104 \times 12/9{\cdot}5 = 1395 \text{ in.}^3.$$

Therefore the section is satisfactory.

Moment of Inertia with 16-in. $\times \frac{5}{8}$-in. flange plates.

Web, as before	$= 4{,}608{\cdot}0$	
Flange angles, as before	$\begin{cases} = 29{\cdot}5 \\ = 12{,}648{\cdot}2 \end{cases}$	
Flange plates, two 16 in. $\times \frac{5}{8}$ in.		
Self inertia, $2 \times 0{\cdot}33$	$= 0{\cdot}7$	p. 653
Ay^2, where $y = 24{\cdot}56$, $= 2 \times 16 \times \frac{5}{8} \times 603{\cdot}2$	$= 12{,}064{\cdot}0$	p. 654
	Gross $I = 29{,}350$ in.^{4}	

Deduct rivet holes

$y = 22$, as before	$= 1589$	
$y = 24{\cdot}25,\ 4 \times \frac{15}{16} \times 1\frac{1}{4} \times 588$	$= 2756$	
		4,345

Net $I = 25{,}005$ in.^{4}

$$Z = \frac{25{,}005}{24{\cdot}875} = 1005 \text{ in.}^3.$$

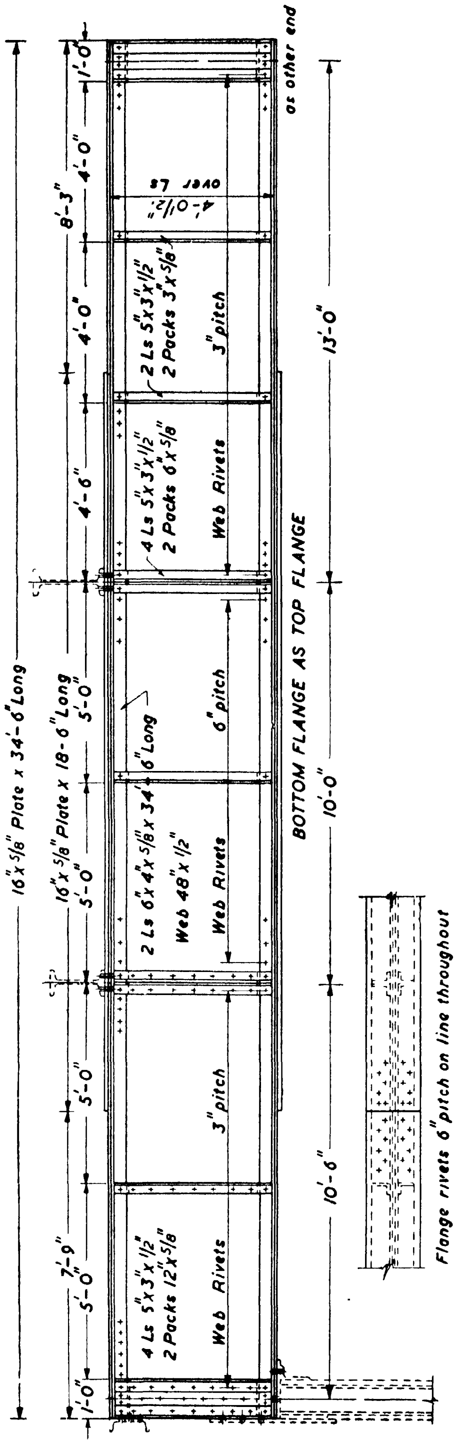

16" x 5/8" Plate x 34'-6" Long
16" x 5/8" Plate x 18-6" Long
8'-3"
7'-9"
1'-0"
4'-0"
4'-0"
4'-6"
5'-0"
5'-0"
5'-0"
5'-0"
1'-0"
4'-0 1/2" over Ls
as other end
2 Ls 5" x 3" x 1/2"
2 Packs 3" x 5/8"
3" pitch
4 Ls 5" x 3" x 1/2"
2 Packs 6" x 5/8"
Web Rivets
6" pitch
6" Long
2 Ls 6" x 4" x 5/8" x 34'
Web 48" x 1/2"
Web Rivets
3" pitch
4 Ls 5" x 3" x 1/2"
2 Packs 12" x 5/8"
Web Rivets
BOTTOM FLANGE AS TOP FLANGE
13'-0"
10'-0"
10'-6"
Flange rivets 6" pitch on line throughout

FIG. 426

This is used in the flange curtailment diagram given in Fig. 428. Note the cut-off points differ slightly from those found by the approximate method,

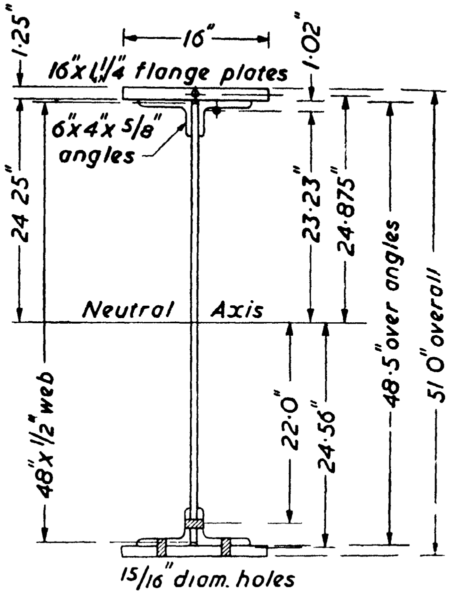


FIG. 427

the reason being that in the latter the effective depth is assumed to remain constant and equal to the depth over the angles, whereas by calculating on

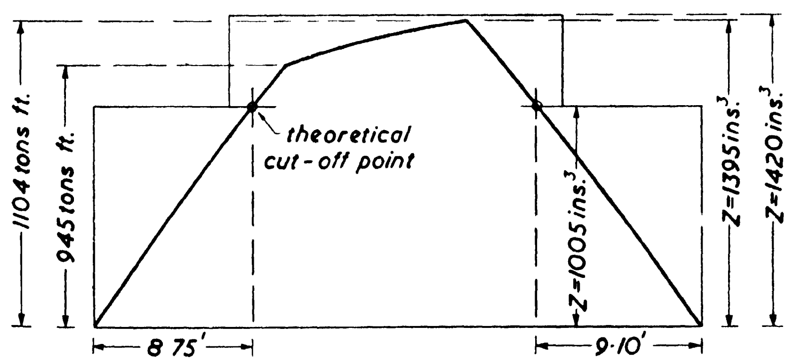


FIG. 428

the moment of inertia allowance is made for the variation in effective depth. The difference is not, however, of any significance.

Rivet pitches are calculated on the more exact formula, based upon the

distribution of horizontal shear. The horizontal shear per lineal inch $= S \cdot a \cdot y/I$, where

S = shear at section,
a = area of section to be connected by rivets,
y = lever arm about NA of area a, and
I = moment of inertia of section about NA

Then, at the left-hand end, for the rivets connecting flange angle to web, we have, as shown in Fig. 429,

$$\begin{aligned} &\text{Flange plate, } ay = 10{\cdot}0 \times 24{\cdot}56 && = 245{\cdot}6 \\ &\text{Flange angles, } ay = 2 \times 5{\cdot}86 \times 23{\cdot}23 && = 272{\cdot}3 \\ & && \overline{517{\cdot}9} \end{aligned}$$

Less rivet holes

$$\begin{aligned} &\text{Flange, } ay = 2 \times \tfrac{15}{16} \times 1\tfrac{1}{4} \times 24{\cdot}25 = 56{\cdot}9 \\ &\text{Web, } ay = \tfrac{15}{16} \times 1\tfrac{3}{4} \times 22{\cdot}0 = 36{\cdot}1 \\ & \qquad\qquad\qquad = \;\; 93{\cdot}0 \\ & \qquad\qquad\qquad\;\; \overline{424{\cdot}9} \end{aligned}$$

Hence, $$\text{shear per inch} = \frac{97{\cdot}2 \times 424{\cdot}9}{25{,}005} = 1{\cdot}65 \text{ tons.}$$

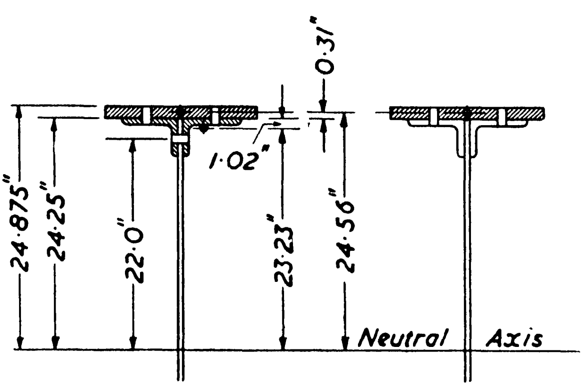


FIG. 429

For the rivets between flange angles and flange plates,

$$\begin{aligned} ay &= 245{\cdot}6 - (2 \times \tfrac{15}{16} \times \tfrac{5}{8} \times 24{\cdot}56) \\ &= 216{\cdot}9. \end{aligned}$$

Hence, $$\begin{aligned} \text{shear per inch} &= \frac{97{\cdot}2 \times 216{\cdot}9}{25{,}005} \\ &= 0{\cdot}844 \text{ ton.} \end{aligned}$$

Both these values are less than those given by the approximate method.

Calculations at the point of cut-off of the outer plate are made in a similar manner.

It will be noted from these comparative figures for section modulus and rivet pitch, that the approximate method is quite sufficient for plate girders

of this type. Calculations for stiffener spacing and stiffeners are the same whether the exact or approximate method is used.

Design of a Welded Plate Girder

General Design Information

1. The girder designed in this example is 36 ft. 0 in. span and is assumed to be simply supported at its ends.

2. The loading on the girder is shown in Fig. 430. In Case 1 loading (Fig.

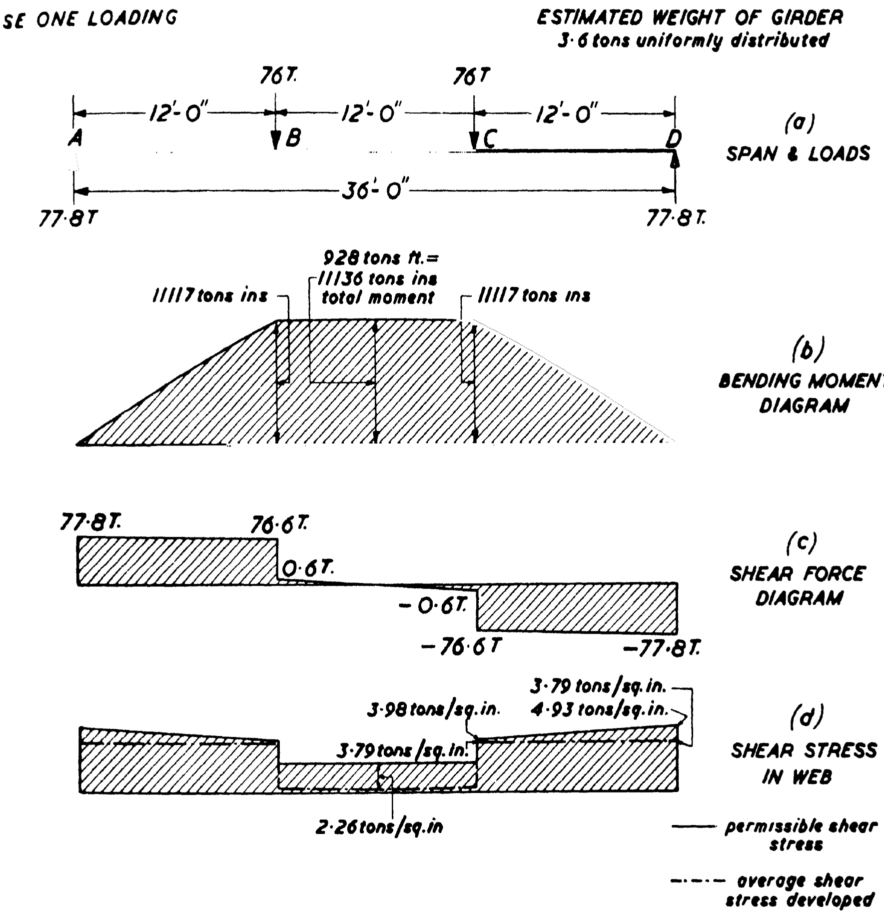


FIG. 430 (a) to (d)

430 (a)), both point loads are shown at their maximum magnitude. In Case 2 loading (Fig. 430 (e)), one of the loads is shown at its maximum magnitude and the other at its simultaneous minimum magnitude; this latter case gives the condition for maximum shear in the centre length of the girder.

The self-weight of the girder, which is first estimated, must eventually be checked when the girder is finally designed.

3. It is assumed that there are effective lateral restraints for the compression flange at positions A, B, C and D (Fig. 430 (a)).

Bending Moments and Shear Forces

The bending moments and shear forces acting on the girder are calculated by simple statics and are shown in Fig. 430 (b), (c), (f) and (g) for the alternative loading conditions.

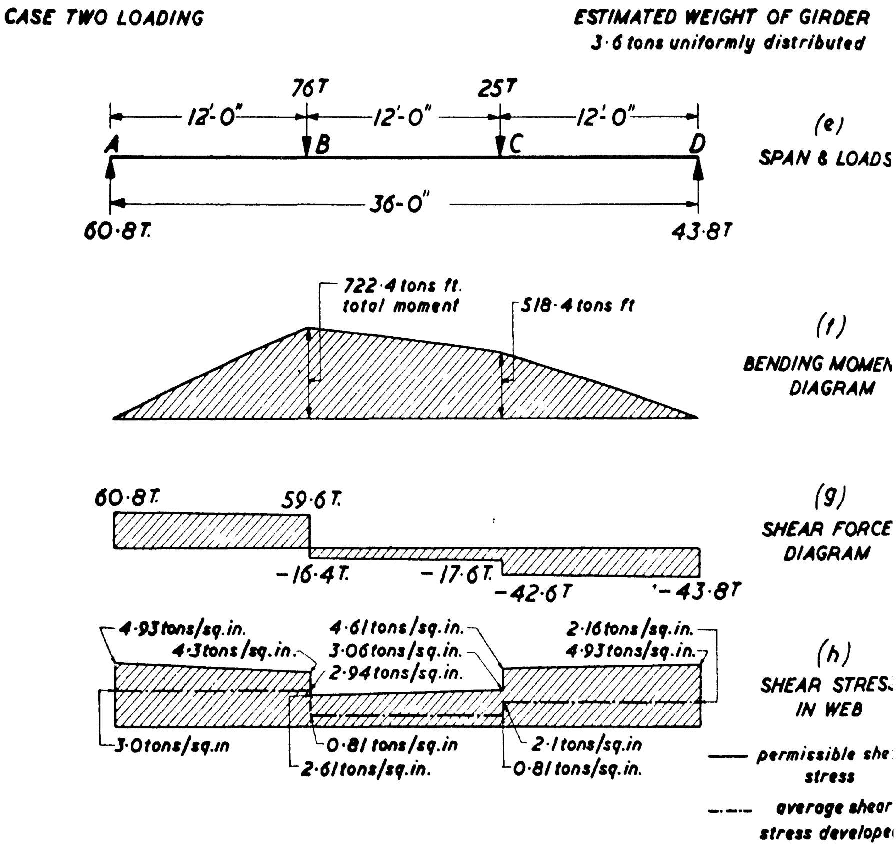


FIG. 430 (e) to (h)

Girder Cross-section

A girder of H-shape cross-section will be used (Fig. 431).

It should be noted that this section is built up by welding directly together the flange and web plates, and there is no need for the flange angles used in riveted construction.

It is first decided that a depth of 54 in. between flange plates and a width of flanges of 15 in. is likely to prove a satisfactory basic dimension for the section. An approximate area of flange plate is obtained by dividing the maximum B.M. by the depth of the girder and using a permissible bending stress of 9·5 tons per sq. in. An approximate required thickness of web is obtained

based on the maximum shear force and a maximum permissible shear stress of 6.5 tons per sq. in. The proposed section shown in Fig. 432 is determined in this manner and should next be checked by detailed calculations to ensure that the section is completely in accordance with the requirements of B.S. 449.

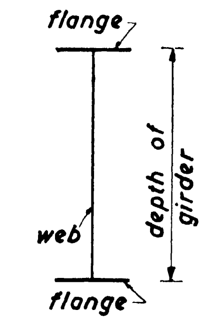


FIG. 431

Flanges

The permissible stress for tension due to bending is 9·5 tons per sq. in. (B.S. 449, Clause 19 (a) (ii)).

The permissible stress for compression due to bending is

$$9{\cdot}5 \text{ tons per sq. in.} \quad \text{(Clause 19 (c) (i))}$$

$$\text{or} \quad \frac{1000}{l/r} \times K_1 \quad \text{(Clause 19 (c) (ii)),}$$

whichever is the lesser, where r is the radius of gyration of the cross-section measured perpendicularly to the plane of bending (i.e. in this case $r_{yy} = 3{\cdot}49$) and l is the length between effective lateral restraints (i.e. 12 ft. 0 in. = 144 in.).

K_1 is obtained from Table 10 in B.S. 449, Clause 19 (c) (ii), and depends on the ratio r_{xx}/r_{yy} which is in this case $24{\cdot}2/3{\cdot}49 = 6{\cdot}94$. As this figure is greater than 5·0, which is the maximum ratio figure in the table, K_1 is taken as unity.

$$\text{Therefore} \quad \frac{1000}{l/r} \times K_1 = \frac{1000 \times 3{\cdot}49}{144} \times 1 = 24{\cdot}2 \text{ tons per sq. in.}$$

Properties of Cross-section

54 in. × $\frac{3}{8}$ in. Web.

15 in. × $1\frac{1}{4}$ in. Flanges.

$$I_{XX} = \frac{3}{8} \times \frac{54^3}{12} + 2 \times \frac{15 \times (1\frac{1}{4})^3}{12} + 2 \times 15 \times 1\frac{1}{4}(27{\cdot}625)^2$$

$$= 33{,}630 \text{ in.}^4.$$

$$I_{YY} = 2 \times 1\tfrac{1}{4} \times \frac{15^3}{12} + \frac{54 \times (\frac{3}{8})^3}{12}$$

$$= 704 \text{ in.}^4.$$

$$\text{Area} = 2 \times 15 \times 1\tfrac{1}{4} + 54 \times \tfrac{3}{8}$$

$$= 57{\cdot}75 \text{ sq. in.}$$

$$r_{XX} = \sqrt{\frac{33{,}630}{57{\cdot}75}} = 24{\cdot}2 \text{ in.}$$

$$r_{YY} = \sqrt{\frac{704}{57{\cdot}75}} = 3{\cdot}49 \text{ in.}$$

$a.y$ at joint of flange and web

$$= 15 \times 1\tfrac{1}{4} \times 27{\cdot}625 = 518 \text{ in.}^3.$$

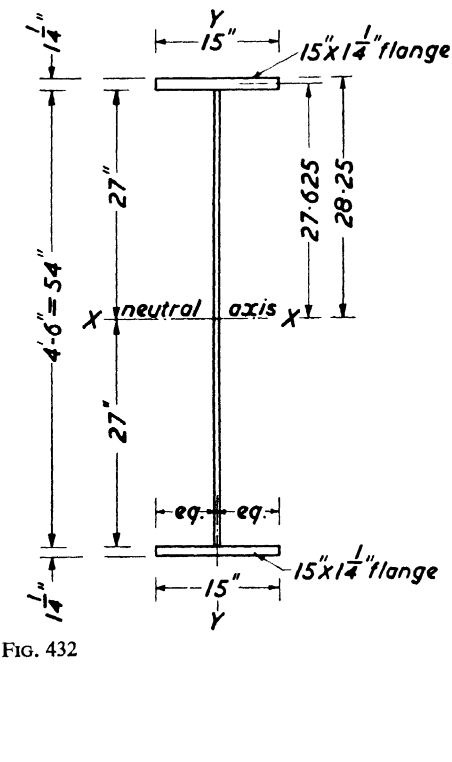


FIG. 432

The maximum permissible compression bending stress is therefore 9·5 tons per sq. in.

From Fig. 430 (b), the maximum B.M. is 928 tons ft.

Therefore, maximum stress (tension or compression) in flanges due to bending

$$= \frac{928 \times 12 \times 28{\cdot}25}{33{,}630}$$

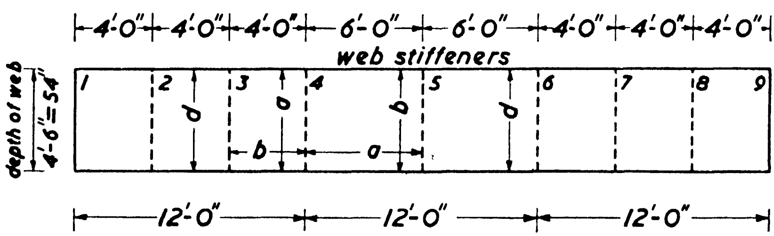
$$= 9{\cdot}38 \text{ tons per sq. in.}$$

which is satisfactory.

It should be noted that if there had not been effective lateral restraints for the compression flange in this example, the permissible stress for compression would have been less than 9·5 tons per sq. in. It would then have been economical to make the compression flange larger than the tension flange.

Web

The ratio d/t of the web is 54 in./$\frac{3}{8}$ in. $= 144$. This ratio is greater than 75, therefore web stiffeners are required according to B.S. 449, Clause 20 (b). The same clause limits the spacing of the web stiffeners to $1\frac{1}{2}d = 1\frac{1}{2} \times 54$ in.



PROPOSED WEB STIFFENER SPACING

FIG. 433

$= 81$ in. $= 6$ ft. 9 in. The web-stiffener spacing employed is shown in Fig. 433.

The permissible average shear stress in the web at the particular cross-section of the girder being considered is given by B.S. 449, Clause 20 (b), where the compression stress due to bending does not exceed 2 tons per sq. in., and by B.S. 449, Clause 20 (b) in conjunction with Clause 22 (c) where the compressive stress due to bending exceeds 2 tons per sq. in.

To avoid confusion in dealing with these two conditions, the permissible average shear stress given by Clause 20 (b) is here referred to as the 'permissible average shear stress for shear only', when it is desired to distinguish it from the 'permissible average shear stress for shear and bending' given by Clauses 20 (b) and 22 (c). The 'permissible shear stress' in the web is the first of those where the compressive stress due to bending does not exceed 2 tons per sq. in., and the second where the compressive stress due to bending exceeds 2 tons per sq. in.

Fig. 430 (d) shows the manner in which the permissible shear stress in the web varies along the length of the girder in the present example, for Case 1 loading; and corresponding information is given in Fig. 430 (h) for Case 2 loading.

The critical web panel for Case 1 loading is panel 3–4, Fig. 433. For this panel $a = 54$ in., $b = 48$ in., $t = \frac{3}{8}$ in., as defined in B.S. 449, Clause 20 (b).

Hence in panel 3–4, permissible shear stress for shear only is (B.S. 449, Clause 20 (b))

$$\left(\frac{225}{b/t}\right)^2\left[1 + \frac{3}{4}\left(\frac{b}{a}\right)^2\right] = \left(\frac{225}{128}\right)^2\left[1 + \frac{3}{4}(0{\cdot}888)^2\right]$$

$$= 4{\cdot}93 \text{ tons per sq. in.}$$

The maximum compression due to bending in panel 3–4 at cross-section immediately adjacent to stiffener 4 (Fig. 433) and at the upper edge of the web is

$$\frac{11{,}114}{33{,}630} \times \frac{27}{1} = 8{\cdot}93 \text{ tons per sq. in.}$$

This is greater than 2 tons per sq. in. Therefore a reduction factor must be applied to the foregoing shear stress of 4·93 tons per sq. in. This reduction factor is:

$$(1 - K_3 f_b^2/100) \quad \text{(B.S. 449, Clause 22 (c))}$$

where $K_3 = [(b/t) \div 170]^5 = (128/170)^5 = 0{\cdot}243,$

therefore $(1 - K_3 f_b^2/100) = 1 - 0{\cdot}243 \times 8{\cdot}93^2/100 = 0{\cdot}807.$

Hence in panel 3–4 at cross-section immediately adjacent to stiffener 4, permissible shear stress for shear and bending

$$= 4{\cdot}93 \times 0{\cdot}807 = 3{\cdot}98 \text{ tons per sq. in.}$$

This is the permissible shear stress in panel 3–4, at cross-section immediately adjacent to stiffener 4.

The actual average shear stress in the web at this same cross-section is given very nearly by the formula

$$\frac{\text{Total shear}}{dt} = \frac{76{\cdot}6}{54} \times \frac{8}{3} = 3{\cdot}79 \text{ tons per sq. in.}$$

This is within the permissible shear stress in the web at this same cross-section. Similar investigations show that the actual average shear stress in the web is within the permissible shear stress in the web at all cross-sections. (See Fig. 430 (d) and Fig. 430 (h).) The proposed web is therefore satisfactory.

The shear taken by the web is slightly less than the total shear, but the error in using the total shear when calculating the average shear stress on the cross-section of the web is here only about 2 per cent and is on the safe side. Fig. 434 shows the shear per inch resisted by the cross-section at various distances from the neutral axis. At any point the shear stress in the web is the shear per inch divided by t. The shear stress in the flanges varies substantially across the width of the flange and requires consideration of the shape of the flange at right angles to the plane of Fig. 434.

Care must be exercised in the foregoing calculations in regard to the directions in which *a*, *b*, *c*, *d*, are measured. Reference to Fig. 433, shows that *d* is measured in the same direction for all panels, but that *b*, being the shorter

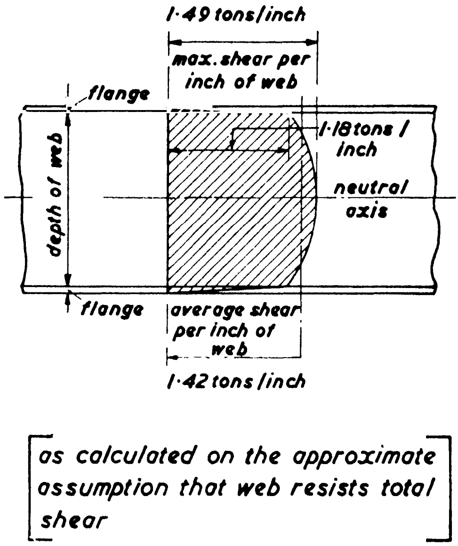


FIG. 434

dimension of each panel, is measured horizontally in the end panels and vertically in the centre panels, in this particular example.

Intermediate Stiffeners: Nos. 2, 3, 5, 7, 8. (Fig. 433)

There is no applied end load on these stiffeners.

It is here decided to use stiffeners consisting of flats arranged in pairs, that is to say, the flats are arranged on opposite sides of the web and directly opposite one another.

By B.S. 449, Clause 47 (a), the minimum outstand of each flat, when so arranged, is 2 in. plus $D/30$ where D is the overall depth of girder,

$$=2+\frac{56\cdot5}{30}=2+1\cdot9=3\cdot9 \text{ in., say 4 in.}$$

The minimum thickness of flat by B.S. 449, Clause 47 (a), is $\frac{4}{16}=\frac{1}{4}$ in., but if exposed to the weather the minimum thickness by B.S. 449, Clause 32, is $\frac{5}{16}$ in., unless special provision against corrosion is made. There is also to be considered the possibility of damage to thin outstanding members during transit and erection.

It is decided here to use flats 4 in. $\times\frac{5}{16}$ in.

Stiffeners under Loads: Nos. 4, 6. (Fig. 433)

The applied end load on these stiffeners is 76 tons. (See Fig. 430 (a).) It is applied in the form of a line load across the top flange in this example.

Try 7-in. $\times\frac{5}{8}$-in. flats arranged in a pair and located directly under the applied line load.

The area for design as a column includes 20 times the thickness of the web, $=20 \times \frac{3}{8}$ in. $=7\frac{1}{2}$ in. on each side of the centre line of the stiffener where available. (See B.S. 449, Clause 47 (b).) The area for design as a column is shown in Fig. 435 (a).

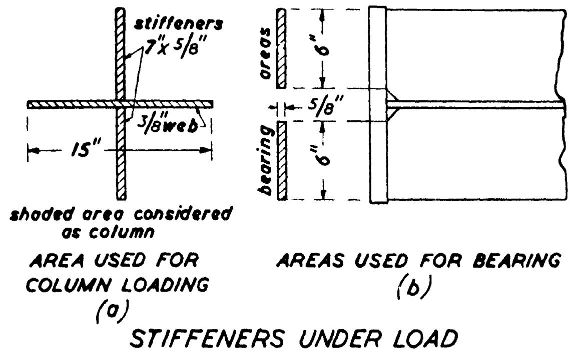


FIG. 435

The effective length of the column is three-quarters of the length of the stiffener,

$$I \text{ about centre line of web} = \frac{\frac{5}{8} \times (14\frac{3}{8})^3}{12} + \frac{15 \times (\frac{3}{8})^3}{12}$$

$$= 154{\cdot}5 \text{ in.}^4.$$

$$\text{Area} = 14 \times \tfrac{5}{8} + 15 \times \tfrac{3}{8} = 14{\cdot}37 \text{ sq. in.}$$

$$r \text{ about centre line of web} = \sqrt{\frac{154{\cdot}5}{14{\cdot}37}} = 3{\cdot}28 \text{ in.}$$

$$\frac{l}{r} = \frac{0{\cdot}75 \times 54}{3{\cdot}28} = 12{\cdot}35.$$

The permissible stress as a column, by B.S. 449, Clause 18 (a), is 8·4 tons per sq. in.

The permissible load as a column, is $14{\cdot}37 \times 8{\cdot}4 = 120$ tons, but check for bearing at top of stiffener, as follows:

The bearing must be taken entirely on the outstanding legs of the stiffener (B.S. 449, Clause 47 (b)). The area for bearing is shown in Fig. 435 (b).

The stiffeners are notched 1 in. to clear the welds connecting the web to the flanges. Therefore, the outstanding width for bearing is 6 in. on each side. The area for bearing is $2 \times 6 \times \frac{5}{8} = 7{\cdot}5$ sq. in. Allowable stress for bearing is 12 tons per sq. in. (B.S. 449, Clause 21).

$$\text{Allowable load for bearing} = 7{\cdot}5 \times 12 = 90 \text{ tons.}$$

Actual load 76 tons. Therefore 7-in. $\times \frac{5}{8}$-in. flats are satisfactory.

Stiffeners at Ends: Nos. 1, 9. (Fig. 433)

The applied end load on these stiffeners is 77·8 tons. (See Fig. 430 (a).) It is applied in the form of a line load across the bottom flange in this example.

Try 15-in. × $\frac{1}{2}$-in. flat arranged across end of web, and located directly above the applied line load.

The area for design as a column includes 20 times the thickness of the web $= 20 \times \frac{3}{8}$ in. $= 7\frac{1}{2}$ in., this being in this case available on one side of the centre line of the stiffener. (See B.S. 449, Clause 47 (b).) The area for design as a column is shown in Fig. 436 (a). The effective length of the column is three-quarters of the length of the stiffener.

$$I \text{ about centre line of web} = \frac{\frac{1}{2} \times 15^3}{12} + \frac{7 \cdot 5 \times (\frac{3}{8})^3}{12} = 141 \text{ in.}^4.$$

$$\text{Area} = 15 \times \tfrac{1}{2} + 7\tfrac{1}{2} \times \tfrac{3}{8} = 10 \cdot 32 \text{ sq. in.}$$

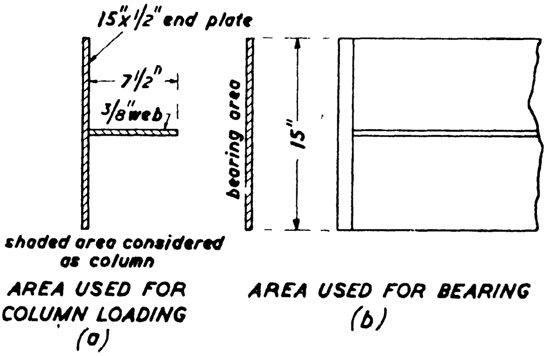
$$r \text{ about centre line of web} = \sqrt{\frac{141}{10 \cdot 32}} = 3 \cdot 7 \text{ in.}$$

$$\frac{l}{r} = \frac{0 \cdot 75 \times 54}{3 \cdot 7} = 10 \cdot 9.$$

Permissible stress as column is 8·46 tons per sq. in. Permissible load as column is 10·32 × 8·46 = 87·5 tons, but check for bearing at bottom of stiffener as follows:

The area for bearing is shown in Fig. 436 (b).

This stiffener, being at the extreme end of the girder, is not notched to clear the weld connecting the web to the flanges.



STIFFENERS AT ENDS OF GIRDER

FIG. 436

The area for bearing is $15 \times \frac{1}{2} = 7 \cdot 5$ sq. in.

Permissible stress for bearing is 12 tons per sq. in.

Permissible load for bearing = 7·5 × 12 = 90 tons. Therefore the permissible load as column, 87·5 tons, is here the deciding factor.

Actual load 77·8 tons. Therefore 15-in. × $\frac{1}{2}$-in. flat is satisfactory.

Welding: Web to Flange

Maximum shear 77·8 tons. (See Fig. 430 (c).)

Shear per linear inch at joint between web and flange is

$$\frac{S.a.y}{I} \text{ tons.}$$

where S is maximum shear in tons on cross-section in question, a is area of that part of the cross-section which is outside the point at which the shear per linear inch is being calculated, in sq. in., y is distance in inches from neutral axis to centroid of area a, I is moment of inertia of complete cross-section in inches^4

$$=\frac{77 \cdot 8 \times 518}{33{,}630}=1 \cdot 20 \text{ tons.}$$

Use two fillet welds, one each side of web. Shear per inch on each weld $=1 \cdot 2/2=0 \cdot 60$ tons.

$\frac{3}{16}$-in. fillet welds would be of sufficient strength but for connection to $1\frac{1}{4}$-in. plate, the minimum permissible size is $\frac{1}{4}$ in.

Therefore:

1. Two $\frac{1}{4}$-in. fillet welds might be used continuous throughout the length of the girder. This would be satisfactory except that the amount of welding would be considerably greater than required for strength in this example; or
2. Intermittent welds of the minimum size (here $\frac{1}{4}$ in.) might be used to reduce the amount of welding.

The minimum ratio of effective length of $\frac{1}{4}$-in. intermittent welds to total length, to suit stress requirements is

$$\frac{\text{Actual shear per linear inch of joint per weld}}{\text{Strength per linear inch of } \frac{1}{4}\text{-in. weld}}$$

$$=\frac{0 \cdot 60}{1 \cdot 14}=0 \cdot 527 \text{ ratio at ends of girder.}$$

Maximum space between intermittent welds for $\frac{3}{8}$-in. web welded to $1\frac{1}{4}$-in. flange is $16 \times \frac{3}{8}=6$ in. at compression edge of web and $24 \times \frac{3}{8}=9$ in. at tension edge of web.

It is seen that 7-in. length of welds with 6-in. spaces would give a ratio of $7/7+6=7/13=0 \cdot 538$, and this, being slightly greater than the required ratio of $0 \cdot 527$, would be satisfactory.

It is therefore decided to adopt 7-in. lengths of $\frac{1}{4}$-in. fillet welds and not more than 6-in. spaces, for the welding connecting the web to the flanges in the end 12 ft. lengths of the girder. Note that the spaces may require to be adjusted to suit the detail dimensions of the structure, and this point must be watched during the preparation of detail drawings.

Consider now the requirements for intermittent $\frac{1}{4}$-in. fillet welds connecting the web to the flanges in the centre 12 ft. length of the girder. The maximum shear here is 17·6 tons. (See Fig. 430 (g).) The minimum ratio of effective length of $\frac{1}{4}$ in. intermittent weld to total length, to suit stress requirements is here

$$\frac{0 \cdot 60}{1 \cdot 14} \times \frac{17 \cdot 6}{77 \cdot 8}=0 \cdot 12 \text{ ratio.}$$

This requirement would be met with 0·8-in. length of welds and 6-in. spaces, but B.S. 449, Clause 45 (f), requires the length of $\frac{1}{4}$-in. fillet weld when carrying

load to be not less than $4 \times \frac{1}{4} = 1{\cdot}0$ in.; moreover in usual practice today on a structure of the size and type of the present example it would be customary to use a minimum length of about 3 in. or 4 in. for intermittent fillet welds when carrying load. It is here decided to adopt 3-in. lengths of $\frac{1}{4}$-in. fillet welds and not more than 6-in. spaces, for the welding connecting the web to the flanges in the centre 12-ft. length of the girder. This provides more welding than required by calculation, but to reduce the amount of welding here would be of doubtful economy or merit of design.

The spaces in the intermittent welding of the web to the bottom tension flange could be increased to 9 in., but it is here decided to use the same spaces as for the top flange. This simplifies detailing and marking-off.

Welding—Intermediate Stiffeners: Nos. 2, 3, 5, 7, 8. (Fig. 433)

There are no external loads applied to these stiffeners, their function being simply to prevent the web from buckling and hold the flanges square to the web. The welds provided in this example comply with the minimum sizes and spacing of welds laid down in B.S. 449, Clause 47 (e).

Three-sixteenth-inch intermittent fillet welds are used for connecting the stiffeners to the web, as shown in Fig. 437.

For the welds connecting the stiffeners to the flanges, $\frac{1}{4}$-in. fillets are used, this being the minimum size in relation to the $1\frac{1}{4}$-in. flange thickness.

Welding—Load-bearing Stiffeners: Nos. 4, 6. (Fig. 433)

Load-bearing stiffeners must be provided with sufficient welds to transmit the whole of the load to the web.

In this case the load per line of welds $= 76/4 = 19$ tons.

If continuous welds were used, the total length of each weld would be 52 in. (allowing for the 1-in. notches at each end) and the load per linear inch of weld would be $19/52 = 0{\cdot}37$ tons.

The minimum size of fillet weld that could be used is $\frac{3}{16}$ in., the strength of which is 0·85 tons per lin. in. It is economical, therefore, to use $\frac{3}{16}$-in. intermittent welds. The lengths and spaces of welds are determined in the manner outlined for the web-to-flange welds, subject to the provisions of B.S. 449, Clause 47 (e), and the arrangement adopted in this example for the stiffener-to-web welds is shown in Fig. 437.

It is assumed in this case that these stiffeners are fitted to bear tightly on the underside of the top flange plate and the welds between the ends of the stiffeners and the flange are not required to transmit load. They are made $\frac{1}{4}$-in. fillets therefore, as for the intermediate stiffeners.

Welding—Load-bearing Stiffeners at Ends of Girder: Nos. 1, 9. (Fig. 433)

Load per line of welds $= 77{\cdot}8/2 = 38{\cdot}9$ tons.

As these stiffeners are not notched at the ends, the total length of each continuous weld is 54 in.

The load per linear inch of weld is therefore $38{\cdot}9/54 = 0{\cdot}72$ tons.

Three-sixteenth-inch fillet welds satisfy the requirements of both minimum

size and strength (0·85 tons per lin. in.) and are adopted for the stiffener-to-web welds.

For the attachment of the ends of the stiffeners to the flanges, butt welds of single-bevel type provide a neat finish at the girder ends and are used as shown in Fig. 437. The bottom flange butt weld is required to transmit the full reaction of 77·8 tons, and must, therefore, be a properly sealed full-strength weld with a throat thickness of ½ in. The weld is then capable of carrying the same load as the stiffener itself.

Notes on Foregoing Design of Plate Girder

The sizes of most of the fillet welds in this example are governed by the 'minimum size' requirements of B.S. 449, Clause 45 (e), Table 18. These 'minimum size' requirements must always be given attention as well as stress requirements.

The question whether, and to what extent, intermittent welds should be used involves consideration of the following points:

1. Intermittent fillet welds are not economical unless the 'minimum size' of weld is being used. A longer length of smaller fillet weld is usually more economical for the same strength. This is because the strength of a fillet weld increases only directly as the size but the weight of weld metal as the square of the size.
2. If automatic welding is to be used the welds should be continuous.
3. If the structure is exposed, the use of continuous welds may be considered preferable as conducive to greater ease of maintenance or longer life of the structure.
4. If severe dynamic loads act on the structure intermittent welds must not be used.
5. Intermittent butt welds may be used to resist shear forces only and, in general, are not to be recommended.

The girder as designed above is shown in Fig. 437.

For comparison, several alternative designs for the same conditions are shown in Fig. 438. In some of these the flange plates have been made in more than one length and reduced in thickness where the B.M. permits. This reduces the weight of parent metal but increases the amount of welding on account of the additional butt joints. Whether overall economy is effected depends on the conditions of each case. In some cases the web plate also can be made in more than one length and reduced in thickness where the shear permits. This again reduces the weight of parent metal but increases the amount of welding.

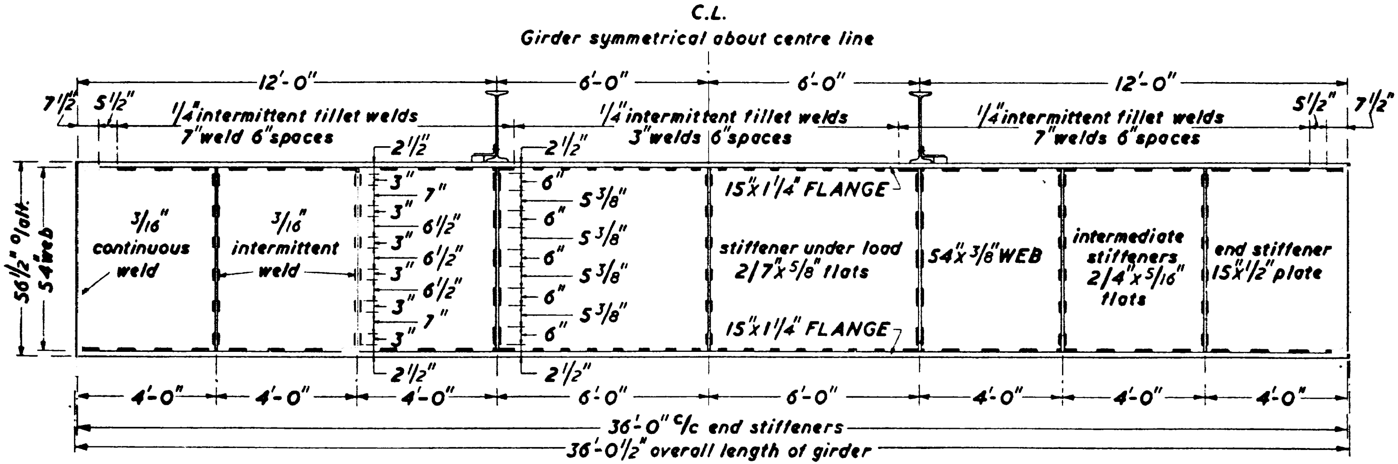


FIG. 437 (a)

NOTE.—The specified lengths of intermittent welds and spaces are based on the effective length of weld required. The actual total length of weld is the effective length plus the weld size added for each free end and the actual space is reduced correspondingly. See typical diagram below:

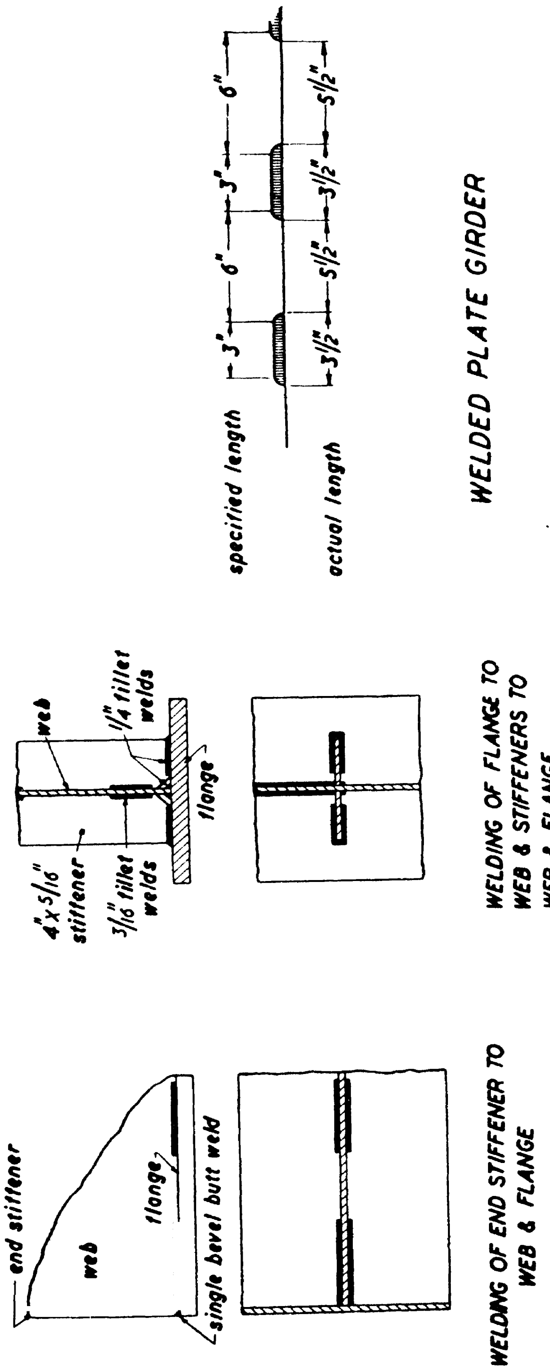
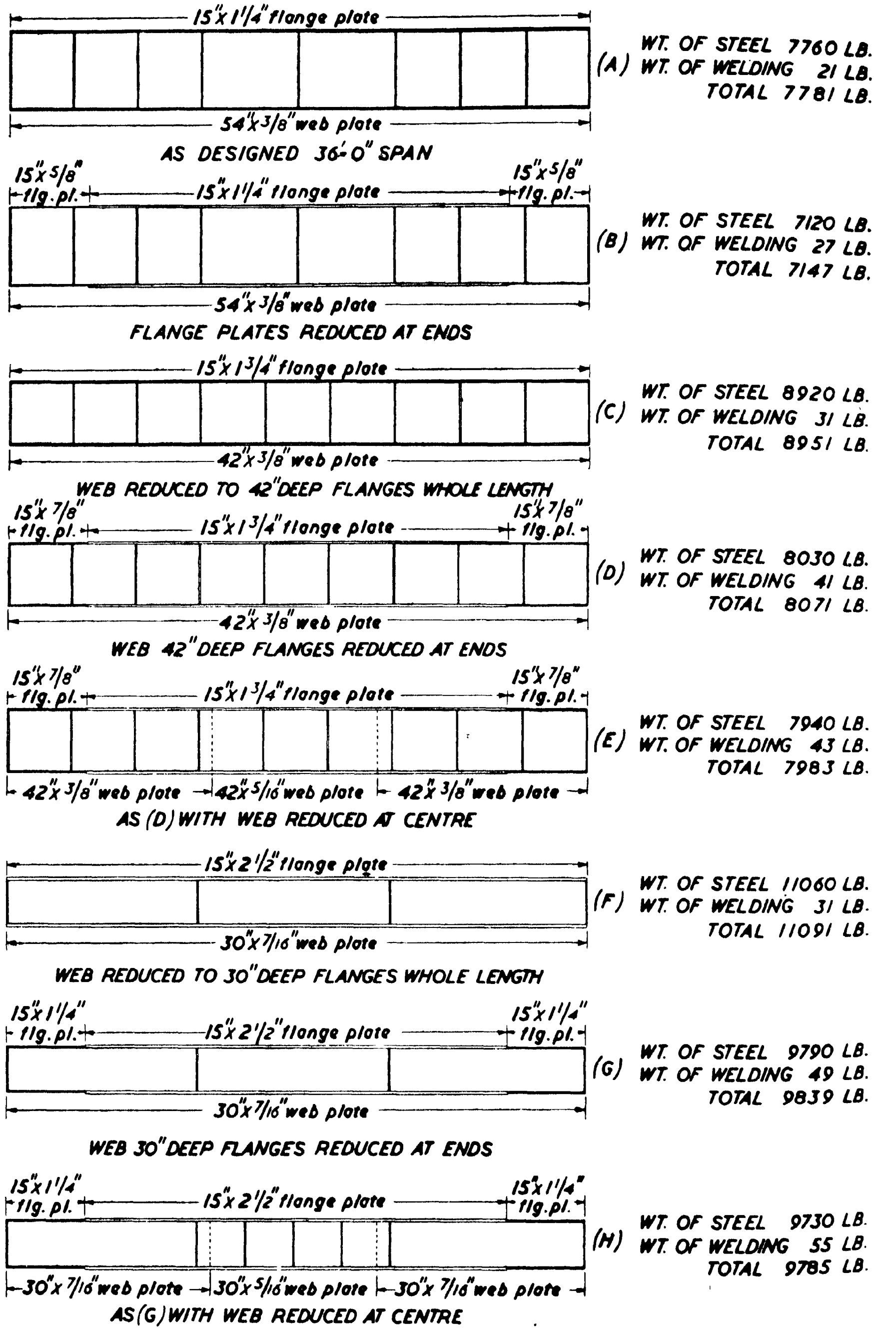

end stiffener
web
flange
single bevel butt weld
WELDING OF END STIFFENER TO WEB & FLANGE
4" x 5/16" stiffener
3/16" fillet welds
web
1/4" fillet welds
flange
WELDING OF FLANGE TO WEB & STIFFENERS TO WEB & FLANGE
specified length
actual length
3"
6"
3"
6"
3 1/2"
5 1/2"
3 1/2"
5 1/2"
WELDED PLATE GIRDER

FIG. 437 (b)



COMPARISON OF ALTERNATIVE DESIGNS OF WELDED PLATE GIRDER

FIG. 438

32. MOMENTS OF INERTIA OF PLATE GIRDERS

THE tables on pages 648 to 663 are designed to reduce the amount of work involved in calculating the moments of inertia of plate girders or other plated sections. Consider, for example, the plate girder shown in Fig. 439.

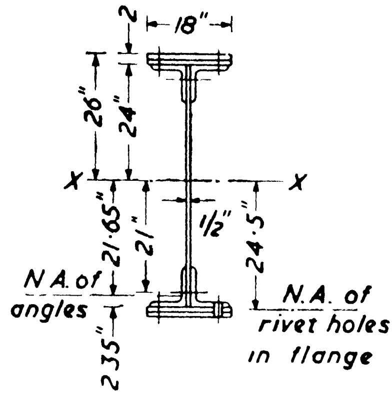


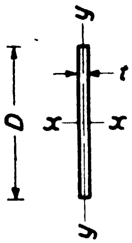
FIG. 439

The web plate is 48 in. deep and $\frac{1}{2}$ in. thick, the flange plates are 18 in. wide and 2 in. deep, while the angles are 8 in. × 8 in. × 1 in. The moment of inertia of this section, allowing for two rivet holes ($\frac{15}{16}$ in. diameter) in the web and two rivet holes in each flange, is calculated as follows:

I_{XX} for Plate Girder in Inch Units

Component	I_{xx}	A	y	y^2	Ay^2	$I_{xx}+Ay^2$
Web plate	4,608	—	—	—	—	4,608
2 Flange plates	24	72	25	625	45,000	45,024
4 Angles	351	60	21·65	468·7	28,122	28,473
−2 Rivet holes −4 Rivet holes	— —	−4·69 −11·25	21·00 24·50	441·0 600·3	−2,067 −6,753	−2,067 −6,753
					$I_{XX}=\Sigma I_{xx}+\Sigma Ay^2=$	69,285

The values of I_{xx}, y^2 and A for four angles are obtained from the tables, the values of y^2 being the same as the values of Ay^2 for unit areas.



MOMENTS OF INERTIA I_{xx} OF WEB PLATES

Depth D in Inches	Plate Thickness t in Inches							
	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$
4	1·33	1·67	2·00	2·33	2·67	3·00	3·33	3·67
5	2·60	3·26	3·91	4·56	5·21	5·86	6·51	7·16
6	4·50	5·63	6·75	7·88	9·00	10·13	11·25	12·38
7	7·15	8·93	10·72	12·51	14·29	16·08	17·86	19·65
8	10·67	13·33	16·00	18·67	21·33	24·00	26·67	29·33
9	15·19	18·98	22·78	26·58	30·38	34·17	37·97	41·77
10	20·83	26·04	31·25	36·46	41·67	46·87	52·08	57·29
11	27·73	34·66	41·59	48·53	55·46	62·39	69·32	76·26
12	36·00	45·00	54·00	63·00	72·00	81·00	90·00	99·00
13	45·77	57·21	68·66	80·1	91·54	103·0	114·4	125·9
14	57·17	71·46	85·75	100·0	114·3	128·6	142·9	157·2
15	70·31	87·89	105·5	123·1	140·6	158·2	175·8	193·4
16	85·33	106·7	128·0	149·3	170·7	192·0	213·3	234·7
17	102·4	127·9	153·5	179·1	204·7	230·3	255·9	281·5
18	121·5	151·9	182·3	212·6	243·0	273·4	303·8	334·1
19	142·9	178·6	214·3	250·1	285·8	321·5	357·2	393 0
20	166·7	208·3	250·0	291·7	333·3	375·0	416·7	458·3
21	192·9	241·2	289·4	337·6	385·9	434·1	482·3	530·6
22	221·8	277·3	332·8	388·2	443·7	499·1	554·6	610·0
23	253·5	316·9	380·2	443·6	507·0	570·3	633·7	697·1
24	288·0	360·0	432·0	504·0	576·0	648·0	720·0	792·0
25	325·5	406·9	488·3	569·7	651·0	732·4	813·8	895·2
26	366·2	457·7	549·3	640·8	732·3	823·9	915·4	1,007
27	410·1	512·6	615·1	717·6	820·1	922·6	1,025	1,128
28	457·3	571·7	686·0	800·3	914·7	1,029	1,143	1,258
29	508·1	635·1	762·2	889·2	1,016	1,143	1,270	1,397
30	562·5	703·1	843·8	984·4	1,125	1,266	1,406	1,547
31	620·7	775·8	931·0	1,086	1,241	1,396	1,552	1,707
32	682·7	853·3	1,024	1,195	1,365	1,536	1,707	1,877
33	748·7	935·8	1,123	1,310	1,497	1,685	1,872	2,059
34	818·8	1,024	1,228	1,433	1,638	1,842	2,047	2,252
35	893·3	1,117	1,340	1,563	1,786	2,010	2,233	2,456
36	972·0	1,215	1,458	1,701	1,944	2,187	2,430	2,673
37	1,055	1,319	1,583	1,847	2,111	2,374	2,638	2,902
38	1,143	1,429	1,715	2,001	2,286	2,572	2,858	3,144
39	1,236	1,545	1,854	2,163	2,472	2,781	3,090	3,398

MOMENTS OF INERTIA I_{xx} OF WEB PLATES

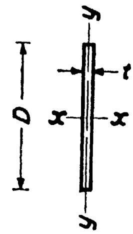
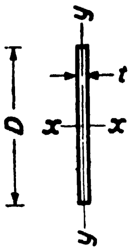


Plate Thickness t in Inches								*Depth D in Inches*
$\frac{3}{4}$	$\frac{13}{16}$	$\frac{7}{8}$	1	$1\frac{1}{8}$	$1\frac{1}{4}$	$1\frac{3}{8}$	$1\frac{1}{2}$	
4·00	4·33	4·67	5·33	6·00	6·66	7·33	8·00	4
7·81	8·46	9·11	10·42	11·72	13·02	14·32	15·63	5
13·50	14·63	15·75	18·00	20·25	22·50	24·75	27 00	6
21·44	23·22	25·01	28·58	32·16	35·73	39·30	42·88	7
32·00	34·67	37·33	42·67	48·00	53·33	58·67	64·00	8
45·56	49·36	53·16	60·75	68·34	75·94	83·53	91·13	9
62·50	67·71	72·92	83·33	93·75	104·2	114·6	125·0	10
83·19	90·12	97·05	110·9	124·8	138·6	152·5	166·4	11
108·0	117·0	126·0	144·0	162·0	180·0	198·0	216·0	12
137·3	148·8	160·2	183·1	206·0	228·4	251·8	274·6	13
171·5	185·8	200·1	228·7	257·2	285·8	314·5	343·0	14
210·9	228·5	246·1	281·3	316·4	351·6	386·8	421·8	15
256·0	277·3	298·7	341·3	384·0	426·6	493·3	512·0	16
307·1	332·7	358·2	409·4	460·6	511·8	562·9	614·2	17
364·5	394·9	425·3	486·0	546·8	607·6	668·3	729·0	18
428·7	464·4	500·1	571·6	643·0	714·4	785·9	857·4	19
500·0	541·7	583·3	666·7	750·0	833·4	916·7	1,000	20
578·8	627·1	675·3	771·8	868·2	964·6	1,061	1,158	21
665·5	721·0	776·4	887·3	998·2	1,109	1,220	1,331	22
760·4	823·8	887·2	1,014	1,141	1,267	1,394	1,521	23
864·0	936·0	1,008	1,152	1,296	1,440	1,584	1,728	24
976·6	1,058	1,139	1,302	1,465	1,628	1,790	1,953	25
1,099	1,190	1,282	1,465	1,648	1,831	2,014	2,197	26
1,230	1,333	1,435	1,640	1,845	2,050	2,255	2,460	27
1,372	1,486	1,601	1,829	2,058	2,287	2,515	2,744	28
1,524	1,651	1,778	2,032	2,286	2,541	2,795	3,048	29
1,688	1,828	1,969	2,250	2,531	2,813	3,094	3,376	30
1,862	2,017	2,172	2,483	2,793	3,103	3,414	3,724	31
2,048	2,219	2,389	2,731	3,072	3,413	3,755	4,096	32
2,246	2,433	2,620	2,995	3,369	3,743	4,118	4,492	33
2,456	2,661	2,866	3,275	3,685	4,094	4,504	4,912	34
2,680	2,903	3,126	3,573	4,020	4,466	4,913	5,359	35
2,916	3,159	3,402	3,888	4,374	4,860	5,346	5,832	36
3,166	3,430	3,693	4,221	4,749	5,276	5,804	6,332	37
3,429	3,715	4,001	4,573	5,144	5,716	6,288	6,858	38
3,707	4,016	4,325	4,943	5,561	6,179	6,797	7,414	39



MOMENTS OF INERTIA I_{xx} OF WEB PLATES

Depth D in Inches	Plate Thickness t in Inches							
	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$
40	1,333	1,667	2,000	2,333	2,667	3,000	3,333	3,667
41	1,436	1,795	2,154	2,513	2,872	3,231	3,590	3,949
42	1,543	1,929	2,315	2,701	3,087	3,473	3,859	4,245
43	1,656	2,070	2,485	2,899	3,313	3,727	4,141	4,555
44	1,775	2,218	2,662	3,106	3,549	3,993	4,437	4,880
45	1,898	2,373	2,848	3,322	3,797	4,272	4,746	5,221
46	2,028	2,535	3,042	3,549	4,056	4,563	5,070	5,577
47	2,163	2,704	3,245	3,785	4,326	4,867	5,407	5,949
48	2,304	2,880	3,456	4,032	4,608	5,184	5,760	6,336
49	2,451	3,064	3,677	4,289	4,902	5,515	6,128	6,741
50	2,604	3,255	3,906	4,557	5,208	5,859	6,510	7,161
51	2,764	3,454	4,145	4,836	5,527	6,218	6,909	7,599
52	2,929	3,662	4,394	5,126	5,859	6,591	7,323	8,056
53	3,102	3,877	4,652	5,428	6,203	6,979	7,754	8,529
54	3,281	4,101	4,921	5,741	6,561	7,381	8,201	9,021
55	3,466	4,333	5,199	6,066	6,932	7,799	8,665	9,532
56	3,659	4,573	5,488	6,403	7,317	8,232	9,147	10,061
57	3,858	4,823	5,787	6,752	7,716	8,681	9,646	10,610
58	4,065	5,081	6,097	7,113	8,130	9,146	10,162	11,178
59	4,279	5,349	6,418	7,488	8,558	9,627	10,697	11,767
60	4,500	5,625	6,750	7,875	9,000	10,125	11,250	12,375
62	4,965	6,206	7,448	8,689	9,930	11,172	12,413	13,654
64	5,461	6,827	8,192	9,557	10,923	12,288	13,653	15,019
66	5,990	7,487	8,984	10,482	11,979	13,476	14,974	16,471
68	6,551	8,188	9,826	11,464	13,101	14,739	16,377	18,014
70	7,146	8,932	10,719	12,505	14,292	16,078	17,865	19,651
72	7,776	9,720	11,664	13,608	15,552	17,496	19,440	21,384
74	8,442	10,553	12,663	14,774	16,884	18,995	21,105	23,216
76	9,145	11,432	13,718	16,004	18,291	20,577	22,863	25,150
78	9,887	12,358	14,830	17,301	19,773	22,245	24,716	27,188
80	10,667	13,333	16,000	18,667	21,333	24,000	26,667	29,333
82	11,487	14,359	17,230	20,102	22,974	25,845	28,717	31,589
84	12,348	15,435	18,522	21,609	24,696	27,783	30,870	33,957
86	13,251	16,564	19,877	23,190	26,502	29,815	33,128	36,441
88	14,197	17,747	21,296	24,845	28,395	31,944	35,493	39,043
90	15,188	18,984	22,781	26,578	30,375	34,172	37,969	41,766
92	16,223	20,278	24,334	28,390	32,445	36,501	40,557	44,612
94	17,304	21,630	25,956	30,282	34,608	38,934	43,260	47,586
96	18,432	23,040	27,648	32,256	36,864	41,472	46,080	50,688

MOMENTS OF INERTIA I_{xx} OF WEB PLATES

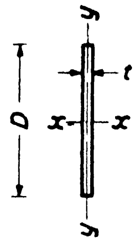
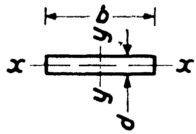


Plate Thickness t in Inches								Depth D in Inches
3/4	13/16	7/8	1	1 1/8	1 1/4	1 3/8	1 1/2	
4,000	4,333	4,667	5,333	6,000	6,667	7,333	8,000	40
4,308	4,667	5,025	5,743	6,461	7,179	7,897	8,615	41
4,630	5,016	5,402	6,174	6,946	7,718	8,489	9,261	42
4,969	5,383	5,797	6,626	7,454	8,282	9,110	9,938	43
5,324	5,768	6,211	7,099	7,986	8,873	9,761	10,648	44
5,695	6,170	6,645	7,594	8,543	9,492	10,442	11,391	45
6,084	6,591	7,097	8,111	9,125	10,139	11,153	12,167	46
6,489	7,030	7,570	8,652	9,733	10,815	11,896	12,977	47
6,912	7,488	8,064	9,216	10,368	11,520	12,672	13,824	48
7,353	7,966	8,579	9,804	11,030	12,255	13,481	14,706	49
7,813	8,464	9,115	10,417	11,719	13,021	14,323	15,625	50
8,291	8,982	9,673	11,054	12,436	13,818	15,199	16,581	51
8,788	9,520	10,253	11,717	13,182	14,647	16,111	17,576	52
9,305	10,080	10,856	12,406	13,957	15,508	17,058	18,610	53
9,842	10,662	11,482	13,122	14,762	16,403	18,042	19,683	54
10,398	11,265	12,132	13,865	15,598	17,331	19,064	20,796	55
10,976	11,891	12,805	14,635	16,464	18,293	20,122	21,952	56
11,575	12,539	13,504	15,433	17,362	19,291	21,220	23,150	57
12,195	13,211	14,227	16,259	18,292	20,324	22,356	24,390	58
12,836	13,906	14,976	17,115	19,254	21,394	23,533	25,672	59
13,500	14,625	15,750	18,000	20,250	22,500	24,750	27,000	60
14,896	16,137	17,378	19,861	22,344	24,826	27,308	29,792	62
16,384	17,749	19,115	21,845	24,576	27,306	30,038	32,768	64
17,969	19,466	20,963	23,958	26,952	29,948	32,942	35,938	66
19,652	21,290	22,927	26,203	29,478	32,754	36,028	39,304	68
21,438	23,224	25,010	28,583	32,156	35,730	39,302	42,876	70
23,328	25,272	27,216	31,104	34,992	38,880	42,768	46,656	72
25,327	27,437	29,548	33,769	37,990	42,210	46,432	50,654	74
27,436	29,722	32,009	36,581	41,154	45,726	50,300	54,872	76
29,660	32,131	34,603	39,546	44,490	49,432	54,376	59,320	78
32,000	34,667	37,333	42,667	48,000	53,334	58,666	64,000	80
34,461	37,332	40,204	45,947	51,690	57,434	63,178	68,922	82
37,044	40,131	43,218	49,392	55,566	61,740	67,914	74,088	84
39,754	43,067	46,379	53,005	59,630	66,256	72,882	79,508	86
42,592	46,141	49,691	56,789	63,888	70,986	78,086	85,184	88
45,563	49,359	53,156	60,750	68,344	75,938	83,532	91,126	90
48,668	52,724	56,779	64,891	73,002	81,114	89,224	97,336	92
51,912	56,238	60,563	69,215	77,868	86,520	95,172	103,824	94
55,296	59,904	64,512	73,728	82,944	92,160	101,376	110,592	96



MOMENTS OF INERTIA I_{xx} OF FLANGE PLATES

Depth d in inches	Plate breadth b in Inches						
	4	5	6	7	8	9	10
3/8	0·018	0·022	0·026	0·031	0·035	0·040	0·044
7/16	0·028	0·035	0·042	0·049	0·056	0·063	0·070
1/2	0·042	0·052	0·062	0·073	0·083	0·094	0·104
9/16	0·059	0·074	0·089	0·104	0·119	0·133	0·148
5/8	0·081	0·102	0·122	0·142	0·163	0·183	0·203
11/16	0·108	0·135	0·162	0·190	0·217	0·244	0·271
3/4	0·141	0·176	0·211	0·246	0·281	0·316	0·352
13/16	0·179	0·223	0·268	0·313	0·358	0·402	0·447
7/8	0·223	0·279	0·335	0·391	0·447	0·502	0·558
15/16	0·275	0·343	0·412	0·481	0·549	0·618	0·687
1	0·333	0·417	0·500	0·583	0·667	0·750	0·833
1 1/8	0·475	0·593	0·712	0·831	0·949	1·068	1·187
1 1/4	0·651	0·814	0·976	1·139	1·302	1·465	1·628
1 3/8	0·867	1·083	1·300	1·517	1·733	1·950	2·166
1 1/2	1·125	1·406	1·687	1·969	2·250	2·531	2·812
1 5/8	1·430	1·788	2·145	2·503	2·861	3·218	3·576
1 3/4	1·786	2·233	2·679	3·126	3·573	4·019	4·466
1 7/8	2·197	2·747	3·296	3·845	4·395	4·944	5·493
2	2·667	3·333	4·000	4·667	5·333	6·000	6·667
2 1/8	3·199	3·998	4·798	5·598	6·397	7·197	7·996
2 1/4	3·797	4·746	5·695	6·645	7·594	8·543	9·492
2 3/8	4·465	5·582	6·698	7·814	8·931	10·047	11·164
2 1/2	5·208	6·510	7·812	9·115	10·417	11·719	13·021
2 5/8	6·029	7·537	9·044	10·551	12·059	13·566	15·073
2 3/4	6·932	8·665	10·398	12·132	13·865	15·598	17·331
2 7/8	7·921	9·902	11·882	13·862	15·842	17·823	19·803
3	9·000	11·250	13·500	15·750	18·000	20·250	22·500

MOMENTS OF INERTIA I_{xx} OF FLANGE PLATES

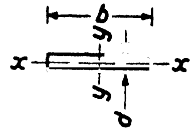
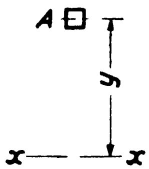
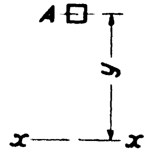


Plate breadth b in Inches							*Depth d in inches*
12	14	16	18	20	22	24	
0·053	0·062	0·070	0·079	0·088	0·097	0·105	$\frac{3}{8}$
0·084	0·098	0·112	0·126	0·140	0·154	0·167	$\frac{7}{16}$
0·125	0·146	0·167	0·187	0·208	0·229	0·250	$\frac{1}{2}$
0·178	0·208	0·237	0·267	0·297	0·326	0·356	$\frac{9}{16}$
0·244	0·285	0·326	0·366	0·407	0·447	0·488	$\frac{5}{8}$
0·325	0·379	0·433	0·487	0·542	0·596	0·650	$\frac{11}{16}$
0·422	0·492	0·562	0·633	0·703	0·774	0·844	$\frac{3}{4}$
0·536	0·626	0·715	0·805	0·894	0·983	1·073	$\frac{13}{16}$
0·670	0·782	0·893	1·005	1·117	1·228	1·340	$\frac{7}{8}$
0·824	0·961	1·099	1·236	1·373	1·511	1·648	$\frac{15}{16}$
1·000	1·167	1·333	1·500	1·667	1·833	2·000	1
1·424	1·661	1·898	2·136	2·373	2·611	2·848	$1\frac{1}{8}$
1·952	2·279	2·604	2·930	3·255	3·580	3·906	$1\frac{1}{4}$
2·600	3·033	3·466	3·899	4·333	4·767	5·199	$1\frac{3}{8}$
3·375	3·937	4·500	5·062	5·625	6·187	6·750	$1\frac{1}{2}$
4·291	5·006	5·721	6·437	7·152	7·867	8·582	$1\frac{5}{8}$
5·359	6·253	7·146	8·039	8·932	9·825	10·719	$1\frac{3}{4}$
6·592	7·690	8·789	9·888	10·986	12·086	13·184	$1\frac{7}{8}$
8·000	9·333	10·667	12·000	13·333	14·667	16·000	2
9·596	11·195	12·794	14·394	15·993	17·593	19·191	$2\frac{1}{8}$
11·391	13·289	15·187	17·086	18·984	20·883	22·781	$2\frac{1}{4}$
13·396	15·629	17·862	20·095	22·327	24·559	26·793	$2\frac{3}{8}$
15·625	18·229	20·833	23·437	26·042	28·646	31·250	$2\frac{1}{2}$
18·088	21·103	24·117	27·132	30·146	33·161	36·176	$2\frac{5}{8}$
20·797	24·263	27·729	31·195	34·661	38·128	41·594	$2\frac{3}{4}$
23·764	27·724	31·685	35·646	39·606	43·567	47·527	$2\frac{7}{8}$
27·000	31·500	36·000	40·500	45·000	49·500	54·000	3



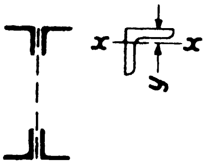
MOMENTS OF INERTIA Ay^2 OF UNIT AREAS

Value of y	0·0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9
5	25·00	26·01	27·04	28·09	29·16	30·25	31·36	32·49	33·64	34·81
6	36·00	37·21	38·44	39·69	40·96	42·25	43·56	44·89	46·24	47·61
7	49·00	50·41	51·84	53·29	54·76	56·25	57·76	59·29	60·84	62·41
8	64·00	65·61	67·24	68·89	70·56	72·25	73·96	75·69	77·44	79·21
9	81·00	82·81	84·64	86·49	88·36	90·25	92·16	94·09	96·04	98·01
10	100·0	102·0	104·0	106·1	108·2	110·3	112·4	114·5	116·6	118·8
11	121·0	123·2	125·4	127·7	130·0	132·3	134·6	136·9	139·2	141·6
12	144·0	146·4	148·8	151·3	153·8	156·3	158·8	161·3	163·8	166·4
13	169·0	171·6	174·2	176·9	179·6	182·3	185·0	187·7	190·4	193·2
14	196·0	198·8	201·6	204·5	207·4	210·3	213·2	216·1	219·0	222·0
15	225·0	228·0	231·0	234·1	237·2	240·3	243·4	246·5	249·6	252·8
16	256·0	259·2	262·4	265·7	269·0	272·3	275·6	278·9	282·2	285·6
17	289·0	292·4	295·8	299·3	302·8	306·3	309·8	313·3	316·9	320·4
18	324·0	327·6	331·2	334·9	338·6	342·3	346·0	349·7	353·4	357·2
19	361·0	364·8	368·6	372·5	376·4	380·3	384·2	388·1	392·0	396·0
20	400·0	404·0	408·0	412·1	416·2	420·3	424·4	428·5	432·6	436·8
21	441·0	445·2	449·4	453·7	458·0	462·3	466·6	470·9	475·2	479·6
22	484·0	488·4	492·9	497·3	501·8	506·3	510·8	515·3	519·8	524·4
23	529·0	533·6	538·2	542·9	547·6	552·3	557·0	561·7	566·4	571·2
24	576·0	580·8	585·6	590·5	595·4	600·3	605·2	610·1	615·0	620·0
25	625·0	630·0	635·0	640·1	645·2	650·3	655·4	660·5	665·6	670·8
26	676·0	681·2	686·4	691·7	697·0	702·3	707·6	712·9	718·2	723·6
27	729·0	734·4	739·8	745·3	750·8	756·3	761·8	767·3	772·8	778·4
28	784·0	789·6	795·2	800·9	806·6	812·3	818·0	823·7	829·4	835·2
29	841·0	846·8	852·6	858·5	864·4	870·3	876·2	882·1	888·0	894·0
30	900·0	906·0	912·0	918·1	924·2	930·3	936·4	942·5	948·6	954·8
31	961·0	967·2	973·4	979·7	986·0	992·3	998·6	1005	1011	1018
32	1024	1030	1037	1043	1050	1056	1063	1069	1076	1082
33	1089	1096	1102	1109	1116	1122	1129	1136	1142	1149
34	1156	1163	1170	1176	1183	1190	1197	1204	1211	1218
35	1225	1232	1239	1246	1253	1260	1267	1274	1282	1289
36	1296	1303	1310	1318	1325	1332	1340	1347	1354	1362
37	1369	1376	1384	1391	1399	1406	1414	1421	1429	1436
38	1444	1425	1459	1467	1475	1482	1490	1498	1505	1513
39	1521	1529	1537	1544	1552	1560	1568	1576	1584	1592
40	1600	1608	1616	1624	1632	1640	1648	1656	1665	1673
41	1681	1689	1697	1706	1714	1722	1731	1739	1747	1756
42	1764	1772	1781	1789	1798	1806	1815	1823	1832	1840
43	1849	1858	1866	1875	1884	1892	1901	1910	1918	1927
44	1936	1945	1954	1962	1971	1980	1989	1998	2007	2016
45	2025	2034	2043	2052	2061	2070	2079	2088	2098	2107
46	2116	2125	2134	2144	2153	2162	2172	2181	2190	2200
47	2209	2218	2228	2237	2247	2256	2266	2275	2285	2294
48	2304	2314	2323	2333	2343	2352	2362	2372	2381	2391
49	2401	2411	2421	2430	2440	2450	2460	2470	2480	2490



MOMENTS OF INERTIA Ay^2 OF UNIT AREAS

Value of *y*	0·0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9
50	2500	2510	2520	2530	2540	2550	2560	2570	2581	2591
51	2601	2611	2621	2632	2642	2652	2663	2673	2683	2694
52	2704	2714	2725	2735	2746	2756	2767	2777	2788	2798
53	2809	2820	2830	2841	2852	2862	2873	2884	2894	2905
54	2916	2927	2938	2948	2959	2970	2981	2992	3003	3014
55	3025	3036	3047	3058	3069	3080	3091	3102	3114	3125
56	3136	3147	3158	3170	3181	3192	3204	3215	3226	3238
57	3249	3260	3272	3283	3295	3306	3318	3329	3341	3352
58	3364	3376	3387	3399	3411	3422	3434	3446	3457	3469
59	3481	3493	3505	3516	3528	3540	3552	3564	3576	3588
60	3600	3612	3624	3636	3648	3660	3672	3684	3697	3709
61	3721	3733	3745	3758	3770	3782	3795	3807	3819	3832
62	3844	3856	3869	3881	3894	3906	3919	3931	3944	3956
63	3969	3982	3994	4007	4020	4032	4045	4058	4070	4083
64	4096	4109	4122	4134	4147	4160	4173	4186	4199	4212
65	4225	4238	4251	4264	4277	4290	4303	4316	4330	4343
66	4356	4369	4382	4396	4409	4422	4436	4449	4462	4476
67	4489	4502	4516	4529	4543	4556	4570	4583	4597	4610
68	4624	4638	4651	4665	4679	4692	4706	4720	4733	4747
69	4761	4775	4789	4802	4816	4830	4844	4858	4872	4886
70	4900	4914	4928	4942	4956	4970	4984	4998	5013	5027
71	5041	5055	5069	5084	5098	5112	5127	5141	5155	5170
72	5184	5198	5213	5227	5242	5256	5271	5285	5300	5314
73	5329	5344	5358	5373	5388	5402	5417	5432	5446	5461
74	5476	5491	5506	5520	5535	5550	5565	5580	5595	5610
75	5625	5640	5655	5670	5685	5700	5715	5730	5746	5761
76	5776	5791	5806	5822	5837	5852	5868	5883	5898	5914
77	5929	5944	5960	5975	5991	6006	6022	6037	6053	6068
78	6084	6100	6115	6131	6147	6162	6178	6194	6209	6225
79	6241	6257	6273	6288	6304	6320	6336	6352	6368	6384
80	6400	6416	6432	6448	6464	6480	6496	6512	6529	6545
81	6561	6577	6593	6610	6626	6642	6659	6675	6691	6708
82	6724	6740	6757	6773	6790	6806	6823	6839	6856	6872
83	6889	6906	6922	6939	6956	6972	6989	7006	7022	7039
84	7056	7073	7090	7106	7123	7140	7157	7174	7191	7208
85	7225	7242	7259	7276	7293	7310	7327	7344	7362	7379
86	7396	7413	7430	7448	7465	7482	7500	7517	7534	7552
87	7569	7586	7604	7621	7639	7656	7674	7691	7709	7726
88	7744	7762	7779	7797	7815	7832	7850	7868	7885	7903
89	7921	7939	7957	7974	7992	8010	8028	8046	8064	8082
90	8100	8118	8136	8154	8172	8190	8208	8226	8245	8263
91	8281	8299	8317	8336	8354	8372	8391	8409	8427	8446
92	8464	8482	8501	8519	8538	8556	8575	8593	8612	8630
93	8649	8668	8686	8705	8724	8742	8761	8780	8798	8817
94	8836	8855	8874	8892	8911	8930	8949	8968	8987	9006
95	9025	9044	9063	9082	9101	9120	9139	9158	9178	9197
96	9216	9235	9254	9274	9293	9312	9332	9351	9370	9390
97	9409	9428	9448	9467	9487	9506	9526	9545	9565	9584
98	9604	9624	9643	9663	9683	9702	9722	9742	9761	9781
99	9801	9821	9841	9860	9880	9900	9920	9940	9960	9980



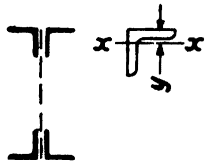
PROPERTIES OF FOUR EQUAL ANGLES

y = distance from the centroid to the back of an angle

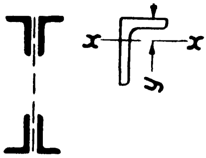
Leg Lengths (*in.*)	*Thicknesses* (*in.*)	$4 \times I_{xx}$ (*in.*^{4})	4 × *Gross Area* (*sq. in.*)	*Distance from Centroid to Back of Angle* (*in.*)
2¼ × 2¼	0·1875	1·52	3·24	0·62
	0·25	1·96	4·24	0·64
	0·3125	2·36	5·24	0·67
	0·375	2·76	6·20	0·69
2½ × 2½	0·25	2·72	4·76	0·70
	0·3125	3·32	5·84	0·73
	0·375	3·84	6·92	0·75
	0·4375	4·36	8·00	0·78
	0·5	4·84	9·00	0·80
3 × 3	0·25	4·80	5·76	0·83
	0·3125	5·88	7·12	0·85
	0·375	6·88	8·44	0·88
	0·4375	7·84	9·72	0·90
	0·5	8·72	11·00	0·92
	0·5625	9·56	12·24	0·95
3½ × 3½	0·25	7·76	6·76	0·95
	0·3125	9·52	8·36	0·97
	0·375	11·20	9·96	1·00
	0·4375	12·80	11·48	1·02
	0·5	14·28	13·00	1·05
	0·5625	15·72	14·48	1·07
	0·625	17·08	15·96	1·09
4 × 4	0·3125	14·44	9·60	1·10
	0·375	17·04	11·44	1·12
	0·4375	19·48	13·24	1·15
	0·5	21·84	15·00	1·17
	0·5625	24·08	16·72	1·20
	0·625	26·24	18·44	1·22
	0·6875	28·32	20·12	1·24
	0·75	30·28	21·76	1·26
4½ × 4½	0·3125	20·84	10·88	1·22
	0·375	24·60	12·96	1·24
	0·4375	28·20	15·00	1·27
	0·5	31·68	17·00	1·29
	0·5625	35·04	19·00	1·32
	0·625	38·24	20·96	1·34
	0·6875	41·36	22·88	1·37
	0·75	44·32	24·76	1·39
5 × 5	0·375	34·12	14·44	1·37
	0·4375	39·24	16·76	1·39
	0·5	44·16	19·00	1·42
	0·5625	48·88	21·24	1·44
	0·625	53·48	23·44	1·47
	0·6875	57·88	25·60	1·49
	0·75	62·16	27·76	1·51

PROPERTIES OF FOUR EQUAL ANGLES

y = distance from the centroid to the back of an angle



Leg Lengths (*in.*)	*Thicknesses* (*in.*)	$4 \times I_{xx}$ (*in.*^{4})	4 × *Gross Area* (*sq. in.*)	*Distance from Centroid to Back of Angle* (*in.*)
6 × 6	0·375	59·80	17·44	1·61
	0·4375	69·00	20·24	1·64
	0·5	77·92	23·00	1·66
	0·5625	86·56	25·72	1·69
	0·625	94·92	28·44	1·71
	0·6875	103·08	31·12	1·74
	0·75	110·96	33·76	1·76
	0·8125	118·60	36·36	1·78
	0·875	126·04	38·92	1·81
7 × 7	0·5	125·68	27·00	1·91
	0·5625	139·92	30·24	1·93
	0·625	153·80	33·44	1·96
	0·6875	167·32	36·60	1·98
	0·75	180·48	39·76	2·01
	0·8125	193·32	42·88	2·03
	0·875	205·80	45·92	2·06
	0·9375	218·00	49·00	2·08
	1·00	229·84	52·00	2·10
8 × 8	0·625	233·04	38·44	2·20
	0·6875	253·92	42·12	2·23
	0·75	274·32	45·76	2·25
	0·8125	294·28	49·36	2·28
	0·875	313·76	52·96	2·30
	0·9375	332·80	56·48	2·33
	1·00	351·40	60·00	2·35



PROPERTIES OF FOUR UNEQUAL ANGLES

Long legs parallel with the web

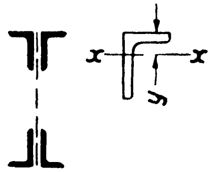
y = distance from the centroid to the back of an angle

Leg Lengths (*in.*)	*Thicknesses* (*in.*)	$4 \times I_{xx}$ (*in.*^{4})	4 × *Gross Area* (*sq. in.*)	*Distance from Centroid to Back of Angle* (*in.*)
2½ × 2	0·1875	1·96	3·24	0·75
	0·25	2·52	4·24	0·77
	0·3125	3·08	5·24	0·80
	0·375	3·56	6·20	0·82
3 × 2	0·1875	3·24	3·60	0·95
	0·25	4·24	4·76	0·98
	0·3125	5·16	5·84	1·00
	0·375	6·00	6·92	1·03
	0·4375	6·84	8·00	1·05
3 × 2½	0·25	4·56	5·24	0·89
	0·3125	5·56	6·48	0·92
	0·375	6·48	7·68	0·94
	0·4375	7·36	8·88	0·97
3½ × 2½	0·25	7·00	5·76	1·09
	0·3125	8·56	7·12	1·12
	0·375	10·04	8·44	1·15
	0·4375	11·44	9·72	1·17
3½ × 3	0·25	7·44	6·24	1·01
	0·3125	9·08	7·72	1·04
	0.375	10·68	9·20	1·07
	0·4375	12·16	10·60	1·09
	0·5	13·60	12·00	1·12
	0·5625	14·96	13·36	1·14
4 × 2½	0·25	10·16	6·24	1·30
	0·3125	12·44	7·72	1·33
	0·375	14·64	9·20	1·36
	0·4375	16·72	10·60	1·38
4 × 3	0·3125	13·20	8·36	1·24
	0·375	15·56	9·96	1·27
	0·4375	17·76	11·48	1·29
	0·5	19·88	13·00	1·32
	0·5625	21·92	14·48	1·34
4 × 3½	0·3125	13·88	9·00	1·16
	0·375	16·36	10·68	1·19
	0·4375	18·72	12·36	1·21
	0·5	20·96	14·00	1·24
	0·5625	23·08	15·60	1·26
	0·625	25·12	17·20	1·29
4½ × 3	C·3125	18·36	9·00	1·44
	0·375	21·64	10·68	1·47
	0·4375	24·76	12·36	1·50
	0·5	27·76	14·00	1·52
	0·5625	30·64	15·60	1·55

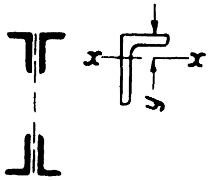
PROPERTIES OF FOUR UNEQUAL ANGLES

Long legs parallel with the web

y = distance from the centroid to the back of an angle



Leg Lengths (*in.*)	*Thicknesses* (*in.*)	$4 \times I_{xx}$ (*in.*^{4})	$4 \times$ *Gross Area* (*sq. in.*)	*Distance from Centroid to Back of Angle* (*in.*)
5 × 3	0·3125	24·56	9·60	1·66
	0·375	29·00	11·44	1·68
	0·4375	33·24	13·24	1·71
	0·5	37·32	15·00	1·73
	0·5625	41·24	16·72	1·76
5 × 3½	0·3125	25·84	10·24	1 56
	0·375	30·52	12·20	1·59
	0·4375	35·04	14·12	1·62
	0·5	39·36	16·00	1·64
	0·5625	43·52	17·88	1·66
	0·625	47·56	19·68	1·69
5 × 4	0·375	31·88	12·96	1·51
	0·4375	36·60	15·00	1·53
	0·5	41·16	17·00	1·56
	0·5625	45·56	19·00	1·58
	0·625	49·76	20·96	1·61
6 × 3	0·3125	40·52	10·88	2·09
	0·375	47·96	12·96	2·12
	0·4375	55·12	15·00	2·15
	0·5	62·04	17·00	2·17
	0·5625	68·72	19·00	2·20
	0·625	75·20	20·96	2·22
6 × 3½	0·3125	42·60	11·48	1·98
	0·375	50·48	13·68	2·01
	0·4375	58·08	15·84	2·04
	0·5	65·44	18·00	2·06
	0·5625	72·52	20·12	2·09
	0·625	79·40	22·20	2·11
6 × 4	0·375	52·84	14·44	1·91
	0·4375	60·84	16·76	1·94
	0·5	68·56	19·00	1·97
	0·5625	76·04	21·24	1·99
	0·625	83·28	23·44	2·02
	0·6875	90·28	25·60	2·04
	0·75	97·04	27·76	2·06
7 × 3½	0·375	77·12	15·20	2·44
	0·4375	88·88	17·60	2·47
	0·5	100·28	20·00	2·50
	0·5625	111·36	22·36	2·53
	0·625	122·12	24·68	2·55
7 × 4	0·5	105·04	21·00	2·39
	0·5625	116·68	23·48	2·42
	0·625	128·00	25·92	2·44
	0·6875	139·00	28·36	2·47
	0·75	149·68	30·76	2·49



PROPERTIES OF FOUR UNEQUAL ANGLES

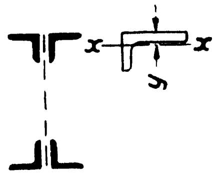
Long legs parallel with the web

y = distance from the centroid to the back of an angle

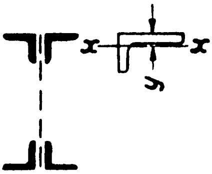
Leg Lengths (*in.*)	*Thicknesses* (*in.*)	$4 \times I_{xx}$ (*in.*^{4})	4 × *Gross Area* (*sq. in.*)	*Distance from Centroid to Back of Angle* (*in.*)
8 × 3½	0·375	111·16	16·68	2·89
	0·4375	128·32	19·36	2·92
	0·5	144·96	22·00	2·95
	0·5625	161·20	24·60	2·97
	0·625	176·96	27·20	3·00
8 × 4	0·5	151·80	23·00	2·83
	0·5625	168·88	25·72	2·86
	0·625	185·48	28·44	2·88
	0·6875	201·68	31·12	2·91
	0·75	217·44	33·76	2·93
8 × 6	0·5	173·88	27·00	2·44
	0·5625	193·72	30·24	2·47
	0·625	213·08	33·44	2·49
	0·6875	231·96	36·60	2·52
	0·75	250·40	39·76	2·54
	0·8125	268·40	42·88	2·57
	0·875	285·96	45·92	2·59
9 × 4	0·5	209·84	25·00	3·27
	0·5625	233·68	28·00	3·30
	0·625	256·92	30·92	3·33
	0·6875	279·64	33·88	3·36
	0·75	301·80	36·76	3·38
	0·8125	323·44	39·60	3·41
	0·875	344·52	42·44	3·43

PROPERTIES OF FOUR UNEQUAL ANGLES
Short legs parallel with the web

y = distance from the centroid to the back of an angle



Leg Lengths (*in.*)	*Thicknesses* (*in.*)	$4 \times I_{xx}$ (*in.*^{4})	$4 \times$ *Gross Area* (*sq. in.*)	*Distance from Centroid to Back of Angle* (*in.*)
2½ × 2	0·1875	1·12	3·24	0·50
	0·25	1·44	4·24	0·53
	0·3125	1·72	5·24	0·55
	0·375	2·00	6·20	0·57
3 × 2	0·1875	1·16	3·60	0·46
	0·25	1·52	4·76	0·48
	0·3125	1·80	5·84	0·51
	0·375	2·12	6·92	0·53
	0·4375	2·36	8·00	0·56
3 × 2½	0·25	2·88	5·24	0·65
	0·3125	3·48	6·48	0·67
	0·375	4·08	7·68	0·70
	0·4375	4·60	8·88	0·72
3½ × 2½	0·25	2·96	5·76	0·60
	0·3125	3·64	7·12	0·63
	0·375	4·24	8·44	0·65
	0·4375	4·80	9·72	0·68
3½ × 3	0·25	5·04	6·24	0·77
	0·3125	6·16	7·72	0·79
	0·375	7·20	9·20	0·82
	0·4375	8·20	10·60	0·84
	0·5	9·12	12·00	0·87
	0·5625	10·04	13·36	0·89
4 × 2½	0·25	3·08	6·24	0·56
	0·3125	3·76	7·72	0·59
	0·375	4·40	9·20	0·61
	0·4375	4·96	10·60	0·64
4 × 3	0·3125	6·36	8·36	0·75
	0·375	7·48	9·96	0·77
	0·4375	8·52	11·48	0·80
	0·5	9·48	13·00	0·82
	0·5625	10·40	14·48	0·84
4 × 3½	0·3125	9·88	9·00	0·92
	0·375	11·64	10·68	0·94
	0·4375	13·28	12·36	0·97
	0·5	14·88	14·00	0·99
	0·5625	16·36	15·60	1·01
	0·625	17·80	17·20	1·04
4½ × 3	0·3125	6·56	9·00	0·70
	0·375	7·68	10·68	0·73
	0·4375	8·76	12·36	0·75
	0·5	9·80	14·00	0·78
	0·5625	10·76	15·60	0·80



PROPERTIES OF FOUR UNEQUAL ANGLES

Short legs parallel with the web

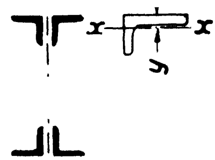
y = distance from the centroid to the back of an angle

Leg Lengths (*in.*)	*Thicknesses* (*in.*)	$4 \times I_{xx}$ (*in.*^{4})	$4 \times$ *Gross Area* (*sq. in.*)	*Distance from Centroid to Back of Angle* (*in.*)
5 × 3	0·3125	6·72	9·60	0·67
	0·375	7·88	11·44	0·69
	0·4375	9·00	13·24	0·72
	0·5	10·04	15·00	0·74
	0·5625	11·04	16·72	0·77
5 × 3½	0·3125	10·48	10·24	0·82
	0·375	12·36	12·20	0·85
	0·4375	14·12	14·12	0·87
	0·5	15·84	16·00	0·90
	0·5625	17·44	17·88	0·92
	0·625	18·96	19·68	0·94
5 × 4	0·375	18·12	12·96	1·01
	0·4375	20·76	15·00	1·04
	0·5	23·32	17·00	1·06
	0·5625	25·76	19·00	1·09
	0·625	28·08	20·96	1·11
6 × 3	0·3125	6·96	10·88	0·61
	0·375	8·20	12·96	0·63
	0·4375	9·36	15·00	0·66
	0·5	10·48	17·00	0·68
	0·5625	11·52	19·00	0·71
	0·625	12·56	20·96	0·73
6 × 3½	0·3125	10·88	11·48	0·75
	0·375	12·84	13·68	0·77
	0·4375	14·72	15·84	0·80
	0·5	16·52	18·00	0·82
	0·5625	18·20	20·12	0·85
	0·625	19·84	22·20	0·87
6 × 4	0·375	18·96	14·44	0·92
	0·4375	21·76	16·76	0·95
	0·5	24·44	19·00	0·97
	0·5625	27·04	21·24	1·00
	0·625	29·48	23·44	1·02
	0·6875	31·84	25·60	1·05
	0·75	34·12	27·76	1·07
7 × 3½	0·375	13·24	15·20	0·71
	0·4375	15·20	17·60	0·74
	0·5	17·08	20·00	0·76
	0·5625	18·84	22·36	0·79
	0·625	20·56	24·68	0·81
7 × 4	0·5	25·32	21·00	0·90
	0·5625	28·00	23·48	0·93
	0·625	30·56	25·92	0·95
	0·6875	33·04	28·36	0·98
	0·75	35·44	30·76	1·00

PROPERTIES OF FOUR UNEQUAL ANGLES

Short legs parallel with the web

y = distance from the centroid to the back of an angle



Leg Lengths (*in.*)	*Thicknesses* (*in.*)	$4 \times I_{xx}$ (*in.*^{4})	4 × *Gross Area* (*sq. in.*)	*Distance from Centroid to Back of Angle* (*in.*)
8 × 3½	0·375	13·56	16·68	0·66
	0·4375	15·56	19·36	0·69
	0·5	17·52	22·00	0·72
	0·5625	19·36	24·60	0·74
	0·625	21·12	27·20	0·77
8 × 4	0·5	26·00	23·00	0·85
	0·5625	28·80	25·72	0·87
	0·625	31·48	28·44	0·90
	0·6875	34·08	31·12	0·92
	0·75	36·56	33·76	0·94
8 × 6	0·5	84·32	27·00	1·45
	0·5625	93·76	30·24	1·47
	0·625	102·96	33·44	1·50
	0·6875	111·88	36·60	1·52
	0·75	120·56	39·76	1·55
	0·8125	128·96	42·88	1·57
	0·875	137·16	45·92	1·60
9 × 4	0·5	26·60	25·00	0·80
	0·5625	29·48	28·00	0·82
	0·625	32·24	30·92	0·85
	0·6875	34·92	33·88	0·87
	0·75	37·48	36·76	0·90
	0·8125	39·96	39·60	0·92
	0·875	42·40	42·44	0·95

33. RIVET GROUPS

THERE are two general cases of moments on rivet groups:

Case I. That in which the moment is applied in the plane of the connection and the centre of rotation is at the C.G. of the rivet group.

Case II. That in which the moment is applied at right angles to the plane of connection. It will be assumed here that the centre of rotation is at the centre of the lowest rivet, although engineers are not unanimous on this point.

Consider the first case, an example of which is shown in Fig. 440.

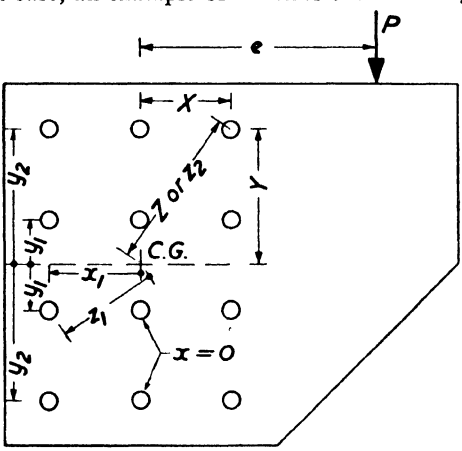


FIG. 440

Let P = any vertical load,

e = the eccentricity of application of P about the C.G. of the rivet group,

n = the number of rivets in the group,

x, y = the co-ordinates of any rivet using the C.G. of the rivet group as origin,

z = the distance of any rivet from the C.G.

$= \sqrt{(x^2 + y^2)}$,

X, Y = the co-ordinates of the rivet farthest from the G.G.

Z = the distance of this rivet from the C.G.

$= \sqrt{(X^2 + Y^2)}$

and F_b = the load on any rivet due to the moment $M = P \cdot e$

$= o \times z$, where o = load due to moment on a rivet at unit distance from the C.G. of the group.

Then the moment resisted by this rivet

$$= \text{load} \times \text{lever arm to C.G.}$$
$$= o \times z^2 = o \times (x^2 + y^2).$$

Hence, the total moment resisted by the group

$$=o(\Sigma x^2+\Sigma y^2)=P\,.\,e$$

i.e.
$$o=\frac{P\,.\,e}{(\Sigma x^2+\Sigma y^2)}.$$

Therefore the load due to the moment on the rivet receiving the maximum load

$$=F_b=o\times Z$$
$$=\frac{P\,.\,e\times Z}{(\Sigma x^2+\Sigma y^2)}=P\,.\,e\times\frac{Z}{(\Sigma x^2+\Sigma y^2)}$$

or
$$F=P\,.\,e\div\frac{(\Sigma x^2+\Sigma y^2)}{Z}$$

Now consider the most heavily loaded rivet, as shown in Fig. 441.

Let F_s = the direct load on any rivet due to the load P
$=P/n$.

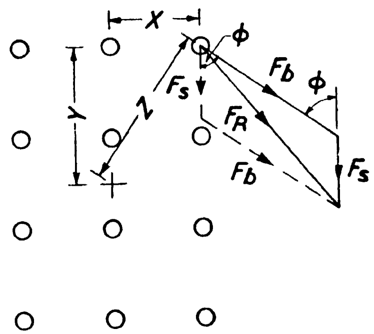


FIG. 441

Then the resultant load F_R on the rivet is a vector quantity derived from the moment load F_b and the direct load F_s. The value of F_R can, of course, be found graphically and it is often convenient to do so.

The horizontal component of $F_b=(Y/Z)\,.\,F_b$, while the vertical component $=(X/Z)\,.\,F_b$ (since the line of action of F_b is at right angles to the lever arm).

Hence
$$F_R=\sqrt{\left[\left(F_s+\frac{X}{Z}\,.\,F_b\right)^2+\left(\frac{Y}{Z}\,.\,F_b\right)^2\right]}$$
$$=\sqrt{\left(F_s^2+\frac{2X\,.\,F_s\,.\,F_b}{Z}+F_b^2\right)}$$
$$=\sqrt{(F_s^2+F_b^2+2F_s\,.\,F_b\cos\phi)}$$

where ϕ is the angle between the line of action of F_b and F_s.

None of the foregoing calculations is difficult, but with groups containing a large number of rivets a considerable amount of time can be spent calculating the value $(\Sigma x^2+\Sigma y^2)/Z$. Consequently, the tables shown on pages 669 to 696 have been prepared which give the values of $(\Sigma x^2+\Sigma y^2)/Z$ for a number of rivet groups which frequently occur in practice. Similarly, the table on page 668 gives values of $\Sigma y^2/Y$ for single vertical rows of rivets. All values are in inch units.

As an example, consider the brackets attached to the twin-stalk compound girder shown in Fig. 442.

It is required to calculate the stress in the most highly stressed rivet, say, the top right-hand rivet in the Figure. There are twelve rivets in a group, while the stanchions have 6-in. flanges and are at 8-in. centres. For 6-in.

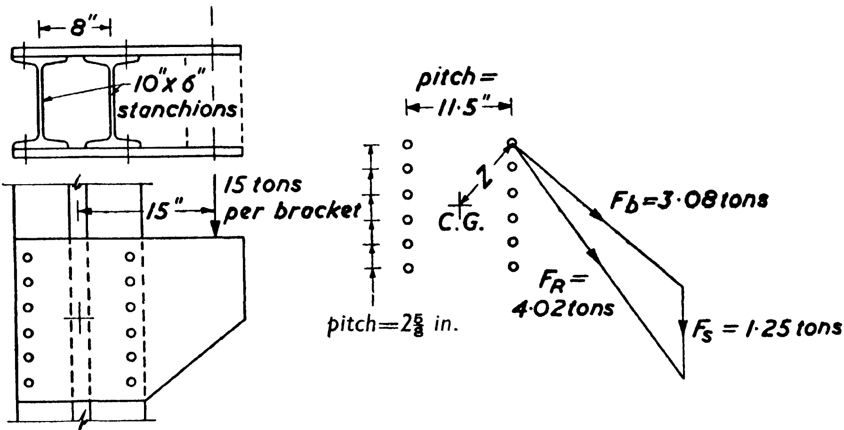


FIG. 442

flanges the spacing of the rivet holes is 3½-in., so that the horizontal pitch for the two vertical rows of rivets is (8 in. + 3½ in.) = 11½ in. The rivets are ⅞ in. in diameter, and they are at 2⅝ in. vertical pitch (i.e. 3 diameters). The value of $P \, . \, e$ for *each* bracket is $15 \times 15 = 225$ tons in.

On page 695 it will be found that the value of $(\Sigma x^2 + \Sigma y^2)/Z$ for this rivet group is 73·11 units. Therefore,

$$F_b = \frac{P \, . \, e \times Z}{(\Sigma x^2 + \Sigma y^2)} = \frac{225}{73 \cdot 11} = 3 \cdot 08 \text{ tons.}$$

Now $\quad F_s = \frac{P}{n} = \frac{15}{12} = 1 \cdot 25$ tons.

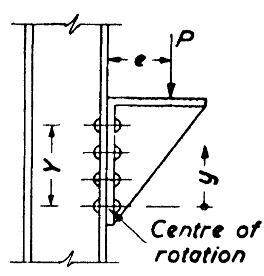


FIG. 443

If the values of F_b and F_s are drawn to scale as shown in Fig. 442, then the value of F_R may be found graphically to be approximately 4·02 tons.

The cross-sectional area of a ⅞-in. diameter rivet, which fills a $\frac{15}{16}$-in. hole, is 0·69 sq. in.

Hence, the stress on the rivet, which is in single shear,

$$= \frac{4 \cdot 02}{0 \cdot 69} = 5 \cdot 83 \text{ tons per sq. in.}$$

Fig. 443 shows an example of the second case of a rivet group where the applied moment is at right angles to the plane of the connection.

By analogy with the foregoing calculations it will be apparent that for one vertical row of rivets the tensile load on the topmost rivet due to the moment $P \, . \, e$ will be

$$m = \frac{P \, . \, e \times Y}{\Sigma y^2},$$

the co-ordinate Y being the distance between the top and bottom rivets, and y being the height of any rivet above the bottom rivet.

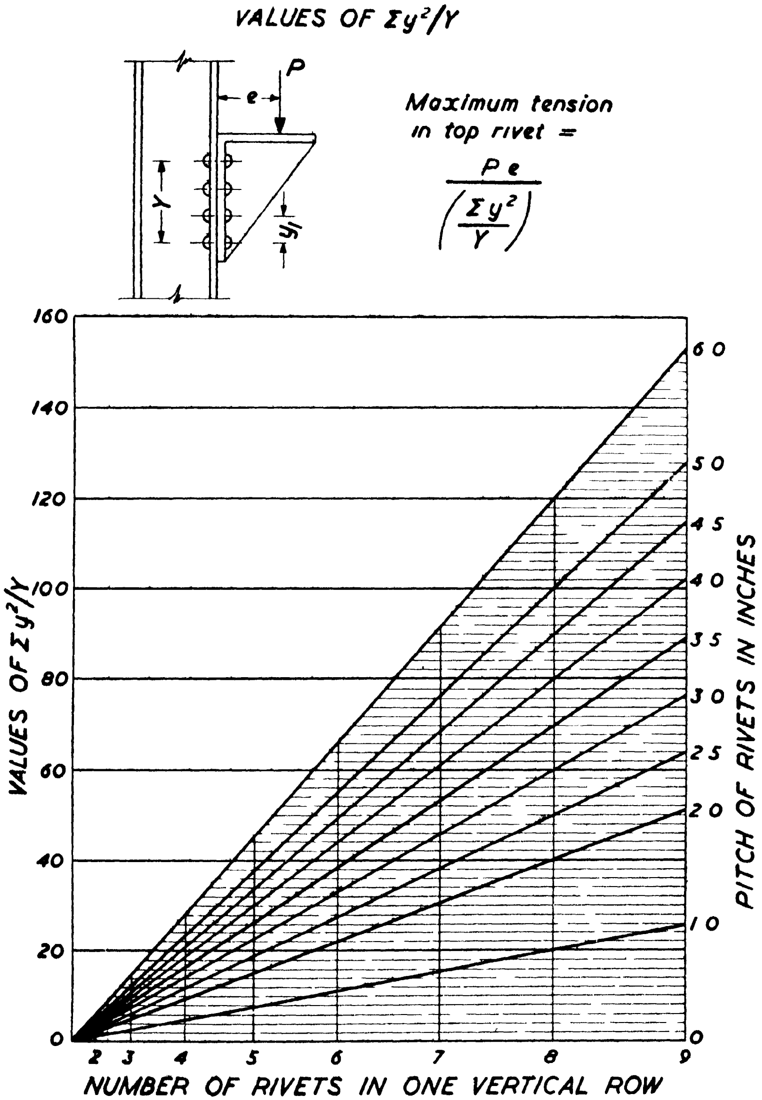
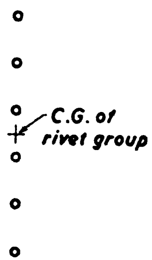


FIG 444

The chart shown in Fig 444 gives the values of $\Sigma y^2/Y$ for single rows of rivets up to nine in number and up to 6 in in pitch

The resultant force F_R on the top rivet is a vector quantity derived from the force F_b (given by the chart) and the direct force F_s (which equals P/n).



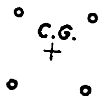
RIVET GROUPS UNDER MOMENT

SINGLE ROW

CENTRE OF ROTATION AT CENTRE OF GROUP

Values of $\frac{\Sigma y^2}{Y}$

Vertical Pitch in inches	Number of Rivets: 2	3	4	5	6	7	8	9
$1\frac{7}{8}$	1·875	3·750	6·250	9·38	13·12	17·50	22·50	28·12
2	2·000	4·000	6·667	10·0	14·00	18·67	24·00	30·00
$2\frac{1}{4}$	2·250	4·500	7·450	11·25	15·75	21·00	27·00	33·75
$2\frac{1}{2}$	2·500	5·000	8·333	12·50	17·50	23·33	30·00	37·50
$2\frac{5}{8}$	2·620	5·250	8·750	13·13	18·37	24·50	31·50	39·37
3	3·000	6·000	10·00	15·00	21·00	28·00	36·00	45·00
$3\frac{1}{2}$	3·500	7·000	11·67	17·50	24·50	32·67	42·00	52·50
4	4·000	8·000	13·33	20·00	28·00	37·33	48·00	60·00
$4\frac{1}{2}$	4·500	9·000	15·00	22·50	31·50	42·00	54·00	67·50
5	5·000	10·00	16·67	25·00	35·00	46·67	60·00	75·00
6	6·000	12·00	20·00	30·00	42·00	56·00	72·00	90·00



RIVET GROUPS UNDER MOMENT

CENTRE OF ROTATION AT CENTRE OF GROUP

4 Rivets (in 2 rows of 2)

Values of $\frac{\Sigma x^2 + \Sigma y^2}{Z}$

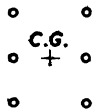
Vertical Pitch in inches	Horizontal Pitch in inches												
	$1\frac{7}{8}$	2	$2\frac{1}{4}$	$2\frac{1}{2}$	$2\frac{5}{8}$	$2\frac{3}{4}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$	$4\frac{3}{4}$	5	6
$1\frac{7}{8}$	5·304	5·480	5·858	6·250	6·450	6·656	7·076	7·940	8·836	9·748	10·21	10·68	12·57
2	5·480	5·656	6·020	6·404	6·600	6·800	7·212	8·064	8·944	9·852	10·31	10·77	12·65
$2\frac{1}{4}$	5·858	6·020	6·364	6·724	6·916	7·108	7·500	8·320	9·176	10·06	10·51	10·96	12·82
$2\frac{1}{2}$	6·250	6·404	6·724	7·068	7·252	7·432	7·812	8·600	9·436	10·30	10·73	11·18	13·00
$2\frac{5}{8}$	6·450	6·600	6·916	7·252	7·424	7·604	7·972	8·748	9·572	10·42	10·86	11·30	13·10
3	7·076	7·212	7·500	7·812	7·972	8·140	8·484	9·220	10·00	10 82	11·24	11·66	13·42
$3\frac{1}{2}$	7·940	8·064	8·320	8·600	8·748	8·904	9·220	9·900	10·63	11·40	11·80	12·20	13·89
4	8·836	8·944	9·176	9·432	9·572	9·708	10·00	10·63	11·31	12·04	12·42	12·81	14·42
$4\frac{1}{2}$	9·748	9·852	10·06	10·30	10·42	10·55	10·82	11·40	12·04	12·73	13·08	13·45	15·00
5	10·68	10·77	10·96	11·18	11·30	11·41	11·66	12 20	12·81	13·45	13·79	14·14	15·62
6	12·57	12·65	12·82	13·00	13·10	13·20	13·42	13·89	14·42	15·00	15·30	15·62	16·97

RIVET GROUPS UNDER MOMENT

CENTRE OF ROTATION AT CENTRE OF GROUP

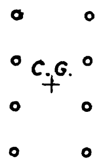
6 Rivets (in 2 rows of 3)

Values of $\dfrac{\Sigma x^2 + \Sigma y^2}{Z}$



Horizontal Pitch in inches (columns); *Vertical Pitch in inches* (rows)

Vertical Pitch in inches	$1\frac{7}{8}$	2	$2\frac{1}{4}$	$2\frac{1}{2}$	$2\frac{5}{8}$	$2\frac{3}{4}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$	$4\frac{3}{4}$	5	6
$1\frac{7}{8}$	9·225	9·441	9·907	10·40	10·66	10·92	11·48	12·65	13·89	15·18	15·83	16·50	19·24
2	9·630	9·839	10·28	10·76	11·00	11·27	11·80	12·94	14·14	15·41	16·05	16·71	19·41
$2\frac{1}{4}$	10·47	10·66	11·07	11·51	11·74	11·98	12·48	13·55	14·70	15·91	16·54	17·17	19·80
$2\frac{1}{2}$	11·34	11·51	11·89	12·30	12·51	12·74	13·21	14·22	15·30	16·47	17·07	17·67	20·23
$2\frac{5}{8}$	11·78	11·95	12·31	12·70	12·91	13·13	13·58	14·56	15·62	16·76	17·35	17·95	20·46
3	13·13	13·28	13·60	13·97	14·15	14·35	14·76	15·66	16·64	17·70	18·25	18·82	21·21
$3\frac{1}{2}$	14·98	15·11	15·39	15·71	15·87	16·05	16·41	17·22	18·11	19·08	19·58	20·11	22·35
4	16·86	16·98	17·23	17·51	17·66	17·81	18·14	18·87	19·68	20·56	21·03	21·52	23·60
$4\frac{1}{2}$	18·77	18·88	19·10	19·14	19·48	19·63	19·92	20·58	21·32	22·01	22·57	23·01	24·96
5	20·69	20·79	20·99	21·22	21·35	21·47	21·74	22·34	23·03	23·78	24·18	24·60	26·41
6	24·58	24·66	24·92	25·03	25·13	25·24	25·46	25·98	26·56	27·21	27·56	27·92	29·52



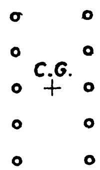
RIVET GROUPS UNDER MOMENT
CENTRE OF ROTATION AT CENTRE OF GROUP

8 Rivets (in 2 rows of 4)

Values of $\frac{\Sigma x^2 + \Sigma y^2}{Z}$

Horizontal Pitch in inches (columns); *Vertical Pitch in inches* (rows)

	$1\frac{7}{8}$	2	$2\frac{1}{4}$	$2\frac{1}{2}$	$2\frac{5}{8}$	$2\frac{3}{4}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$	$4\frac{3}{4}$	5	6
$1\frac{7}{8}$	14·22	14·46	14·95	15·49	15·77	16·06	16·68	18·01	19·46	21·00	21·81	22·63	26·06
2	14·97	15·18	15·64	16·16	16·43	16·70	17·29	18·58	19·98	21·47	22·25	23·05	26·40
$2\frac{1}{4}$	16·46	16·65	17·07	17·54	17·79	18·04	18·58	19·76	21·06	22·47	23·20	23·96	27·15
$2\frac{1}{2}$	17·99	18·16	18·55	18·97	19·20	19·44	19·94	21·03	22·23	23·56	24·25	24·96	28·00
$2\frac{5}{8}$	18·76	18·93	19·30	19·71	19·93	20·15	20·63	21·68	22·85	24·12	24·81	25·50	28·47
3	21·11	21·26	21·58	21·95	22·14	22·34	22 77	23·72	24·78	25·93	26·56	27·19	29·95
$3\frac{1}{2}$	24·28	24 41	24·70	25·01	25·19	25·36	25·73	26·47	27·51	28·54	29·09	29·66	32·16
4	27·50	27·62	27·87	28·14	28·29	28·45	28·78	29·52	30·36	31·29	31·79	32·31	34·59
$4\frac{1}{2}$	30·75	30·85	31·07	31·32	31·45	31·59	31·89	32 56	33·01	34·17	34·61	35·08	37·16
5	34·01	34·10	34·30	34·52	34·64	34·77	35·04	35·64	36 33	37·10	37·51	37·95	39·86
6	40·56	40·64	40·81	41·00	41·10	41·20	41·43	41 94	42·52	43·17	43·52	43·22	45·54



RIVET GROUPS UNDER MOMENT
CENTRE OF ROTATION AT CENTRE OF GROUP

10 Rivets (in 2 rows of 5)

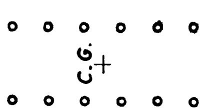
Values of $\frac{\Sigma x^2 + \Sigma y^2}{Z}$

Vertical Pitch in inches	Horizontal Pitch in inches												
	1⅞	2	2¼	2½	2⅝	2¾	3	3½	4	4½	4¾	5	6
1⅞	20·46	20·69	21·19	21·74	22·04	22·34	22·98	24·40	25·96	27·66	28·55	29·47	33·40
2	21·66	21·83	22·30	22·82	23·10	23·38	23·99	25·34	26·83	28·46	29·32	30·21	34·00
2¼	23·94	24·16	24·56	25·03	25·27	25·54	26·10	27·31	28·69	30·19	30·99	31·81	35·36
2½	26·30	26·45	26·85	27·28	27·51	27·75	28·26	29·37	30·64	32·04	32·77	33·54	36·87
2⅝	27·49	27·65	28·02	28·43	28·65	28·88	29·36	30·44	31·66	32·99	33·71	34·45	37·67
3	31·09	31·23	31·56	31·92	32·11	32·32	32·74	33·70	34·78	35·99	36·63	37·31	40·25
3½	35·93	36·06	36·35	36·66	36·82	37·00	37·36	38·74	39·15	39·53	40·77	41·37	43·99
4	40·82	40·93	41·17	41·45	41·60	41·75	42·08	42·82	43·66	44·59	45·11	45·63	47·99
4½	45·74	45·83	46·05	46·29	46·42	46·56	46·85	47·52	48·26	49·11	49·57	50·05	52·18
5	50·67	50·75	50·96	51·15	51·31	51·43	51·68	52·28	52·94	53·72	54·12	54·56	56·51
6	60·53	60·63	60·80	61·00	61·08	61·17	61·41	61·93	62·45	63·11	63·48	63·82	65·48

RIVET GROUPS UNDER MOMENT
CENTRE OF ROTATION AT CENTRE OF GROUP

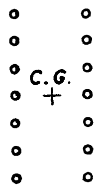
12 Rivets (in 2 rows of 6)

Values of $\frac{\Sigma x^2 + \Sigma y^2}{Z}$



Horizontal Pitch in inches (columns); *Vertical Pitch in inches* (rows)

Vertical Pitch in inches	$1\frac{7}{8}$	2	$2\frac{1}{4}$	$2\frac{1}{2}$	$2\frac{5}{8}$	$2\frac{3}{4}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$	$4\frac{3}{4}$	5	6
$1\frac{7}{8}$	27·95	28·18	28·68	29·23	29·53	29·83	30·49	31·94	33·62	35·35	36·30	37·28	41·52
2	29·59	29·81	30·27	30·80	31·08	31·37	31·99	33·36	34·91	36·62	37·52	38·46	42·53
$2\frac{1}{4}$	32·92	33·11	33·53	34·00	34·25	34·51	34·71	36·32	37·72	39·28	40·10	40·97	44·73
$2\frac{1}{2}$	36·28	36·45	36·84	37·26	37·49	37·73	38·23	39·37	40·65	42·07	42·84	43·63	47·13
$2\frac{5}{8}$	37·97	38·14	38·50	38·91	39·12	39·35	39·83	40·92	42·15	43·52	44·25	45·03	48·39
3	43 07	43·22	43·53	43·89	44·09	44·29	44·71	45·67	46·77	47·99	48·64	49·33	52·36
$3\frac{1}{2}$	49·92	50·05	50·32	50·63	50·80	50·97	51·33	52·17	53·12	54·18	54·76	55·36	58·03
4	56·83	56·91	57·18	57·42	57·61	57·75	58·06	58·79	59·61	60·56	61·06	61·59	63·98
$4\frac{1}{2}$	63·71	63·78	64·01	64·27	64·44	64·56	64·82	65·45	66·21	67·03	67·52	68·03	70·11
5	70·56	70·79	71·04	71·16	71·25	71·41	71·64	72·25	72·91	73·68	74·11	74·51	76·44
6	84·53	84·63	84·79	84·97	85·09	85·17	85·34	85·88	86·45	87·06	87·40	87·77	89·41



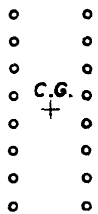
RIVET GROUPS UNDER MOMENT

CENTRE OF ROTATION AT CENTRE OF GROUP

14 Rivets (in 2 rows of 7)

Values of $\frac{\Sigma x^2 + \Sigma y^2}{Z}$

Vertical Pitch in inches	Horizontal Pitch in inches: $1\frac{7}{8}$	2	$2\frac{1}{4}$	$2\frac{1}{2}$	$2\frac{5}{8}$	$2\frac{3}{4}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$	$4\frac{3}{4}$	5	6
$1\frac{7}{8}$	36·68	36·91	37·40	37·96	38·2[illegible]	38·56	39·23	40·70	42·36	44·20	45·18	46·19	50·65
2	38·91	39·12	39·59	40·12	40·40	40·69	41·31	42·70	44·27	46·02	46·95	47·92	52·18
$2\frac{1}{4}$	43·40	43·60	44·02	44·49	44·74	44·99	45·55	46·81	48·22	49·81	50·66	51·54	55·44
$2\frac{1}{2}$	47·68	48·16	48·49	48·90	49·13	49·37	49·88	51·01	52·31	53·75	54·53	55·34	58·93
$2\frac{5}{8}$	50·20	50·37	50·73	51·14	51·36	51·58	52·07	54·06	55·67	57·42	58·30	59·30	51·52
3	57·06	57·21	57·52	57·88	58·07	58·27	58·69	59·65	60·74	61·97	62·63	63·32	66·41
$3\frac{1}{2}$	66·25	66·35	66·64	66·91	67·12	67·34	67·69	68·50	69·41	70·54	71·09	71·69	74·36
4	75·44	75·58	75·83	76·11	76·23	76·43	76·72	77·46	78·22	79·19	79·72	80·22	82·62
$4\frac{1}{2}$	84·72	84·79	85·00	85·24	85·41	85·58	85·89	86·47	87·18	88·08	88·54	89·03	91·11
5	93·97	94·08	94·26	94·48	94·63	94·72	94·93	95·55	96·23	96·96	97·24	97·80	99·74
6	112·6	112·6	112·7	112·9	113·0	113·2	113·3	113·9	114·4	115·0	115·4	115·7	117·4

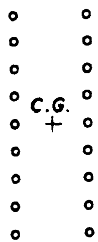


RIVET GROUPS UNDER MOMENT
CENTRE OF ROTATION AT CENTRE OF GROUP

16 Rivets (in 2 rows of 8)

Values of $\frac{\Sigma x^2 + \Sigma y^2}{Z}$

Vertical Pitch in inches	Horizontal Pitch in inches: $1\frac{7}{8}$	2	$2\frac{1}{4}$	$2\frac{1}{2}$	$2\frac{5}{8}$	$2\frac{3}{4}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$	$4\frac{3}{4}$	5	6
$1\frac{7}{8}$	46·69	46·90	47·22	47·95	48·24	48·55	49·21	50·64	52·37	54·25	55·24	56·30	60·88
2	49·56	49·78	50·25	50·77	51·05	51·35	51·96	53·36	54·94	56·72	57·66	58·66	63·02
$2\frac{1}{4}$	55·39	55·59	56·00	56·47	56·72	56·98	57·54	58·80	60·21	61·82	62·60	63·57	67·55
$2\frac{1}{2}$	61·26	61·43	61·81	62·23	62·46	62·69	63·19	64·33	65·63	67·08	67·86	68·68	73·32
$2\frac{5}{8}$	64·20	64·36	64·72	65·12	65·33	65·56	66·04	67·12	68·36	69·75	70·50	71·30	74·79
3	73·06	73·17	73·51	73·82	74·06	74·31	74·72	75·66	76·49	78·01	78·65	79·33	82·42
$3\frac{1}{2}$	84·87	85·03	85·30	85·62	85·76	85·91	86·30	87·15	88·07	89·16	89·75	90·32	93·02
4	96·80	96·87	97·10	97·37	97·48	97·67	98·01	98·72	99·58	100·5	101·0	101·5	103·9
$4\frac{1}{2}$	108·7	108·8	109·0	109·2	109·3	109·5	109·8	110·4	111·2	111·9	112·4	112·9	115·1
5	120·6	120·7	120·9	121·1	121·2	121·4	121·6	122·2	122·8	123·6	124·0	124·5	126·3
6	144·5	144·6	144·7	144·9	145·0	145·4	145·4	145·8	146·3	147·0	147·4	147·8	149·4



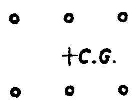
RIVET GROUPS UNDER MOMENT
CENTRE OF ROTATION AT CENTRE OF GROUP

18 Rivets (in 2 rows of 9)

Values of $\frac{\Sigma x^2 + \Sigma y^2}{Z}$

Horizontal Pitch in inches (columns); *Vertical Pitch in inches* (rows)

	$1\frac{7}{8}$	2	$2\frac{1}{4}$	$2\frac{1}{2}$	$2\frac{5}{8}$	$2\frac{3}{4}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$	$4\frac{3}{4}$	5	6
$1\frac{7}{8}$	57·69	58·14	58·63	59·18	59·48	59·79	60·45	61·93	63·63	65·52	66 53	67·59	72·28
2	61·55	61·77	62·23	62·75	63·03	63·33	63·95	65·35	66·94	68·72	69·69	70·69	75·14
$2\frac{1}{4}$	68·89	69·05	69·49	69·96	70·20	70·47	71·02	72·28	73·70	75·31	76·17	77·08	81·11
$2\frac{1}{2}$	76·28	76·42	76·82	77·27	77·48	77·70	78·19	79·32	81·22	82·06	82·83	83·66	87·36
$2\frac{5}{8}$	79·95	80·08	80·46	80·89	81·08	81·37	81·83	82·89	84·09	85·55	86·28	87·06	90·56
3	91·01	91·20	91·52	91·88	92·05	92·22	92·68	93·66	94·66	95·91	96·61	97·27	100·4
$3\frac{1}{2}$	105·9	106·0	106·2	106·5	106·7	106·9	107·3	108·1	109·0	110·1	110·7	111·3	114·0
4	120·8	120·9	121·1	121·4	121·6	121·7	122·0	122·7	123·6	124·4	124·9	125·5	127·9
$4\frac{1}{2}$	135·7	135·8	136·0	136·2	136·3	136·5	136·7	137·4	138·2	139·0	139·4	139·8	142·0
5	150·6	150·7	150·9	151·2	151·3	151·4	151·6	152·2	152·8	153·6	154·1	154·6	156·4
6	180·5	180·6	180·7	180·9	181·1	181·2	181·4	181·8	182·4	183·0	183·4	183·8	185·3



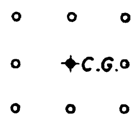
RIVET GROUPS UNDER MOMENT

CENTRE OF ROTATION AT CENTRE OF GROUP

6 Rivets (in 3 rows of 2)

Values of $\frac{\Sigma x^2 + \Sigma y^2}{Z}$

Vertical Pitch in inches	Horizontal Pitch in inches										
	1⅞	2	2¼	2½	2⅝	3	3½	4	4½	5	6
1⅞	9·225	9·630	10·47	11·34	11·78	13·13	14·98	16·86	18·77	20·69	24·58
2	9·441	9·839	10·66	11·51	11·95	13·29	15·11	16·98	18·88	20·79	24·66
2¼	9·907	10·29	11·07	11·89	12·31	13·61	15·39	17·23	19·10	20·99	24·84
2½	10·40	10·76	11·51	12·30	12·70	13·96	15·71	17·51	19·35	21·22	25·02
2⅝	10·66	11·01	11·74	12·51	12·91	14·15	15·87	17·66	19·48	21·35	25·13
3	11·47	11·80	12·48	13·21	13·58	14·76	16·41	18·14	19·92	21·74	25·46
3½	12·65	12·94	13·55	14·22	14·56	15·66	17·22	18·87	20·58	22·34	25·98
4	13·89	14·14	14·71	15·30	15·62	16·64	18·11	19·68	21·32	23 02	26·56
4½	15·17	15·41	15·91	16·47	16·76	17·70	19·08	20·56	22·13	23·78	27·21
5	16·49	16·71	17·17	17·67	17·95	18·82	20·11	21·52	23·02	24·60	27·92
6	19·24	19·41	19·80	20·23	20 46	21·21	22·35	23·60	24·96	26·41	29·52



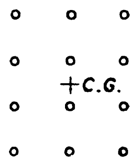
RIVET GROUPS UNDER MOMENT

CENTRE OF ROTATION AT CENTRE OF GROUP

9 Rivets (in 3 rows of 3)

Values of $\frac{\Sigma x^2 + \Sigma y^2}{Z}$

Vertical Pitch in inches	Horizontal Pitch in inches: $1\frac{7}{8}$	2	$2\frac{1}{4}$	$2\frac{1}{2}$	$2\frac{5}{8}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$	5	6
$1\frac{7}{8}$	15·91	16·45	17·58	18·75	19·35	21·22	23·82	26·50	29·25	32·03	37·71
2	16·45	16·97	18·06	19·21	19·80	21·63	24·19	26·83	29·55	32·31	37·94
$2\frac{1}{4}$	17·58	18·06	19·10	20·18	20·95	22·50	24·97	27·54	30·19	32·90	38·45
$2\frac{1}{2}$	18·75	19·21	20·18	21·21	21·76	23·43	25·81	28·30	30·89	33·54	39·00
$2\frac{5}{8}$	19·35	19·80	20·75	21·76	22·27	23·92	26·25	28·71	31·26	33·88	39·30
3	21·22	21·63	22·50	23·43	23·92	25·45	27·66	30·00	32·45	34·98	40·25
$3\frac{1}{2}$	23·82	24·19	24 97	25·81	26·25	27·66	29·70	31·89	34·20	36·62	41·67
4	26·50	26·83	27·54	28·30	28·71	30·00	31·89	33·94	36·12	38·42	43·27
$4\frac{1}{2}$	29·25	29·55	30·19	30·89	31·26	32·45	34·20	36·12	38·18	40·36	45·00
5	32·03	32·31	32·90	33·54	33·88	34 98	36·62	38·42	40·36	42·43	46·86
6	37·71	37·94	38·45	39·00	39·30	40·25	41·67	43·27	45·00	46·86	50·91



RIVET GROUPS UNDER MOMENT

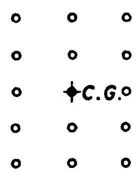
CENTRE OF ROTATION AT CENTRE OF GROUP

12 Rivets (in 3 rows of 4)

Values of $\frac{\Sigma x^2+\Sigma y^2}{Z}$

Horizontal Pitch in inches (columns); *Vertical Pitch in inches* (rows)

	1⅞	2	2¼	2½	2⅝	3	3½	4	4½	5	6
1⅞	23·92	24·55	25·88	27·30	28·04	30·33	33·57	36·96	40·46	44·05	51·52
2	24·90	25·51	26·80	28 17	28·88	31·11	34 28	37·60	41·04	44·59	51·88
2¼	26·95	27·51	28·71	29·98	30·66	32·76	35·77	38·96	42·31	45·74	52·87
2½	29·07	29·59	30·71	31 89	32·52	34·52	37·39	40·45	43·66	47·00	53·96
2⅝	30·15	30 65	31·71	32·89	33·50	35 43	38·22	41·23	44·38	47·67	54 54
3	33·46	33·91	34·88	35·94	36 49	38·27	40·87	43·68	46·67	49·80	56·40
3½	38·00	38·41	39·25	40·20	40·70	42·29	44·65	47·23	50·00	52·93	59·17
4	42·65	43·00	43·77	44 61	45 06	46·51	48·65	51·03	53·60	56·34	62·23
4½	47·37	47·69	48 39	49·15	49·55	50·87	52·83	55·03	57·41	59·97	65·52
5	52·14	52·43	53·06	53·87	54·12	55·33	57·15	59·18	61·40	63·79	69·03
6	61·80	62·04	62·57	63·16	63·48	64·51	66·07	67·82	69·78	71·84	76·52

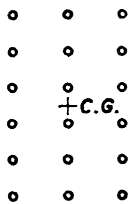


RIVET GROUPS UNDER MOMENT
CENTRE OF ROTATION AT CENTRE OF GROUP

15 Rivets (in 3 rows of 5)

Values of $\frac{\Sigma x^2 + \Sigma y^2}{Z}$

Vertical Pitch in inches	Horizontal Pitch in inches										
	$1\frac{7}{8}$	2	$2\frac{1}{4}$	$2\frac{1}{2}$	$2\frac{5}{8}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$	5	6
$1\frac{7}{8}$	33·55	34·23	35·70	37·27	38·09	40·71	44·45	48·42	52·57	56·87	65·79
2	35·12	35·78	37·18	38·69	39·49	42·00	45·62	49·50	53·56	57·78	66·56
$2\frac{1}{4}$	38·36	38·97	40·25	41·64	42·38	44·72	48·13	51·80	55·68	59·74	68·25
$2\frac{1}{2}$	41·69	42·25	43·44	44·72	45·41	47·59	50·79	54·27	57·97	61·87	70·10
$2\frac{5}{8}$	43·39	43·92	45·05	46·30	46·96	49·07	52·17	55·56	59·18	63·00	71·08
3	48·54	49·01	50·03	51·15	51·75	53·67	56·50	59·63	63·00	66·58	74·25
$3\frac{1}{2}$	55·56	55·97	56·87	57·85	58·37	60·07	62·61	65·43	68·49	71·78	78·90
4	62·69	63·06	63·85	64·72	65·20	66·71	69·00	71·56	74·35	77·38	84·00
$4\frac{1}{2}$	69·91	70·23	70·94	71·73	72·15	73·52	75·59	77·93	80·52	83·25	89·42
5	77·20	77·45	78·11	78·81	79·20	80·46	82·39	84·49	86·91	89·44	94·34
6	91·86	92·03	92·60	93·19	93·48	94·58	96·20	98·02	100·0	102·3	107·3



RIVET GROUPS UNDER MOMENT

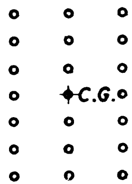
CENTRE OF ROTATION AT CENTRE OF GROUP

18 Rivets (in 3 rows of 6)

Values of $\frac{\Sigma x^2 + \Sigma y^2}{Z}$

Horizontal Pitch in inches (columns); *Vertical Pitch in inches* (rows)

	$1\frac{7}{8}$	2	$2\frac{1}{4}$	$2\frac{1}{2}$	$2\frac{5}{8}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$	5	6
$1\frac{7}{8}$	44·91	45·64	47·19	48·86	49·74	52·57	56·68	61·11	65·80	70·71	80·98
2	47·22	47·91	49·39	50·98	51·83	54·54	58·50	62·78	67·34	72·13	82·20
$2\frac{1}{4}$	51·93	52·56	53·90	55·37	56·14	58·63	62·31	66·33	70·63	75·18	84·85
$2\frac{1}{2}$	56·75	57·32	58·54	59·88	60·60	62·91	66·33	70·10	74·15	78·48	87·74
$2\frac{5}{8}$	59·19	59·72	60·91	62·20	62·86	65·10	68·41	72·06	76·00	80·21	89·27
3	66·57	67·06	68·10	69·25	69·86	71·86	74·85	78·18	81·81	85·70	94·17
$3\frac{1}{2}$	76·59	77·00	77·91	78·91	79·45	81·20	83·84	86·80	90·05	93·56	101·3
4	86·74	87·06	87·88	88·75	89·23	90·80	93·20	95·82	98·81	102·0	109·1
$4\frac{1}{2}$	96·96	97·21	98·33	98·79	99·20	100·5	102·7	105·1	107·8	110·7	117·3
5	107·2	107·5	108·1	108·8	109·2	110·4	112·4	114·7	117·0	119·8	125·8
6	127·8	128·1	128·6	129·2	129·5	130·6	132·3	134·1	136·2	138·5	143·7



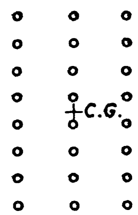
RIVET GROUPS UNDER MOMENT
CENTRE OF ROTATION AT CENTRE OF GROUP

21 Rivets (in 3 rows of 7)

Values of $\frac{\Sigma x^2 + \Sigma y^2}{Z}$

Horizontal Pitch in inches (columns); *Vertical Pitch in inches* (rows)

	$1\frac{7}{8}$	2	$2\frac{1}{4}$	$2\frac{1}{2}$	$2\frac{5}{8}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$	5	6
$1\frac{7}{8}$	58·10	58·85	60·45	62·19	63·12	66·09	70·46	75·24	80·36	85·74	97·19
2	61·27	61·98	63·49	65·15	66·04	68·87	73·05	77·66	82·60	87·84	99·00
$2\frac{1}{4}$	67·72	68·36	69·74	71·23	72·04	74·63	78·48	82·75	87·36	90·77	102·9
$2\frac{1}{2}$	74·27	74·85	76·10	77·47	78·20	80·59	84·16	88·12	92·44	97·07	107·1
$2\frac{5}{8}$	77·58	78·13	79·33	80·65	81·34	83·64	87·06	90·89	95·07	99·57	109·4
3	87·59	88·07	89·13	90·30	90·93	92·97	96·04	99·50	103·3	107·4	116·4
$3\frac{1}{2}$	101·0	101·5	102·5	103·5	104·0	105·8	108·4	111·6	114·9	118·6	126·8
4	114·8	115·0	115·9	116·8	117·2	118·8	121·2	123·9	126·9	130·3	137·7
$4\frac{1}{2}$	128·4	128·7	129·5	130·3	130·7	132·1	134·2	136·7	139·5	142·4	149·3
5	142·1	142·5	143·1	143·8	144·2	145·5	147·5	149·7	152·2	155·0	161·1
6	169·9	170·1	170·6	171·1	171·5	173·5	174·2	176·1	178·2	180·6	186·0



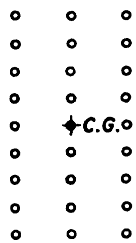
RIVET GROUPS UNDER MOMENT

CENTRE OF ROTATION AT CENTRE OF GROUP

24 Rivets (in 3 rows of 8)

Values of $\frac{\Sigma x^2 + \Sigma y^2}{Z}$

Vertical Pitch in inches	Horizontal Pitch in inches										
	1⅞	2	2¼	2½	2⅝	3	3½	4	4½	5	6
1⅞	70·21	73·89	75 53	77·32	78·27	81·34	85·92	90·95	96·39	102·2	114·6
2	77·31	78·02	79·57	81·26	82·16	85·08	89·44	94·27	99·49	105·1	117·1
2¼	85·75	86·38	87·77	89·31	90 11	92 78	96·76	101·2	106·0	111·3	122·6
2½	94 29	94·87	96·14	97 53	98·29	100·7	104·4	108·5	113·0	117·8	128·5
2⅝	98 59	99 14	100·3	101·7	102·4	104·7	108 2	112·2	116·3	121·2	131·6
3	111·5	112 1	113·2	114 4	115·0	117 0	120·1	123·8	127·7	131·9	141·4
3½	129·1	129·5	130·5	131·5	132·0	133·8	136·5	139·7	143·1	146·9	155·4
4	146·7	147·1	147·9	148·8	149·3	150 8	153·3	156·0	159·1	162 5	170·2
4½	164·4	164 8	165·5	166 2	166·7	168·1	170·3	172 8	175·5	178·7	185·6
5	182·2	182·4	183·1	184 0	184 3	185·5	187 6	189·8	192·2	195·0	200·7
6	217·8	218 0	218·6	219 3	219·3	220 6	222 4	224·1	226·4	228·6	234·1



RIVET GROUPS UNDER MOMENT

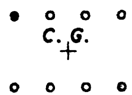
CENTRE OF ROTATION AT CENTRE OF GROUP

27 Rivets (in 3 rows of 9)

Values of $\frac{\Sigma x^2 + \Sigma y^2}{Z}$

Horizontal Pitch in inches (columns); *Vertical Pitch in inches* (rows)

	$1\frac{7}{8}$	2	$2\frac{1}{4}$	$2\frac{1}{2}$	$2\frac{5}{8}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$	5	6
$1\frac{7}{8}$	90·04	90·80	92·46	94·27	95·24	98·39	103·1	108·3	114·0	120·1	133·3
2	95·32	96·05	97·60	99·32	100·2	103·2	107·7	112·7	118·1	124·0	136·8
$2\frac{1}{4}$	106·0	106·6	108·0	109·6	110·4	113·1	117·2	121·8	126·8	132·2	144·1
$2\frac{1}{2}$	116·8	117·3	118 6	120 0	120·8	123·3	127·0	131·2	135·9	140·9	152·0
$2\frac{5}{8}$	122·2	122·8	124·1	125·4	126·1	128·4	132·0	136·1	140·5	145·3	156·2
3	138·6	139·0	140·1	141·3	141·9	144·1	147·2	150·8	154·8	159·2	169·0
$3\frac{1}{2}$	160·5	161·0	161·9	163·0	163·5	165·3	168·1	171·2	174·7	178·5	187·3
4	182·7	183·1	183·9	184·7	185·2	186 8	189·3	192·1	195·2	198·7	206·4
$4\frac{1}{2}$	205·0	205·2	206·0	206·7	207·2	208·6	210·8	213·7	216·0	219·2	226·3
5	227·2	227·5	228·2	228·9	229·2	230·6	232·5	234·7	237·4	240·1	246·5
6	271·8	272·1	272·7	273·3	273 6	274·6	276·3	278·2	280·3	282·6	288·1



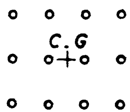
RIVET GROUPS UNDER MOMENT

CENTRE OF ROTATION AT CENTRE OF GROUP

8 Rivets (in 4 rows of 2)

Values of $\frac{\Sigma x^2 + \Sigma y^2}{Z}$

Vertical Pitch in inches	Horizontal Pitch in inches										
	$1\frac{7}{8}$	2	$2\frac{1}{4}$	$2\frac{1}{2}$	$2\frac{5}{8}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$	5	6
$1\frac{7}{8}$	14·22	14·97	16·46	17·99	18·76	21·11	24·28	27·50	30·75	34·01	40·56
2	14·46	15·18	16·65	18·16	18·93	21·26	24·41	27·62	30·85	34·10	40·64
$2\frac{1}{4}$	14·95	15·64	17·07	18·55	19·30	21·58	24·70	27·87	31·07	34 30	40·81
$2\frac{1}{2}$	15·49	16·16	17·54	18·97	19·71	21·95	25·01	28·14	31·32	34·52	41·00
$2\frac{5}{8}$	15·77	16·43	17·79	19·20	19·93	22·14	25·19	28·29	31·45	34·64	41·10
3	16·68	17·29	18·58	19·94	20·63	22·77	25·73	28·78	31·89	35·04	41·43
$3\frac{1}{2}$	18·01	18·58	19·76	21·03	21·68	23·72	26·47	29·52	32·56	35·64	41·94
4	19·46	19·98	21·06	22·23	22·85	24·78	27·51	30·36	33·01	36·33	42·52
$4\frac{1}{2}$	21·00	21·47	22·47	23·56	24·12	25·93	28·54	31·29	34·17	37·10	43·17
5	22·63	23·05	23·96	24·96	25·50	27·19	29·66	32·31	35·08	37·95	43·22
6	26·06	26·40	27·15	28·00	28·47	29·95	32·16	34·59	37·16	39·86	45·54



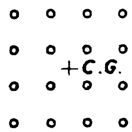
RIVET GROUPS UNDER MOMENT

CENTRE OF ROTATION AT CENTRE OF GROUP

12 Rivets (in 4 rows of 3)

Values of $\frac{\Sigma x^2 + y^2}{Z}$

Vertical Pitch in inches	Horizontal Pitch in inches										
	$1\frac{7}{8}$	2	$2\frac{1}{4}$	$2\frac{1}{2}$	$2\frac{5}{8}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$	5	6
$1\frac{7}{8}$	23·92	24·90	26·95	29·07	30·15	33·46	38·00	42·65	47·37	52·14	61·80
2	24·55	25·51	27·51	29·59	30·65	33·91	38·41	43·00	47·69	52·43	62·04
$2\frac{1}{4}$	25·88	26·80	28·71	30·71	31·71	34·88	39·25	43·77	48·39	53·06	62·57
$2\frac{1}{2}$	27·30	28·17	29·98	31·89	32·89	35·94	40·20	44·61	49·15	53·87	63·16
$2\frac{5}{8}$	28·04	28·88	30·66	32·52	33·50	36·49	40 70	45·06	49·55	54·12	63·48
3	30·33	31·11	32·76	34·52	35·43	38·27	42·29	46·51	50·87	55·33	64·51
$3\frac{1}{2}$	33·57	34·28	35·77	37·39	38·22	40·87	44·65	48·65	52 83	57·15	66·07
4	36·96	37·60	38·96	40·45	41·23	43·68	47·23	51·03	55·03	59·18	67·82
$4\frac{1}{2}$	40·46	41·04	42·31	43·66	44·38	46·67	50·00	53·60	57·41	61·40	69·78
5	44·05	44·59	45·74	47·00	47·67	49·80	52·93	56·34	59·97	63·79	71·84
6	51·52	51·88	52·87	53·96	54·54	56·40	59·17	62·23	65·52	69·03	76·52



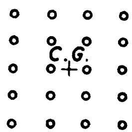
RIVET GROUPS UNDER MOMENT

CENTRE OF ROTATION AT CENTRE OF GROUP

16 Rivets (in 4 rows of 4)

Values of $\frac{\Sigma x^2 + \Sigma y^2}{Z}$

Vertical Pitch in inches	Horizontal Pitch in inches: $1\frac{7}{8}$	2	$2\frac{1}{4}$	$2\frac{1}{2}$	$2\frac{5}{8}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$	5	6
$1\frac{7}{8}$	35·36	36·53	39·07	41·65	42·99	47·17	52·93	58·91	64·99	71·20	83·81
2	36·53	37·71	40·13	42·69	44·00	48·08	53·76	59·63	65·68	71·81	84·32
$2\frac{1}{4}$	39·07	40·13	42·43	44·83	46·11	50·00	55·47	61·17	67·09	73·09	85·47
$2\frac{1}{2}$	41·65	42·69	44·83	47·12	48·35	52·08	57·33	62·91	68·64	74·56	86·64
$2\frac{5}{8}$	42·99	44·00	46·11	48·35	49·49	53·15	58·32	63·81	69·47	75·31	87·31
3	47·17	48·08	50·00	52·08	53·15	56·56	61·47	66·67	72·13	77·73	89·44
$3\frac{1}{2}$	52·93	53·76	55·47	57·33	58·32	61·47	66·00	70·88	76·03	81·36	92·59
4	58·91	59·63	61·17	62·91	63·81	66·67	70·88	75·41	80·27	85·39	96·16
$4\frac{1}{2}$	64·99	65·68	67·09	68·64	69·47	72·13	76·03	80·27	84·88	89·68	100·0
5	71·20	71·81	73·09	74·56	75·31	77·73	81·36	85·39	89·68	94·29	104·1
6	83·81	84·32	85·47	86·64	87·31	89·44	92·59	96·16	100·0	104·1	113·1



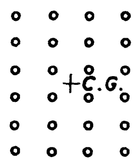
RIVET GROUPS UNDER MOMENT
CENTRE OF ROTATION AT CENTRE OF GROUP

20 Rivets (in 4 rows of 5)

Values of $\frac{\Sigma x^2 + \Sigma y^2}{Z}$

Horizontal Pitch in inches (columns); *Vertical Pitch in inches* (rows)

	$1\frac{7}{8}$	2	$2\frac{1}{4}$	$2\frac{1}{2}$	$2\frac{5}{8}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$	5	6
$1\frac{7}{8}$	48·74	50·11	52 96	55·98	57·55	62·41	69·26	76·41	83·78	91·30	106·7
2	50·69	52·00	54·75	57·69	59·21	63·92	70·64	77·66	84·92	92·35	107·6
$2\frac{1}{4}$	54·72	55·92	58·51	61·24	62·68	67·17	73·57	80·33	87·36	94·61	109·6
$2\frac{1}{2}$	58·90	60·02	62·43	65·00	66·35	70·61	76·72	83·23	90·03	97·07	111·6
$2\frac{5}{8}$	61·03	62·12	64·44	66·94	68·27	72·40	78·38	84·73	91·44	98·37	112·8
3	67·58	68·57	70·68	72·97	74·17	78·00	83·56	89·57	95·92	102·5	116·4
$3\frac{1}{2}$	76·60	77·47	79·34	81·38	82·46	85·92	91·01	96·53	102·4	108·7	121·9
4	85·84	86·61	88·28	90·12	91·10	94·24	98·89	104·0	109·6	115·4	127·9
$4\frac{1}{2}$	95·22	95·92	97·44	99·10	100·0	102·9	107·1	111·8	117·0	122·5	134·3
5	104·8	105·4	106·7	108·3	109·1	111·8	115·6	120·1	124·9	130·0	141·3
6	123·9	124·5	125·6	127·0	127·6	129·9	133·3	137·1	141·3	145·9	156·0



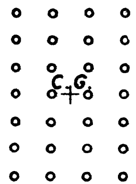
RIVET GROUPS UNDER MOMENT
CENTRE OF ROTATION AT CENTRE OF GROUP

24 Rivets (in 4 rows of 6)

Values of $\frac{\Sigma x^2 + \Sigma y^2}{Z}$

Horizontal Pitch in inches (columns); *Vertical Pitch in inches* (rows)

	$1\frac{7}{8}$	2	$2\frac{1}{4}$	$2\frac{1}{2}$	$2\frac{5}{8}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$	5	6
$1\frac{7}{8}$	64·32	65·78	68·90	72·24	73·95	79·42	87·18	95·36	103·9	112·6	130·6
2	67·19	68·60	71·60	74 80	76·48	81·76	89·31	97·31	105·6	114·3	132·0
$2\frac{1}{4}$	73·12	74·41	77·18	80·16	81·72	86 68	93·82	101·5	109·5	117·8	135·2
$2\frac{1}{2}$	79·23	80·42	82·98	85·76	87·21	91·86	98·63	105·9	113·6	121·6	138·6
$2\frac{5}{8}$	82·34	83·47	85·95	88·63	90·03	94·55	101·1	108·2	115·8	123·6	140·2
3	91·82	92·84	95·07	97·48	98·77	102·9	109·0	115·6	122·6	130·2	146·0
$3\frac{1}{2}$	104·8	105·7	107·6	109·8	110 9	114·6	120·1	126·1	132·6	139·5	154·4
4	118·1	118·8	120·4	122·4	123·5	126·8	131·6	137·2	143·2	149·6	163·6
$4\frac{1}{2}$	131·3	131·8	133·6	135·4	136·2	139·3	143·8	148·8	154·5	160·3	177·3
5	144·8	145·4	147·0	148·5	149·2	152·0	156·2	160·8	165·9	171·5	183·8
6	172·0	172·5	173·8	175·1	175·8	178·2	181·7	185·6	190·1	195·0	205·8



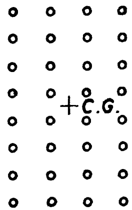
RIVET GROUPS UNDER MOMENT

CENTRE OF ROTATION AT CENTRE OF GROUP

28 Rivets (in 4 rows of 7)

Values of $\frac{\Sigma x^2 + \Sigma y^2}{Z}$

Vertical Pitch in inches	Horizontal Pitch in inches										
	$1\frac{7}{8}$	2	$2\frac{1}{4}$	$2\frac{1}{2}$	$2\frac{5}{8}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$	5	6
$1\frac{7}{8}$	82·17	83·72	87·05	90·61	92·47	98·40	106·9	116·0	125·5	135·3	155·9
2	86·17	87·66	90·82	94·24	96·04	101·7	110·0	118·8	128·1	137·7	157·9
$2\frac{1}{4}$	94·37	95·72	98·62	101·8	103·4	108·7	116·4	124·8	133·6	142·9	162·4
$2\frac{1}{2}$	102·7	104·0	106·7	109·6	111·1	116·0	123·3	131·2	139·6	148·6	167·4
$2\frac{5}{8}$	107·0	108·2	110·7	113·5	115·0	119·8	126·8	134·5	142·8	151·5	169·9
3	119·9	121·0	123·3	125·8	127·2	131·5	137·9	144·9	152·6	160·8	178·2
$3\frac{1}{2}$	137·5	138·5	140·4	142·7	143·9	147·7	153·5	159·8	166·7	174·2	190·3
4	155·3	156·2	157·9	160 0	161·0	164·3	169·5	175·3	181·6	188·5	203·5
$4\frac{1}{2}$	173·4	174·1	175·8	177·5	178·3	181·5	186·1	191·5	197·1	203·4	217·4
5	191·5	192·2	193·7	195·3	196·1	198·9	203·2	207·9	213·3	214·3	220·9
6	228·1	228·6	229·9	231·3	232·0	234·2	237·9	242·1	246·5	251·6	263·0



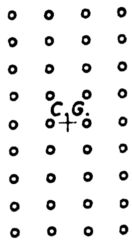
RIVET GROUPS UNDER MOMENT

CENTRE OF ROTATION AT CENTRE OF GROUP

32 Rivets (in 4 rows of 8)

Values of $\frac{\Sigma x^2 + \Sigma y^2}{Z}$

Vertical Pitch in inches	Horizontal Pitch in inches: 1⅞	2	2¼	2½	2⅝	3	3½	4	4½	5	6
1⅞	102·3	103·9	107·4	111·1	113·1	119·4	128·5	138·3	148·7	159·5	182·2
2	107·6	109·2	112·5	116·1	118·0	124·0	132·8	142·3	152·4	163·0	185·3
2¼	118·4	119·9	122 9	126·2	127·9	133·5	141·6	150·6	160·1	170·2	191·5
2½	129·5	130·8	133·6	136·6	138·1	143·3	151·0	159·3	168·3	178·0	198·4
2⅝	135·1	136·3	139·0	141·8	143·4	148·3	155·7	163·9	172·6	181·9	201·8
3	152·0	153·1	155·4	158·0	159·5	163·9	170·7	178·0	186·1	194·7	213·4
3½	174·9	175·9	177·8	180·2	181·3	185·3	191·3	197·8	205·1	212·9	230·1
4	198·0	198·9	200·7	202·8	203·7	207·2	212·6	218·5	225·1	232·2	248·1
4½	221·4	222·2	223·9	225·6	226·4	229·7	234·5	239 9	245·7	252·3	266·9
5	244·8	245·5	247·0	248·6	249·5	252·3	256·7	261·6	267·2	273·1	286·6
6	292·0	292·7	293·9	295·3	296·0	298·5	302·1	306·2	310·9	316·2	327·7



RIVET GROUPS UNDER MOMENT

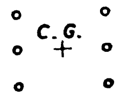
CENTRE OF ROTATION AT CENTRE OF GROUP

36 Rivets (in 4 rows of 9)

Values of $\frac{\Sigma x^2 + \Sigma y^2}{Z}$

Vertical Pitch in inches	Horizontal Pitch in inches										
	$1\frac{7}{8}$	2	$2\frac{1}{4}$	$2\frac{1}{2}$	$2\frac{5}{8}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$	5	6
$1\frac{7}{8}$	125·1	126·7	130·3	134·1	136·2	142·8	152·4	162·8	173·9	185·7	210·3
2	131·9	133·4	136·8	140·5	142·4	148·7	157·9	168·0	178·9	190·2	214·3
$2\frac{1}{4}$	145·6	147·0	150·1	153·5	155·2	161·0	169·5	178·8	189·0	199·8	222·7
$2\frac{1}{2}$	159·7	160·9	163·6	166·8	168·5	173·8	181·5	190·4	199·9	210·0	232·0
$2\frac{5}{8}$	166·7	167·9	170·6	173·5	175·2	180·3	188·0	196·3	205·5	215·4	236·7
3	188·0	189·2	191·5	194·2	195·6	200·0	207·0	214·6	223·0	232·1	252·0
$3\frac{1}{2}$	217·0	217·9	220·0	222·3	223·4	227·4	233·5	240·3	247·8	256·0	274·0
4	246·0	246·9	248·8	250·8	251·8	255·4	260·8	266·8	273·5	281·0	297·4
$4\frac{1}{2}$	275·6	276·2	277·9	279·7	280·7	283·7	288·6	294·1	300·1	306·9	322·1
5	305·0	305·6	307·1	308·7	309·6	312·6	316·9	312·8	327·4	333·7	347·5
6	364·0	364·6	365·8	367·3	368·0	370·5	374·1	378·3	383·1	388·4	400·3





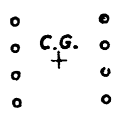
RIVET GROUPS UNDER MOMENT FOR TWIN-STALK COLUMNS

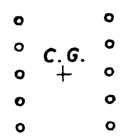
CENTRE OF ROTATION AT CENTRE OF GROUP

Two vertical rows of rivets

Values of $\frac{\Sigma x^2 + \Sigma y^2}{Z}$

			Horizontal Pitch in inches												
			8¾	9¾	10½	11½	10¾	11¾	12	13	12½	13½	13¾	14¾	16¾
Four Rivets	*Vertical Pitch in inches*	2⅝	18·27	20·20	21·64	23·59	22·13	24·08	24·57	26·52	25·54	27·50	28·00	29·96	33·91
		3	18·50	20·40	21·47	23·77	22·32	24·25	24·74	26·68	25·71	27·66	28·15	30·10	34·03
		3½	18·85	20·72	22·21	24·04	22·61	24·52	25·00	26·93	25·96	27·89	28·38	30·32	34·22
		4	19·24	21·11	22·47	24·35	22·94	24·83	25·30	27·20	26·25	28·16	28·64	30·56	34·44
Six Rivets		2⅝	27·91	30·73	32·87	35·74	33·58	36·47	37·19	40·09	38·64	41·55	42·28	45·21	51·09
		3	28·43	31·20	33·30	36·13	34·01	36·85	37·57	40·44	39·00	41·89	42·60	45·51	51·35
		3½	29·24	31·93	33·97	36·75	34·66	37·45	38·15	40 98	39·56	42·39	43·08	45·97	51·76
		4	30·17	32·76	34·75	37·46	35 42	38·14	38·83	41·60	40·21	43·00	43·70	46·52	52·24
Flange width		. . .	5	5	6	6	6½	6½	7	7	7½	7½	8	8	8
Joist centres		. . .	6	7	7	8	7	8	8	9	8	9	9	10	12





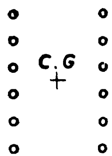
RIVET GROUPS UNDER MOMENT FOR TWIN-STALK COLUMNS

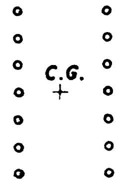
CENTRE OF ROTATION AT CENTRE OF GROUP

Two vertical rows of rivets

Values of $\frac{\Sigma x^2 + \Sigma y^2}{Z}$

	Vertical Pitch in inches	Horizontal Pitch in inches: 8¾	9¾	10½	11½	10¾	11¾	12	13	12½	13½	13¾	14¾	16¾
Eight Rivets	2⅝	37·73	41·33	44·10	47·84	45·03	48·79	49·74	53·55	51·63	55·46	56·43	60·29	68·08
	3	38·74	42·22	44·90	48·55	45·80	49·47	50·40	54·14	52·26	56·02	56·97	60·78	68·48
	3½	40·33	43·63	46·20	49·70	47·07	50·59	51·49	55·11	53·30	57·35	57·87	61·60	69·16
	4	42·17	45·29	47·72	51·08	48·55	51·86	52·80	56·30	54·54	58·08	58·97	62·59	70·01
Ten Rivets	2⅝	48·17	52·40	55·69	60·16	56·79	61·30	62·44	67·05	64·74	69·40	70·61	75·30	84·91
	3	50·01	54·02	57·15	61·44	58·21	62·54	63·64	68·11	65·87	70·38	71·53	76·14	85·57
	3½	52·87	56·58	59·51	63·55	60·50	64·58	65·62	69·87	67·73	72·05	73·14	77·57	86·75
	4	56·09	59·52	62·24	66 04	63·18	67·01	68·00	72·22	70·01	74·08	75·13	79·40	88·20
Flange width	. . .	5	5	6	6	6½	6½	7	7	7½	7½	8	8	8
Joist centres	. . .	6	7	7	8	7	8	8	9	8	9	9	10	12





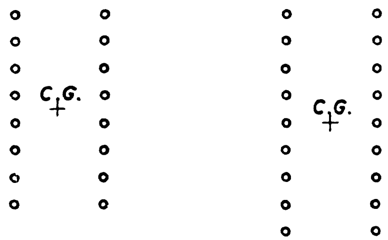
RIVET GROUPS UNDER MOMENT FOR TWIN-STALK COLUMNS

CENTRE OF ROTATION AT CENTRE OF GROUP

Two vertical rows of rivets

Values of $\frac{\Sigma x^2 + \Sigma y^2}{Z}$

	Vertical Pitch in inches	*Horizontal Pitch in inches* $8\frac{3}{4}$	$9\frac{3}{4}$	$10\frac{1}{2}$	$11\frac{1}{2}$	$10\frac{3}{4}$	$11\frac{3}{4}$	12	13	$12\frac{1}{2}$	$13\frac{1}{2}$	$13\frac{3}{4}$	$14\frac{3}{4}$	$16\frac{3}{4}$
Twelve Rivets	$2\frac{5}{8}$	59·70	64·39	68·05	73·11	69·31	74·41	75·71	81·00	78·33	83·69	85·05	90·54	101·8
	3	62·73	67·10	70·53	75·31	71·71	76·53	77·77	82·82	80·28	85·40	86·66	91·98	102·9
	$3\frac{1}{2}$	67·30	71·32	74·46	78·84	75·50	79·97	81·13	85·85	83·48	88·36	89·48	94·53	104·9
	4	72·38	75·94	78·83	82·98	79·88	83 98	85·08	89·44	87·26	91·77	92·93	97·56	107·5
Fourteen Rivets	$2\frac{5}{8}$	72·58	77·59	81·55	87·04	82·89	88·46	89·89	95·73	91·81	98·72	100·2	106·3	118·9
	3	77·20	81·71	85·40	90·53	86·69	91·92	93·16	98·69	95·97	101·5	103·0	108·7	120·9
	$3\frac{1}{2}$	83·83	87·97	91·38	95·98	92·49	97·19	98·43	103·4	100·9	106·1	107·4	112·8	124·2
	4	91·15	94·88	97·85	102·2	98·97	103·2	104·3	109·0	106·6	111·4	112·6	117·7	128·4
Flange width	. . .	5	5	6	6	$6\frac{1}{2}$	$6\frac{1}{2}$	7	7	$7\frac{1}{2}$	$7\frac{1}{2}$	8	8	8
Joist centres	. . .	6	7	7	8	7	8	8	9	8	9	9	10	12



RIVET GROUPS UNDER MOMENT FOR TWIN-STALK COLUMNS CENTRE OF ROTATION AT CENTRE OF GROUPS

Two vertical rows of rivets

Values of $\frac{\Sigma x^2 + \Sigma y^2}{Z}$

		Horizontal Pitch in inches												
		8¾	9¾	10½	11½	10¾	11¾	12	13	12½	13½	13¾	14¾	16¾
Sixteen Rivets — *Vertical Pitch in inches*	2⅝	86·94	92·22	96·39	102·2	97·84	103·8	105·3	111·5	108·3	114·7	116·3	123·0	136·8
	3	93·26	98·12	102·0	107·3	103·3	108·7	110·2	115·9	113·0	119·0	120·5	126·7	139·8
	3½	102·6	106·9	110·4	115·1	111·5	116·4	117·7	122·9	120·3	125·7	127·1	132·9	144·9
	4	112·6	116·3	119·3	123·8	120·4	124·8	126·1	130·8	128·4	133·4	134·6	140·0	151·2
Eighteen Rivets — *Vertical Pitch in inches*	2⅝	102·9	108·3	112·8	118·8	114·2	120·4	122·0	128·5	125·2	132·0	133·7	140·8	155·6
	3	111·5	116·4	120·3	125·9	121·8	127·3	128·8	134·8	131·8	138·0	139·6	146·2	160·1
	3½	123·8	128·0	131·5	136·5	132·7	137·7	139·1	144·5	141·7	147·4	148·8	154·8	167·5
	4	136·6	140·4	143·5	147·9	144·5	149·1	150·3	155·2	152·6	157·7	159·1	164·5	176·2
Flange width	. . .	5	5	6	6	6½	6½	7	7	7½	7½	8	8	8
Joist centres	. . .	6	7	7	8	7	8	8	9	8	9	9	10	12

34. TABLES OF RIVETS AND BOLTS

THE tables on pages 698, 699 and 700 are extracted from *Safe Load Tables,* prepared and issued jointly by the British Constructional Steelwork Association and the British Steel Makers. They are based on B.S. 449 : 1948, the relevant data being as follows:

Permissible Stresses

Permissible stresses on gross areas of rivets and bolts:

	Single Shear *tons per sq. in.*	*Bearing* *tons per sq. in.*
Shop-driven rivets or turned and fitted bolts	6	12
Field-driven rivets	5	10
Black bolts	4	8

Multiple Shear

For rivets and bolts in double or quadruple shear, the area to be assumed shall be two or four times respectively the area for single shear.

Where the rivets or turned and fitted bolts are in double shear, the permissible bearing stress on the central thickness of metal may be increased by 25 per cent.

Critical Values

Bearing values printed in ordinary type are less than single shear. In these cases, the bearing values are the determining factors.

Bearing values printed in prominent type are greater than single, and less than double shear, so that in the case of

(a) single shear, the shearing value is the criterion;
(b) double shear, the bearing value is the criterion.

Bearing values printed in light type are greater than double shear. In these cases, the shearing values are the criterion.

SHEARING and BEARING VALUES FOR RIVETS AND BOLTS

BASED ON B.S. 449 REVISED 1948

Effective Diameter of Rivet or Bolt in inches	Area in square inches	Shearing Value @ 6 tons/inch² Single Shear	Double Shear	Simple Bearing Value @ 12 tons/inch² on Thickness in inches of plate passed through: 1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	7/8	1
3/8	0·1104	0·66	1·33	**1·13**	1·41	1·69								
7/16	0·1503	0·90	1·80	**1·31**	**1·64**	1·97	2·30							
1/2	0·1964	1·18	2·36	**1·50**	**1·88**	**2·25**	2·63	3·00						
9/16	0·2485	1·49	2·98	**1·69**	**2·11**	**2·53**	**2·95**	3·38	3·80					
5/8	0·3068	1·84	3·68	**1·88**	**2·34**	**2·81**	**3·28**	3·75	4·22					
11/16	0·3712	2·23	4·45	2·06	**2·58**	**3·09**	**3·61**	**4·13**	4·64	5·16				
3/4	0·4418	2·65	5·30	2·25	**2·81**	**3·38**	**3·94**	**4·50**	**5·06**	5·63	6·19			
13/16	0·5185	3·11	6·22	2·44	3·05	**3·66**	**4·27**	**4·88**	**5·48**	**6·09**	6·70	7·31		
7/8	0·6013	3·61	7·22	2·63	3·28	**3·94**	**4·59**	**5·25**	**5·91**	**6·56**	7·22	7·88		
15/16	0·6903	4·14	8·28	2·81	3·52	**4·22**	**4·92**	**5·63**	**6·33**	**7·03**	**7·73**	8·44	9·84	
1	0·7854	4·71	9·42	3·00	3·75	4·50	**5·25**	**6·00**	**6·75**	**7·50**	**8·25**	**9·00**	10·50	12·00
1 1/16	0·8866	5·32	10·64	3·19	3·98	4·78	**5·58**	**6·38**	**7·17**	**7·97**	**8·77**	**9·56**	11·16	12·75

Effective Diameter of Rivet or Bolt in inches	Area in square inches	Shearing Value @ 5 tons/inch² Single Shear	Double Shear	Simple Bearing Value @ 10 tons/inch² on Thickness in inches of plate passed through: 1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	7/8	1
3/8	0·1104	0·55	1·10	**0·94**	1·17	1·41								
7/16	0·1503	0·75	1·50	**1·09**	**1·37**	1·64	1·91							
1/2	0·1964	0·98	1·96	**1·25**	**1·56**	**1·88**	2·19	2·50						
9/16	0·2485	1·24	2·49	**1·41**	**1·76**	**2·11**	**2·46**	2·81	3·16					
5/8	0·3068	1·53	3·07	**1·56**	**1·95**	**2·34**	**2·73**	3·13	3·52					
11/16	0·3712	1·86	3·71	1·72	**2·15**	**2·58**	**3·01**	**3·44**	3·87	4·30				
3/4	0·4418	2·21	4·42	1·88	**2·34**	**2·81**	**3·28**	**3·75**	**4·22**	4·69	5·16			
13/16	0·5185	2·59	5·18	2·03	2·54	**3·05**	**3·55**	**4·06**	**4·57**	**5·08**	5·59	6·09		
7/8	0·6013	3·01	6·01	2·19	2·73	**3·28**	**3·83**	**4·38**	**4·92**	**5·47**	6·02	6·56		
15/16	0·6903	3·45	6·90	2·34	2·93	**3·52**	**4·10**	**4·69**	**5·27**	**5·86**	**6·45**	7·03	8·20	
1	0·7854	3·93	7·85	2·50	3·13	3·75	**4·38**	**5·00**	**5·63**	**6·25**	**6·88**	**7·50**	8·75	10·0[illegible]
1 1/16	0·8866	4·43	8·87	2·66	3·32	3·98	**4·65**	**5·31**	**5·98**	**6·64**	**7·30**	**7·97**	9·30	10·6[illegible]

For explanations of tables, see page 697

BASED ON B.S. 449 REVISED 1948

SHEARING and BEARING VALUES FOR RIVETS AND BOLTS

Effective Diameter of Rivet or Bolt in inches	Area in square inches	Shearing Value @ 6 tons/inch²		Enclosed Bearing Value @ 12 tons/inch² + 25% on Thickness in inches of enclosed plate								
		Single Shear	Double Shear	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4
3/8	0·1104	0·66	1·33	1·41	1·76							
7/16	0·1503	0·90	1·80	**1·64**	2·05	2·46						
1/2	0·1964	1·18	2·36	**1·88**	**2·34**	2·81	3·28					
9/16	0·2485	1·49	2·98	**2·11**	**2·64**	3·16	3·69					
5/8	0·3068	1·84	3·68	**2·34**	**2·93**	**3·52**	4·10	4·69				
11/16	0·3712	2·23	4·45	**2·58**	**3·22**	**3·87**	4·51	5·16				
3/4	0·4418	2·65	5·30	**2·81**	**3·52**	**4·22**	**4·92**	5·63	6·33			
13/16	0·5185	3·11	6·22	3·05	**3·81**	**4·57**	**5·33**	**6·09**	6·86	7·62		
7/8	0·6013	3·61	7·22	3·28	**4·10**	**4·92**	**5·74**	**6·56**	7·38	8·20		
15/16	0·6903	4·14	8·28	3·52	**4·39**	**5·27**	**6·15**	**7·03**	**7·91**	8·79	9·67	
1	0·7854	4·71	9·42	3·75	4·69	**5·63**	**6·56**	**7·50**	**8·44**	**9·38**	10·31	11·25
1 1/16	0·8866	5·32	10·64	3·98	4·98	**5·98**	**6·97**	**7·97**	**8·96**	**9·96**	10·96	11·95

Effective Diameter of Rivet or Bolt in inches	Area in square inches	Shearing Value @ 5 tons/inch²		Enclosed Bearing Value @ 10 tons/inch² + 25% on Thickness in inches of enclosed plate								
		Single Shear	Double Shear	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4
3/8	0·1104	0·55	1·10	1·17	1·46							
7/16	0·1503	0·75	1·50	**1·37**	1·71	2·05						
1/2	0·1964	0·98	1·96	**1·56**	**1·95**	2·34	2·73					
9/16	0·2485	1·24	2·49	**1·76**	**2·20**	2·64	3·08					
5/8	0·3068	1·53	3·07	**1·95**	**2·44**	**2·93**	3·42	3·91				
11/16	0·3712	1·86	3·71	**2·15**	**2·69**	**3·22**	3·76	4·30				
3/4	0·4418	2·21	4·42	**2·34**	**2·93**	**3·52**	**4·10**	4·69	5·27			
13/16	0·5185	2·59	5·18	2·54	**3·17**	**3·81**	**4·44**	**5·08**	5·71	6·35		
7/8	0·6013	3·01	6·01	2·73	**3·42**	**4·10**	**4·79**	**5·47**	6·15	6·84		
15/16	0·6903	3·45	6·90	2·93	**3·66**	**4·39**	**5·13**	**5·86**	**6·59**	7·32	8·06	
1	0·7854	3·93	7·85	3·13	3·91	**4·69**	**5·47**	**6·25**	**7·03**	**7·81**	8·59	9·38
1 1/16	0·8866	4·43	8·87	3·32	4·15	**4·98**	**5·81**	**6·64**	**7·47**	**8·30**	9·13	9·96

For areas to be deducted from a bar for one hole, see page 700

SHEARING and BEARING VALUES FOR BLACK BOLTS

BASED ON B.S. 449 REVISED 1948

Diameter of Bolt in inches	Area in square inches	Shearing Value @ 4 tons/inch²		Bearing Value @ 8 tons/inch² on Thickness in inches of plate passed through										
		Single Shear	Double Shear	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	7/8	1
3/8	0.1104	0.44	0.88	**0.75**	0.94	1.13								
7/16	0.1503	0.60	1.20	**0.88**	**1.09**	1.31	1.53							
1/2	0.1964	0.79	1.57	**1.00**	**1.25**	**1.50**	1.75	2.00						
9/16	0.2485	0.99	1.99	**1.13**	**1.41**	**1.69**	**1.97**	2.25	2.53					
5/8	0.3068	1.23	2.45	**1.25**	**1.56**	**1.88**	**2.19**	2.50	2.81					
11/16	0.3712	1.48	2.97	1.38	**1.72**	**2.06**	**2.41**	**2.75**	3.09	3.44				
3/4	0.4418	1.77	3.53	1.50	**1.88**	**2.25**	**2.63**	**3.00**	**3.38**	3.75	4.13			
13/16	0.5185	2.07	4.15	**1.63**	2.03	**2.44**	**2.84**	**3.25**	**3.66**	**4.06**	4.47	4.88		
7/8	0.6013	2.41	4.81	1.75	2.19	**2.63**	**3.06**	**3.50**	**3.94**	**4.38**	4.81	5.25		
15/16	0.6903	2.76	5.52	1.88	2.34	**2.81**	**3.28**	**3.75**	**4.22**	**4.69**	**5.16**	5.63	6.56	
1	0.7854	3.14	6.28	2.00	2.50	3.00	**3.50**	**4.00**	**4.50**	**5.00**	**5.50**	**6.00**	7.00	8.00
1 1/16	0.8866	3.55	7.09	2.13	2.66	3.19	**3.72**	**4.25**	**4.78**	**5.31**	**5.84**	**6.38**	7.44	8.50

AREAS IN SQUARE INCHES TO BE DEDUCTED FOR ONE HOLE THROUGH A MEMBER

Diameter of hole in inches	Thickness of member at hole in inches										
	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	7/8	1
3/8	0.094	0.117	0.141	0.164	0.188						
7/16	0.109	0.137	0.164	0.191	0.219	0.246					
1/2	0.125	0.156	0.188	0.219	0.250	0.281	0.313				
9/16	0.141	0.176	0.211	0.246	0.281	0.316	0.352	0.387			
5/8	0.156	0.195	0.234	0.273	0.313	0.352	0.391	0.430	0.469		
11/16	0.172	0.215	0.258	0.301	0.344	0.387	0.430	0.473	0.516	0.602	
3/4	0.188	0.234	0.281	0.328	0.375	0.422	0.469	0.516	0.563	0.656	0.750
13/16	0.203	0.254	0.305	0.356	0.406	0.457	0.508	0.559	0.609	0.711	0.813
7/8	0.219	0.273	0.328	0.383	0.438	0.492	0.547	0.602	0.656	0.766	0.875
15/16	0.234	0.293	0.352	0.410	0.469	0.527	0.586	0.645	0.703	0.820	0.938
1	0.250	0.313	0.375	0.438	0.500	0.563	0.625	0.688	0.750	0.875	1.000
1 1/16	0.266	0.332	0.398	0.465	0.531	0.598	0.664	0.731	0.797	0.930	1.063

35. OVERHEAD CRANES

B.S. 466, Electric Overhead Travelling Cranes, stipulates the minimum requirements for power-driven cranes of all sizes intended for use in factories and workshops.

The names and addresses of crane-makers may be obtained from the Association of Crane Makers, 52, Lincoln's Inn Fields, London, W.C.2.

Overhead cranes vary greatly in design with each individual maker, and it is not possible to give approximate dimensions and loads which are applicable to all cranes. However, the data shown on pages 702 and 703 give typical details of cranes manufactured by Messrs. Herbert Morris Ltd. of Loughborough, England, to whom grateful acknowledgment is made.

Loads and clearances are given to enable a designer to prepare preliminary estimates for crane girders and columns. Other dimensions and information should be obtained directly from the crane-makers.

The allowances to be made for the dynamic effect, including impact, of a crane on a structure, are laid down in Clause 15 of B.S. 449:1948.

General data for girders and columns are given in the following pages. The design for an Engineering Workshop, given on pages 706 to 726, includes crane gantry girders.

Crane Gantry Girders

The following information relating to gantry girders applies to girders which are simply supported. If it is desired to employ continuous spans, then the influence lines embodied in Figs. 109 to 121 should be consulted.

LOADS ON GANTRY GIRDERS



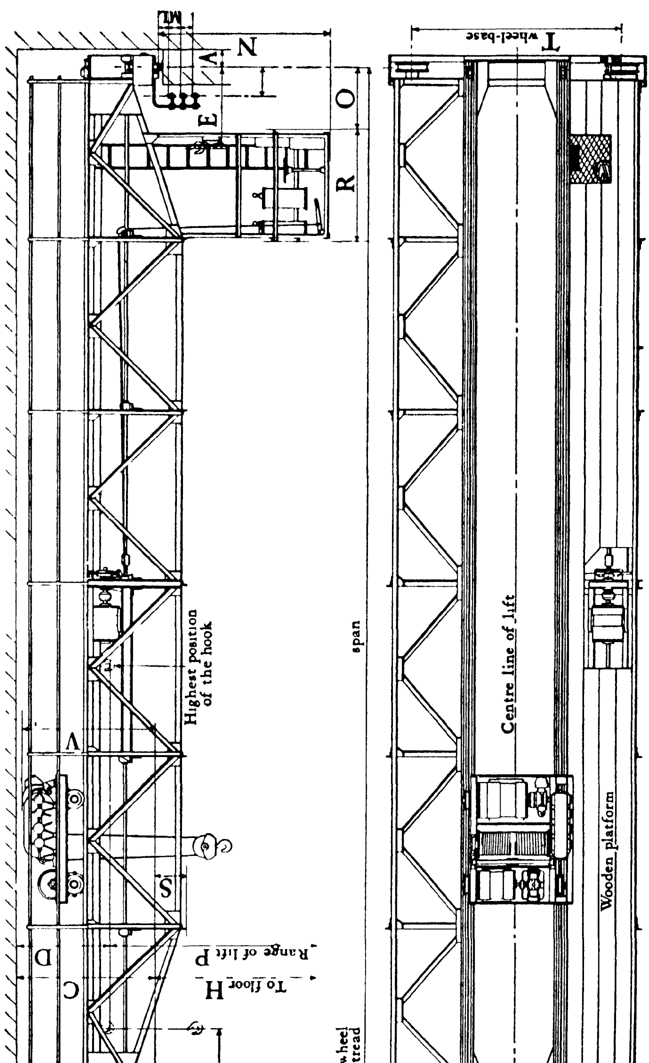
(a) One Crane (b) Two Cranes coupled

FIG. 445

Bending Moments

Consider Figs. 445 (a) and (b) which show respectively a pair of crane gantry loads W and two similar pairs coupled together. Then,

(a) The maximum B.M. under *any wheel* occurs when its axis and the centre of gravity of the load system are equidistant from the centre of the span.



N
M
L
A
E
O
R
V
S
D
C
P Range of lift
H To floor
Highest position of the hook
span
A
B wheel tread
T wheel-base
Centre line of lift
Wooden platform
J
G

Working Load	*Tons*	1	2	3	5	7½	10	15	20	25	30	40	50	60	80	100
All spans P=35′ 0″	*D*	3′ 0″	3′ 4″	3′ 7″	4′ 3″	5′ 3″	5′ 6″	7′ 0″	7′ 2″	9′ 0″	9′ 6″	10′ 0″	11′ 0″	12′ 0″	12′ 9″	13′ 6″
	E	2′ 0″	2′ 3″	2′ 4″	2′ 10″	3′ 10″	3′ 10″	3′ 6″	3′ 9″	4′ 3″	4′ 6″	4′ 9″	5′ 0″	5′ 3″	5′ 6″	5′ 9″
	F	2′ 0″	2′ 0″	2′ 0″	2′ 3″	2′ 9″	2′ 9″	3′ 2″	3′ 2″	3′ 3″	3′ 3″	3′ 5″	4′ 0″	4′ 3″	6′ 0″	7′ 0″
20′ 0″ span	*A*	7″	7″	7″	8″	8″	8″	9″	10″	11″	11″	11″	1′ 0″	1′ 0″	1′ 2″	1′ 3″
	C	4′ 2″	4′ 9″	4′ 10″	5′ 6″	6′ 4″	6′ 8″	7′ 2″	7′ 6″	7′ 6″	7′ 9″	7′ 9″	8′ 3″	8′ 9″	10′ 0″	11′ 0″
	T	7′ 8″	7′ 8″	7′ 8″	10′ 0″	10′ 0″	10′ 0″	11′ 0″	12′ 6″	13′ 3″	13′ 6″	14′ 6″	15′ 6″	15′ 6″	17′ 6″	11′ 0″*
Total weight of crane	tons	4	4½	5	6	7	8	11	13	16	18	21	25	29	35	46
Maximum load per wheel	tons	2	2½	3½	5	6½	8	11	14	17	20	26	32	37	49	31*
40′ 0″ span	*A*	7″	7″	7″	8″	8″	8″	9″	10″	11″	11″	1′ 0″	1′ 0″	1′ 1″	1′ 3″	1′ 4″
	C	4′ 6″	5′ 0″	5′ 3″	6′ 0″	7′ 0″	7′ 3″	7′ 4″	7′ 6″	7′ 6″	7′ 9″	7′ 9″	8′ 3″	8′ 9″	10′ 0″	11′ 0″
	T	9′ 0″	9′ 0″	9′ 0″	10′ 0″	10′ 0″	10′ 0″	11′ 0″	12′ 6″	13′ 3″	13′ 6″	14′ 6″	15′ 6″	15′ 6″	17′ 6″	11′ 0″*
Total weight of crane	tons	7	8	9	10	11	12	15	18	21	23	27	32	38	44	56
Maximum load per wheel	tons	3	3½	4½	5½	7	9	13	16	19	22	29	34	40	50	32*
60′ 0″ span	*A*	8″	8″	8″	9″	9″	9″	10″	10″	11″	11″	1′ 0″	1′ 0″	1′ 1″	1′ 4″	1′ 5″
	C	5′ 4″	5′ 9″	5′ 10″	6′ 3″	7′ 2″	7′ 3″	7′ 4″	7′ 6″	7′ 8″	8′ 0″	8′ 6″	8′ 9″	9′ 3″	11′ 0″	12′ 0″
	T	12′ 0″	12′ 0″	12′ 0″	12′ 0″	12′ 0″	12′ 0″	12′ 0″	12′ 6″	13′ 6″	13′ 6″	14′ 6″	16′ 0″	16′ 6″	12′ 0″*	12′ 0 *
Total weight of crane	tons	10	12	13	14	16	17	20	23	26	29	34	40	45	54	70
Maximum load per wheel	tons	4	5	5½	7	8½	10	14	18	21	24	31	37	43	28*	35*
80′ 0″ span	*A*	9″	9″	9″	9″	10″	10″	10″	11″	1′ 0″	1′ 0″	1′ 1″	1′ 1″	1′ 2″	1′ 4″	1′ 6″
	C	6′ 0″	6′ 4″	6′ 5″	6′ 8″	7′ 4″	7′ 4″	7′ 5″	7′ 6″	7′ 8″	8′ 0″	8′ 6″	8′ 9″	9′ 6″	11′ 0″	13′ 0″
	T	13′ 3″	13′ 3″	13′ 3″	13′ 3″	13′ 6″	13′ 6″	13′ 6″	13′ 6″	13′ 6″	14′ 0″	15′ 6″	16′ 6″	16′ 6″	14′ 0″*	14′ 0″*
Total weight of crane	tons	16	17	18	20	22	24	28	31	34	38	44	50	56	65	84
Maximum load per wheel	tons	5	6	7	9	11	13	16	20	24	27	34	40	48	30*	38*
100′ 0″ span	*A*	10″	10″	10″	10″	10″	10″	11″	11″	1′ 0″	1′ 0″	1′ 1″	1′ 1″	1′ 2″	1′ 4″	1′ 6″
	C	6′ 8″	7′ 0″	7′ 0″	7′ 3″	7′ 6″	7′ 6″	7′ 6″	7′ 8″	7′ 10″	8′ 3″	8′ 9″	9′ 0″	9′ 9″	12′ 0″	14′ 0″
	T	16′ 9″	16′ 9″	16′ 9″	16′ 9″	17′ 0″	17′ 0″	17′ 0″	17′ 0″	17′ 0″	17′ 0″	17′ 0″	17′ 0″	17′ 0″*	17′ 0″*	17′ 0″*
Total weight of crane	tons	20	22	24	26	30	32	35	39	43	47	53	60	67	78	98
Maximum load per wheel	tons	7	8	10	11	13	15	19	23	26	30	37	44	26*	33*	40*

F = Minimum approach of crane-hook to gantry rail.

* There are four wheels in each end carriage. Dimension *T* is the centres of the two-wheeled bogies, these wheels being approximately 5′ 6″ apart.

(b) The maximum B.M. at *any point* in the span is found by moving the loads across the span, the maximum B.M. being given when $W_1/x - W_2/y$ changes sign when the load at the point is included first in the total load W_1 on section x and then in the total load W_2 on section y. (See Fig. 446.)

(c) For one pair of wheel loads, the maximum B.M. on span L occurs with one wheel load only on the span for values of the wheel base a equal to or greater than $0{\cdot}586L$ and for two wheel loads on the span for values of a equal to or less than $0{\cdot}586L$.

Reactions at Supports of Girders

(1) For one crane the maximum load on an end or intermediate column is equal to the maximum shear force in the span.

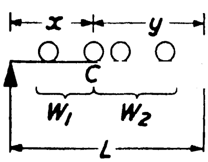


FIG. 446

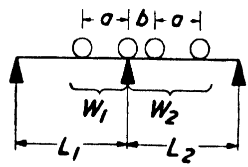


FIG. 447

LOADS ON GANTRY GIRDERS

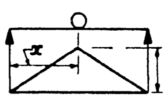
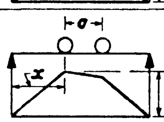
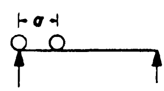
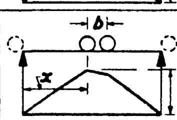
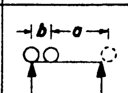
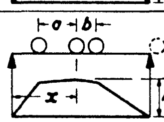
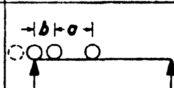
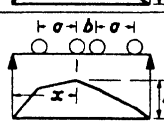
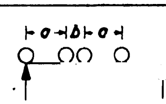
Loading Conditions: No. of Wheels on span	Loading Conditions: No. of Cranes on span	Bending Moment Each Load = W Span = L Position for max. M		Shear Force Each Load = W Span = L Position for max. S.F.	
1	1 See note (c) in text		$x = \frac{L}{2}$ $M = \frac{WL}{4}$		$S = W$
2	1 See note (c) in text		$x = \left(\frac{L}{2} - \frac{a}{4}\right)$ $M = \frac{2Wx^2}{L}$		$S = \frac{W(2L-a)}{L}$
2	2		$x = \left(\frac{L}{2} - \frac{b}{4}\right)$ $M = \frac{2Wx^2}{L}$		$S = \frac{W(2L-b)}{L}$
3	2		$x = \frac{3L+a-b}{6}$ $M = \frac{3Wx^2}{L} - Wa$		$S = \frac{W(3L-a-2b)}{L}$
4	2		$x = \left(\frac{L}{2} - \frac{b}{4}\right)$ $M = \frac{4Wx^2}{L} - Wa$		$S = \frac{W(4L-4a-2b)}{L}$

FIG. 448

(2) For two cranes on an end span the maximum load on an end column is equal to the maximum shear force in the span.

(3) For two cranes over an intermediate support the maximum load on the column is given when $W_1/L_1 - W_2/L_2$ changes sign when the load over the column is included first in the load W_1 on span L_1 and then in the load W_2 on span L_2. (See Fig. 447.)

Tabular Data

Fig. 448 shows the values of the maximum B.M. and S.F. in the girders while Fig. 449 shows the values of the maximum column loads under various loading conditions.

GANTRY GIRDERS

END AND INTERMEDIATE COLUMN LOADS

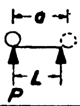
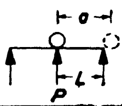
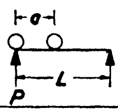
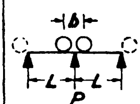
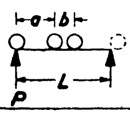
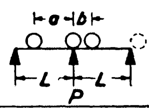
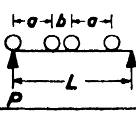
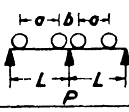
No. of Wheels on Girder	End Column Maximum Load P		Intermediate Column Maximum Load P	
1		$P=W$		$P=W$
2		$P=\frac{W(2L-a)}{L}$		$P=\frac{W(2L-b)}{L}$
3		$P=\frac{W(3L-2a-b)}{L}$		$P=\frac{W(3L-a-b)}{L}$
4		$P=\frac{W(4L-4a-2b)}{L}$		$P=\frac{W(4L-2a-2b)}{L}$

FIG. 449

36. ENGINEERING WORKSHOP DESIGN

THE following calculations for a steel-framed workshop with electric overhead travelling crane are such as would be done in the project stage, for preparing an estimate of cost, and would be subject to final check for detail compliance with B.S. 449.

Dimensions of building:

Length	150 ft. 0 in.
Span, centres of side stanchions	60 ft. 0 in.
Clear height to underside of roof	36 ft. 0 in.
Rise of roof	one-fifth of span

Roof covering:

Corrugated galvanised steel sheets, 10/3-in. corrugations, 20 gauge, double corr. lap.

Side covering:

Corrugated galvanised steel sheets, 10/3-in. corrugations, 22 gauge, $1\frac{1}{2}$ corr. lap.

One side to be designed for future 60-ft. span roof and 15-ton e.o.t. crane. One end to be designed for future extension.

Wind pressure:

Windward side, +10 lb. per sq. ft. full height.

Leeward side, −10 lb. per sq. ft. full height.

Superimposed load on roof: 15 lb. per sq. ft. plan area.

Gantry to carry one electric overhead travelling crane:

Crane capacity	15 tons
Minimum distance from face of side stanchion to gantry rail	1 ft. 0 in.
Approx. distance from centre of side stanchion to gantry rail	1 ft. 6 in.
Approx. centres of gantry rails	57 ft. 0 in.
Approx. minimum approach of crane hook to gantry rail	3 ft. 0 in.
Clear height from gantry rail to underside of roof	7 ft. 0 in.
Height from floor level to gantry rail	29 ft. 0 in.
Centres of end carriage wheels (2 wheels per carriage)	12 ft. 0 in.
Weight of crane, excluding crab	18 tons
Weight of crab	5 tons

Maximum vertical load per end carriage wheel,

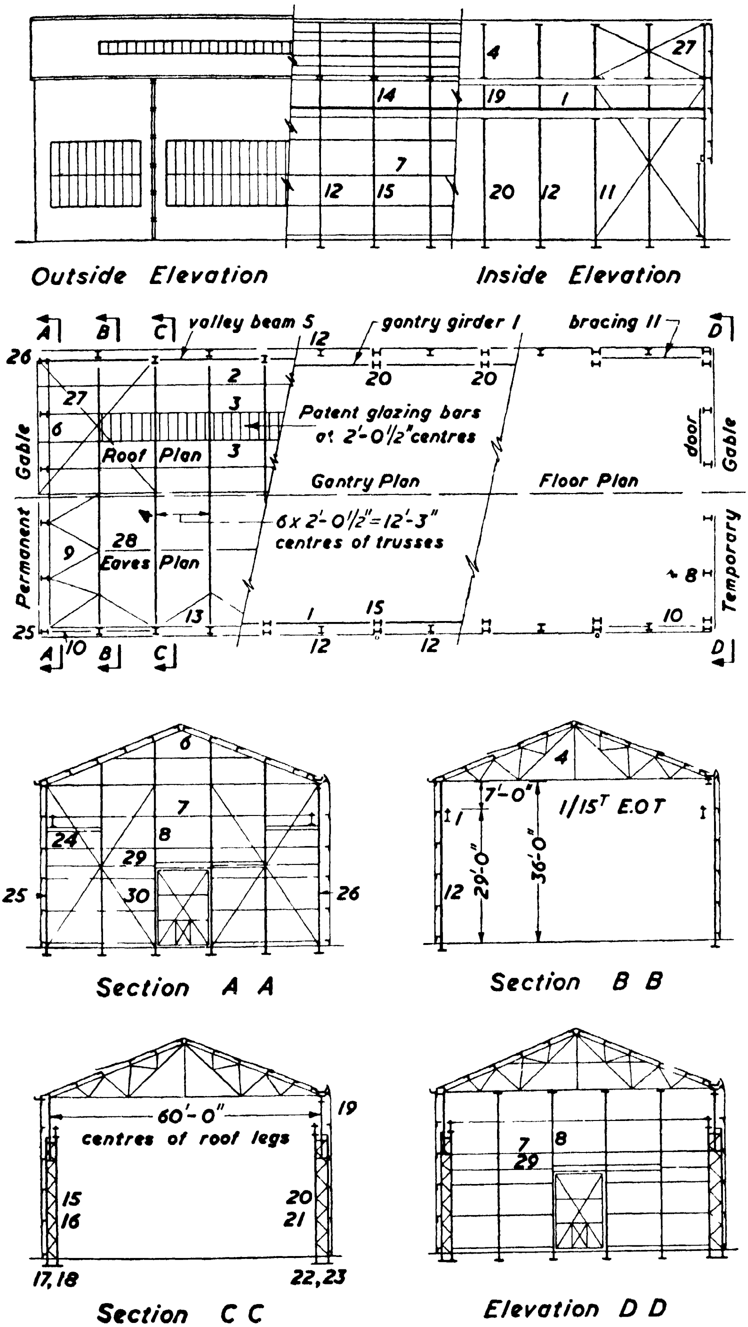
$$= \frac{18{\cdot}0}{4} + \frac{54{\cdot}0}{57{\cdot}0}\left(\frac{5{\cdot}0 + 15{\cdot}0}{2}\right) = 4{\cdot}5 + 9{\cdot}5$$

$$= 14{\cdot}0 \text{ tons}$$

add for impact, etc.,

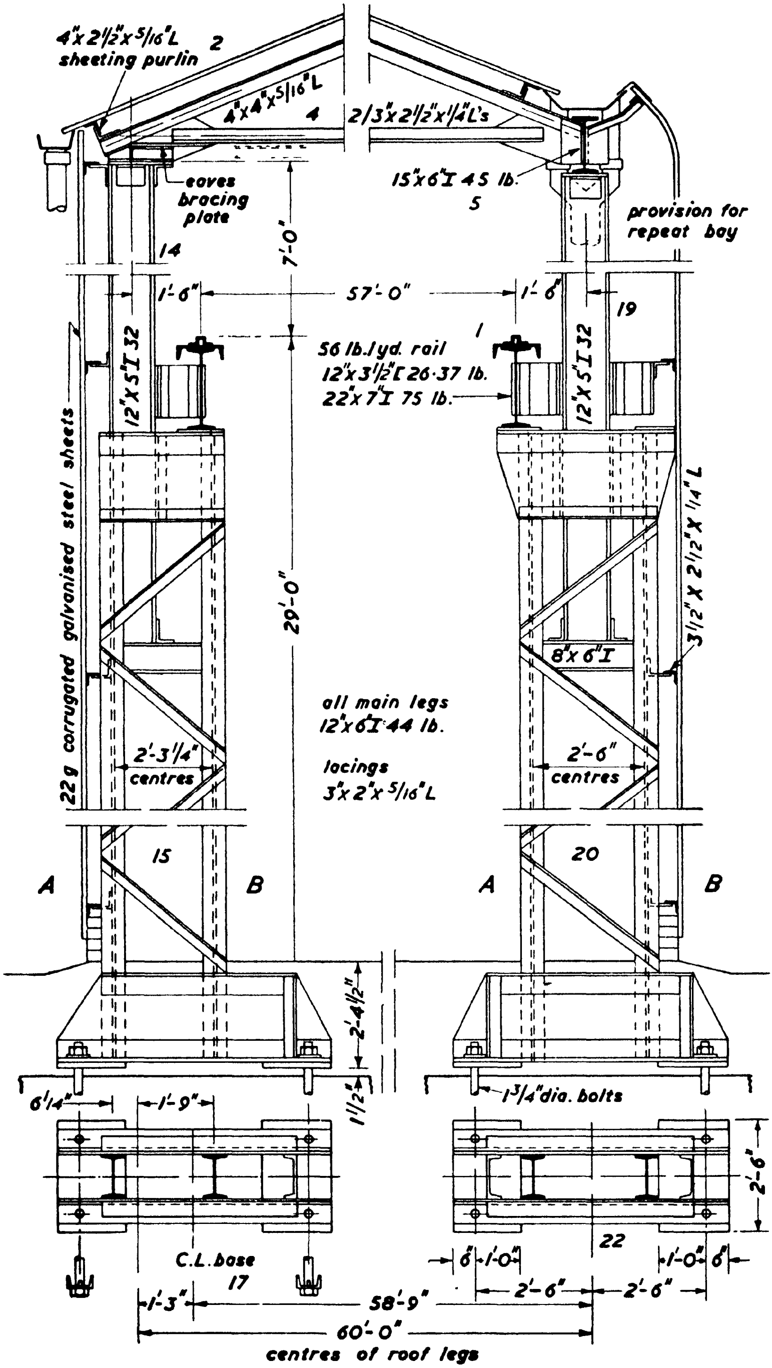
25 per cent $= 3{\cdot}5$

$17{\cdot}5$ tons



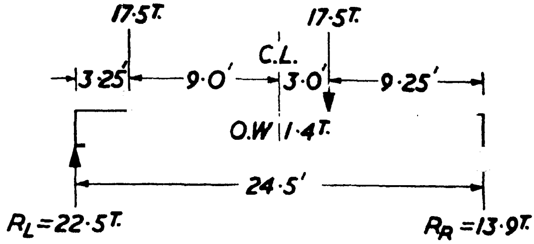
GENERAL ARRANGEMENT OF WORKSHOP BUILDING

FIG. 450



DETAILS OF MAIN STANCHIONS

FIG. 451



LOADING DIAGRAM FOR MAXIMUM BENDING MOMENT IN GANTRY GIRDER

FIG. 452

In the following calculations the numbers of the sub-headings are the same as for the appropriate members in Figs. 450 and 451.

1. Gantry Girder

Span, 24 ft. 6 in.

Section, 56 lb. per yd. rail.
12-in. × 3½-in. channel.
22-in. × 7-in. 'I'.

Z_{xx} top flange, 269·0 in.^{3}.
bottom flange, 169·0 in.^{3}.

Z_{yy} top flange, 30·0 in.^{3}.

Permissible stress, B.S. 449:1948 = 11·0 tons per sq. in.

Vertical loads, 1·4 + 2 × 17·5 = 36·4 tons.

For M_{max},
$$R_R = 0{\cdot}7 + \frac{35 \times 9{\cdot}25}{24{\cdot}5} = 13{\cdot}9 \text{ tons}$$

$$M_{max} = 13{\cdot}9 \times 9{\cdot}25 \text{ ft.} - \frac{1{\cdot}4 \times 9{\cdot}25^2}{24{\cdot}5 \times 2} = 126{\cdot}3 \text{ tons ft.}$$

Transverse load on all four crane wheels,
= 10 per cent of (5·0 tons crab + 15·0 tons lift)
= 2 tons,

hence transverse load per wheel
= 0·5 tons, i.e. one-thirty-fifth of vertical load,

hence, transverse B.M.
= 126·3/35 = 3·6 tons ft.

Maximum bending stress in top flange,
$$= \frac{126{\cdot}3 \times 12}{269} + \frac{3{\cdot}6 \times 12}{30{\cdot}0} = 7{\cdot}1 \text{ tons per sq. in.}$$

Maximum bending stress in bottom flange
$$= \frac{126{\cdot}3 \times 12}{169} = 9{\cdot}0 \text{ tons per sq. in.}$$

With one wheel at end of span, the vertical shear

$$=0{\cdot}7+17{\cdot}5+\frac{17{\cdot}5\times 12{\cdot}5}{24{\cdot}5}$$

$$=27{\cdot}1 \text{ tons.}$$

$$\text{Shear stress}=\frac{27{\cdot}1}{22 \text{ in.}\times 0{\cdot}5 \text{ in.}}=2{\cdot}5 \text{ tons per sq. in.}$$

Vertical load on stanchion from two gantry spans

$$=27{\cdot}1+0{\cdot}7=27{\cdot}8 \text{ tons.}$$

Transverse load on stanchion from two end carriage wheels

$$=0{\cdot}5+\frac{0{\cdot}5\times 12{\cdot}5}{24{\cdot}5}=0{\cdot}76 \text{ ton.}$$

Purlins

$$\text{Minimum depth required}=\frac{\text{Span}}{45}=\frac{12{\cdot}25 \text{ ft.}\times 12}{45}=3{\cdot}3 \text{ in.}$$

$$\text{Minimum width required}=\frac{\text{Span}}{60}=\frac{12{\cdot}25 \text{ ft.}\times 12}{60}=2{\cdot}5 \text{ in.}$$

2. *Sheeting purlin*

4-in. × $2\frac{1}{2}$-in. × $\frac{5}{16}$-in. angle ($Z_{xx}=1{\cdot}17$ in.^{3}).
Spacing, 6 ft. 6 in. up slope, 6 ft. 0 in. horizontal.

Dead load of purlin, 12·25 ft. × 6·58 lb.	= 81 lb.
Dead load of sheeting, 12·25 ft. × 6·5 ft. × 2·3 lb. per sq. ft.	= 183
Superimposed load, 12·25 ft. × 6·0 ft. × 15 lb. per sq. ft.	= 1103
	1367 lb.

$$Z_{xx} \text{ required}=\frac{1367\times 12{\cdot}25\times 12}{2240\times 90}=1{\cdot}0 \text{ ins.}^3$$

3. *Glazing purlin*

4-in. × 3-in. × $\frac{5}{16}$-in angle ($Z_{xx}=1{\cdot}20$ in.^{3}) with 2-in. × 2-in. × $\frac{1}{4}$-in. angle shelf.

Dead load of purlin, 12·25 × 10·3 lb.	= 127 lb.
Dead load of sheeting, 12·25 × 3·25 × 2·3	= 92
Dead load of glazing, 12·25 × 3·25 × 6·0	= 240
Superimposed load, 12·25 × 6·0 × 15	= 1103
	1562 lb.

$$Z_{xx} \text{ required}=\frac{1562\times 12{\cdot}25\times 12}{2240\times 90}=1{\cdot}14 \text{ ins.}^3$$

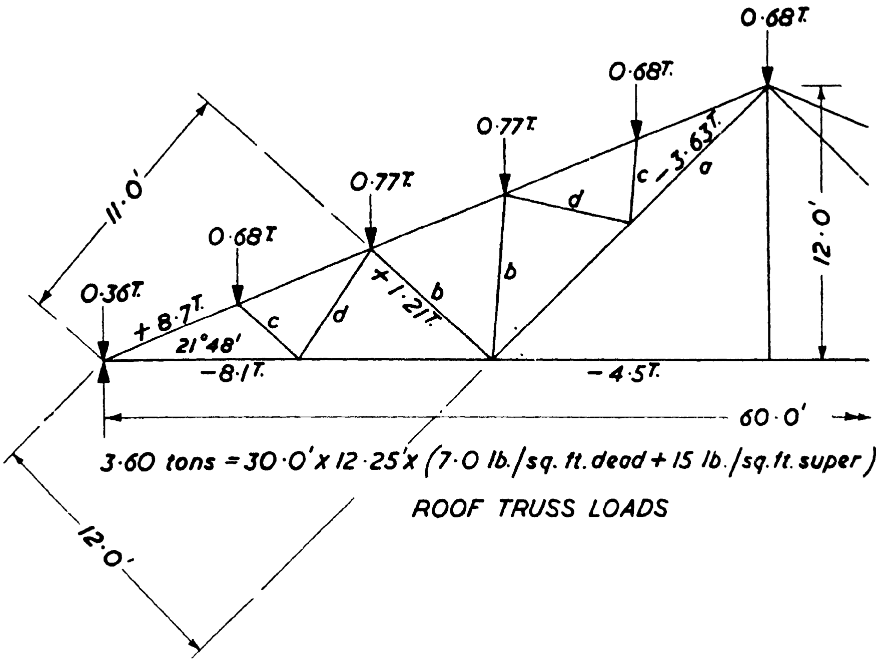


FIG. 453

4. Roof Truss

Span, 60 ft. 0 in.; spacing, 12 ft. 3 in.

Approx. total weight 0·7 tons.

For the purpose of this design the stresses due to wind are not calculated, because by inspection they would be less than those calculated for 15 lb. per sq. ft. superimposed load.

Panel point loads, including superimposed load:

Sheeting purlin, 1367 lb. $= 0{\cdot}61$ ton

Dead weight of truss $= \dfrac{0{\cdot}7}{10} = 0{\cdot}07$

$\underline{0{\cdot}68}$ ton

Glazing purlin, 1562 lb. $= 0{\cdot}70$ ton

Dead weight of truss $= \underline{0{\cdot}07}$

$\underline{0{\cdot}77}$ ton

Rafter one 4-in. × 4-in. × $\frac{5}{16}$-in. angle.

$$\text{Load} = \frac{3{\cdot}60 - 0{\cdot}36}{\sin 21°48'} = 8{\cdot}7 \text{ tons.}$$

$\dfrac{L}{r} = \dfrac{6{\cdot}5 \times 12}{0{\cdot}78} = 100$; $F_a = 4{\cdot}13$ tons per sq. in. (from Table 7, B.S. 449: 1948. see p. 570.)

$$f_a = \frac{8{\cdot}7}{2{\cdot}4} = 3{\cdot}63 \text{ tons per sq. in.}$$

Main tie one 3½-in. × 2½-in. × ¼-in. angle, net area = 0·94 sq. in.

$$\text{Load}=\frac{3{\cdot}60-0{\cdot}36}{\tan 21^\circ\ 48'}=8{\cdot}1 \text{ tons}$$

$$f_t=\frac{8{\cdot}1}{0{\cdot}94}=8{\cdot}6 \text{ tons per sq. in.}$$

Centre section of main tie :

One 2½-in. × 2½-in. × ¼-in. angle; net area = 0·70 sq. in.

$$\text{Load}=\frac{3{\cdot}6\times 15{\cdot}0 \text{ ft.}}{12{\cdot}0 \text{ ft.}}=4{\cdot}5 \text{ tons.}$$

$$f_t=\frac{4{\cdot}5}{0{\cdot}70}=6{\cdot}5 \text{ tons per sq. in.}$$

The single-angle sections shown for the main and centre ties are suitable for intermediate trusses, but will be doubled-up for trusses in line with main stanchions, to provide adequate transverse stiffness to the building.

Diagonal tie (*a*):

One 2½-in. × 2½-in. × ¼-in. angle; net area = 0·70 sq. in.

Load (by method of sections)

$$=\frac{2(0{\cdot}68+0{\cdot}77)\times 15 \text{ ft.}}{12 \text{ ft.}}=3{\cdot}63 \text{ tons.}$$

$$f=\frac{3{\cdot}63}{0{\cdot}70}=5{\cdot}2 \text{ tons per sq. in.}$$

Struts (*b*), one 2½-in × 2½-in × ¼-in. angle.

$$\text{Load}=\frac{0{\cdot}68\times 6 \text{ ft.}+0{\cdot}77\times 12 \text{ ft.}}{11 \text{ ft.}}=1{\cdot}21 \text{ tons}$$

$\frac{L}{r}=\frac{7{\cdot}0 \text{ ft.}\times 12}{0{\cdot}49}=172$; $F_c2=1{\cdot}58$ tons per sq. in. (from Table 8, B.S. 449: 1948, see p. 570.)

$$f=\frac{1{\cdot}21}{1{\cdot}19}=1{\cdot}02 \text{ tons per sq. in.}$$

Struts (*c*) and ties (*d*), one 2½-in. × 2-in. × ¼-in. angle.

5. Valley Beams

15-in. × 6-in. 'I', 45 lb.; span, 24 ft. 6 in.

Truss reaction, including super load	= 3·6 tons
,, ,, from future bay	= 3·6
Dead weight of beams, gutter, etc.	= 1·0
	8·2 tons

$$M=\frac{7{\cdot}2\times 24{\cdot}5}{4}+\frac{1{\cdot}0\times 24{\cdot}5}{8}=47{\cdot}2 \text{ tons ft.}$$

$$f=\frac{47{\cdot}2\times 12}{65{\cdot}6}=8{\cdot}7 \text{ tons per sq. in.}$$

6. Gable Rafters

12-ft. 0-in. horizontal span; 6-in. × 3-in. channel.

Load = 12·0 ft. × 7·0 ft. × 22 lb. per sq. ft., say 1·0 ton.

$$f=\frac{1\cdot0\times12\times12}{8\times7\cdot09}=2\cdot5 \text{ tons per sq. in.}$$

7. Side Rails

$3\frac{1}{2}$-in × $2\frac{1}{2}$-in. × $\frac{1}{4}$-in. angle; span, 12 ft. 3 in.; spacing, 7 ft. 0 in.

Vertical load:

22-gauge sheeting 12·25 × 7·0 × 2·0 lb.	= 172 lb.
Dead weight of rail	= 60
	232 lb.

Horizontal load: Wind, 12·25 ft. × 7·0 ft. × 10 lb. per sq. ft. = 860 lb.

Assume that rail is restrained by the sheeting, and calculate stresses on the *yy* and *xx* axes.

Min. Z_{yy} = 0·39 in.³ Max. Z_{yy} = 1·23 in.³

Min. Z_{xx} = 0·73 in.³ Max. Z_{xx} = 1·01 in.³

$$\text{Max. } f_{yy}=\frac{232\times12\cdot25\times12}{2240\times10\times0\cdot39}=3\cdot91 \qquad \text{Min. } f_{yy}=1\cdot24 \text{ tons per sq. in.}$$

$$\text{Max. } f_{xx}=\frac{860\times12\cdot25\times12}{2240\times10\times0\cdot73}=7\cdot74 \qquad \text{Min. } f_{xx}=3\cdot52 \text{ tons per sq. in.}$$

Max. combined stress = 1·24 + 7·74 = 8·98 tons per sq. in.

Permissible stress = 10 + 25 per cent = 12·5 tons per sq. in.

Side rails carrying glazing, use $3\frac{1}{2}$-in. × 3-in. × $\frac{5}{16}$-in. angle.

8. Gable Stanchion

Spacing, 12-ft. centres.

12-in. × 5-in. 'I', 32 lb.; area = 9·45 sq. in.; Z_{xx} = 36·84 in.³

r_{xx} = 4·84 in.; r_{yy} = 1·01 in.; K_1 = 1·05 (Table 10, B.S. 449: 1948.)

Height from floor level to eaves = 36·0 ft.

Tied on *yy* axis by sheeting rails at 7-ft. centres and by mid-height bracing.

Held in position at both ends.

$$\frac{L}{r_{xx}}=\frac{36\times12}{4\cdot84}=90.$$

$$\frac{L_1}{r_{yy}}=\frac{18\times12\times0\cdot75}{1\cdot01}=160; \quad F_a=2\cdot06 \text{ tons per sq. in.}$$

$$F_{bc}=\frac{1000\times1\cdot01\times1\cdot05}{18\times12}=4\cdot91 \text{ tons per sq. in. plus 25 per cent}$$

$$=6\cdot14 \text{ tons per sq. in.}$$

Vertical load:

Roof 12·0 ft. × 6·5 ft. × 20 lb. per sq. ft. inclusive	= 0·70 tons
Sheeting and rails 12·0 × 46·0 ft. × 3·0 lb. per sq. ft.	= 0·74
Dead weight of stanchion, say	= 0·76
	2·20 tons

$$f_a = \frac{2 \cdot 2}{9 \cdot 45} = 0 \cdot 233 \text{ tons per sq. in.}; \quad \frac{f_a}{F_a} = \frac{0 \cdot 233}{2 \cdot 06} = 11 \cdot 3 \text{ per cent.}$$

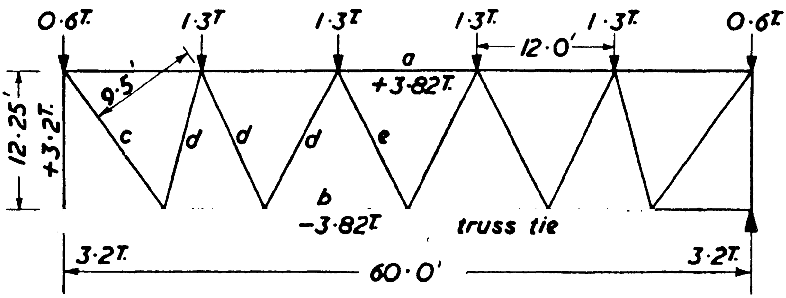
Bending load:

Wind from floor level to eaves,

12·0 ft. × 36·0 ft. × 10 lb. per sq. ft. = 2·0 tons

$$f_b = \frac{2 \times 36 \times 12}{8 \times 36 \cdot 84} = 2 \cdot 93 \text{ tons per sq. in.}; \quad \frac{f_b}{F_{bc}} = \frac{2 \cdot 93}{6 \cdot 14} = 47 \cdot 7 \text{ per cent.}$$

Total = 11·3 + 47·7 = 59 per cent.



GABLE WIND GIRDER AT EAVES LEVEL
(Reverse all loads and forces for leeward gable)

FIG. 454

9. Gable Wind Girder

60-ft. span

Panel point loads from windward gable stanchions,

= 12·0 ft. × 23·0 ft. × 10 lb. per sq. ft. = 1·3 tons.

Girder to be capable of taking pressure or suction.

Boom (*a*), one 4-in. × 4-in. × $\frac{5}{16}$-in. angle.

$$\frac{L}{r} = \frac{12 \text{ ft.} \times 12}{0 \cdot 78} = 185; \quad F_a = 1 \cdot 60 + 25 \text{ per cent}$$

$$= 2 \cdot 00 \text{ tons per sq. in.}$$

$$\text{Load} = \frac{(3 \cdot 2 - 0 \cdot 6)(30 \text{ ft.} - 12 \text{ ft.})}{12 \cdot 25 \text{ ft.}} = 3 \cdot 82 \text{ tons}$$

$$f = \frac{3 \cdot 82}{2 \cdot 40} = 1 \cdot 60 \text{ tons per sq. in.}$$

Boom (b), bottom member of truss:

Gable wind load	$= +3{\cdot}82$	or	$-3{\cdot}82$
Dead-load tension in roof truss $= 4{\cdot}5 \text{ tons} \times \dfrac{7 \text{ lb.}}{22 \text{ lb.}}$	$= -1{\cdot}43$	or	$-1{\cdot}43$
Superimposed load tension $= 4{\cdot}5 \times \dfrac{15}{22} =$			$-3{\cdot}07$
	$+2{\cdot}39$	or	$-8{\cdot}32$

Two $2\frac{1}{2}$-in. $\times$ $2\frac{1}{2}$-in. $\times$ $\frac{1}{4}$-in. angles.

$$\frac{L}{r} = \frac{12{\cdot}5 \text{ ft.} \times 12}{0{\cdot}76} = 200; \quad F_a = 1{\cdot}39 \text{ tons per sq. in.}$$

$$f_a = \frac{2{\cdot}39}{2{\cdot}38} = 1{\cdot}00 \text{ ton per sq. in.}$$

$$f_t = \frac{8{\cdot}32}{2{\cdot}0} = 4{\cdot}16 \text{ tons per sq. in.}$$

Web member (c):

$$\text{Max. load} = (3{\cdot}2 - 0{\cdot}6) \times \frac{12{\cdot}0 \text{ ft.}}{9{\cdot}5 \text{ ft.}} = \pm 3{\cdot}3 \text{ tons}$$

Two 3-in. $\times$ 3-in. $\times$ $\frac{1}{4}$-in angles.

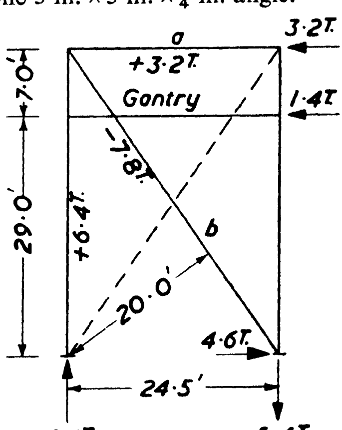
$$\frac{L}{r} = \frac{15 \text{ ft.} \times 12}{0{\cdot}91} = 200.$$

$$F_e 2 = 1{\cdot}25 + 25 \text{ per cent} = 1{\cdot}56 \text{ tons per sq. in.}$$

$$f_e = \frac{3{\cdot}3}{2{\cdot}88} = 1{\cdot}15 \text{ tons per sq. in.}$$

Web members (d), two 3-in. $\times$ 3-in. $\times$ $\frac{1}{4}$-in. angles.

Web member (e), one 3-in. $\times$ 3-in. $\times$ $\frac{1}{4}$-in. angle.



VERTICAL BRACING IN PERMANENT SIDE (Double all loads and forces for temporary side)

FIG. 455

10. Vertical Bracing in Permanent Side

Load from gable wind girder = 3·2 tons.

Horizontal force from crane = 5 per cent of 28 tons static wheel loads = 1·4 tons.

Strut (*a*), one 4-in. × 4-in. × $\frac{5}{16}$-in. angle.

$$\frac{L}{r}=\frac{12\cdot25\times12}{78}=190.$$

$$F_e2=1\cdot35+25\text{ per cent}=1\cdot7.$$

$$f_e=\frac{3\cdot2}{2\cdot4}=1\cdot33\text{ tons per sq. in.}$$

Bracing (*b*):

One 3-in. × 2½-in. × ¼-in angle.

Effective area = 0·82 sq. in.

$$\text{Load}=\frac{3\cdot2\times36\text{ ft.}+1\cdot4\times29\text{ ft.}}{20\text{ ft.}}=7\cdot8\text{ tons.}$$

$$f_t=\frac{7\cdot8}{0\cdot82}=9\cdot5\text{ tons per sq. in.}$$

11. Vertical Bracing in Temporary Side

Members (*a*) and (*b*) to be doubled up to provide for future bay.

12. Intermediate Stanchions in Permanent Side

(Intermediate stanchions in temporary side to be similar.)

12-in. × 5-in. 'I', 32 lb. F_a as gable stanchion.

Vertical loads:

From roof truss	= 3·60 tons
Sheeting and rails 12·25 × 36·0 × 3	= 0·60
Dead weight of stanchion	= 0·60
	4·80 tons

$$f_a=\frac{4\cdot8}{9\cdot45}=0\cdot51\text{ ton per sq. in.}$$

$$\frac{f_a}{F_a}=\frac{0\cdot51}{2\cdot06}=24\cdot8\text{ per cent.}$$

Bending load:

Wind from floor level to eaves

12·25 ft. × 36·0 ft. × 10 lb. per sq. ft. = 2·0 tons.

$$f_b=\frac{2\times36\times12}{8\times36\cdot84}=2\cdot93\text{ tons per sq. in.}$$

$$\frac{f_{bc}}{F_{bc}}=\frac{2\cdot93}{6\cdot14}=47\cdot7\text{ per cent.}$$

Total = 24·8 + 47·7 = 72·5 per cent.

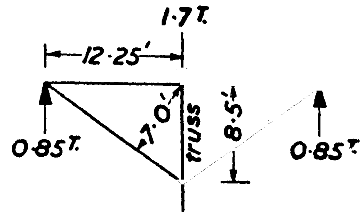


FIG. 456

13. Eaves Bracing

Span, 24 ft. 6 in.

Load from intermediate stanchion	= 1·0 ton
Wind shear from truss, 12·25 ft. × 13·0 ft. × 10 lb. per sq. ft.	= 0·7
	1·7 tons

Load in diagonal brace

$$= \frac{0·85 \times 12·25 \text{ ft.}}{7·0 \text{ ft.}} = 1·5 \text{ tons}$$

One $3\frac{1}{2}$-in. × $2\frac{1}{2}$-in. × $\frac{1}{4}$-in. angle

$$f = \frac{1·5}{0·72} = 2·1 \text{ tons per sq. in.}$$

Load in eaves strut

$$= \frac{0·85 \times 12·25 \text{ ft.}}{8·5 \text{ ft.}} = 1·23 \text{ tons}$$

One 4-in. × 4-in. × $\frac{5}{16}$-in. angle, as for end bay.

14. Roof Leg of Gantry Stanchion in Permanent Side

12-in × 5-in. 'I', 32 lb.; area 9·45 sq. in.; $Z_{xx} = 36·84$ in.^{3}

$r_{xx} = 4·84$ in.; $r_{yy} = 1·01$ in.; $K_1 = 1·05$.

Height from point of connection with lower leg to eaves = 9 ft. 0 in.

$$\frac{L}{r_{xx}} = \frac{9·0 \times 12 \times 1·5}{4·84} = 34.$$

$$\frac{L}{r_{yy}} = \frac{9·0 \times 12}{1·01} = 107.$$

$$F_a = 3·8 \text{ tons per sq. in.}$$

$$F_{bc} = \frac{1000 \times 1·01 \times 1·05}{9 \text{ ft.} \times 12} = 9·94 + 25 \text{ per cent} = 12·3 \text{ tons per sq. in.}$$

Vertical load from truss	= 3·60 tons
Sheeting and rails, 12·25 ft. × 9 ft. × 3 lb.	= 0·15
Dead weight of leg, say	= 0·25
	4·00 tons

$$f_a = \frac{4 \cdot 0}{9 \cdot 45} = 0 \cdot 424; \quad \frac{f_a}{F_a} = \frac{0 \cdot 424}{3 \cdot 8} = 11 \cdot 2 \text{ per cent}$$

Horizontal load on top of stanchion

Wind from eaves bracing, 2 × 0·85 = 1·7 tons

Wind from truss = 0·7

2·4 tons

Horizontal load at mid-height

Side wind, 12·25 ft. × 9 ft. × 10 lb. = 0·5 ton.

B.M. = 2·4 × 9 + 0·5 × 4·5 = 23·9 tons ft.

$$f_b = \frac{23 \cdot 9 \times 12}{36 \cdot 84} = 7 \cdot 8; \quad \frac{f_{bc}}{F_{bc}} = \frac{7 \cdot 8}{12 \cdot 3} = 63 \cdot 5 \text{ per cent.}$$

Total = 11·2 + 63·5 = 74·7 per cent.

15. Main Legs of Gantry Stanchion in Permanent Side

Load		*Case No.*							
		(1)		(2)		(3)		(4)	
Roof		max		max		min		min	
Crane gantry		max		max		min		min	
Surge		L–R		R–L		nil		nil	
Wind		L–R		R–L		L–R		R–L	
	Leg	A	B	A	B	A	B	A	B
Roof leg:									
Dead	1·5	+1·1	+ 0·4			+ 1·1	+0·4		
Super	2·5	+1·9	+ 0·6						
Total	4·0	+3·0	+ 1·0						
Gantry	27·8	+3·0	+24·8			+ 0·2	+1·2		
Sheeting and rails	0·6	+ 0·6				+ 0·6			
Stanchion	1·6	+ 0·8	+ 0·8			+ 0·8	+0·8		
Total	34·0	+ 8·4	+25·6	+ 8·4	+25·6	+ 2·7	+ 2·4	+ 2·7	+ 2·4
Surge: 0·76 × 31 ft./2·25 ft.		− 10·5	+ 10·5	+ 10·5	− 10·5				
Eaves wind: 2·4 × 38 ft./2·25 ft.		− 40·5	+ 40·5	+ 40·5	− 40·5	− 40·5	+ 40·5	+ 40·5	− 40·5
Side wind: 2·0 × 20 ft./2·25 ft.		− 17·8	+ 17·8	+ 17·8	− 17·8	− 17·8	+ 17·8	+ 17·8	− 17·8
		− 61·4	+ 95·4	+ 76·2	− 42·2	− 55·6	+ 60·7	+ 61·0	− 55·9

Gantry leg B, 12-in. × 6-in. 'I', 44 lb.

$$\frac{L}{r_{xx}} = \frac{29 \times 12 \times 0 \cdot 85}{4 \cdot 94} = 60; \quad \frac{L}{r_{yy}} = \frac{4 \cdot 5 \times 12}{1 \cdot 3} = 42.$$

$F_a = 6 \cdot 09 + 25$ per cent $= 7 \cdot 6$ tons per sq. in.

f_a, case (1), $= \frac{95 \cdot 4}{13 \cdot 0} = 7 \cdot 4$ tons per sq. in.

Outside leg A, 12-in. × 6-in. 'I', 44 lb.

f_a, case (2), $= \frac{76 \cdot 2}{13 \cdot 0} = 5 \cdot 9$ tons per sq. in.

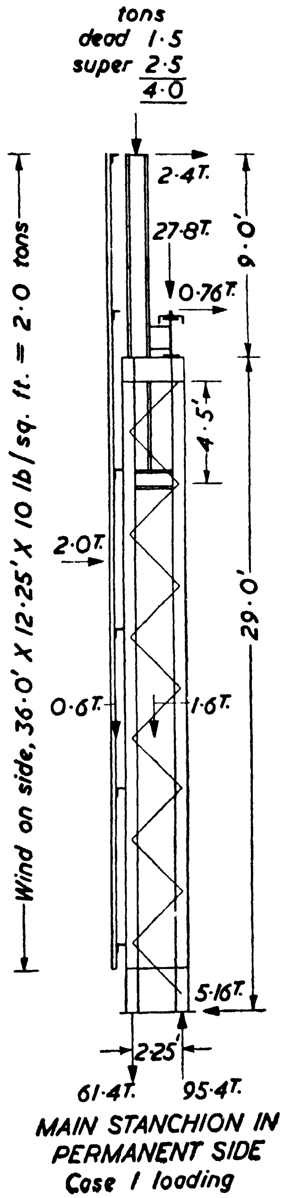

tons
dead 1·5
super 2·5
4·0
2·4T.
27·8T.
0·76T.
9·0'
4·5'
29·0'
2·0T.
0·6T.
1·6T.
5·16T.
2·25'
61·4T.
95·4T.
Wind on side, 36·0' X 12·25' X 10 lb/sq. ft. = 2·0 tons
MAIN STANCHION IN
PERMANENT SIDE
Case 1 loading

FIG. 457

16. Lacings

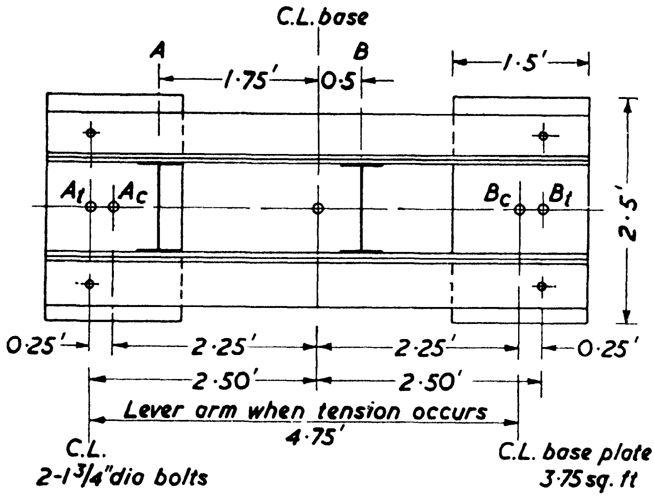
Total horizontal shear $=0\cdot76+2\cdot4+2\cdot0+2\tfrac{1}{2}$ per cent of 34 tons
$=6\cdot01$ tons.

$$\text{Force per lace}=\frac{6\cdot01\times\sqrt{2}}{2}=4\cdot3 \text{ tons.}$$

3-in × 2-in. × $\frac{5}{16}$-in. angle,

$$\frac{L}{r}=\frac{3\cdot2\times12}{0\cdot43}=90.$$

$$F_c2=3\cdot4;\quad f=\frac{4\cdot3}{1\cdot46}=3\cdot0 \text{ tons per sq. in.}$$



BASE OF MAIN STANCHION IN PERMANENT SIDE

FIG. 458

17. Base of Gantry Stanchion in Permanent Side

Case (1)

Load $A=-62\cdot0$; load $B=+96\cdot0$.

Reactions:

$$A_t=\frac{-62\times4+96\times1\cdot75}{4\cdot75}=\frac{-80}{4\cdot75}=-17$$

$$B_c=\frac{-62\times0\cdot75+96\times3\cdot0}{4\cdot75}=\frac{+242}{4\cdot75}=+51$$

Case (2)

Load $A=+76\cdot0$; load $B=-42\cdot0$.

Reactions:

$$A_c=\frac{+76\times4\cdot25-42\times2\cdot0}{4\cdot75}=\frac{+239}{4\cdot75}=+51.$$

$$B_t=\frac{+76\times0\cdot5-42\times2\cdot75}{4\cdot75}=\frac{-78}{4\cdot75}=-17.$$

Case (3)

Load $A = -56{\cdot}0$; load $B = +61{\cdot}0$.

Reactions:

$$A_t = \frac{-56\times 4+61\times 1{\cdot}75}{4{\cdot}75} = \frac{-117}{4{\cdot}75} = -25.$$

$$B_c = \frac{-56\times 0{\cdot}75+61\times 3{\cdot}0}{4{\cdot}75} = \frac{+141}{4{\cdot}75} = +30.$$

Case (4)

Load $A = +61{\cdot}0$; load $B = -56{\cdot}0$.

$$A_c = \frac{+61\times 4{\cdot}25-56\times 2{\cdot}0}{4{\cdot}75} = \frac{+147}{4{\cdot}75} = +31.$$

$$B_t = \frac{+61\times 0{\cdot}5-56\times 2{\cdot}75}{4{\cdot}75} = \frac{-124}{4{\cdot}75} = -26.$$

$$\text{Max. pressure on concrete} = \frac{51}{3{\cdot}75} = 13{\cdot}6 \text{ tons per sq. ft.}$$

18. Anchor Bolts ($1\frac{3}{4}$ in. diameter)

$$F_t = 6{\cdot}0 + 25 \text{ per cent} = 7{\cdot}5 \text{ tons per sq. in.}$$

$$f_t = \frac{26}{2\times 1{\cdot}928} = 6{\cdot}75 \text{ tons per sq. in.}$$

19. Roof Leg of Double Gantry Stanchion

Section 12-in. × 5-in. 'I', 32 lb.; area $= 9{\cdot}45$ sq. in.; $Z_{xx} = 36{\cdot}84$ in.^{3}

$$r_{xx} = 4{\cdot}84 \text{ in.},\quad r_{yy} = 1{\cdot}01 \text{ in.};\quad K_1 = 1{\cdot}05.$$

$$\frac{L}{r_{xx}} = \frac{9\times 12\times 1{\cdot}5}{4{\cdot}84} = 34.$$

$$\frac{L}{r_{yy}} = \frac{9\times 12}{1{\cdot}01} = 107.\quad F_a = 3{\cdot}8 \text{ tons per sq. in.}$$

$$F_{bc} = \frac{1000\times 1{\cdot}01\times 1{\cdot}05}{9\times 12} = 9{\cdot}84 + 25 \text{ per cent} = 12{\cdot}3 \text{ tons per sq. in.}$$

With future span constructed,

Vertical load from valley beams	=	8·2 tons
,, ,, ,, trusses	=	7·2
Dead weight of roof leg	=	0·2
		15·6 tons

$$f_a = \frac{15{\cdot}6}{9{\cdot}45} = 1{\cdot}65;\quad \frac{f_a}{F_a} = \frac{1{\cdot}65}{3{\cdot}8} = 43{\cdot}5 \text{ per cent.}$$

Horizontal load at top of stanchion

Wind from eaves girders = 0·7 tons
„ „ truss = 0·7

1·4

„ „ future roof 50 per cent = 0·7

2·1 tons

$$\text{B.M.} = 2{\cdot}1 \times 9 = 18{\cdot}9 \text{ tons ft.}$$

$$f_b = \frac{18{\cdot}9 \times 12}{36{\cdot}84} = 6{\cdot}15; \quad \frac{f_{bc}}{F_{bc}} = \frac{6{\cdot}15}{12{\cdot}3} = 50{\cdot}0 \text{ per cent.}$$

$$\text{Total} = 43{\cdot}5 + 50 = 93{\cdot}5 \text{ per cent.}$$

This case, with the future span constructed, gives a higher total stress than one span with the temporary side.

20. Main Legs of Double Gantry Stanchion

Load		*Case No.*			
		(1) *With Future Bay*		(2) *Without Future Bay*	
		Leg A	*Leg* B	*Leg* A	*Leg* B
Roof leg:					
Dead	5·6	+ 2·8	+ 2·8	+ 1·4	+ 1·4
Super	10·0	+ 5·0	+ 5·0	+ 2·5	+ 2·5
Total	15·6				
Gantry	27·8	+27·8		+27·8	
„	27·8		+27·8		0·0
Sheeting					+ 0·6
Stanchion	1·8	+ 0·9	+ 0·9	+ 0·9	+ 0·9
Total	73·0	+36·5	+36·5	+32·6	+ 5·4
Surge: $\frac{2 \times 0{\cdot}76 \times 31 \text{ ft.}}{2{\cdot}5 \text{ ft.}}$		+19·0	− 19·0	+ 9·5	− 9·5
Roof wind: 2·1 × 38 ft./2·5 ft.		+32·0	− 32·0		
or 1·4 × 38 ft./2·5 ft.				+21·3	−21·3
Side wind: 1·0 × 38 ft./2·5 ft.				+15·2	−15·2
2·0 × 20 ft./2·5 ft.				+16·0	−16·0
		+87·5	− 14·5	+94·6	−56·6
For anchor-bolt design deduct: Roof super-load				− 2·5	− 2·5
				+92·1	−59·1

12-in. × 6-in. 'I', 44 lb.

$F_a = 7{\cdot}6$ tons per sq. in. as in previous calculations.

$$f_a = \frac{94{\cdot}6}{13{\cdot}0} = 7{\cdot}3 \text{ tons per sq. in.}$$

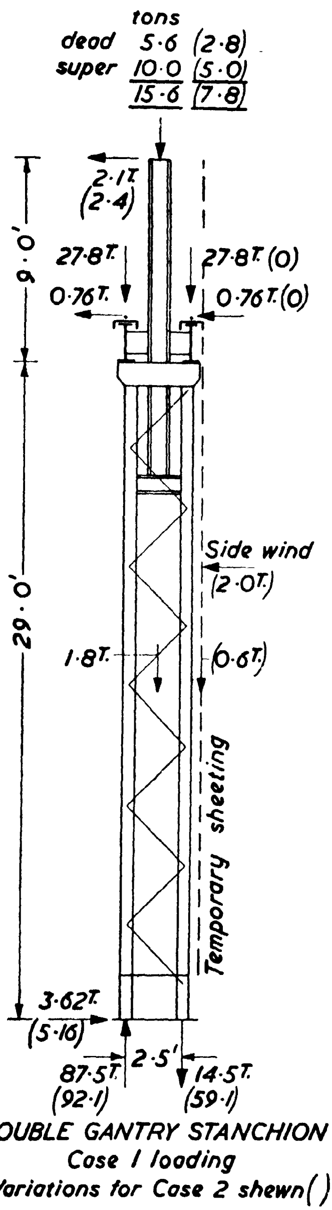

tons
dead 5·6 (2·8)
super 10·0 (5·0)
15·6 (7·8)
2·1T. (2·4)
9·0'
27·8T.
0·76T.
27·8T. (0)
0·76T. (0)
Side wind (2·0T.)
29·0'
1·8T.
(0·6T.)
Temporary sheeting
3·62T. (5·16)
2·5'
87·5T. (92·1)
14·5T. (59·1)
DOUBLE GANTRY STANCHION
Case 1 loading
Variations for Case 2 shewn ( )

FIG. 459

21. Lacings

Total horizontal shear

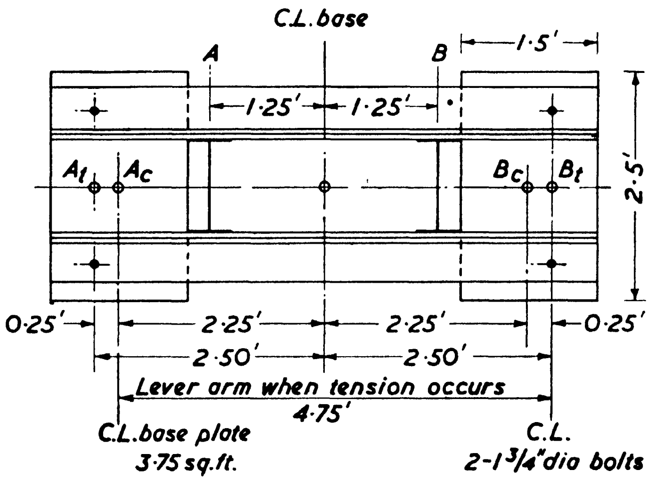
	Case 1	*Case 2*
Surge	1·52	0·76
Roof wind	2·10	1·40
Side wind		3·00
$2\frac{1}{2}$ per cent of 73 tons	1·83	
$2\frac{1}{2}$ per cent of 38 tons		0·95
	5·45	6·11

$$\text{Force per lace} = \frac{6{\cdot}11 \times \sqrt{2}}{2} = 4{\cdot}4 \text{ tons}$$

3-in. × 2-in. × $\frac{5}{16}$-in. angle.

$$\frac{L}{r} = \frac{3{\cdot}5 \times 12}{0{\cdot}43} = 98.$$

$$F_c 2 = 3{\cdot}18 + 25 \text{ per cent} = 4{\cdot}0; \quad f = \frac{4{\cdot}4}{1{\cdot}46} = 3{\cdot}0 \text{ tons per sq. in.}$$



BASE OF DOUBLE GANTRY STANCHION

FIG. 460

22. Base

Case (1)

Load $A = +87{\cdot}5$; load $B = -14{\cdot}5$.

Reactions:

$$A_c = \frac{+87{\cdot}5 \times 3{\cdot}5 - 14{\cdot}5 \times 1{\cdot}0}{4{\cdot}5} = \frac{+292}{4{\cdot}5} = +65.$$

$$B_c = \frac{+87{\cdot}5 \times 1{\cdot}0 - 14{\cdot}5 \times 3{\cdot}5}{4{\cdot}5} = \frac{+36}{4{\cdot}5} = +8.$$

Case (2)

Load $A = +92{\cdot}0$; load $B = -59{\cdot}0$.

$$A_c = \frac{+92\times3{\cdot}75-59\times1{\cdot}25}{4{\cdot}75} = \frac{+271}{4{\cdot}75} = +57.$$

$$B_t = \frac{+92\times1{\cdot}0-59\times3{\cdot}5}{4{\cdot}75} = \frac{-114}{4{\cdot}75} = -24.$$

$$\text{Max. pressure on concrete} = \frac{57}{3{\cdot}75} = 15{\cdot}2 \text{ tons per sq. ft.}$$

23. Anchor Bolts ($1\frac{3}{4}$ in. diameter)

$$F_t = 6+25 \text{ per cent} = 7{\cdot}5 \text{ tons per sq. in.}$$

$$f_t = \frac{24}{2\times1{\cdot}928} = 6{\cdot}2 \text{ tons per sq. in.}$$

24. Beam in Permanent Gable, Supporting Gantry

12-in. × 6-in. 'I', 44 lb.; span 12·0 ft.

Point load of 27·1 tons at 1 ft. 6 in. from R_L.

$$R_L = \frac{27{\cdot}1\times10{\cdot}5}{12{\cdot}0} = 23{\cdot}8 \text{ tons}$$

$$\text{B.M.} = 23{\cdot}8\times1{\cdot}5 \text{ ft.} = 36 \text{ tons ft.}$$

$$f_b = \frac{36\times12}{52{\cdot}79} = 8{\cdot}2 \text{ tons per sq. in.}$$

$$f_s = \frac{23{\cdot}8}{12\times0{\cdot}4} = 5{\cdot}0 \text{ tons per sq. in.}$$

To resist web buckling, provide double angle stiffeners under point load and at left-hand bearing.

25. Stanchion at Junction of Permanent Side and Permanent End

Vertical loads

Roof, 6·0 ft. × 7·0 ft. × 22 lb. per sq. ft.	= 0·4 ton
Sheeting and rails, 12·0 ft. × 36·0 ft. × 3 lb. per sq. ft.	= 0·6
Gantry support	= 24·0
Dead weight of stanchion	= 1·0
	26·0 tons

10-in. × 8-in. 'I', 55 lb., tied at mid-height.

$$\frac{L}{r} = \frac{18 \text{ ft.}\times12}{1{\cdot}84} = 118. \qquad K_1 = 1{\cdot}5$$

$$F_a = 3{\cdot}34 \text{ tons per sq. in.}$$

$$f_a = \frac{26}{16{\cdot}18} = 1{\cdot}61; \quad \frac{f_a}{F_a} = \frac{1{\cdot}61}{3{\cdot}34} = 48 \text{ per cent.}$$

$$F_{bc} = \text{the lesser of } \left[\frac{1000}{118}\times1{\cdot}5\right] \text{ or } 10 = 10 \text{ tons per sq. in.}$$

$$f_b = \frac{24\times1 \text{ in.}}{13{\cdot}69} = 1{\cdot}76; \quad \frac{f_{bc}}{F_{bc}} = \frac{1{\cdot}76}{10} = 18 \text{ per cent.}$$

$$\text{Total} = 48+18 = 66 \text{ per cent.}$$

The anchor bolts for this stanchion have to resist an uplift of 6·4 tons from the side bracing. Provide four $\frac{7}{8}$-in. diameter bolts, 2 ft. 0 in. long, and a concrete foundation weighing not less than 9·6 tons for 50 per cent stability margin.

26. Stanchion at Junction of Permanent Gable and Temporary Side, with Provision for future Bay

Vertical loads

Valley beam	4·1 tons
Gable rafters	1·0
Sheeting and rails	0·6
Gantry supports	48·0
Dead weight of stanchion	1·0
	54·7 tons

10-in × 8-in. 'I', 55 lb., tied at mid-height.

$$\frac{L}{r}=\frac{18\times 12}{1\cdot 84}=118.$$

$$F_a=3\cdot 34.$$

$$f_a=\frac{54\cdot 7}{16\cdot 18}=3\cdot 38 \text{ tons per sq. in.}$$

Anchor bolts, four $\frac{7}{8}$-in. diam. × 2 ft. 0 in. long.

Minimum weight of foundation 19·0 tons to provide 50 per cent stability margin against uplift from side-bracing.

27. Rafter Bracing

One 3-in. × $2\frac{1}{2}$-in. × $\frac{1}{4}$-in angle; clipped to purlins.

28. Longitudinal Ties to Bottom Members of Trusses

One 3-in. × $2\frac{1}{2}$-in. × $\frac{1}{4}$-in. angle; 3-in. leg vertical.

29. Mid-height Ties to Gable Stanchion

One 3-in. × $2\frac{1}{2}$-in. × $\frac{1}{4}$-in. angle; 3-in. leg vertical.

30. Vertical Bracing in Permanent Gable

Force from eaves bracing = 0·85 ton.

Surge from crane = 0·76.

$$\text{Load in brace}=\frac{0\cdot 85\times 36+0\cdot 76\times 29}{20}=2\cdot 6 \text{ tons.}$$

One $2\frac{1}{2}$-in. × $2\frac{1}{2}$-in. × $\frac{1}{4}$-in. angle.

37. JOINTS IN RIGID-FRAME STRUCTURES

ALTHOUGH the general principles on which the design of rigid joints is based are fairly well established, the details offer considerable scope for ingenuity. While these joints may be riveted, bolted or welded, the theory to be developed in the following pages will, for simplicity, be largely associated with welding.

A variety of sections may be used in rigid frames, but the most common are R.S.J.s, B.F.B.s, or built-up plate sections.

Knees for Rectangular Frames

Consider a simple knee for a rectangular frame, as shown in Fig. 461. It does not matter, in principle, whether the cross-beam butts against the column or rests on top of it, but it is assumed that there are two sets of stiffeners, so that the web of the joint has a rectangular frame around it.

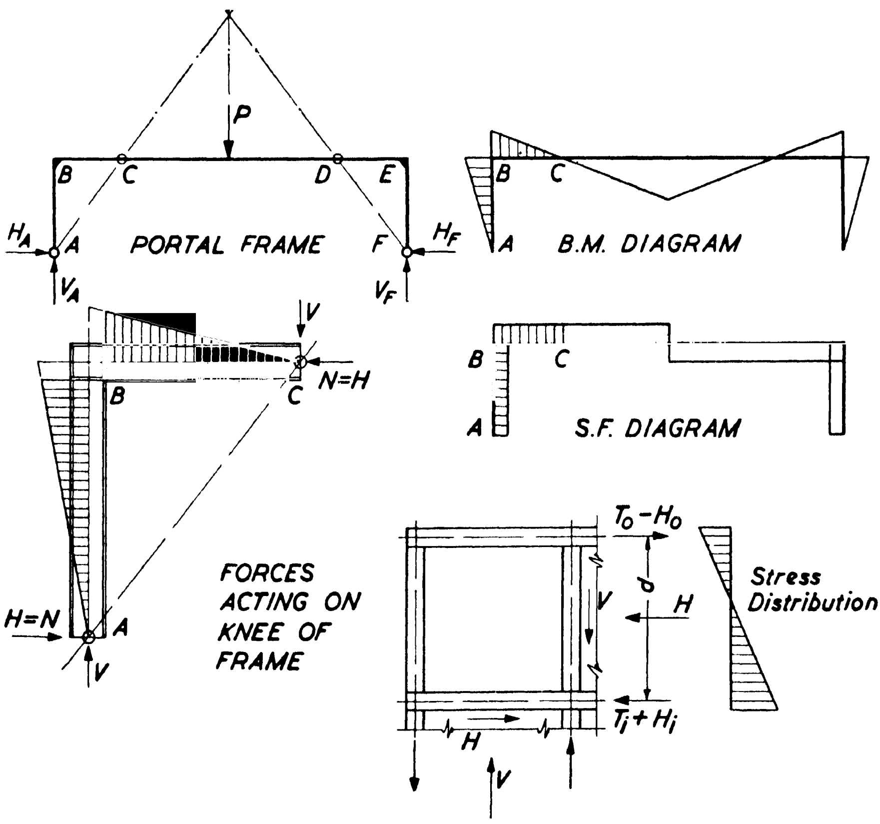


FIG. 461

At any section of a member, the stresses may be found from the normal expression of stress derived in the chapter on Bending and Axial Stresses:

$$f=\frac{P}{A}\pm\frac{M_{xx}y}{I_{xx}},$$

where f = the stress in any fibre,

P = the longitudinal thrust (i.e. N or H in this case),

A = the cross-sectional area of the member,

M_{xx} = the B.M. at the section,

y = the distance from the neutral axis to the fibre being considered

and I_{xx} = the moment of inertia of the member.

Whilst the maximum B.M. in the frame occurs at the intersection of the neutral axes of the girder and column, the B.M. taken for design purposes can be that at the limits of the knee, i.e. in line with the inside flange of the girder or column.

The shear stress may be found in the strictly accurate manner, giving the distribution shown in Fig. 462 or, in accordance with B.S. 449, by dividing the shear force by the gross web area. Considering the forces applied to the joint, if

f_o and f_i = the average bending stresses in the outside and inside flanges respectively;

A_o and A_i = the cross-sectional areas of the outside and inside flanges respectively;

T_o and T_i = the forces in the outside and inside flanges respectively; while

H_o and H_i = the components of the horizontal or normal thrust H (or N) in the outside and inside flanges respectively; then

$$T_o=A_of_o \quad \text{and} \quad T_i=A_if_i$$

$$H_o=\frac{A_o}{A}H \quad \text{and} \quad H_i=\frac{A_i}{A}H.$$

Consequently, the girder will impose a tensile force of T_o-H_o in the outside flange and a compressive force of T_i+H_i in the inside flange at the boundaries of the knee.

The foregoing analysis is theoretical and can safely be simplified by assuming that the flanges of the girder take the whole of the B.M. and transmit the whole of the thrusts, while the web transmits only the shear.

Then, $T_o=T_i=\frac{M}{d}$ (where d is the depth of the girder)

$$H_o=\frac{A_o}{A_o+A_i}\cdot H$$

$$H_i=\frac{A_i}{A_o+A_i}\cdot H.$$

In rolled sections, where the flanges are equal in size, $H_o=H_i$.

As for the knee itself, experimental evidence shows that there is no tensile stress at the extreme corner as the load takes a direct path across the web.

It is possible to assume, therefore, that the tensile forces in the outer flanges vary uniformly from a maximum at points in line with the inside flanges of the frame, to zero at the outside corner, as shown in Fig. 462.

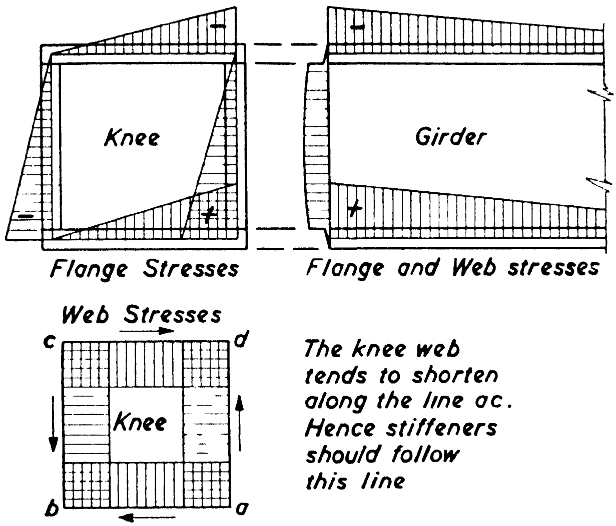


FIG. 462

Each of the flange loads is transmitted into the knee web plate within the lengths of its sides, and this plate is the only means by which the B.M. is transferred from the girder to the column. Consequently, there are heavy shear forces in the knee.

Considering the shear, if

L = the length of the side of the web plate being considered,

t = the web thickness

and T = the total thrust in the flange,

then, the shear per unit length of plate $= \dfrac{T}{L}$,

while the shear stress $= \dfrac{T}{L \times t}$.

In welded knees, the load per unit length of fillet weld (one each side of the web plate) is $T/2L$.

In the top edge of the web plate the thrust T is equal to $T_o - H_o$, while in the bottom edge it is equal to $(T_i + H_i) - H$, as shown in Fig. 461. But $(T_i + H_i) - H = (T_i + H_i) - (H_i + H_o) = T_i - H_o = T_o - H_o$. Therefore the shear forces in the top and bottom edges of the web plate are equal. Similarly, those in the outside and inside vertical edges are equal.

If the forces in the web tend to cause overstressing, the web may be increased in thickness or provided with suitable stiffeners. The normal procedure for simple knees is to use diagonal stiffeners.

In his papers (see references 1, 2 and 3 in the bibliography) Professor A. W. Hendry described experiments which he carried out on frames with members set at right angles and provided with various types of knee. The members consisted of small joist sections, small built-up plated sections and plastic models. Among his conclusions were the following:

Sharp-Cornered Knees.

1. Sharp-cornered frames built up from 'I'-sections of the usual proportions will, as a rule, fail by instability of the compression flange if unstiffened and unsupported laterally, the failing load in such cases being generally much below the possible strength of the members connected. If the compression flange is restrained laterally the web will buckle at the knee after part of it has become plastic.
2. A satisfactory design for a knee joint will be achieved if a diagonal stiffener is provided, proportioned to carry 40 per cent of the load due to the change in direction of the flange stress. Such a connection will develop the full strength of the members connected. It appears to make little difference whether the beam and column members are butt-welded and stiffener plates added or whether these members are fillet-welded to the stiffener plate.

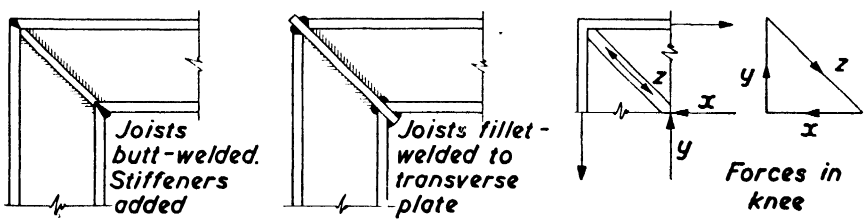


FIG. 463

Typical knees, following Hendry's recommendations, are shown in Fig. 463. The flange loads are resolved in the normal way, so that in the case of members of equal depth, the diagonal load equals $\sqrt{2}$ times that in either flange.

It should be noted that the foregoing principles can be applied to the knees of frames with pitched roofs.

With regard to Hendry's contention that it makes little difference whether the members are butt welded together or fillet welded to the stiffener plate, it is true that failure might occur in the latter case if the stiffener plate is laminated and, to avoid this possibility, the main members should be butt welded together.

Hendry's rule for the proportioning of diagonal stiffeners should be used only for joist sections or substantially similar small plated sections. For large members a precise analysis is desirable.

American Research on Rectangular Knees

Much of the experimental work carried out on the knees of portal frames has been done in America, particularly by the American Bureau of Standards

and at Lehigh University. One of the more easily adaptable groups of formulæ for rectangular web plates, attributed to Osgood, was published in Research Paper, R.P. 1130 (reference 4). The theory and example which follow are based on extracts from this paper.

Consider a flat rectangular plate of uniform thickness t, loaded by forces and couples, as shown in Fig. 464. For equilibrium,

$$M_x - a(2F_{xy} + F_y) = M_y - b(2F_{yx} + F_x).$$

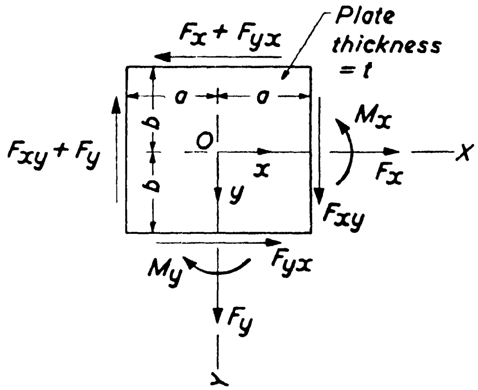


FIG. 464

It is required to determine the stress conditions in the plate, assuming that the normal stresses f_x and f_y along the boundaries $x = a$ and $y = b$ respectively are uniformly varying and along the boundaries $x = -a$ and $y = -b$ are everywhere zero.

Such a condition may be derived from the Airy Stress Function:

$$\phi = b_2xy + \tfrac{1}{6}(3c_3 + d_4y)(a + x)y^2 + \tfrac{1}{6}(3b_3 + b_4x)(b + y)x^2.$$

It can be shown that:

$$b_2 = \frac{1}{4abt}\left[M_x - a(2F_{xy} + F_y) - \frac{3}{2}(M_x + M_y)\right]$$

$$b_3 = \frac{F_y}{4abt}, \quad c_3 = \frac{F_x}{4abt}$$

$$b_4 = \frac{3M_y}{4a^3bt}, \quad d_4 = \frac{3M_x}{4ab^3t},$$

from which it may be derived that:

$$f_x = \frac{1}{4abt}\left(F_x + \frac{3M_x}{b^2}y\right)(a + x)$$

$$f_y = \frac{1}{4abt}\left(F_y + \frac{3M_y}{a^2}x\right)(b + y)$$

$$v_{xy} = -\frac{1}{4abt}\left[M_x - a(2F_{xy} + F_y) + F_y \, . \, x + F_x \, . \, y - \frac{3M_y}{2}\left(1 - \frac{x^2}{a^2}\right) - \frac{3M_x}{2}\left(1 - \frac{y^2}{b^2}\right)\right]$$

where $\quad v_{xy}$ = the shear stress

$$F_x = -(1-k-j)H - \left(\frac{m}{1-k+j} - \frac{n}{1+k-j}\right)\frac{M}{b}$$

$$F_y = -(1-2p)V$$

$$F_{xy} = V - pV - \frac{r}{p}(M_o - Hb)$$

$$F_{yx} = H - jH - \frac{nM}{(1+k-j)b}$$

$$M_x = -\left(1 - \frac{m}{1-k+j} - \frac{n}{1+k-j}\right)M$$

$$M_y = -(1-2r)(M_O - Hb).$$

These formulæ have been quoted in their original form, but the stresses f_x and f_y will be negative if compressive and positive if tensile, the signs being opposite to those given elsewhere in this section.

In the foregoing formulæ p and r are the proportions of V and M respectively, which are taken by each flange of the column at the edge $y = b$. If a flange is not wholly continuous at the knee, as in riveted construction, it will transmit stress only partially across the discontinuous section. Consequently, k and j are the proportions of H, and m and n are the proportions of M, which are taken by the top and bottom flanges of the beam portion of the knee at the edge $x = a$. Had there been no discontinuity, k would equal j and m would equal n. It is further assumed that the flanges carry no transverse shear.

For a welded frame, as in Fig. 465,

$$M_O = V(a+A) = H(b+B), \qquad M = M_O - Va$$

$$M_x = -M(1-2n), \qquad M_y = -(M_O - Hb)(1-2p)$$

$$F_x = -(1-2j)H, \qquad F_y = -(1-2p)V$$

$$F_{xy} = V - pV - \frac{r}{a}(M_O - Hb)$$

$$F_{yx} = H - jH - \frac{nM}{b}$$

$$j = \frac{\text{Area of one flange of beam}}{\text{Total sectional area of beam}},$$

$$n = \frac{\text{Moment of inertia } I \text{ of one flange of beam}}{\text{Total } I \text{ for the beam}}, \text{ and}$$

p and r = the corresponding quantities for the column.

Principal Stresses and Greatest Shear Stresses

Although not required by B.S. 449 : 1948, the principal stresses in the knee web of a frame can be computed from the usual formula:

$$f = \frac{f_x + f_y}{2} \pm \sqrt{\left[\left(\frac{f_x - f_y}{2}\right)^2 + v_{xy}^2\right]}.$$

Normally the greatest stress occurs at the inside corner of the knee where $x = +a$, $y = +b$.

The greatest shear stress in the web occurs at the point:

$$x=-\frac{b_3}{b_4},\quad y=\frac{c_3}{d_4},\quad \text{i.e.}\quad x=-\frac{F_y \,.\, a^2}{3M_y},\quad y=-\frac{F_x \,.\, b^2}{3M_x},$$

the maximum stress being computed from the formula:

$$v_{\max}=\sqrt{\left[\left(\frac{f_x-f_y}{2}\right)^2+v_{xy}^2\right]}.$$

It will be seen that it is necessary to compute two sets of coefficients, first to obtain the principal stresses, and then to obtain the greatest shear stress. Now the point where the greatest shear stress occurs is very near the centre of the web. Noting this fact, a Canadian engineer, Dr. D. T. Wright, has produced a formula for the maximum shear stress which gives results within about 2 per cent of the exact figures derived from Osgood's formula, viz.:

$$v=\frac{M_c}{4abt}\left(1+\frac{a^2t}{3Z_a}+\frac{b^2t}{3Z_b}\right),$$

where $M_c=M_O-H\,.\,b-V\,.\,a$ = the moment of the inside corner of the frame.

Z_a = the section modulus of the knee along a horizontal axis, the section including the vertical flanges as well as the web plate, and Z_b = the corresponding section modulus along a vertical axis.

It is often quicker to use this formula than that evolved by Osgood.

The various formulæ will be demonstrated by an example.

Example 1. Fig. 465 shows a square knee for a rectangular portal frame in which the flanges are 12 in. × 1 in. in cross-section, and the web plate is 40 in. × 40 in. × ½ in. thick.

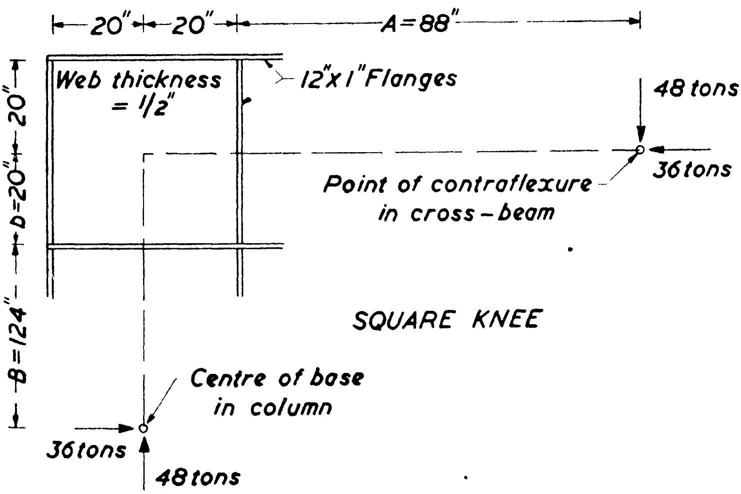


FIG. 465

Using Osgood's formulæ,

$a=b=20$ in.	$t=0{\cdot}5$ in.
$A=88$ in.	$B=124$ in.
$V=48$ tons.	$H=36$ tons.

The proportion of the reaction V taken by each flange of the column,

$$p = \frac{\text{Area of one flange}}{\text{Area of whole section}} = \frac{A_f}{2A_f + A_w}$$

(where f and w refer to flange and web respectively)

$$= \frac{(12 \times 1)}{(2 \times 12 \times 1) + (40 \times 0{\cdot}5)} = 0{\cdot}2727.$$

The proportion of the moment M taken by each flange

$$r = \frac{I_f}{2I_f + I_w} = \frac{12 \times 1 \times 20{\cdot}5^2}{2(12 \times 1 \times 20{\cdot}5^2) + (0{\cdot}5 \times 40^3)/12}$$

$$= \frac{5043}{12{,}753} = 0{\cdot}3955.$$

The proportion of the thrust H taken by each flange of the beam,

$$k = j = 0{\cdot}2727 \quad \text{(as above for } p\text{)}.$$

The proportion of the moment M taken by each flange,

$$m = n = 0{\cdot}3955 \quad \text{(as above for } r\text{)}.$$

The moment about the centre of gravity of the knee web,

$$M_O = V(a + A) = H(b + B)$$
$$= 48(1{\cdot}67 + 7{\cdot}33)$$
$$= 432 \text{ tons ft.}$$

The moment at the junction with the beam (where $x = a$),

$$M = M_O - Va = 432 - (48 \times 1{\cdot}67)$$
$$= 352 \text{ tons ft.}$$
$$F_x = -(1 - 2j)H = -16{\cdot}36 \text{ tons.}$$
$$F_y = -(1 - 2p)V = -21{\cdot}82 \text{ tons.}$$

$$F_{xy} = V - pV - \frac{r}{a}(M_O - Hb)$$
$$= 48 - (0{\cdot}2727 \times 48) - \frac{0{\cdot}3955(432 \times 12 - 36 \times 20)}{20}$$
$$\doteq -53{\cdot}37 \text{ tons.}$$

$$F_{yx} = H - jH - \frac{nM}{b}$$
$$= 36 - (0{\cdot}2727 \times 36) - \frac{0{\cdot}3955 \times 352 \times 12}{20}$$
$$= -57{\cdot}35 \text{ tons.}$$

$$M_x = -M(1 - 2m)$$
$$= -(352 \times 12)(1 - 0{\cdot}7910)$$
$$= -882{\cdot}82 \text{ tons in.}$$
$$M_y = -(M_O - Hb)(1 - 2r)$$
$$= -(432 \times 12 - 36 \times 20)(1 - 0{\cdot}7910)$$
$$= -932{\cdot}98 \text{ tons in.}$$

Checking for equilibrium,

$$M_x - a(2F_{xy} + F_y) = M_y - b(2F_{yx} + F_x)$$

$$-882{\cdot}82 - 20(-106{\cdot}73 - 21{\cdot}82) = -932{\cdot}98 - 20(-114{\cdot}69 - 16{\cdot}36)$$

or

$$1688{\cdot}18 = 1688{\cdot}10,$$

which satisfies the expression $\Sigma M = 0$ with reasonable accuracy.

Stresses

The greatest compressive stresses occur at the inside corner where $x = a$, $y = b$.

Considering the stresses at this point,

$$f_x = \frac{1}{4abt}\left(F_x + \frac{3M_x \,.\, y}{b^2}\right)(a + x)$$

$$= \frac{1}{4 \times 20 \times 20 \times 0{\cdot}5}\left(-16{\cdot}36 + \frac{-3 \times 882{\cdot}82 \times 20}{20^2}\right)(20 + 20)$$

$$= -7{\cdot}44 \text{ tons per sq. in.}$$

$$f_y = \frac{1}{4abt}\left(F_y + \frac{3M_y \,.\, x}{a^2}\right)(b + y)$$

$$= \frac{1}{4 \times 20 \times 20 \times 0{\cdot}5}\left(-21{\cdot}82 + \frac{-3 \times 932{\cdot}98 \times 20}{20^2}\right)(20 + 20)$$

$$= -8{\cdot}088 \text{ tons per sq. in.}$$

$$v_{xy} = -\frac{1}{4abt}[M_x - a(2F_{xy} + F_y) + F_y \,.\, x + F_x \,.\, y + 0 + 0]$$

$$= -\frac{1}{4 \times 20 \times 20 \times 0{\cdot}5}[-882{\cdot}82 - 20(-106{\cdot}7 - 21{\cdot}82) - (21{\cdot}82 \times 20) - (16{\cdot}36 \times 20)]$$

$$= -1{\cdot}156 \text{ tons per sq. in.}$$

The shear stress has been computed as it is needed for the calculation of the principal stresses, but this stress is not the greatest shear stress in the knee. This will be computed later. The maximum principal stress, in compression, is:

$$f_{\max} = \frac{f_x + f_y}{2} - \sqrt{\left[\left(\frac{f_x - f_y}{2}\right)^2 + v_{xy}^2\right]}$$

$$= \frac{-7{\cdot}44 - 8{\cdot}088}{2} - \sqrt{\left[\left(\frac{-7{\cdot}44 + 8{\cdot}088}{2}\right)^2 + 1{\cdot}156^2\right]} = -7{\cdot}764 - 1{\cdot}200$$

$$= -8{\cdot}964 \text{ tons per sq. in.}$$

The formula evolved by Dr. Wright will be used to find the greatest shear stress:

$$v = \frac{M_c}{4abt}\left(1 + \frac{a^2 t}{3Z_a} + \frac{b^2 t}{3Z_b}\right).$$

Now

$$M_c = M_O - H \,.\, b - V \,.\, a$$

$$= (432 \times 12) - (36 \times 20) - (48 \times 20)$$

$$= 3504 \text{ tons in.}$$

$$Z_a = Z_b = \text{Total } I/a = 12{,}753/20 \text{ in.}^3.$$

Hence,

$$v = \frac{3504}{4 \times 20 \times 20 \times 0{\cdot}5}\left(1 + \frac{2 \times 20^2 \times 0{\cdot}5 \times 20}{3 \times 12{,}753}\right)$$

$$= 5{\cdot}59 \text{ tons per sq. in.}$$

Knees for Rigid Frames with Pitched Roofs

Professor Hendry (references 1, 2 and 3) included the following in his conclusions:

> Reinforcement of the Knees of Portal Frames.
>
> Very considerable increases in strength may be achieved by reinforcing the knees of portal frames using either the curved flange or bracket type of knee.

It is quite common to haunch the knees of frames with pitched roofs, as shown in Fig. 466, or by curving the inner flange, a method of treatment to be described later.

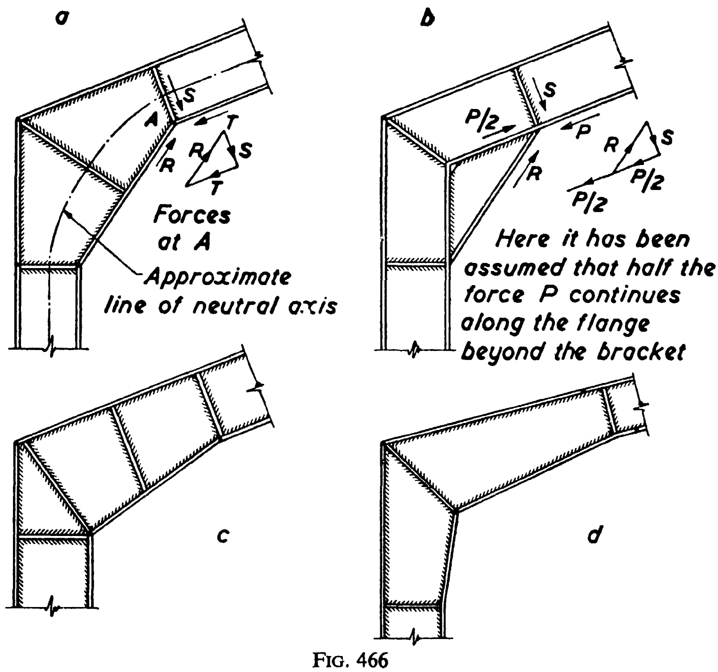


FIG. 466

There is ample experimental evidence to show that the neutral axis of stress in haunched knees moves towards the inside of the frame, as shown in Fig. 466 (a). Consequently the approximate force in the inside flange may be found by assuming that the neutral axis occurs at the third point along the diagonal from the inside flange to the outside corner of the frame. Alternatively the force in the flange may be resolved from the triangle of forces shown in Fig. 466 (a), the known force being that in the inside flange

of the rafter. In the type of joint shown in Fig. 466 (b) it is often assumed that the whole of the inside flange forces are resolved into the bracket flange, but, as an alternative, it may be assumed that, say, one-half of the force continues along the flange of the main member, as shown in the diagram.

The stresses in knees of the type shown in Fig. 466 (a), (c) and (d) can be found by using Vierendeel's Tapered Beam formulæ or Olander's formulæ which are described later. See Figs. 479 and 481. Considering the knee in Fig. 467, all the sections between AA and BB, except in the hatched areas can be analysed by the above formulæ. However, if the knee is not overstressed elsewhere, it is very unlikely to be so in the hatched zones.

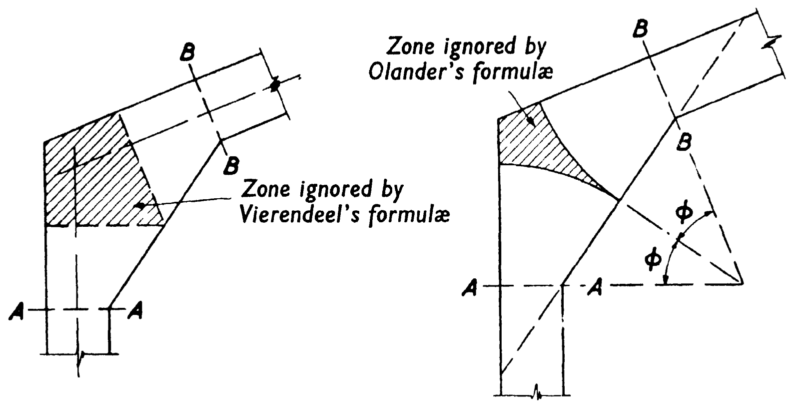


FIG. 467

The forces in the stiffeners at the limits of the knee are found by a resolution of forces, as at point A in Fig. 466 (a) where S is the appropriate stiffener force, and the stiffeners and welds are designed accordingly. The remaining stiffeners inside the knee can be of nominal size, their primary function being to prevent local buckling of the web and lateral failure of the inner flange.

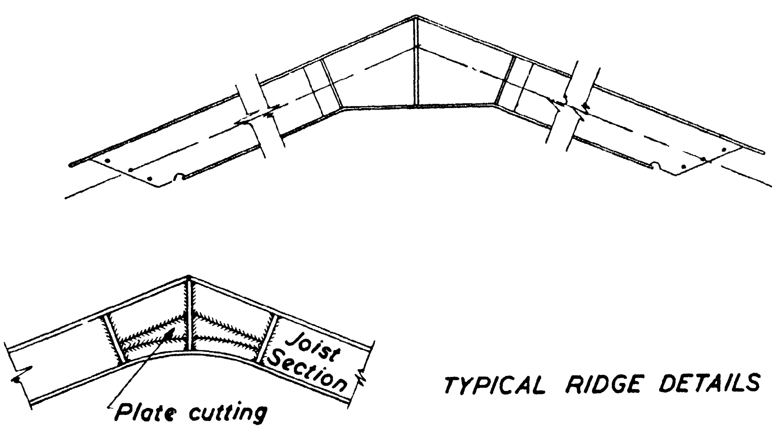


FIG. 468

Ridges in Pitched Roofs

Ridges in pitched roofs are designed in precisely the same manner as obtuse-angled knees. Normally they present less difficulty than knee joints as the angle between the rafters is very obtuse and the forces are much less than in knees. If joist sections are employed it may be unnecessary to add brackets, while in lightly loaded structures it is sufficient to butt weld the ends of the rafters. Some typical joints are shown in Fig. 468.

Knees with Curved Flanges

It is probable that joints with curved flanges will always be the subject of some controversy but here it is proposed to describe and illustrate the articles published or research work carried out both in this country and abroad.

It is of some interest to consider a formula, known as the Winkler-Résal formula, which can be used for an *initially* curved bar with parallel flanges, such as that shown in Fig. 469.

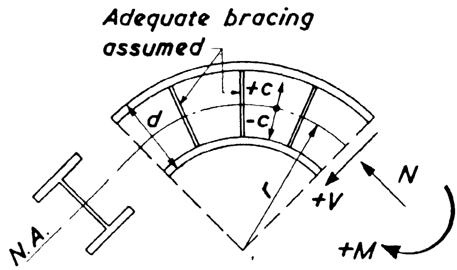


FIG. 469

The formula for the stress in any fibre of the bar is

$$f=\frac{N}{A}-\frac{M}{r\,.\,A}-\frac{M\,.\,c}{U}\times\frac{r}{r+c},$$

where N = the normal thrust,

A = the cross-sectional area of the bar,

M = the applied B.M.

r = the initial radius of curvature of the bar taken to the neutral axis N.A. of the section,

c = the distance from the N.A. to the fibre being considered, being positive when measured away from the centre of curvature and negative when measured towards it

and U = a figure analogous to the moment of inertia I and which may be replaced by I when the value of r is greater than twice the depth d of the bar.

When r is less than twice d and the section is composed of rectangles,

$$U=r^2(2{\cdot}30258r\Sigma b\,.\,\log w_1/w_2-A),$$

where the symbols have the significance shown in Fig. 470.

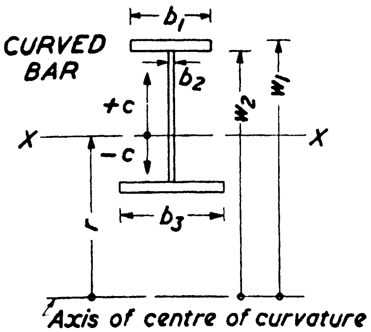


FIG. 470

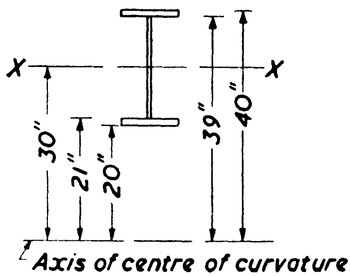


FIG. 471

Suppose the member is of the section shown in Fig. 471, the properties being as follows:

Flanges = 10 in. × 1 in. Web = 18 in. × 0·5 in.

Area A = 29 sq. in. r = 30 in. ($r/d = 1{\cdot}5$).

Then, $$U = r^2(2{\cdot}30258 r \Sigma b \,.\, \log w_1/w_2 - A).$$

Now,
$$10 \log 40/39 = 0{\cdot}10995$$
$$0{\cdot}5 \log 39/21 = 0{\cdot}13442$$
$$10 \log 21/20 = 0{\cdot}21189$$
$$\Sigma b \,.\, \log w_1/w_2 = 0{\cdot}45626$$

Therefore,
$$U = 30^2(2{\cdot}30258 \times 30 \times 0{\cdot}45626 - 29)$$
$$= 2265{\cdot}53 \text{ in.}^4.$$

It is interesting to note that the moment of inertia $I = 10 \times 20^3/12 - 9{\cdot}5 \times 18^3/12 = 2049$ in.^{4}.

It will be observed that the general form of the stress equation resembles the normal stress formula, i.e. $N/A \pm M/Z$.

The Winkler-Résal formula should be used for curved members when r/d is less than 2·5. When this ratio exceeds 2·5, the normal formula can be used with safety.

Example 2. Considering the member shown in Fig. 471, and assuming that it is subjected to a B.M. of 1900 tons in.,

$$f = \frac{N}{A} - \frac{M}{r\,.\,A} - \frac{M\,.\,c}{U} \times \frac{r}{r+c} = \frac{N}{A} - \frac{M}{r\,.\,A} - \frac{M\,.\,r}{U} \times \frac{c}{r+c}$$
$$= 0 - \frac{1900}{30 \times 29} - \frac{1900 \times 30}{2266} \times \frac{c}{30+c}$$
$$= -2{\cdot}184 - 25{\cdot}160 \times \frac{c}{30+c}$$

If values are plotted for various depths c from the neutral axis, the stress diagram shown in Fig. 472 is obtained.

It should be noted that the Winkler-Résal formula makes suitable allowance for the shift of the neutral plane of bending from the N.A. of the section towards the inner flange.

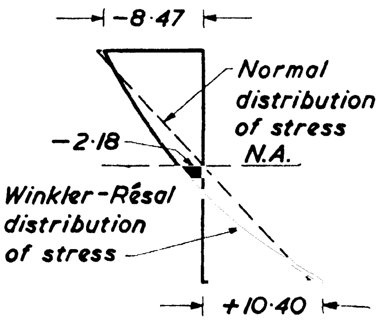


FIG. 472

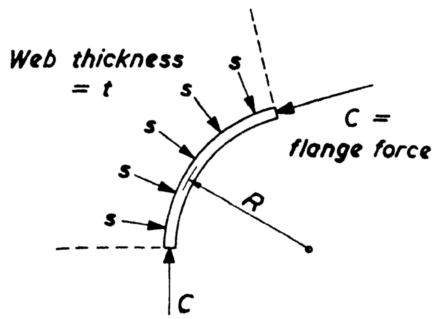


FIG. 473

Now the change in direction of the force in the flange of a curved member induces radial stresses in the web which can be calculated from the following formula, due to Professor Campus of Liège:

$$s = C/Rt, \quad \text{(see Fig. 473)}$$

where s = the unit radial stress,

C = the total flange force,

R = the radius of curvature of the flange being considered

and t = web thickness.

Professor Magnel (reference 6) states that the radial stress should be added to that due to the shear across the web.

The radial force is applied at the junction of web and flange and causes cross-bending, the edges of the flanges moving away from the centre of curvature when the flange is compressed and towards it when it is in tension, as shown in Fig. 474.

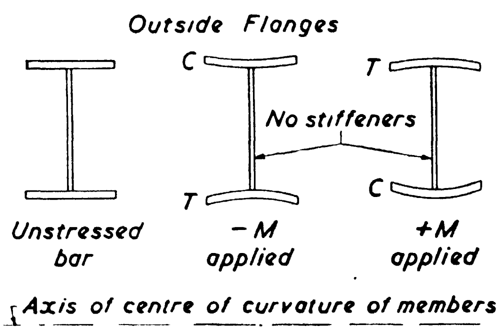


FIG. 474

Dr. Hans H. Bleich (reference 7) investigated the effects of this phenomenon and produced two coefficients ν and μ (nu and mu), the first being associated with the longitudinal stresses in the flanges and the second the transverse stresses.

If f is the mean stress derived from the Winkler-Résal formula, then max. $f=f/\nu$ and $f'=\mu$ max. f, where ν and μ have the following values with respect to the expression b^2/Rt, the symbols for which are shown in Fig. 475:

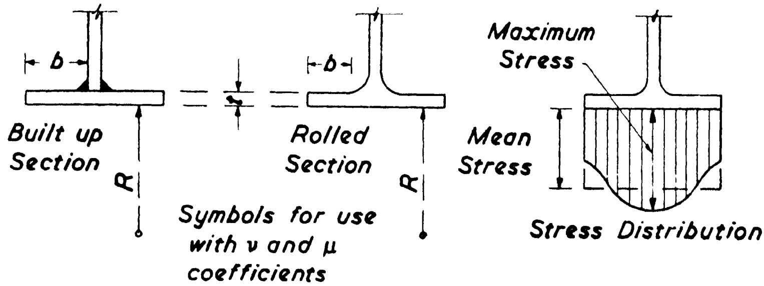


FIG. 475

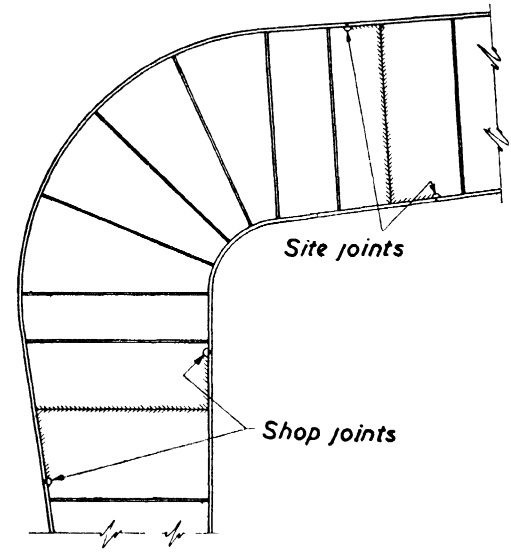
Bleich's Coefficients

$\frac{b^2}{Rt}$	0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9
ν	1·000	0·994	0·977	0·950	0·915	0·878	0·838	0·800	0·762	0·726
μ	0	0·297	0·580	0·836	1·056	1·238	1·382	1·495	1·577	1·636
$\frac{b^2}{Rt}$	1·0	1·1	1·2	1·3	1·4	1·5	2·0	3·0	4·0	5·0
ν	0·693	0·663	0·636	0·611	0·589	0·569	0·495	0·414	0·367	0·334
μ	1·677	1·703	1·721	1·728	1·732	1·732	1·707	1·671	1·680	1·700

If the radial forces tend to lead to overstressing, the flanges must be braced either by entire web stiffeners or by small gussets, the spacing of which is a matter for engineering judgment as no rules, mathematical or empirical, have been derived.

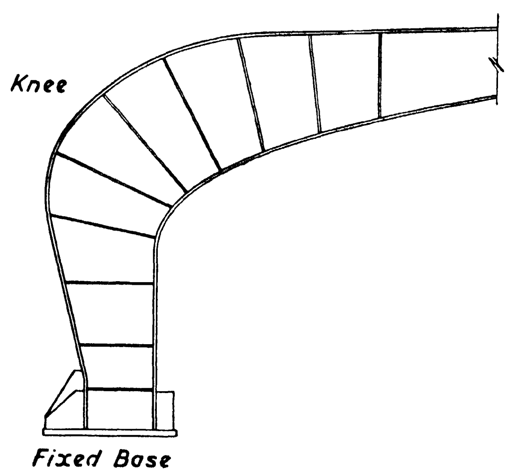
Some practical examples of curved knees closely resembling curved bars are shown in Figs. 476 and 477, the latter being described in reference 8, as well as in other papers.

Generally knees are not shaped like curved bars, the majority being of the type shown in Figs. 478 (a) and (b) where the outside flange is straight. The rapid change of section at the knee and the curvature of the centre-line affect both the magnitude and the distribution of the fibre stresses.



Portal Knee
for Bus Garage

FIG. 476



Portal Knee
for Bridge

FIG. 477

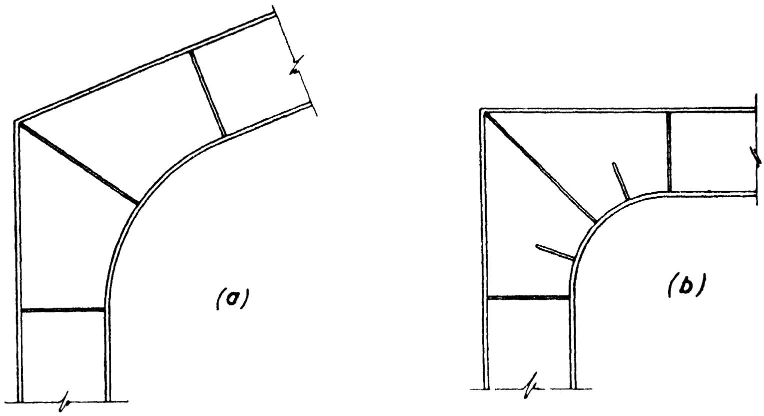


FIG. 478

Professor Vierendeel devised Tapered Beam formulæ for the knee shown in Fig. 479, the formulæ being as follows:

On any section AA,

$$f_o = \frac{P}{A} - \frac{Ma_o}{I}$$

$$f_i = \frac{P}{A} + \frac{Ma_i}{I}$$

$$v = \frac{1}{bd}(V + f_i F_i \sin \phi),$$

where f_o and f_i = the mean stresses in the outside and inside flanges, respectively,

a_o and a_i = the distances of the centroids of the outside and inside flanges, respectively, from the axis shown,

v = the shear stress

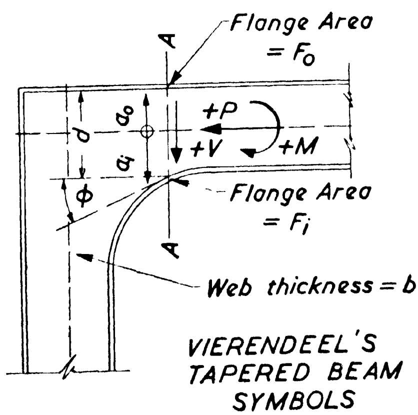


FIG. 479

M, V and P = the bending moment, shear force and thrust at the section AA,

b = the web thickness,

d = the web depth,

$A = bd + F_o + F_i \cos\phi$

and $I = bd^3/12 + F_o a_o^2 + F_i \cos\phi \, a_i^2$.

These formulæ are logical and give reasonable results.

When the mean flange stresses have been calculated, the maximum stresses may be computed using Dr. H. H. Bleich's coefficients and suitable stiffeners added where necessary.

It should be noted that the Tapered Beam formulæ can be applied to knees of the type shown in Fig. 466.

In the U.S.A. a number of investigators have produced formulæ for curved knees, using circular sections as shown in Fig. 480.

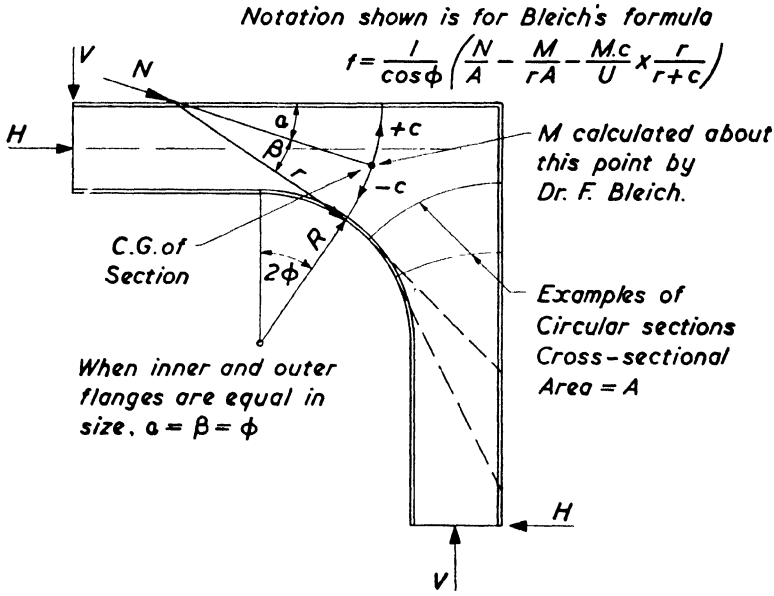


FIG. 480

Osgood's formulæ (reference 5) are mathematically exact but are lengthy and cumbersome for use in design and are not quoted here.

Dr. Friedrich Bleich (reference 9) adapted the Winkler-Résal formula, adjusting it by multiplying the whole expression by $1/\cos\phi$. It will be noted from Fig. 480 that r and c are measured in a different way, but the principles of calculation are identical with those for Example 1.

The amount of work involved in using Bleich's method is less than with Osgood's method but greater than that with a recent method (reference 10) due to Harvey C. Olander, which gives reasonable results and can be recommended for use in design.

Olander states that the method is simply to take circular sections that cut the extreme fibres at right angles, such as section AB, Fig. 481, develop the section as shown and obtain the cross-sectional area A and moment of inertia I of the developed section. Next, resolve all forces to the right of section AB into the values P_O and M_O about the point O, the centre of the arc. P_O passes through the centre of gravity of section AB, and M_O is the moment of the forces about O. Then, with these values, the stresses are

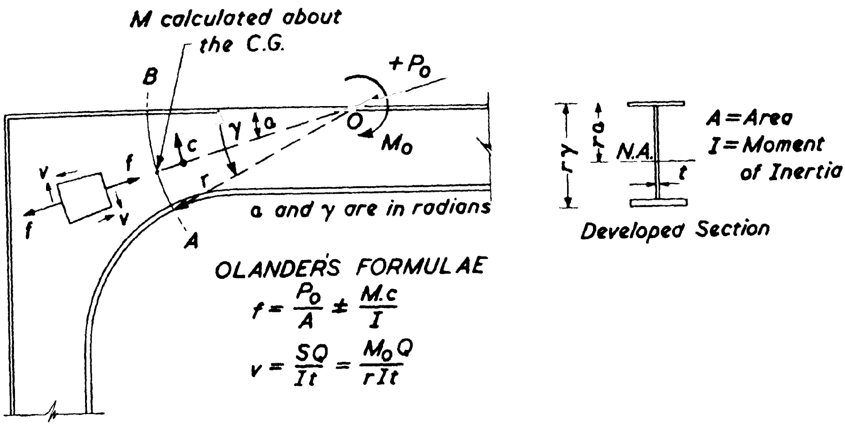


FIG. 481

calculated as for an ordinary beam, except that the shear is determined from M_O. The total shear on section $AB = S = M_O/r$. Then the unit shear along the section,

$$v = \frac{SQ}{It} = \frac{M_O Q}{rIt}$$

where Q is the statical moment of the area of the section about the point being considered. (Cf. British notation, $v = S.a.y/It$.)

The stresses normal to the section,

$$f = \frac{P_O}{A} \pm \frac{M.c}{I}$$

where M is the B.M. at the C.G. of the section AB.

An example will help to explain the method.

Example 3. Fig. 482 shows a curved knee joining a 16-in. × 12-in. × 110-lb. B.F.B. to a 24-in. × 12-in. × 165-lb. B.F.B. The properties of the two B.F.B.s are as follows:

B.F.B. (*in.*)	*Flange Thickness* (*in.*)	*Web Thickness* (*in.*)	*Area A* (*sq. in.*)	Z_{xx} (*in.*^{3})
16 × 12	1·00	0·55	32·35	188·53
24 × 12	1·36	0·70	48·53	404·79

The cross-beam is chosen to resist the B.M. in the centre of the beam and will be understressed at its junction with the knee. If the radius of the inner flange is 48 in., then the radius will equal twice the depth of the larger section or three times that of the smaller section.

It is convenient to continue the flange thicknesses of the smaller B.F.B. around the knee, i.e. to use a 12-in. × 1-in. plate. The web of the knee will be 0·625 in. thick, i.e. some convenient thickness between those for the two B.F.B.s.

The knee can be divided into any number of circular sections. As, however, the greatest stress is situated just inside the knees, these areas should be investigated. Consequently, sections are usually chosen at 15° or 18° intervals along the inside of the knee.

If 18° intervals are used in this case, there will be six cross-sections to consider, as shown in Fig. 482.

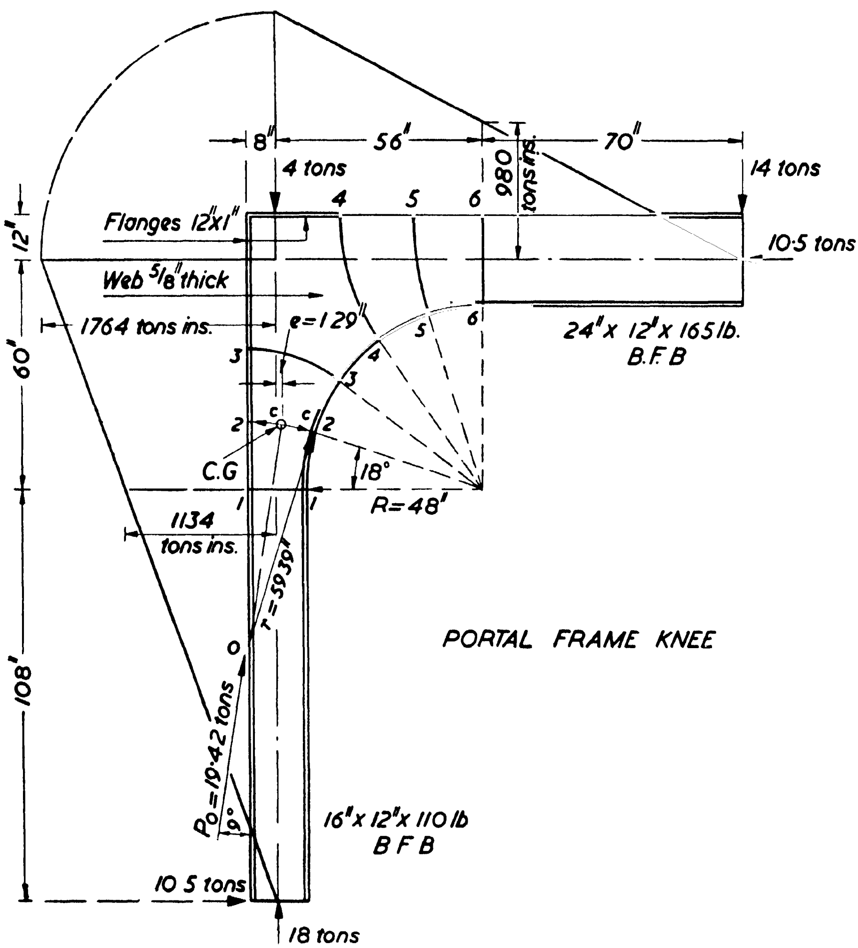


Fig. 482

It is advisable to check that sections 1–1 and 6–6 are in order first, before embarking upon the knee proper.

Considering section 1–1,

$$P = 18 \text{ tons.} \qquad A = 32{\cdot}35 \text{ sq. in.}$$
$$M_{xx} = 1134 \text{ tons in.} \qquad Z_{xx} = 188{\cdot}53 \text{ in.}^3.$$

Then
$$f = \frac{P}{A} \pm \frac{M_{xx}}{Z_{xx}}$$
$$= \frac{18}{32{\cdot}35} \pm \frac{1134}{188{\cdot}53}$$
$$= 6{\cdot}56 \text{ or } -5{\cdot}46 \text{ tons per sq. in.}$$

Using the normal B.S. 449 procedure for joists, the shear stress

$$v = \frac{\text{load}}{\text{gross web area}} = \frac{10{\cdot}5}{16 \times 0{\cdot}55}$$
$$= 1{\cdot}19 \text{ tons per sq. in.}$$

Considering section 6–6, the knee section should be taken, as the properties of the adjoining B.F.B. are greater in magnitude than those for the knee.

Then
$$I_{xx} = \frac{12 \times 24^3}{12} - \frac{11{\cdot}375 \times 22^3}{12}$$
$$= 3731 \text{ in.}^4.$$

and
$$Z_{xx} = \frac{3731}{12} = 311 \text{ in.}^3.$$
$$A = 2(12 \times 1) + (22 \times 0{\cdot}625)$$
$$= 37{\cdot}75 \text{ sq. in.}$$

Therefore
$$f = \frac{P}{A} \pm \frac{M_{xx}}{Z_{xx}}$$
$$= \frac{10{\cdot}5}{37{\cdot}75} \pm \frac{980}{311}$$
$$= 3{\cdot}43 \text{ or } -2{\cdot}87 \text{ tons per sq. in.}$$

and
$$v = \frac{14}{24 \times 0{\cdot}625}$$
$$= 0{\cdot}93 \text{ ton per sq. in.}$$

Consequently, the stresses at sections 1–1 and 6–6 are acceptable.

Although Olander's method is quicker than some other methods of analysis, a certain amount of time is taken calculating the geometrical properties of the circular sections. It is desirable to devise some system of tabulation for the calculations.

Fig. 483 shows the symbols which have to be calculated for each section.

$$r = \frac{h + R(1 - \cos 2\phi)}{\sin 2\phi}$$
$$a = R \,.\, \sin 2\phi$$
$$(a + b) = r \,.\, \cos 2\phi$$
$$d = r \,.\, \cos \phi$$
$$c = \phi \,.\, r \quad (\phi \text{ in radians})$$
$$(g + e) = r \,.\, \sin \phi,$$

where h, R, ϕ and g are known quantities.

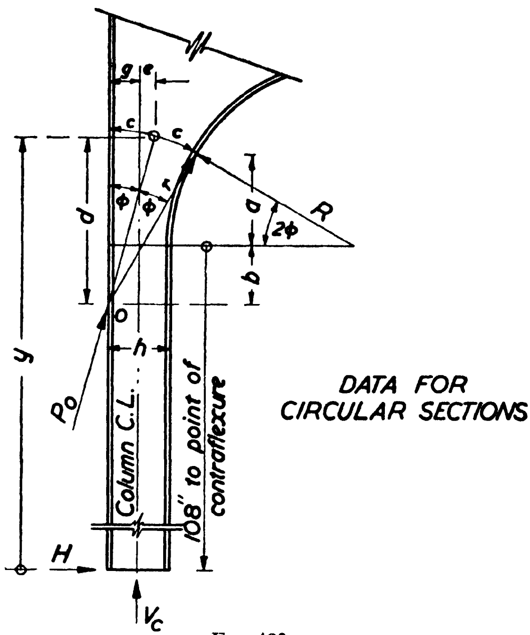


FIG. 483

For values of 2ϕ not exceeding 45° the radius r is measured along the column, but between 45° and 90° it is measured along the cross-beam. (The angle subtended by the inner flange should always be bisected in this way, even if the angle is acute as in the case of a frame with a pitched roof.)

The area and moment of inertia of the 'I'-sections traced by each circular section are found in the normal way, the total depth being the arc length.

For sections 2–2 and 3–3,

$$P_O = V_c \cos\phi + H \sin\phi$$
$$M = H \cdot y - V_c \cdot e.$$

For sections 5–5 and 4–4,

$$P_O = H \cos\phi + V_b \sin\phi$$
$$M = V_b \cdot x - H \cdot e.$$

where y is the vertical distance between the bottom of the column and the C.G. of the section, x is the horizontal distance from the end of the beam to the C.G. of the section and V_c and V_b refer to the column and beam respectively.

Proceeding, the relevant properties of, and the stresses in, the four sections inside the knee are as follows, the units being tons and inches:

Section	P_O	A	M	c	I_{xx}	$\frac{P_O}{A} \pm \frac{M.c}{I_{xx}}$
2–2	19·42	34·41	1289	9·33	2112	+6·26 or −5·13
3–3	20·38	39·57	1400	13·46	4833	+4·41 or −3·38
4–4	14·31	44·88	1430	17·70	9041	+3·18 or −2·48
5–5	12·56	39·49	1217	13·40	4481	+3·92 or −3·36

The complete flange stresses have been plotted in Fig. 484.

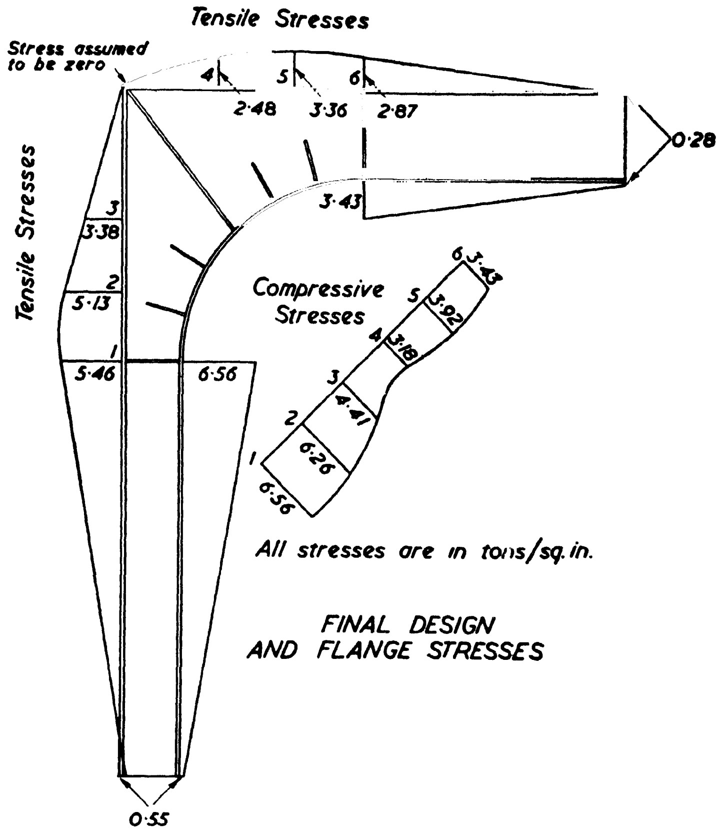


FIG. 484

The stresses given for the inner flange presuppose that it is properly stiffened to resist cross-bending. If unstiffened, the stresses can be calculated by Dr. H. H. Bleich's formulæ. (See p. 741.)

Considering section 2–2,

$$\max. f = \frac{f}{\nu}.$$

The value of b^2/Rt for the flange

$$= \left(\frac{12-0\cdot625}{2}\right)^2 \times \frac{1}{48\times 1}$$

$$= 0\cdot67.$$

Hence $\nu = 0\cdot81$ (by interpolation)

and

$$\max. f = \frac{f}{\nu}$$

$$= \frac{6\cdot26}{0\cdot81}$$

$$= 7\cdot73 \text{ tons per sq. in.}$$

Now the cross-bending stress

$$f' = \mu \text{ max.} f$$

$$\mu = 1{\cdot}46 \text{ (by interpolation).}$$

Therefore $$f' = 1{\cdot}46 \times 7{\cdot}73$$

$$= 11{\cdot}29 \text{ tons per sq. in.}$$

As this exceeds the permissible stress, stiffeners are required.

The radial stress in the web is greatest where the flange stress is greatest, i.e. somewhere between sections 1–1 and 2–2. Taking the flange stress at 1–1 and using Campus's formula,

$$s = \frac{C}{Rt} = \frac{\text{Area} \times \text{unit stress}}{Rt}$$

$$= \frac{12 \times 1 \times 6{\cdot}56}{48 \times 0{\cdot}625}$$

$$= 2{\cdot}62 \text{ tons per sq. in.}$$

Even if the shear stress at 1–1, 1·19 tons per sq. in., is added to this stress, the total stress is still low.

Olander's method of obtaining the shear stresses in the knee can be demonstrated for section 2–2.

Now $$v = \frac{M_O Q}{rIt}.$$

Considering Fig. 483, the value of the distance b for section 2–2 = 41·65 in., and the vertical distance from O to the bottom of the column is 66·35 in.

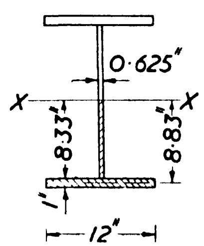


Fig. 485

$$M_O = (10{\cdot}5 \times 66{\cdot}35) + (18 \times 8)$$

$$= 841 \text{ tons in.}$$

Considering Fig. 485,

$$Q = (0{\cdot}625 \times 8{\cdot}33)\frac{8{\cdot}33}{2} + (1 \times 12 \times 8{\cdot}83)$$

$$= 127{\cdot}7 \text{ in.}^3.$$

Hence, $$v = \frac{841 \times 127{\cdot}7}{59{\cdot}39 \times 2112 \times 0{\cdot}625}$$

$$= 1{\cdot}37 \text{ tons per sq. in.}$$

This stress is low and those at other sections are lower still.

It follows that the stiffeners required to prevent cross-bending of the flange can be nominal in size. A thickness of ⅜ in. would be suitable.

Professor Hendry's Tests

Among Professor Hendry's conclusions on portal knees were the following:

Curved Flange Knees

Unstiffened curved flange knees will usually fail by lateral deflection of the compression flange if this is unsupported. Failure in such a case takes place when the flange has become plastic. If the curved flange is prevented from deflecting laterally, collapse will in general be due to

web buckling. The forces tending to produce buckling are, however, very much reduced as the radius is increased, and the collapse load may approach the maximum possible strength, which is limited by the strength of the members connected.

To produce a connection with a curved inner flange, where the ratio of radius of curvature to depth of member is between 1 and 2, which will develop the full strength of the member connected, a diagonal stiffener should be provided, proportioned to carry 30 per cent of the total load due to the change in direction of the flange stress.

Connections between members whose axes intersect at an angle greater than 90° can be dealt with in the same manner as suggested above.

It will be obvious that as the radius of curvature to depth of member increases, the curved flange stress decreases. Hard and fast rules cannot be laid down for the design of curved flanges, but it is reasonable, in cases where the angle between the members is not greater than 120°, to proportion the knee so that the ratio of diagonal depth of knee to depth of member is not less than 2.

Valley Joints

The principles involved in the design of the Y-shaped valley joints in multi-bay construction are the same as those for knee joints. It is quite reasonable to design the inner flange of each rafter section as though the joint were a knee, the other rafter being ignored, as shown in Fig. 486. The detailing should be as simple as possible. Two examples showing haunched and curved inner flanges are shown in Figs. 487 and 488 (references 11 and 12 respectively).

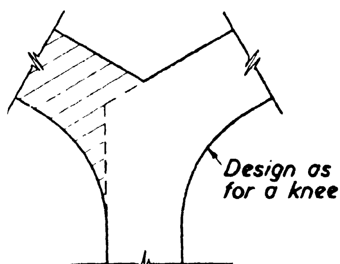


FIG. 486

To reduce the cost of heating the building or for æsthetic reasons roofs are often made of low pitch, and practical difficulties can arise in accommodating the valley gutter. The problem may be overcome as shown in Fig. 488, the

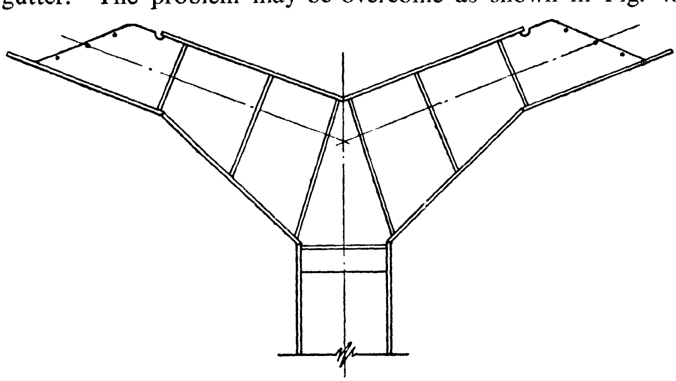


FIG. 487

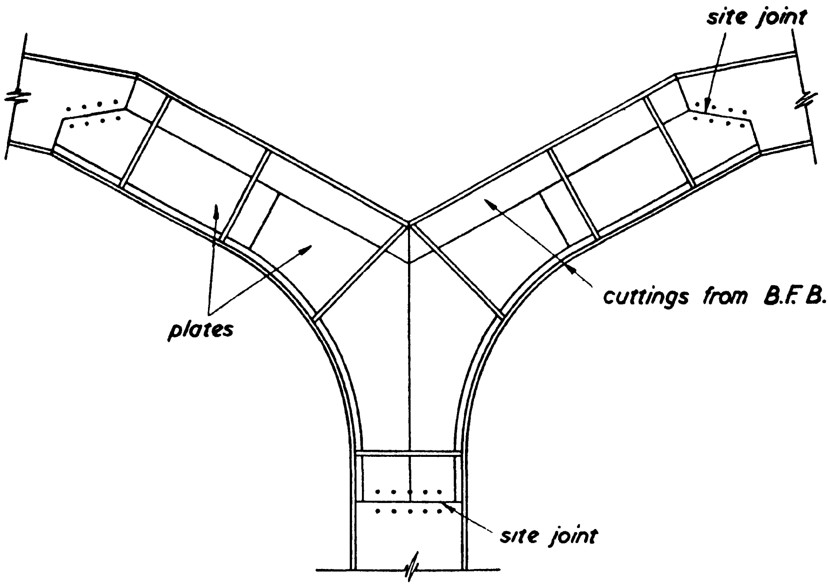


FIG. 488

pitch being fairly steep at the feet of the rafters and low over the central portion of the roof.

Splice Connections

Splice connections in rigid frames should be arranged at or near the dead-load points of contraflexure. Figs. 468, 476, 487 and 488 all incorporate splice connections. Some typical joints for a column and cross-beam are shown in Fig. 489, but it should be noted that it is not essential that joints should occur at stiffeners. Further details may be found in reference 13.

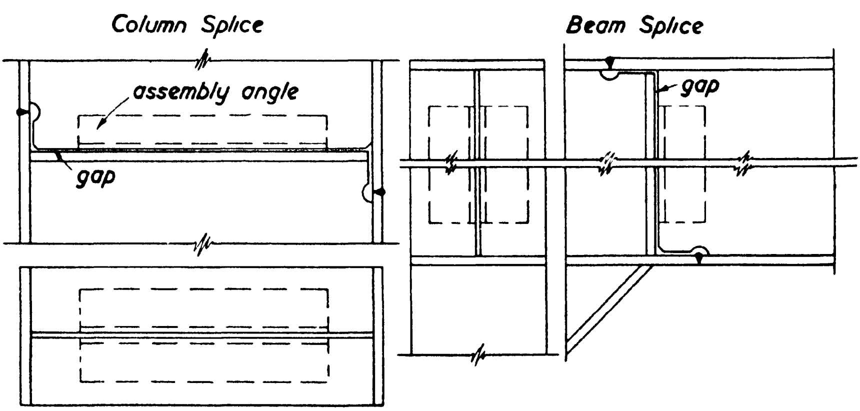


FIG. 489

Rigid Joints in Multi-storey Buildings

Although some of the calculations involved in designing a joint in a multi-storey building may be lengthy, the underlying principles on which the work is based are comparatively elementary.

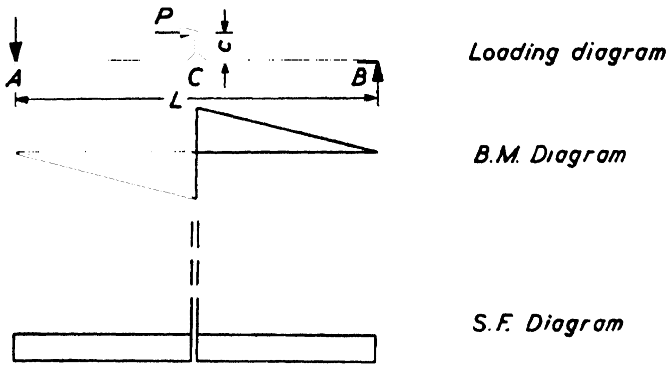


FIG. 490

Consider the simply supported prismatic beam shown in Fig. 490. When the load P is applied, a moment M is induced at C which is distributed between the portions of the beam AC and CB as shown. The reactions at the supports are:

$$R_A = \frac{-M}{L} \quad \text{and} \quad R_B = \frac{+M}{L}.$$

Now the shear in AC and CB may be obtained by calculating the slope of the B.M. diagram. Hence, the value of the shear $= M/L$, which is, of course, equal to the reactions at the supports A or B.

At C the moment diagram undergoes an abrupt vertical change of moment, equal to M, and the corresponding shear is infinite. This results from the assumption that the moment is applied at a point, as drawn in Fig. 490.

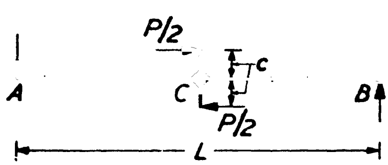


FIG. 491

Much the same state would result if the moment M were applied through a couple as shown in Fig. 491.

If, however, the moment M were to be applied through a couple formed by two equal and opposite forces of value P as shown in Fig. 492, then the change of moment at C would be less abrupt and the S.F. between the two loads could be calculated.

Thus, $$V = \frac{M_{CA}+M_{CB}}{c} = \frac{P(L-c)}{L}.$$

As before, the shear on either side of the applied moment would be of constant value, being equal to cP/L.

Now it is proposed to give a practical example to demonstrate how such joints can be treated.

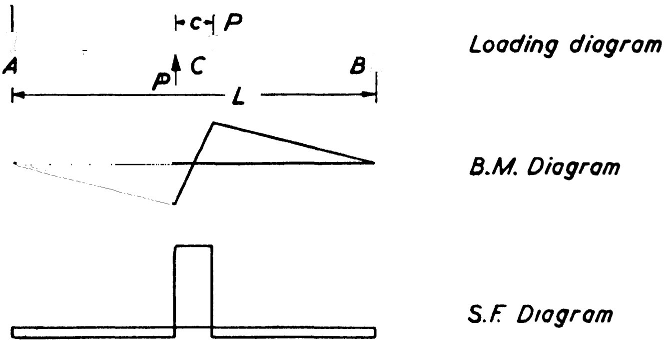


FIG. 492

Example 4. Fig. 493 shows a portion of a rigid-frame building, consisting of a section of girder BD and sections of two columns AB and BC; A, C and D being the points of contraflexure in the B.M. diagram for the worst conditions of loading on this portion of the frame.

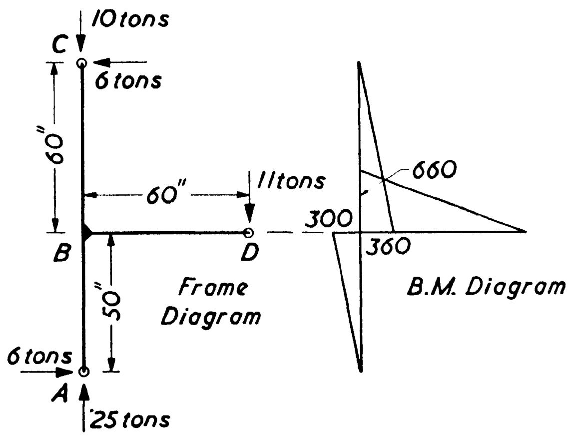


FIG. 493

The maximum B.M.s at the junction of the neutral axes of the girder and columns are as follows:

$$M_{BC} = +360 \text{ tons in.}$$
$$M_{BA} = -300 \text{ tons in.}$$
$$M_{BD} = -660 \text{ tons in.}$$

It is not proposed to design the columns or cross-beam. It will be assumed that the sections shown in Fig. 494 are adequate.

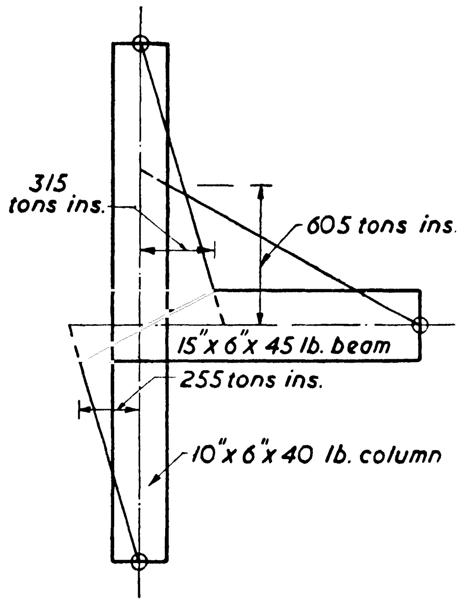


FIG. 494

Considering the joint at B, the shear across B is calculated from the slope of the B.M. diagram.

Hence $$V=\frac{315\cdot 0+255\cdot 0}{15}$$
$$=38 \text{ tons.}$$

Now the web thickness of a 10-in. × 6-in. × 40-lb. column is 0·36 in. and the gross area of the web = 10-in. × 0·36-in. = 3·6 sq. in.

Therefore the shear stress in the web,

$$v=\frac{38}{3\cdot 6}=10\cdot 56 \text{ tons per sq. in.}$$

This stress exceeds the maximum permissible shear stress of 6·5 tons per sq. in. A number of expedients may be adopted to increase the resistance to shear.

One of the most common methods is to weld two rectangular plates to the web of the column. The clear web depth of a 10-in. × 6-in. × 40-lb. column is 7·13 in. If two plates, $7\frac{1}{8}$ in. wide and $\frac{1}{4}$ in. thick, were welded, one each side of the web, as shown in Fig. 495, then the shear stress would be,

$$v=\frac{38}{10\times 0\cdot 86}=4\cdot 54 \text{ tons per sq. in.}$$

which would be acceptable.

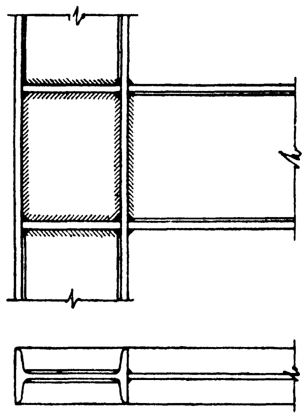


FIG. 495

It will be noticed that two sets of stiffeners have been introduced at the level of the flanges of the girder. These should be of approximately the same section as the girder flanges and, in this case, are made $\frac{3}{4}$ in. thick.

Instead of thickening up the web some kind of triangulated system of stiffeners may be used, such as those shown in Fig. 496. In such cases the diagonal or oblique stiffeners may supplement the strength of the web, or the web may be entirely ignored, when the stiffeners form a truss framework and are designed accordingly.

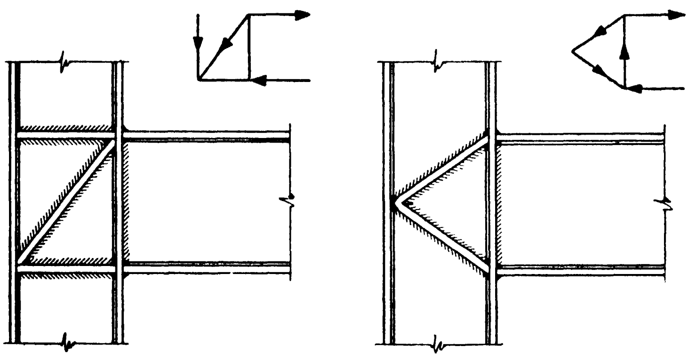


FIG. 496

Another method to avoid overstressing the web of the column is to deepen the end of the girder or to provide brackets at the junction of the girder and column, the object being to reduce the steep slope of the B.M. diagram, i.e. to reduce the shear force in the joint. Various expedients are shown in Fig. 497.

Open-frame or Vierendeel Girders

The most obvious characteristic of open-frame girders, some examples of which are shown in Fig. 498, is the complete absence of diagonal members in

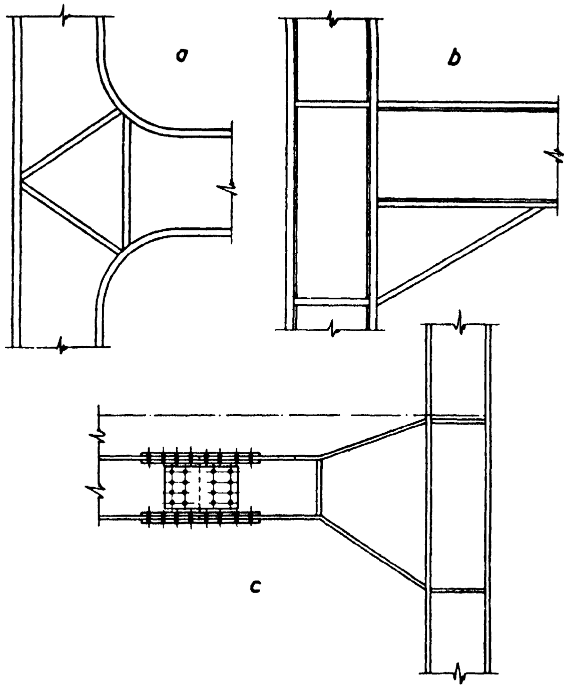


FIG. 497

the panels, the girders depending on the rigidity of the joints for their stability.

The open-frame girder is not commonly used in Great Britain, but many examples exist in Europe, particularly in Belgium, where they are associated with the pioneer work and development of the late Professor Vierendeel after whom they are usually named.

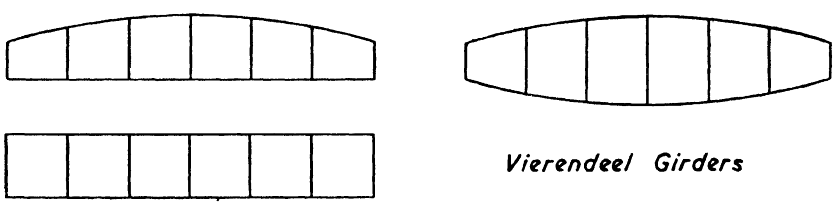


FIG. 498

Although there are many Vierendeel bridges in Belgium, only one such bridge, for pedestrians, has been built in Great Britain. Examples in structures taking static loads have usually resulted from a demand for a free unobstructed space where the use of diagonals has been precluded. Consequently, Vierendeel girders can be used for clerestory lighting in churches and other structures or for spanning any gap where a plate girder or truss would be used but for the fact that the web would provide an obstruction.

Many foreign authors have devoted whole books to the analysis of Vierendeel girders, but, unfortunately, the literature relating to the design of joints is comparatively scanty. Here, it is proposed to deal with girders with parallel top and bottom booms. Provided that the booms are parallel, analysis by Slope Deflection is possible but lengthy, but if the top and bottom booms are of identical section in each panel the girders may be analysed quickly and accurately by Naylor's application of Moment Distribution, as demonstrated in Example 15 in the section on that method of analysis. See page 212.

The analysis and design of a joint for a multi-storey building frame has just been given. Now a single-bay, multi-storey building of the type described in Examples 9 and 11 in the section on Moment Distribution is really a Vierendeel girder erected vertically, so that the principles underlying the design have already been described. However, it is usual for the axial forces in the members of Vierendeel girders to be very great compared with those in the analogous members in building frames.

Most of the loads applied to Vierendeel girders are applied at the panel points but sometimes the booms take comparatively light loads, usually uniformly distributed, when a design by Moment Distribution will incorporate the devices associated with inter-panel loading (c.f. Example 16 in Moment Distribution). Inter-panel loading can also be treated by Slope Deflection.

Formulæ for Joints

Vierendeel evolved a number of formulæ for different types of joint. For the 'T'-joint shown in Fig. 499, the formulæ resemble the Tapered Beam formulæ given earlier in this section, viz.:

$$f_1 = \frac{P}{A} - \frac{Ma_1}{I}$$

$$f_2 = \frac{P}{A} + \frac{Ma_2}{I}$$

$$v = \frac{1}{bd}[V - f_1 F_1 \sin \phi_1 + f_2 F_2 \sin \phi_2] \text{ (using appropriate signs for } f_1 \text{ and } f_2\text{)}$$

$$A = bd + F_1 \cos \phi_1 + F_2 \cos \phi_2$$

$$I = \frac{bd^3}{12} + F_1 \cos \phi_1 a_1^2 + F_2 \cos \phi_2 a_2^2.$$

These formulæ may be used for a joint in a multi-storey building where the radius of the top flange is small so that the flange does not project above floor level. Normally in Vierendeel girders $\phi_1 = \phi_2$.

This type of joint is not used in Belgium for bridges or other structures taking dynamic loading. It is the invariable practice to employ the type shown in Fig. 500, where the posts are planted on the booms and the radii of curvature may be as much as one-third of the panel length.

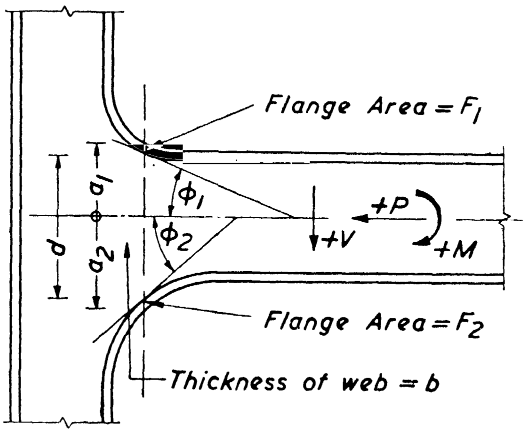


FIG. 499

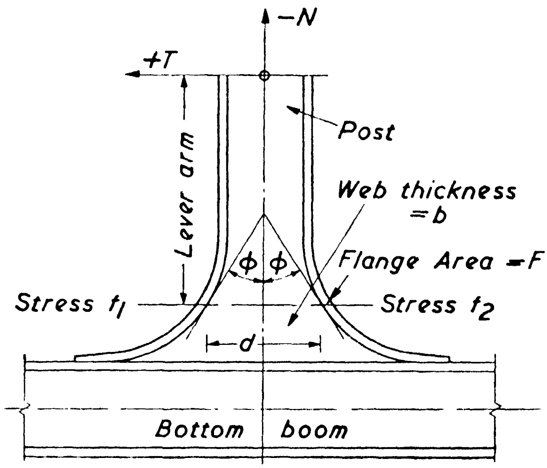


FIG. 500

In this case the post is symmetrical about the vertical axis, and the flange stresses *due to the moment only* are:

$$f_1 = -f_2 = \frac{M}{d(F\cos\phi + bd/6)}.$$

M equals T times the lever arm about the section considered.

At the junction of post and boom, as $\cos\phi = 0$, the moment is taken by the web only.

The stresses due to $-N = -N/A$. The values of v and A are found as before.

The B.M. diagram for a boom is drawn as shown in Fig. 501. The lines between the tangent points of the post are parabolas.

Professor Magnel of Ghent (reference 6) quotes the following formulæ for the section shown in Fig. 502:

$$T = P_3 + P_1 \sin\phi - P_2 \sin\theta$$
$$T\,.\,OX - M = P_3\,.\,OX$$
$$N = P_1\cos\phi + P_2\cos\theta,$$

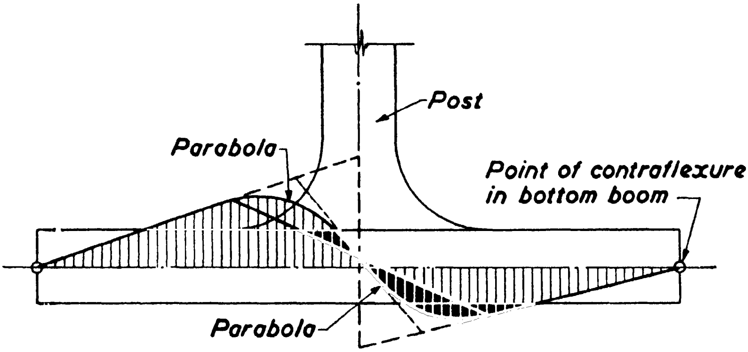


FIG. 501

from which

$$P_1 = \frac{N \cdot \sin\theta + \frac{M}{OX}\cos\theta}{\sin(\phi+\theta)}$$

$$P_2 = \frac{N \cdot \sin\phi - \frac{M}{OX}\cos\phi}{\sin(\phi+\theta)}$$

$$P_3 = T - \frac{M}{OX}$$

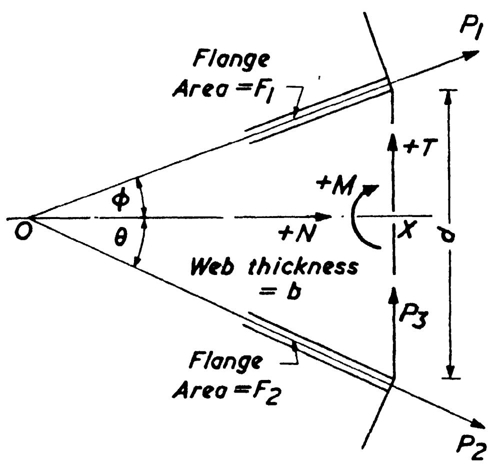


FIG. 502

The member shown has straight sloping sides, but the same formulæ could be used for curved flanges although Magnel states that the formulæ should not be used when $(\phi+\theta)$ exceeds 90°. However, he quotes experiments carried out by Professor Campus in which he proved that in joints such as that shown in Fig. 500 the critical sections were those in which $(\phi+\theta)$ varied from 0 to 90°.

The stresses derived from Magnel's formulæ are as follows:

$$f_1 = \frac{P_1}{F_1};\quad f_2 = \frac{P_2}{F_2};\quad v = \frac{P_3}{bd}.$$

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13. ANON. 'Welded Portal Frame Retort House' *Welding and Metal Fabrication* (April 1953).

38. WELDING PRACTICE

British Standards

The following British Standards are associated with welding:

B.S. 15. *Structural Steel.*

B.S. 968. *High Tensile (Fusion Welding Quality) Structural Steel for Bridges, etc., and General Building Construction.*

B.S. 639. *Covered Electrodes.*

B.S. 1856. *General Requirements for the Metal-Arc Welding of Mild Steel.*

B.S. 938. *Metal-Arc Welding as applied to Tubular Steel Structural Members.*

B.S. 449. *The Use of Structural Steel in Building.*

B.S. 499 (Section 7). *Scheme of Symbols for Welding.*

Types of Weld

Fillet and butt welds are normally used in structural steelwork, the appropriate nomenclature and details of typical welds being shown in Fig. 503.

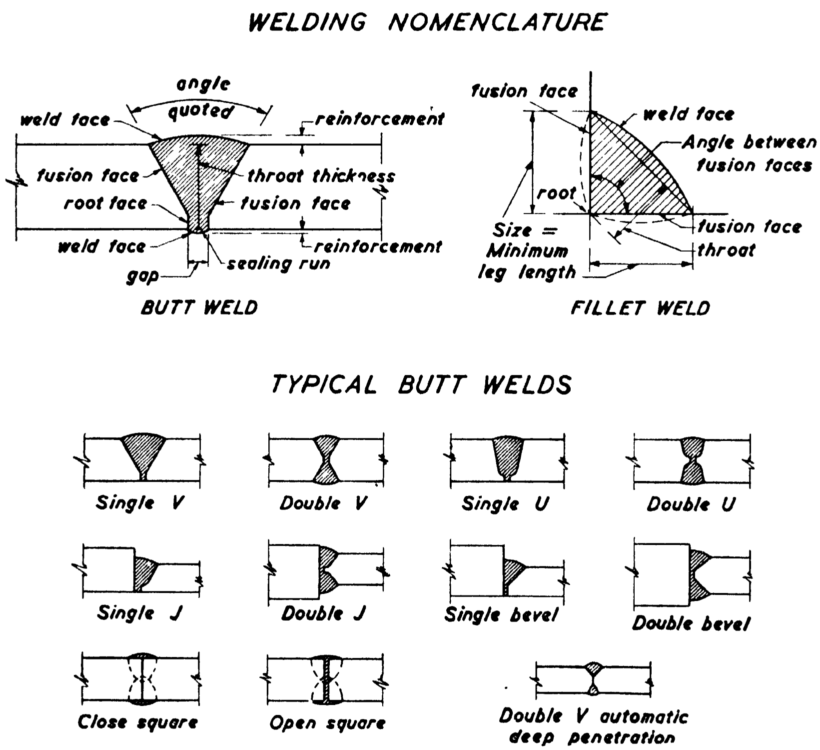


FIG. 503

Butt Welds

Notes on Butt Welds are given in Appendix A of B.S. 1856.

The size of a butt weld is specified by its throat thickness, which is taken as the thickness of the thinner plate when the plate thicknesses vary and as the plate thickness when the plates are equal in thickness. For full stress butt welds must be reinforced and sealed as shown in Fig. 503 although the additional thickness of weld involved is ignored in calculations. Where a sealing run cannot be employed, say in a single 'V' butt weld where the back of the plates is inaccessible, the effective throat thickness is taken as five-eighths of the thinner plate thickness.

The permissible stresses in mild-steel butt welds, calculated on an area equal to the length of the weld times the throat thickness, must not exceed the permissible tensile, compressive and shear stresses of the parent metal. Consequently there is normally no need to make calculations for complete penetration welds.

Fillet Welds

The size of a normal penetration fillet weld shall be taken as the minimum leg length of a convex or flat fillet weld or 1·41 times the effective throat thickness of a concave fillet weld, as shown in Fig. 504.

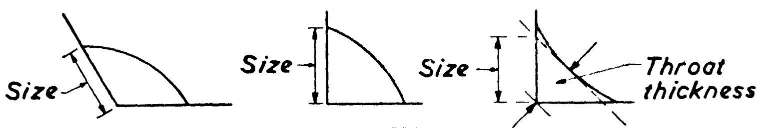


FIG. 504

A deep penetration weld is one in which the depth of penetration beyond the root is $\frac{3}{32}$ in. or over. Consequently, the size of a deep penetration weld is taken as the minimum leg length plus $\frac{3}{32}$ in. in the case of a convex or flat fillet weld and as 1·41 times the effective throat thickness plus $\frac{3}{32}$ in in the case of a concave fillet weld.

Notes on fillet welds are given in Appendix B of B.S. 1856. The following are of interest here:

> *Angle between fusion faces.* Fillet welds connecting parts, the fusion faces of which form an angle of more than 120° or less than 60°, should not be relied upon to transmit calculated loads at the full working stresses unless permitted to do so by the standard for the particular application.
>
> It is appreciated that full penetration to the root of a 60° fillet weld may not be attained, but this point is allowed for in determining the effective throat thickness (see Table A).

TABLE A.—EFFECTIVE THROAT THICKNESS OF FLAT OR CONVEX FILLET WELDS

Angle between fusion faces	60–90°	91–100°	101–106°	107–113°	114–120°
Factor by which fillet size is multiplied to give throat thickness	0·70	0·65	0·60	0·55	0·50

End returns. Fillet welds terminating at the ends or sides of parts, or members, should be returned continuously around the corners for a distance not less than twice the size of the weld. This provision should apply particularly to side and end fillet welds connecting brackets, seats and similar connections at the tension side of such connections.

Size at toe of rolled section. Where a fillet weld is applied to a rounded toe of a rolled section, the specified size of the weld should generally not exceed 75 per cent of the thickness of the section at the toe.

The stress in a fillet weld must not exceed the permissible shear stress in the parent metal, e.g. for mild steel the maximum permissible stress is 6·5 tons per sq. in. The following table gives the strengths of 60–90° angle fillet welds, i.e. based on a throat thickness of 0·7 times the leg length. For different angles between the fusion faces the values of the strength per lineal inch must be varied directly with the factor given in Table A.

TABLE B.—STRENGTH OF FILLET WELDS IN TONS PER LINEAL INCH FOR 60–90° ANGLES BETWEEN FUSION FACES

Fillet Size in Inches	*Throat Thickness in Inches*	*Strength per Lineal Inch at 6·5 tons per sq. in.*
$\frac{3}{16}$	0·131	0·85
$\frac{1}{4}$	0·175	1·14
$\frac{5}{16}$	0·219	1·42
$\frac{3}{8}$	0·263	1·71
$\frac{7}{16}$	0·306	1·99
$\frac{1}{2}$	0·350	2·28
$\frac{5}{8}$	0·438	2·85
$\frac{3}{4}$	0·525	3·41
$\frac{7}{8}$	0·613	3·99
1	0·700	4·55

Clause 22a of B.S. 1856 states:

Minimum size of first run. On parts $\frac{3}{8}$ in. and over in thickness the minimum size of a single-run fillet weld or the first run in a multi-run fillet weld shall be as specified in Table C. On parts under $\frac{3}{8}$ in. thick the minimum size of fillet welds shall be not less than the thickness of the thinner part joined.

TABLE C.—MINIMUM SIZE OF SINGLE-RUN FILLET WELDS

Thickness of Thicker Part in Inches	**Minimum Size of Single Run Fillet Welds in Inches*
$\frac{3}{8}$ up to and including $\frac{3}{4}$	$\frac{3}{16}$
Over $\frac{3}{4}$ up to and including $1\frac{1}{4}$	$\frac{1}{4}$
Over $1\frac{1}{4}$ (see below)	$\frac{5}{16}$

* When the minimum size of the fillet weld specified in the table is greater than the thickness of the thinner part, the minimum size of the weld shall be equal to the thickness of the thinner part.

Where the thicker part is more than 2 in. thick, special precautions shall be taken to ensure weld soundness.

Symbols

The type and size of weld is indicated on drawings in one of the following ways:

(a) By the use of the symbols in B.S. 499, Section 7.

(b) By the use of the following letter symbols with the appropriate dimension:

Fillet Weld	F.W.
Single V Butt Weld	S.V.B.W.
Double V Butt Weld	D.V.B.W.
Single U Butt Weld	S.U.B.W.
Double U Butt Weld	D.U.B.W.
Single J Butt Weld	S.J.B.W.
Double J Butt Weld	D.J.B.W.
Single Bevel Butt Weld	S.B.B.W.
Double Bevel Butt Weld	D.B.B.W.

(c) By a complete description, e.g. $\frac{3}{8}$-in. fillet weld.

Deep-penetration welds must be separately and clearly defined.

Design Examples

Tables 1 to 6 on pages 774 to 779 give values of moments of inertia of various weld groups suitable for use in the design of brackets and other members. The tables have been reproduced from the *Handbook of Welded Structural Steelwork* by kind permission of the Institute of Welding. Examples of their use will be found in this section.

Example 1. Calculate the size of welds required for the joist cutting used as a bracket in Fig. 505.

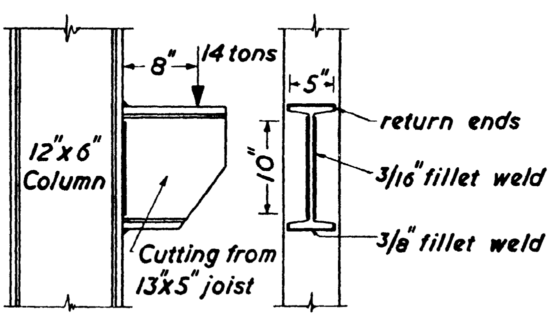


FIG. 505

Assuming that the web welds are half the size of the flange welds and that the latter are of unit throat thickness, the length of weld of unit throat thickness is

$$(2 \times 5) + \tfrac{1}{2}(2 \times 10) = 20 \text{ lineal in.}$$

$$I_{xx} \text{ for flange welds} = 2 \times 5 \times 6{\cdot}5^2 = 422{\cdot}50 \text{ in.}^4$$

$$I_{xx} \text{ for web welds} = \frac{2 \times \frac{1}{2} \times 10^3}{12} = 83{\cdot}33 \text{ in.}^4$$

$$\text{Total } I_{xx} = 505{\cdot}83 \text{ in.}^4.$$

Alternatively, Tables 1 and 2 could be used to find the value of I_{xx}.

Considering Fig. 505,

$m = 5$ in.; $n = 13$ in. (flange welds)

$n_1 = 10$ in. (web welds).

Total length of welds $= 2m + 2n_1/2 = 20$ lineal in.

Using Table 1, $I_{xx} = 423$ in.^{4}

Using Table 2, (and remembering that the web welds are half the size of the flange welds) $I_{xx} = \frac{167}{2} = 83$ in.^{4}

Total $I_{xx} = 506$ in.^{4}

Flange Welds

Direct load per lineal inch,

$$F_s = \frac{P}{2m + n_1} = \frac{14}{20} = 0{\cdot}70 \text{ ton.}$$

Load per lineal inch due to bending,

$$F_b = \frac{P \, . \, e}{Z_{xx}} = \frac{14 \times 8 \times 6{\cdot}5}{506} = 1{\cdot}44 \text{ tons.}$$

Resultant load per lineal inch,

$$F_R = \sqrt{(0{\cdot}70^2 + 1{\cdot}44^2)} = 1{\cdot}60 \text{ tons.}$$

Use $\frac{3}{8}$-in. fillet welds (1·71 tons per lin. in.).

Web Welds (half size of flange welds)

$$F_s = \frac{P}{2(2m + n_1)} = \frac{14}{2 \times 20} = 0{\cdot}35 \text{ ton}$$

$$F_b = \frac{P \, . \, e}{Z_{xx}} = \frac{14 \times 8 \times 5}{2 \times 505{\cdot}83} = 0{\cdot}55 \text{ ton}$$

$$F_R = \sqrt{(0{\cdot}35^2 + 0{\cdot}55^2)} = 0{\cdot}65 \text{ ton.}$$

Use $\frac{3}{16}$-in. fillet welds (0·85 ton per lin. in.).

Example 2. Calculate the size of welds required for the bracket shown in Fig. 506.

The force P on each plate = 12 tons.

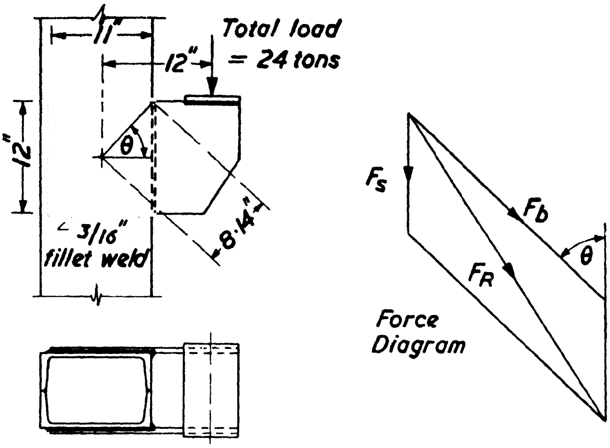


FIG. 506

Considering Table 3 and assuming a unit throat thickness, the loads are calculated as follows:

$$F_s=\frac{P}{2(m+n)}=\frac{12}{2(11+12)}=0{\cdot}26 \text{ ton}$$

$$F_b=\frac{P\,.\,e\,.\,r}{I_p}=\frac{12\times 12\times 8{\cdot}14}{2028}=0{\cdot}58 \text{ ton}$$

$$F_R=\sqrt{(F_s{}^2+F_b{}^2+2F_s\,.\,F_b\,.\cos\theta)}.$$

Now $\cos\theta=\dfrac{5{\cdot}5}{8{\cdot}14}=0{\cdot}676.$

Hence, $F_R=\sqrt{[0{\cdot}26^2+0{\cdot}58^2+(2\times 0{\cdot}26\times 0{\cdot}58\times 0{\cdot}676)]}$
$=0{\cdot}78$ ton.

Use $\frac{3}{16}$-in. fillet welds throughout.

Example 3. Calculate the size of welds required if the brackets in Example 2 are welded on three sides only as shown in Fig. 507.

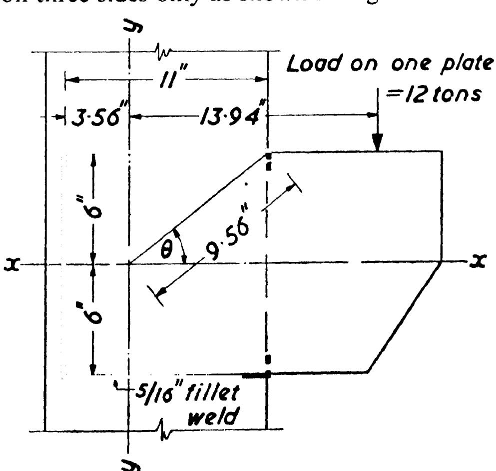


FIG. 507

First find the position of the centroid of the welds by taking moments about the left-hand edge of the plate. Assume welds of unit throat thickness,

$$\bar{x}=\frac{(2\times 11\times 5{\cdot}5)+(12\times 0)}{(2\times 11)+12}=3{\cdot}56 \text{ in.}$$

then
$$e=(12+5{\cdot}5-3{\cdot}56)=13{\cdot}94 \text{ in.}$$

$$I_{xx}=(2\times 11\times 6^2)+\frac{12^3}{12}$$
$$=936 \text{ in.}^4.$$

$$I_{yy}=\left[2\times\frac{11^3}{12}\right]+[2\times 11\times 1{\cdot}94^2]+[12\times 3{\cdot}56^2]$$
$$=457 \text{ in.}^4.$$

$$I_p=I_{xx}+I_{yy}$$
$$=1393 \text{ in.}^4.$$

$$\text{Moment}=12\times 13{\cdot}94$$
$$=167{\cdot}3 \text{ tons in.}$$

$$\text{Maximum } r=\sqrt{(6^2+7{\cdot}44^2)}$$
$$=9{\cdot}56 \text{ in.}$$

Now $\cos\theta=\frac{7{\cdot}44}{9{\cdot}56}=0{\cdot}778.$

$$F_s=\frac{12}{(2\times 11+12)}=0{\cdot}35 \text{ tons per sq. in.}$$

$$F_b=\frac{167{\cdot}3\times 9{\cdot}56}{1393}=1{\cdot}15 \text{ tons per sq. in.}$$

$$F_R=\sqrt{[0{\cdot}35^2+1{\cdot}15^2+(2\times 0{\cdot}35\times 1{\cdot}15\times 0{\cdot}778)]}$$
$$=1{\cdot}44 \text{ tons per lin. in.}$$

Although two points would be slightly overstressed, $\frac{5}{16}$-in. welds could be used (1·42 tons per lin. in.).

Example 4. Calculate the size of welds required for the seating cleat shown in Fig. 508.

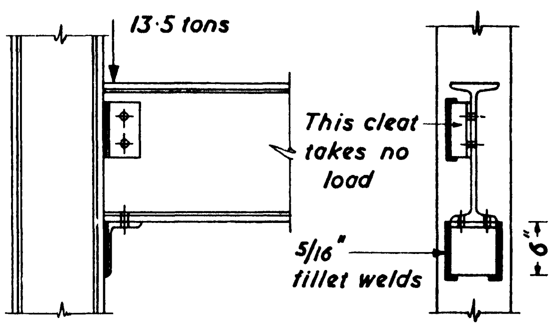


FIG. 508

If the cleat were stiffened and the eccentricity of the beam reaction increased, the welds would have to be calculated as in Example 1, but in this case there is direct load only.

The reaction = 13·5 tons.

Effective length of fillet welds connecting cleat to stanchion = say, 11·25 in.

$$\text{Load per lineal inch} = \frac{13\cdot5}{11\cdot25}$$
$$= 1\cdot20 \text{ tons.}$$

Use $\frac{5}{16}$-in. fillet welds (1·42 tons per lin. in.).

Example 5. Design a gusseted base for the 18-in. × 8-in. × 80-lb. column shown in Fig. 509, which is subjected to an axial load of 80 tons and a B.M. of 600 tons in.

The loading system can be replaced by a single equivalent load of 80 tons with an eccentricity about the vertical axis of the column of 600/80 = $7\frac{1}{2}$ in., as shown in the figure.

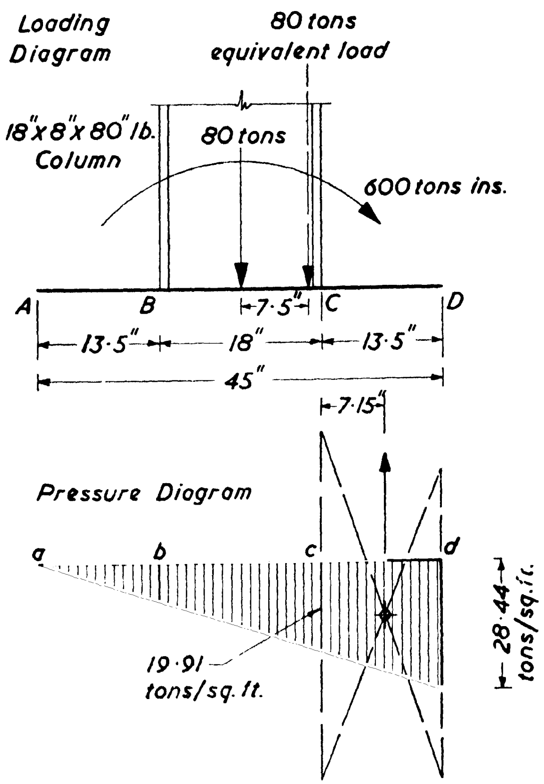


FIG. 509

If the base is made 45 in. long, then the equivalent load is placed 15 in., or one-third of the base length, from the end *D* and the pressure diagram will

be triangular as shown, varying from zero at a to a maximum at d, the value being $2P/A$, where P is the axial load and A the area of the base.

A convenient base width is 18 in. Checking the maximum pressure on the concrete foundation,

$$f=\frac{2P}{A}=\frac{2\times 80\times 144}{45\times 18}$$

$$=28{\cdot}44 \text{ tons per sq. ft.}$$

The base will be of the form shown in Fig. 510.

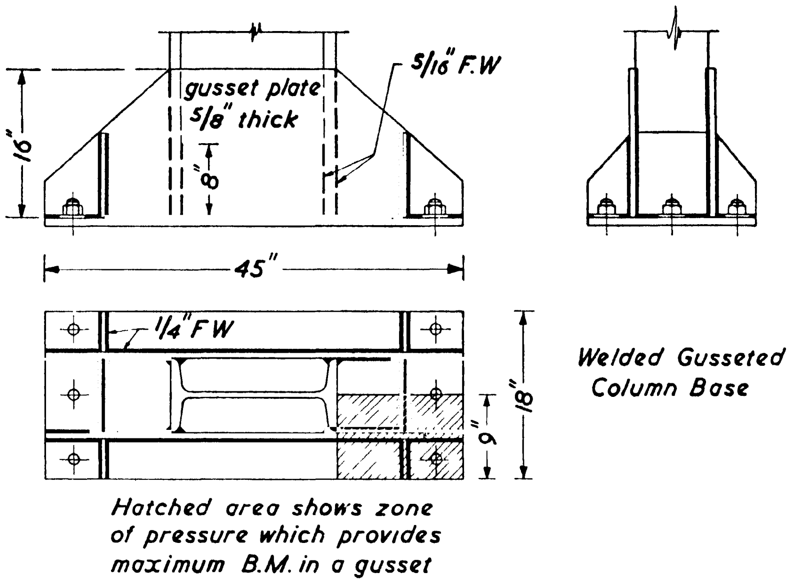


FIG. 510

Gusset Plates

Assume that the two gusset plates are 16 in. high and $\frac{5}{8}$ in. thick.

The critical points for B.M. in a gusset are at the faces of the column flanges. In this case the maximum B.M. is at C in Fig. 509. The load consists of the upward pressure on the base plate in the zone hatched in Fig. 510. The intensity of pressure is shown in the pressure diagram in Fig. 509 and the lever arm, which may be obtained by calculation or graphically as shown, is 7·15 in. Therefore,

$$M_{\max}=\text{load}\times\text{lever arm}$$

$$=\left(\frac{28{\cdot}44+19{\cdot}91}{2}\times\frac{13{\cdot}5\times 9}{144}\right)\times 7{\cdot}15$$

$$=20{\cdot}4\times 7{\cdot}15$$

$$=145{\cdot}8 \text{ tons in.}$$

At C the section modulus

$$Z=\frac{bd^2}{6}=\frac{\frac{5}{8}\times 16^2}{6}$$

$$=26{\cdot}67 \text{ ins.}^3$$

Hence, the maximum stress

$$f_b=\frac{M}{Z}=\frac{145{\cdot}8}{26{\cdot}67}$$

$$=5{\cdot}47 \text{ tons per sq. in.}$$

As the gusset plates are unstiffened along the compression edge, this stress is not unduly low and it would not be prudent to reduce the height of the gusset plates.

The maximum shear stress

$$f_s=\frac{3}{2}\times\frac{20{\cdot}4}{\frac{5}{8}\times 16}$$

$$=3{\cdot}06 \text{ tons per sq. in.}$$

This is satisfactory.

Welds

The most vital welds are those connecting the shaft to the gusset plates.

B.S. 449 : 1948, Clause 50c, Stanchion and column bases, (1) Gusseted bases, states:

> "In riveted and welded construction for stanchions with gusseted bases, the gusset plates, angle cleats, stiffeners, fastenings, etc., in combination with the bearing area of the shaft, all fabricated flush for bearing, shall be sufficient to take the loads, bending moments and reactions to the base plate without exceeding specified stresses, but not less than 60 per cent of the axial load shall be calculated as taken by the fastenings.
>
> In the case of bending moments sufficient rivets or welding shall be provided to transmit the full bending moment to the base.
>
> Where the end of the stanchion shaft and the gusset plates are not faced for complete bearing, the fastenings connecting them to the base plate shall be sufficient to transmit all the forces to which the base is subjected."

The forces are assumed to be imposed by the column through its flanges. Then the force at C, due to the direct load, assuming flush bearing

$$F_s=\frac{80}{2}\times 60\%=24 \text{ tons,}$$

while the force at C due to B.M.

$$F_b=\frac{\text{B.M.}}{\text{Column depth}}=\frac{600}{18}$$

$$=33{\cdot}33 \text{ tons.}$$

Therefore the total load in the flange at $C=24+33{\cdot}33=57{\cdot}33$ tons, and the force applied to one gusset plate $=57{\cdot}33/2=28{\cdot}67$ tons.

The maximum length of weld which can be laid on the inside face of the flange is about 8 in. Allowing 8 in. inside and 16 in. outside and deducting, say, 2 in. for end craters, the length available for welding = 22 in.

Hence, the force per lineal inch

$$=\frac{28\cdot67}{22}=1\cdot31 \text{ tons.}$$

Use $\frac{5}{16}$-in. fillet welds (1·42 tons per lin. in.).

When there is tension in the base plate the welds connecting the gusset plates to the base plates must be capable of carrying the same load as the holding-down bolts. In this case no tension exists and the whole of the remaining welds can be of nominal size sufficient to resist handling and erection stresses, $\frac{1}{4}$-in. fillet welds being suitable.

It should be noted that further details of bases are given in the sections on Stanchions, Engineering Workshop Design and Foundations.

Typical Roof-truss Connections

Fig. 511 illustrates some typical node connections for welded roof trusses.

The connection between the rafter and tie at the shoe consists of a single 'V' butt joint, while at the apex joints are butt or fillet welded. Everywhere else lap joints are used. Lap joints are usually welded all round to resist corrosion. Clause 45 (j) in B.S. 449 states that the minimum amount of lap shall be four times the thickness of the thinner part, while Clause 45 (e) states that the minimum size of single-run fillet weld shall be not less than $\frac{3}{16}$ in. The requirements of B.S. 1856 are the same.

Having decided the amount of overlap to be used it is simple to calculate the length of weld available all round the joint. Then using the allowable stress of 6·5 tons per sq. in. for all fillet welds and factor of 0·7 for the throat thickness, the size of weld is calculated from the following formula:

$$\text{Size}=\frac{\text{Force in member}}{\text{Length of weld}\times 6\cdot5\times 0\cdot7}.$$

Considering the central member shown in Fig. 511 (c) and assuming that the force in the member is 3360 lb. and the length of weld all round is 10 in., then the size of weld required

$$=\frac{3360}{10\times 6\cdot5\times 0\cdot7\times 2240}=0\cdot033 \text{ in.}$$

This calculated size is much less than the $\frac{3}{16}$ in. which is the minimum allowable size.

If 12 in. of $\frac{1}{4}$-in. butt weld are used for the connection in Fig. 511 (a), then the maximum permissible force along the line of weld is

$$12\times 0\cdot25\times 6\cdot5=19\cdot5 \text{ tons.}$$

This is a much greater force than that which might be found in the tie or rafter of a truss.

From the foregoing it will be understood why the welds for such roof trusses are not usually calculated. The shoe members are butt jointed and special arrangements made for site joints, but otherwise $\frac{3}{16}$-in. lap joints are frequently sufficient.

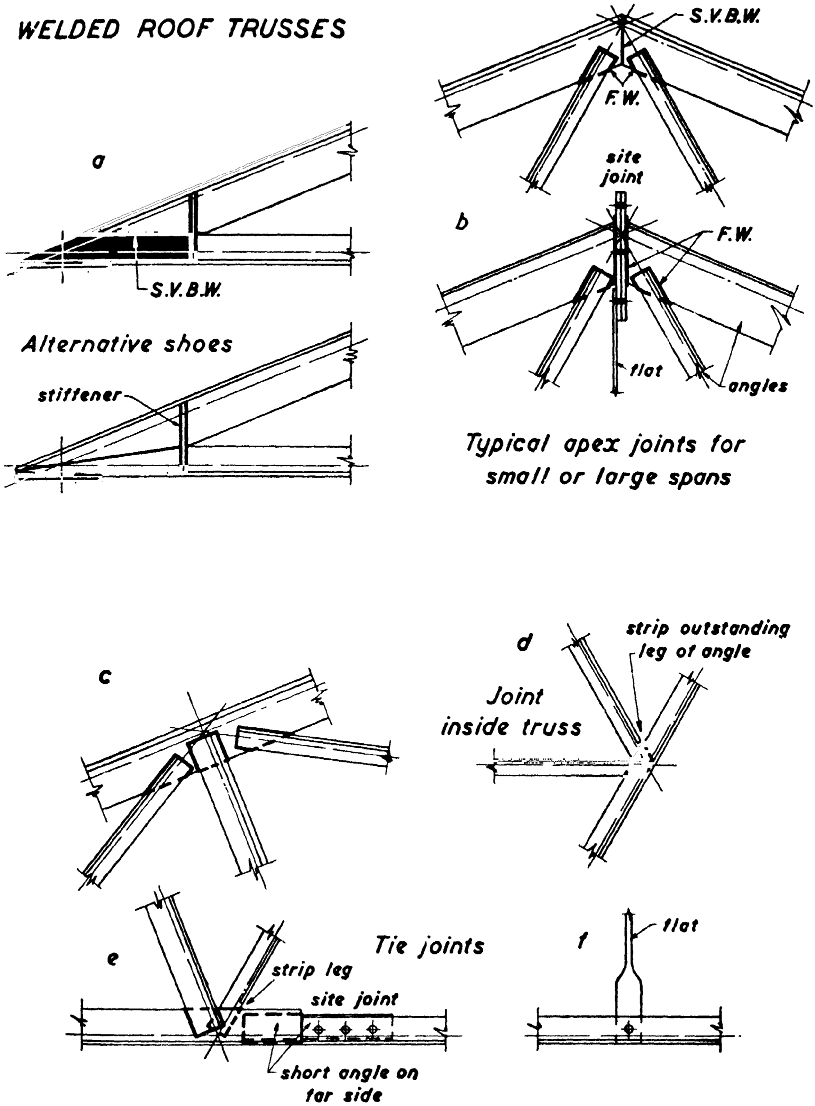
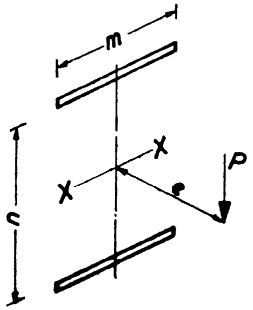


FIG. 511

TABLE 1.—WELDS NOT IN THE PLANE OF THE FORCE



$$F_s=\frac{P}{2m}$$

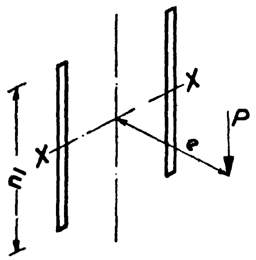
$$F_b=\frac{P\,.\,e\,.\,n}{2I_{XX}}=\frac{P\,.\,e}{Z_{XX}}$$

$$F_R=\sqrt{(F_s{}^2+F_b{}^2)}$$

VALUES OF I_{XX} FOR 1 IN. THROAT THICKNESS

	Values of m in Inches											
Values of n in Inches	1	2	3	4	5	6	7	8	9	10	11	12
3	4·5	9	13·5	18	22·5	27	31·5	36	40·5	45	49·5	54
4	8	16	24	32	40	48	56	64	72	80	88	96
5	12·5	25	37·5	50	62·5	75	87·5	100	112·5	125	137·5	150
6	18	36	54	72	90	108	126	144	162	180	198	216
7	24·5	49	73·5	98	122·5	147	171·5	196	220·5	245	269·5	294
8	32	64	96	128	160	192	224	256	288	320	352	384
9	40·5	81	121·5	162	202·5	243	283·5	324	364·5	405	445·5	486
10	50	100	150	200	250	300	350	400	450	500	550	600
11	60·5	121	181·5	242.	302·5	363	423·5	484	544·5	605	665·5	726
12	72	144	216	288	360	432	504	576	648	720	792	864
13	84·5	169	254	338	423	507	592	676	760	845	930	1014
14	98	196	294	392	490	588	686	784	882	980	1078	1176
15	112·5	225	338	450	563	675	788	900	1013	1125	1238	1350
16	128	256	384	512	640	768	896	1024	1152	1280	1408	1536
17	144·5	289	434	578	723	867	1012	1156	1301	1445	1589	1734
18	162	324	486	648	810	972	1134	1296	1458	1620	1782	1944
19	180·5	361	541	722	902	1083	1263	1444	1624	1805	1985	2166
20	200	400	600	800	1000	1200	1400	1600	1800	2000	2200	2400
21	220·5	441	661	882	1102	1323	1543	1764	1984	2205	2425	2646
22	242	484	726	968	1210	1452	1694	1936	2178	2420	2662	2904
23	264·5	529	793	1058	1322	1587	1851	2116	2380	2645	2909	3174
24	288	576	864	1152	1440	1728	2016	2304	2592	2880	3168	3456

TABLE 2.—WELDS NOT IN THE PLANE OF THE FORCE



$$F_s = \frac{P}{2n_1}$$

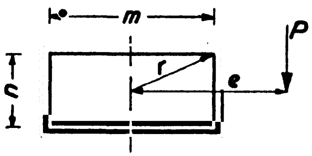
$$F_b = \frac{P \cdot e \cdot n_1}{2I_{XX}} = \frac{P \cdot e}{Z_{XX}}$$

$$F_R = \sqrt{(F_s^2 + F_b^2)}$$

VALUES OF I_{XX}
FOR 1 IN. THROAT THICKNESS

n_1 (in.)	I_{XX}
3	4·5
4	10·7
5	21
6	36
7	57
8	85
9	122
10	167
11	222
12	288
13	366
14	456
15	562
16	682
17	819
18	972
19	1144
20	1333
21	1543
22	1775
23	2028
24	2304

TABLE 3.—WELDS IN THE PLANE OF THE FORCE



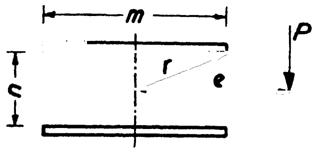
$$F_s = \frac{P}{2(m+n)}$$

$$F_b = \frac{P \cdot e \cdot r}{I_p}$$

Values of I_p for 1 in. Throat Thickness

Values of n in Inches	Values of m in Inches: 2	3	4	5	6	7	8	9	10	11	12
2	10·7	21	36	57	85	121	167	222	288	366	457
3	21	36	57	85	121	167	222	288	366	457	562
4	36	57	85	121	167	222	288	366	457	562	683
5	57	85	121	167	222	288	366	457	562	683	819
6	85	121	167	222	288	366	457	562	683	819	972
7	121	167	222	288	366	457	562	683	819	972	1143
8	167	222	288	366	457	562	683	819	972	1143	1333
9	222	288	366	457	562	683	819	972	1143	1333	1543
10	288	366	457	562	683	819	972	1143	1333	1543	1775
11	366	457	562	683	819	972	1143	1333	1543	1775	2028
12	457	562	683	819	972	1143	1333	1543	1775	2028	2304
13	562	683	819	972	1143	1333	1543	1775	2028	2304	2604
14	683	819	972	1143	1333	1543	1775	2028	2304	2604	2929
15	819	972	1143	1333	1543	1775	2028	2304	2604	2929	3280
16	972	1143	1333	1543	1775	2028	2304	2604	2929	3280	3659
17	1143	1333	1543	1775	2028	2304	2604	2929	3280	3659	4065
18	1333	1543	1775	2028	2304	2604	2929	3280	3659	4065	4500
19	1543	1775	2028	2304	2604	2929	3280	3659	4065	4500	4965
20	1775	2028	2304	2604	2929	3280	3659	4065	4500	4965	5461
21	2028	2304	2604	2929	3280	3659	4065	4500	4965	5461	5989
22	2304	2604	2929	3280	3659	4065	4500	4965	5461	5989	6551
23	2604	2929	3280	3659	4065	4500	4965	5461	5989	6551	7146
24	2929	3280	3659	4065	4500	4965	5461	5989	6551	7146	7776

TABLE 4.—WELDS IN THE PLANE OF THE FORCE



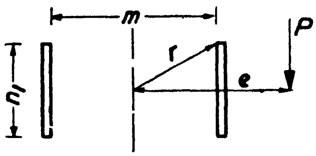
$$F_s = \frac{P}{2m}$$

$$F_b = \frac{P \, . \, e \, . \, r}{I_p}$$

VALUES OF I_p FOR 1 IN. THROAT THICKNESS

Values of n in Inches	Values of m in Inches: 2	3	4	5	6	7	8	9	10	11	12
2	5·33	10·5	18·7	31	48	71	101	140	187	244	312
3	10·3	18	28·7	43·5	62	89	121	163	212	272	342
4	17·3	28·5	42·7	61	84	113	149	194	247	310	384
5	26·3	42	60·7	83·5	111	145	185	235	292	360	438
6	37·3	58·5	82·7	111	144	183	229	284	347	420	504
7	50·3	78	109	144	183	229	281	343	412	492	582
8	65·3	100·5	139	181	228	281	341	410	487	574	672
9	82·3	126	173	224	279	341	409	487	572	668	774
10	101·3	154·5	211	271	336	407	485	572	667	772	888
11	122·3	186	253	324	399	481	569	667	772	888	1014
12	145·3	220·5	299	381	468	561	661	770	887	1014	1152
13	170·3	258	349	444	543	649	761	883	1012	1152	1302
14	197·3	299	403	511	624	743	869	1004	1147	1300	1464
15	226·3	342	461	584	711	845	985	1135	1292	1460	1638
16	257·3	389	523	661	804	953	1109	1274	1447	1630	1824
17	290·3	438	589	744	903	1069	1241	1423	1612	1812	2022
18	325·3	491	659	831	1008	1191	1381	1580	1787	2004	2232
19	362·3	546	733	924	1119	1321	1529	1747	1972	2208	2454
20	401·3	605	811	1021	1236	1457	1685	1922	2167	2422	2688
21	442·3	666	893	1124	1359	1601	1849	2107	2372	2648	2934
22	485·3	731	979	1231	1488	1751	2021	2300	2587	2884	3192
23	530·3	798	1069	1344	1623	1909	2201	2503	2812	3132	3462
24	577·3	869	1163	1461	1764	2073	2389	2714	3047	3390	3744

TABLE 5.—WELDS IN THE PLANE OF THE FORCE



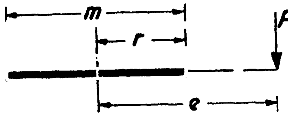
$$F_s = \frac{P}{2n_1}$$

$$F_b = \frac{P \, . \, e \, . \, r}{I_p}$$

VALUES OF I_p FOR 1 IN. THROAT THICKNESS

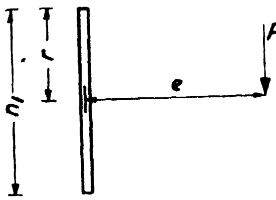
Values of n in Inches	Values of m in Inches										
	2	3	4	5	6	7	8	9	10	11	12
2	5·33	10·3	17·3	26·3	37·3	50·3	65·3	82·3	101·3	122·3	145·3
3	10·5	18	28·5	42	58·5	78	100·5	126	154·5	186	220·5
4	18·7	28·7	42·7	60·7	82·7	109	139	173	211	253	299
5	31	43·5	61	83·5	111	144	181	224	271	324	381
6	48	62	84	111	144	183	228	279	336	399	468
7	71	89	113	145	183	229	281	341	407	481	561
8	101	121	149	185	229	281	341	409	485	569	661
9	140	163	194	235	284	343	410	487	572	667	770
10	187	212	247	292	347	412	487	572	667	772	887
11	244	272	310	360	420	492	574	668	772	888	1014
12	312	342	384	438	504	582	672	774	888	1014	1152
13	392	425	470	528	600	684	782	892	1016	1152	1302
14	485	520	569	632	709	800	905	1024	1157	1304	1465
15	592	629	682	749	832	929	1042	1169	1312	1469	1642
16	715	755	811	883	971	1075	1195	1331	1483	1651	1835
17	853	896	955	1032	1125	1236	1363	1508	1669	1848	2043
18	1008	1053	1116	1197	1296	1413	1548	1701	1872	2061	2268
19	1181	1229	1295	1381	1485	1608	1751	1912	2093	2292	2511
20	1373	1423	1493	1583	1693	1823	1973	2143	2333	2543	2773
21	1585	1638	1711	1806	1921	2057	2215	2393	2593	2813	3055
22	1819	1874	1951	2050	2171	2314	2479	2666	2875	3106	3359
23	2074	2132	2212	2316	2442	2592	2764	2959	3178	3419	3684
24	2352	2412	2496	2604	2736	2892	3072	3276	3504	3756	4032

TABLE 6.—WELDS IN THE PLANE OF THE FORCE



$$F_s = \frac{P}{m}$$

$$F_b = \frac{P \cdot e \cdot r}{I_p}$$



$$F_s = \frac{P}{n_1}$$

VALUES OF I_p
FOR 1 IN. THROAT THICKNESS

m or n_1 (in.)	I_p
3	2·25
4	5·33
5	10·4
6	18
7	28·5
8	42·5
9	61
10	83·5
11	111
12	144
13	183
14	228
15	281
16	341
17	409
18	486
19	572
20	667
21	772
22	887
23	1014
24	1152

39. EXPANSION JOINTS

WHEN structures are very long or when they are constructed in areas subject to large ranges of temperature, it may be necessary to make allowance for expansion joints. Non-monolithic structures, such as factory-type buildings consisting of columns, roof trusses and sheeted roof and side coverings, are capable of absorbing small movements without special provision being made, but when there are concrete roofs and floors and brick walls, expansion joints are usually necessary.

In long buildings twin columns and cross-beams can be employed where breaks are required in the building. The main problem is to make the intervening joint waterproof, but although features can be made of joints, it may be desirable to conceal the joint altogether both outside and inside the building.

Suitable joints for roofs, floors and walls are shown in Fig. 512. The most usual medium for waterproofing a roof or wall joint is a copper strip, which is backed by fibreboard packing. Bronze or stainless steel can be used for the sliding joint in floors.

The Brabazon Hangar at Bristol is a large steel structure in which special provision had to be made for differential movement of the three main bays. In plan the building covers a 'T'-shaped area, as shown in Fig. 513, the stem of which consists of a centre bay having a length of 400 ft. and an uninterrupted span of 331 ft. The side projections constitute east and west bays of the same clear span, but having a length of 250 ft., although provision has been made for extension.

The main components of the steel skeleton of the building consist of cambered portal frames having a tie at haunch-level as shown in elevation in Fig. 514.

The frames are made up of box-section lattice girders which rest upon hinged bases at foundation level. The width of the girders in the centre bay is 5 ft. 6 in. and in the outside bays 7 ft. 6 in. By interlocking the internal pairs of legs as shown in Fig. 516, it has been possible to allow movement between the bays and also to utilise the floor space to the best possible extent.

It was necessary to make special provision for expansion along the valleys *AB* and *CD* and particularly at the re-entrant angles *A* and *C*. The maximum calculated lateral wind sway of the outer bays and of the rear of the centre bay is ± 5 in., but at the same time the front of the centre bay remains substantially static as it is subjected only to uniform suction on the roof.

Some of the arrangements made for expansion in the valleys are shown in Figs. 515 and 517. In addition, the devices shown in Fig. 518 are required to allow for expansion in the continuous glazing strip which is a feature of the rear of the building.

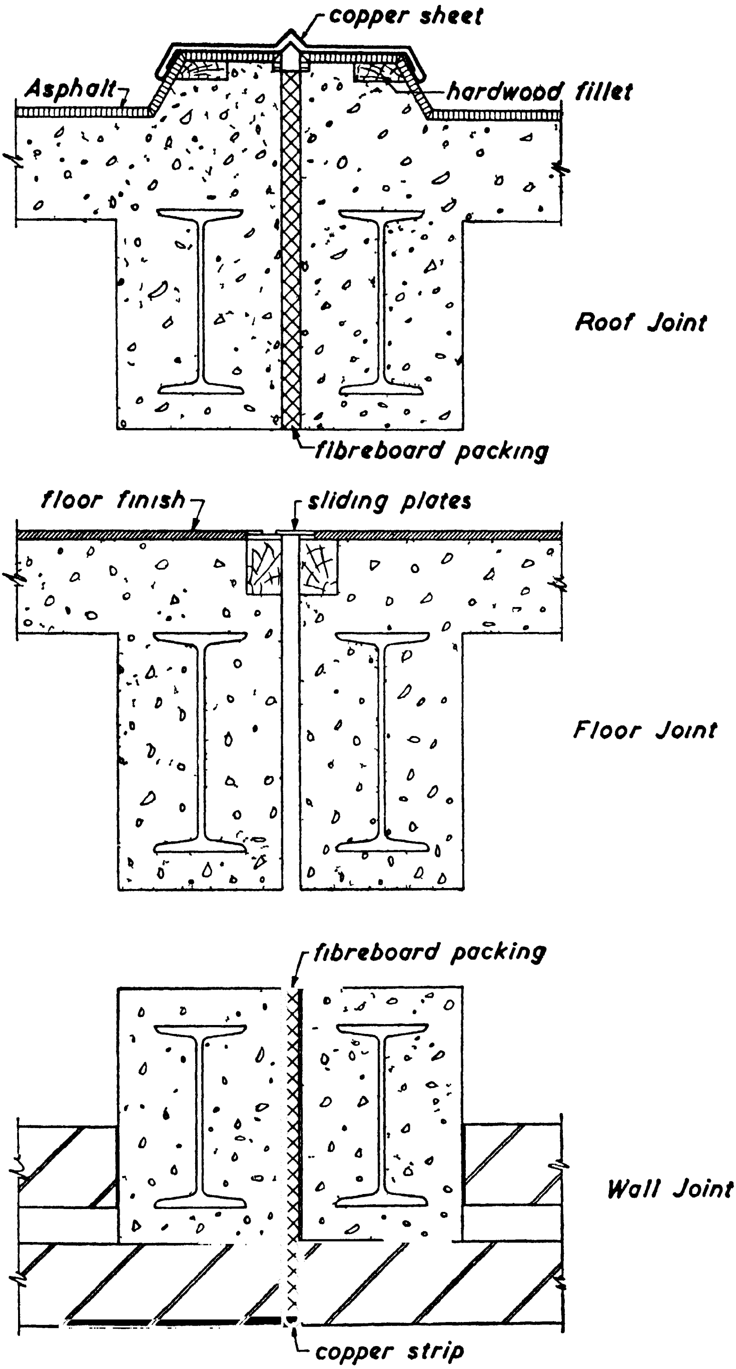

copper sheet
Asphalt
hardwood fillet
Roof Joint
fibreboard packing
floor finish
sliding plates
Floor Joint
fibreboard packing
Wall Joint
copper strip

FIG. 512

The Consulting Engineers for the Brabazon Hangar were Messrs. Brian Colquhoun and Partners, to whom grateful acknowledgement is made for permission to reproduce this material.

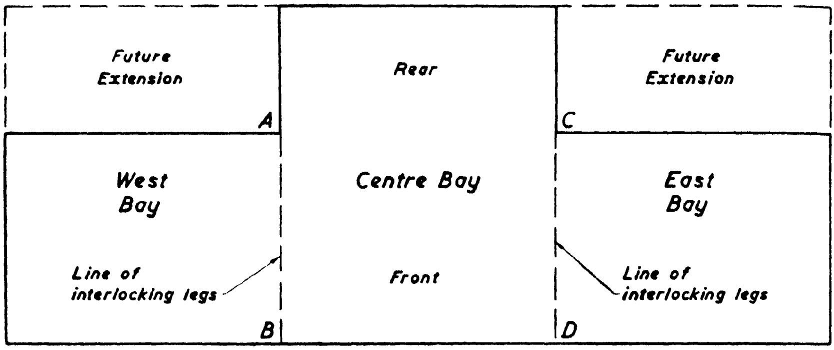


FIG. 513

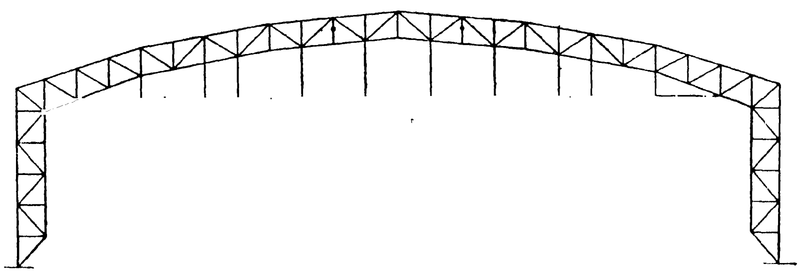


FIG. 514

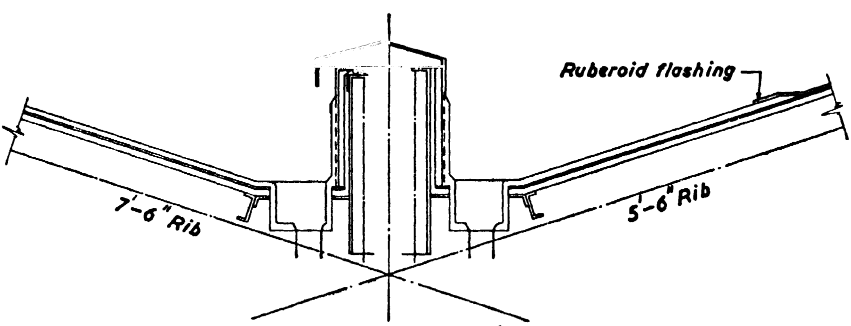


FIG. 515

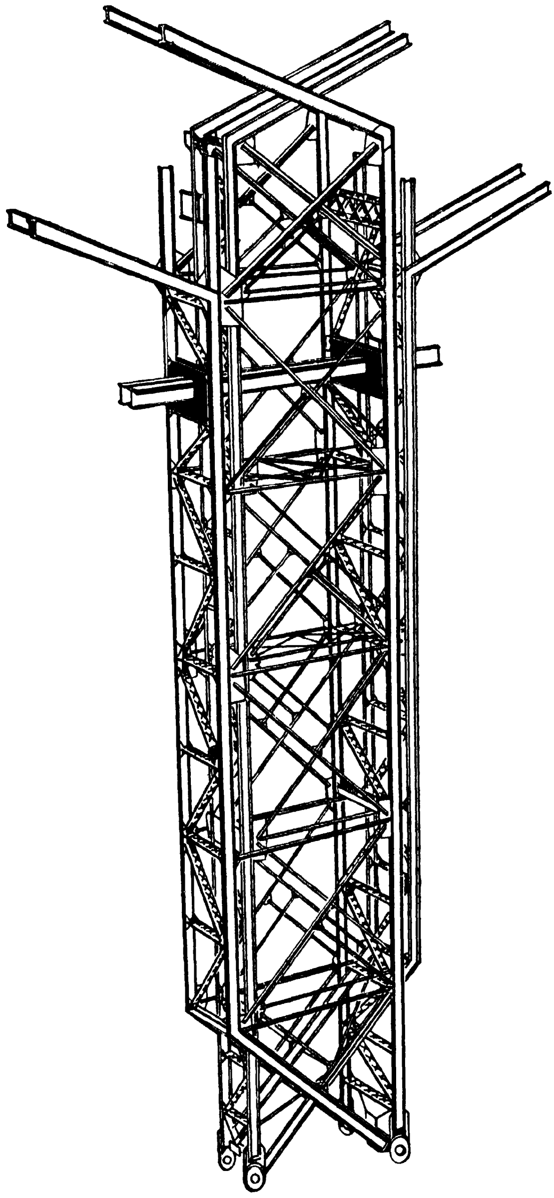
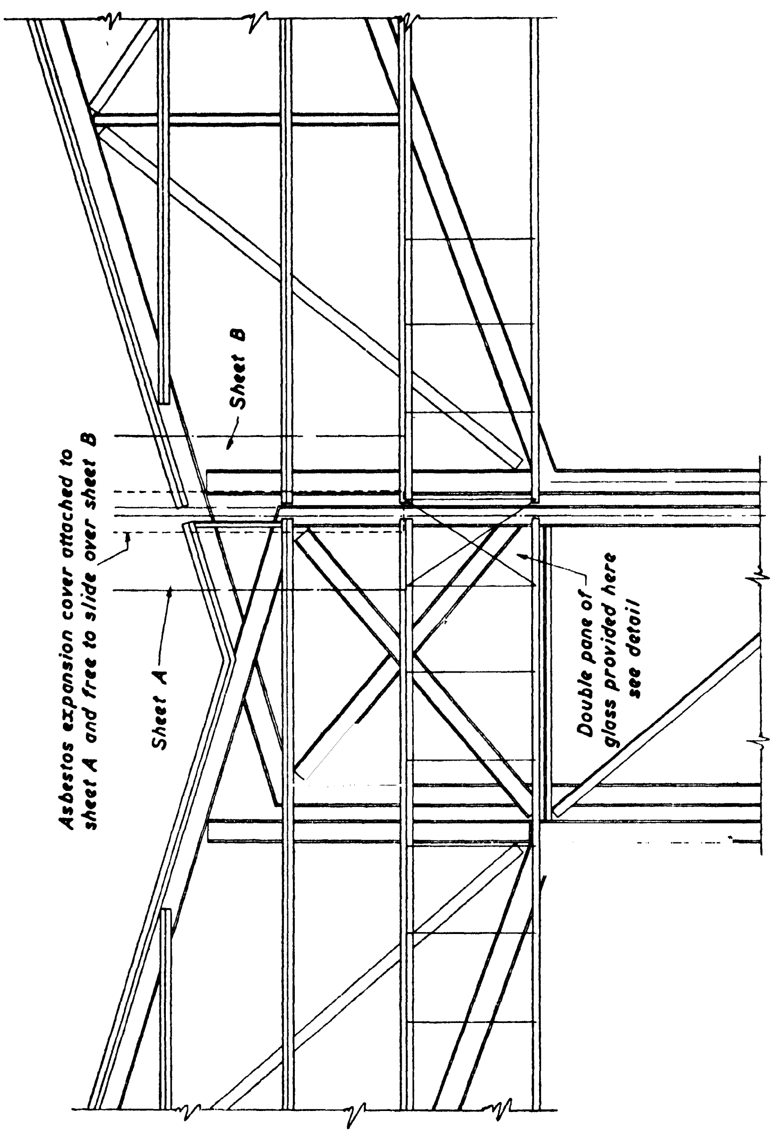
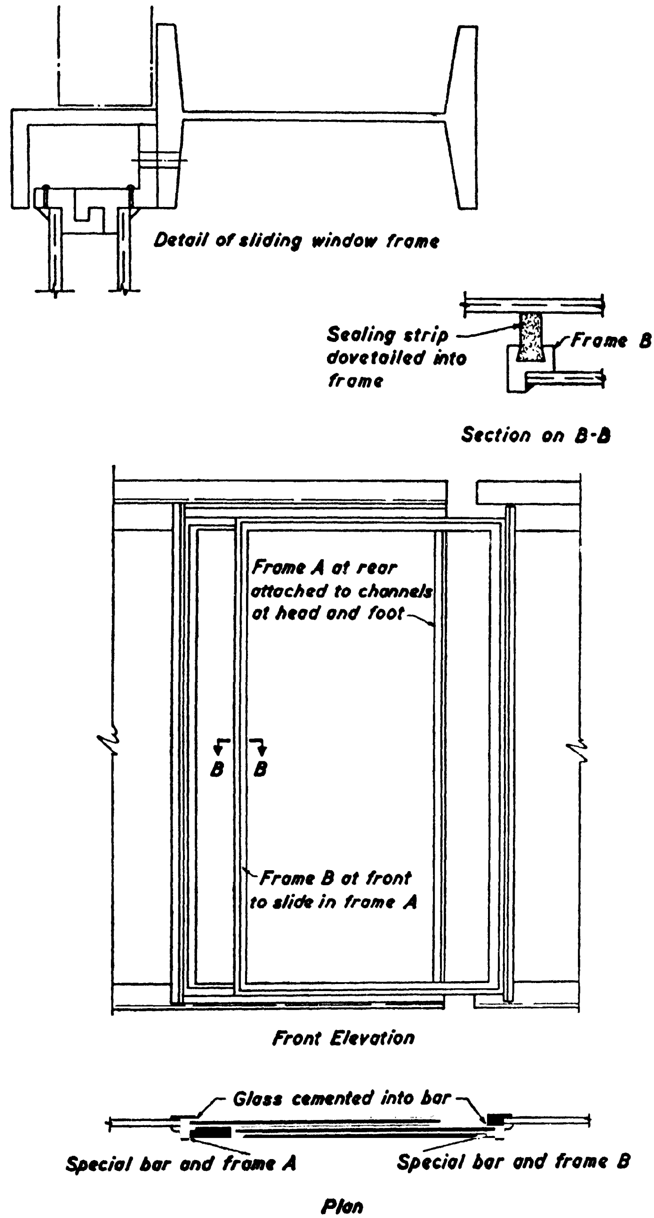


FIG. 516



DETAIL AT VALLEY

FIG. 517



DETAIL OF DOUBLE PANE EXPANSION JOINT

FIG. 518

40. ROOF AND SIDE COVERINGS, GALVANISED CORRUGATED STEEL SHEETING

General

Used as a covering for the roofs and sides of buildings, corrugated steel sheets are strong, light in weight, easy to erect and adapt, and do not deteriorate in any way when properly protected from corrosion. The phrase 'corrugated iron sheets' is frequently heard but is incorrect, for the material is mild steel produced in either an open-hearth or Bessemer furnace.

Corrugated sheets can be supplied curved over the whole or part of the length and to any required radius not less than about 15 in. The curving is done after corrugating.

British Standard

The current Standard for corrugated galvanised steel sheets is B.S. 798, the more important stipulations of which are summarised below:

Material.—Good quality mild steel

Galvanising.—A 'hot process' coating of virgin spelter of good commercial brand containing not less than 97 per cent pure zinc. The weight of the coating shall be not less than 1¾ oz. per sq. ft. (This means ⅞ oz. per sq. ft. per side measured flat before corrugating.)

Tolerance on weight of black steel sheet: plus or minus 7½ per cent all gauges.

No standards are laid down for the sizes and depths of corrugations.

Gauge

The gauge referred to in connection with corrugated steel sheets is the Birmingham Gauge for Hoops and Sheets which became operative in 1914. Makers' lists show a range of sheets from as thick as 14 BG (0·0785 in.) to as thin as 32 BG (0·0098 in.), but it does not follow that such extremes are generally available or desirable as claddings to buildings, for which purpose the limits should be regarded as 16 BG (0·0625 in.) and 24 BG (0·02476 in.), the latter being commonly used where the need for economy is greater than strength. The savings which arise from the use of the lighter gauges are not proportional to the weight, as account must be taken of the cost per ton, which is greater the lighter the gauge.

TABLE 1

Gauge BG	*Thickness in.*	*Weight per sq. ft. of Flat Sheet lb.*
16	0·0625	2·55
18	0·0495	2·02
20	0·0392	1·60
22	0·03125	1·28
24	0·02476	1·01

Length

Where possible the spacing of supports should be arranged for sheets of the same length throughout, but this need not be taken to override other considerations. Sheet lengths from 4 ft. to 10 ft., rising by 6 in., are in regular production. Lengths intermediate to the foregoing could be supplied, but if the quantity is small it would mean cutting from standard lengths, incurring extra cost in labour and waste. To save end laps, longer lengths than 10 ft. can be obtained from some makers but the economy of this may be offset by an extra for length and an extra on the erection cost, due to handling the longer and heavier sheets.

Where the ends of sheets are not square such as in the top strake of a gable, and the bevel can be predetermined, the sheets may be ordered to diagram and the cutting done more cheaply at the makers' works than on the site.

Width of Sheet and Number of Corrugations

Some makers list a variety of sheet widths and corrugation sizes, not all of which are readily obtainable and should only be considered for special cases involving a tonnage sufficient to justify an interruption of the regular lines which are the first two in Table 2. The second two are in production but less frequently.

TABLE 2

Number of corrugations	8	10	5	6
Width of one complete corrugation	3 in.	3 in.	5 in.	5 in.
Approximate depth of corrugation	¾ in.	¾ in.	1¼ in.	1¼ in.
Overall width of sheet after corrugating	2 ft. 2 in.	2 ft. 8 in.	2 ft. 4 in.	2 ft. 9 in.
Nominal width of flat sheet before corrugating	30 in.	36 in.	32 in.	38 in.

The depth of corrugation given in the table is the approximate maximum and tends to be slightly less, if anything, according to the gauge and quality of the material.

It will be noted that the corrugation profile is the same for the first and second types, also for the third and fourth. The variation is in the overall width only, and as, in use, the number of fastenings is the same for sheets of similar type and lap, it follows that the wider sheets are more economical and should always be used unless very good reasons dictate otherwise.

It should be noted also that in all types the corrugation arrangement is symmetrical about the centre of the sheet, i.e. both edges turn down from the crown of the outermost corrugation. With a side lap of one or two corrugations all sheets are laid with the edges turning down so that the joint is properly sealed by the exposed edge.

Weight

The approximate weight in pounds of one corrugated sheet is given in Table 3.

TABLE 3

Type	*Gauge*	*Length of Sheet*												
		ft. in. 5 0	ft. in. 5 6	ft. in. 6 0	ft. in. 6 6	ft. in. 7 0	ft. in. 7 6	ft. in. 8 0	ft. in. 8 6	ft. in. 9 0	ft. in. 9 6	ft. in. 10 0	ft. in. 11 0	ft. in. 12 0
10/3 in.	16	38	42	46	49	53	57	61	65	68	72	76	84	91
	18	31	34	37	40	43	46	49	52	55	58	61	67	73
	20	24	26	29	31	33	35	38	40	42	45	48	52	56
	22	20	22	24	26	27	29	31	33	35	37	39	43	47
	24	16	18	20	21	23	24	26	27	29	31	32	36	39
6/5 in.	16	40	44	48	52	56	60	64	68	72	76	80	88	96
	18	33	36	39	42	45	49	52	55	58	61	64	71	77
	20	25	27	30	33	35	37	40	42	44	47	50	55	59
	22	21	23	25	27	29	31	33	35	37	39	41	45	49
	24	17	19	21	22	24	25	27	29	31	33	34	38	41

The weights for design purposes may be taken from the table below:

TABLE 4

DEAD WEIGHT OF GALVANISED CORRUGATED STEEL SHEETING, INCLUDING SIDE AND END LAPS

Gauge	*10/3-in. Sheets with 2 Corrugations Side Lap*	*6/5-in. Sheets with 1½ Corrugations Side Lap*
	lb. per sq. ft.	lb. per sq. ft.
16	3·6	3·8
18	2·9	3·1
20	2·3	2·4
22	1·9	2·0
24	1·5	1·6

Strength and Stiffness

The resistance of the sheet to bending varies with the gauge and depth of corrugation. The strength of the sheet in terms of the extreme fibre stress is a function of the Section Modulus, whilst the deflection is a function of the Moment of Inertia. With the profiles given in Table 2, it may be noted that in each case the approximate maximum depth of corrugation is one-fourth of the width and by calculation the approximate values of the properties are as shown in Table 5.

TABLE 5

Number and Width of Corrugations	*Gauge*	*Moment of Inertia I, per foot width, inch⁴ units*	*Section Modulus Z, per foot width, inch³ units*
10/3 in.	16	0·064	0·170
	18	0·051	0·135
	20	0·040	0·107
	22	0·032	0·085
	24	0·025	0·068
6/5 in.	16	0·177	0·284
	18	0·140	0·225
	20	0·111	0·178
	22	0·089	0·142
	24	0·070	0·113

Superimposed Loads

Sheeting on the sides of a building is normally subject only to wind loads, the greatest intensity of which should be estimated for the locality. For guidance in this respect, see B.S. 449, or the British Standard Code of Practice, C.P.3, Chapter V (1952), which give basic wind pressures for various degrees of exposure and effective heights of building. Under certain conditions referred to in these documents, the side wall covering may be called upon to resist the full intensity and in either direction.

In the case of roof slopes, regard must be paid to loads due to wind, snow, maintenance men and equipment. Both documents give detailed recommendations relating to the slope of the roof and make special reference to sheeted construction.

Spacing of Supports

The spacing of side rails (sometimes referred to as girts) and roof purlins is governed by:

(i) The total sheeted height of the side or length of rafter.
(ii) The uniformity of sheet lengths, if this is important, or the need to avoid local bending in the truss rafter.
(iii) The superimposed loads which it is assumed will have to be carried.
(iv) The maximum permissible extreme fibre stress in the sheet, or the limit of deflection necessary to ensure a leak-proof covering.

For optimum economy all these factors should be considered in deciding the type and gauge of sheet to use, but for normal cases the thinner gauges are generally adopted.

The maximum permissible stress for corrugated steel sheeting does not figure in any by-law or code. Limiting values have been quoted of 6 tons per sq. in. for 24 gauge and 7 tons per sq. in. for 16 gauge, with intermediate values interpolated. These values form the basis of Table 6 which follows.

Similarly, no limit of deflection is laid down, but it is considered reasonable

Table 6

MAXIMUM PERMISSIBLE SPANS OF CORRUGATED STEEL SHEETING IN FEET FOR LIMITING STRESSES AND DEFLECTIONS

No. and Width of Corrugations	*Gauge*	*Bending Stress (tons per sq. in.)*	*Deflection Span*	*Superimposed Load in lb. per sq. ft.*				
				10	15	20	25	30
				ft. in.	ft. in.	ft. in.	ft. in.	ft. in.
10/3 in.	16	7·0	1/100	9 0	8 3	7 6	7 0	6 9
			1/150	8 0	7 0	6 6	6 3	6 0
			1/200	7 3	6 6	6 0	5 6	5 3
10/3 in.	18	6·75	1/100	8 6	7 6	7 0	6 6	6 3
			1/150	7 6	6 9	6 0	5 9	5 6
			1/200	6 9	6 0	5 6	5 3	5 0
10/3 in.	20	6·5	1/100	8 0	7 3	6 6	6 0	5 6
			1/150	7 0	6 3	5 9	5 3	5 0
			1/200	6 3	5 9	5 3	4 9	4 6
10/3 in.	22	6·25	1/100	7 6	6 9	6 0	5 3	5 0
			1/150	6 6	5 9	5 3	5 0	4 9
			1/200	6 0	5 3	4 9	4 6	4 3
10/3 in.	24	6·0	1/100	7 0	6 0	5 3	4 9	4 3
			1/150	6 0	5 6	5 0	4 6	4 3
			1/200	5 6	5 0	4 6	4 3	4 0
6/5 in.	16	7·0	1/100	12 9	11 6	10 6	10 0	9 3
			1/150	11 0	10 0	9 3	8 6	8 3
			1/200	10 0	9 0	8 3	7 9	7 6
6/5 in.	18	6·75	1/100	12 0	10 9	9 9	9 0	8 3
			1/150	10 6	9 3	8 6	8 0	7 6
			1/200	9 6	8 6	7 9	7 3	7 0
6/5 in.	20	6·5	1/100	11 3	10 0	8 9	8 0	7 3
			1/150	9 9	8 9	8 0	7 6	7 0
			1/200	9 0	8 0	7 3	6 9	6 6
6/5 in.	22	6·25	1/100	10 6	8 9	7 9	7 0	6 3
			1/150	9 3	8 3	7 6	7 0	6 3
			1/200	8 3	7 6	6 9	6 3	6 0
6/5 in.	24	6·0	1/100	9 3	7 9	6 9	6 0	5 6
			1/150	8 6	7 6	6 9	6 0	5 6
			1/200	7 9	6 9	6 3	6 0	5 6

NOTES:

(1) Spans enclosed by heavy lines are governed by stress values. The remainder are governed by deflection values.

(2) Allowance has been made for the dead load of the sheeting.

to recommend that for side-wall sheets the limit should be 1/100th part of the span and for roof sheets 1/150th part provided the side laps are close bolted. A limit of 1/200th part of the span would be conservative for any roof.

These limits of deflection for roof sheeting apply particularly to wind and snow loads, which are more or less uniformly distributed. Corrugated steel sheeting has considerable transverse strength, which enables it to withstand a concentrated load by transferring part of it to adjacent corrugations. On long spans, as the deflection of the loaded corrugation increases with increments of load, the sideways spread is also increased. With the 200 lb. concentrated load given in B.S. 449 and in the Code of Practice, the bending stress in steel sheeting properly designed for wind and snow loads and with side laps well secured, will not be excessive. Roofs subject to heavy or frequent traffic should be designed for these special conditions.

Side Laps

The different arrangements of side lap and the net cover given by each are shown in Fig. 519. In the case of the 8/3-in. and 10/3-in. sheets a side lap of what is known as '1 corrugation' gives an actual lap of 2 in. and is used for side-wall sheeting not subject to exceptional exposure. It is not sufficient for roof sheeting. A so-called '2 corrugation' side lap of 5 in. is normally recommended but a compromise is effected if alternate sheets are turned over and laid underneath both adjoining sheets to give an actual lap of 3½ in., referred to as '1½ corrugation' (Fig. 520). There is no difference in the two surfaces of a galvanised sheet.

For the 5/5-in. and 6/5-in. sheets with the greater depth of water channel, a lap of one corrugation should be sufficient for most roofs provided the seam bolts are not spaced too far apart, but a 1½-corrugation side lap is usually adopted.

With a side lap of 1 or 1½ corrugations, only one crown of each sheet can be seam-bolted whereas with a 2-corrugation lap two corrugations can be connected.

End Laps

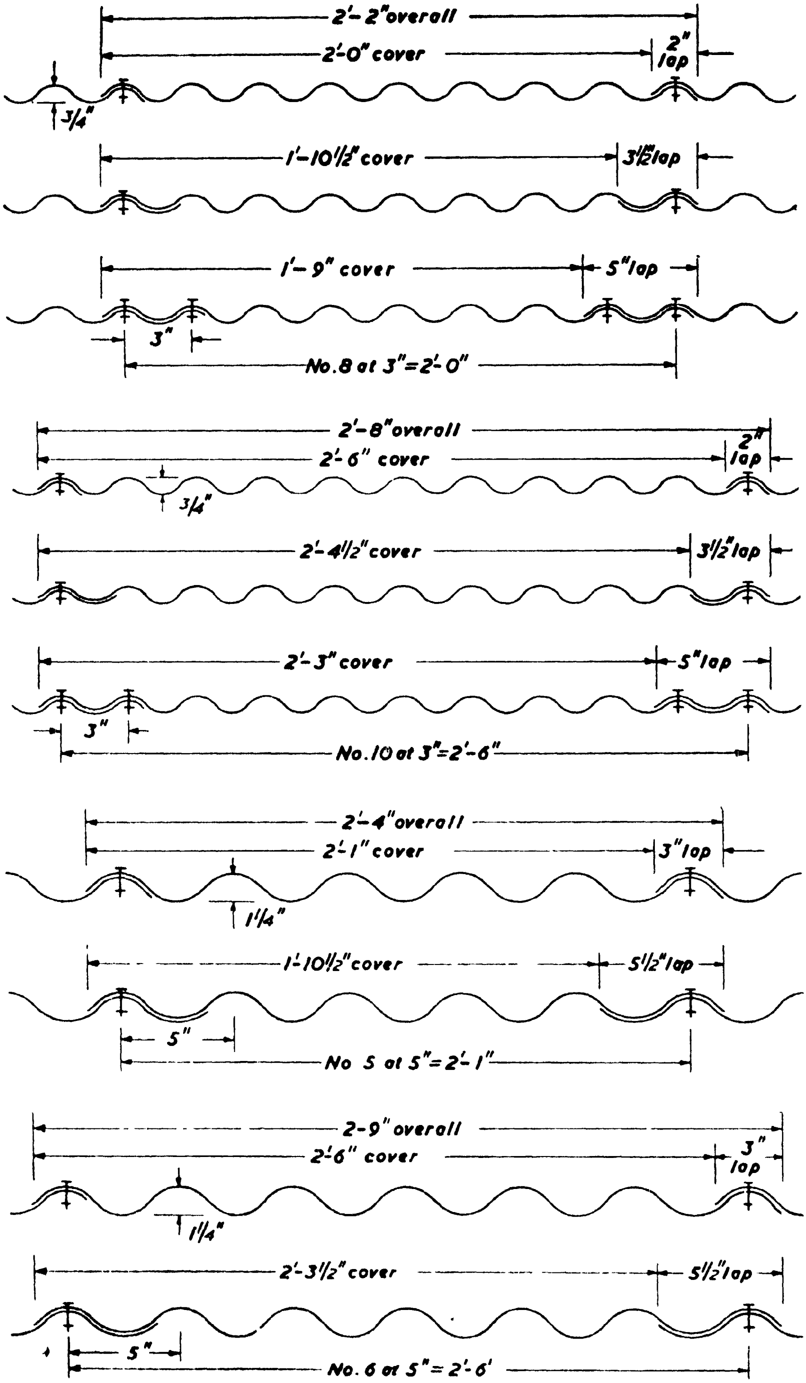
The usual practice is as follows:

Side-wall sheeting. A minimum of 4 in. when the lap is made on a sheeting rail, otherwise 6 in.

Roof sheeting. A minimum of 6 in. for slopes of 20° and over. Flatter slopes, especially on wide buildings and those exposed to driving rain, necessitate an increased end lap and possibly sealing by a bed of mastic.

Fastenings

Laps are usually secured by means of ¼-in. diam. galvanised steel bolts, although self-tapping screws are coming into favour for this purpose. The sheets are fastened to the purlins or rails by $\frac{5}{16}$-in. diam. hook bolts or by



TYPICAL SHEETS AND SIDE LAPS

Fig. 519

clips as in Fig. 521, which also gives the recommended pitches. All bolts must pass through the crown of the corrugation and must be provided with steel and either lead or bitumen felt washers, the latter being much cheaper and fully effective in most cases. For additional protection in special cases the washers are bedded in mastic. All holes for bolts must be punched from the top surface of the sheet.

Layout of Sheets

Practice in regard to the layout of sheets is largely one of individual experience and there are no universally accepted rules.

Fig. 520 shows various layouts for roof and side-wall sheeting. Layout *A* gives the most regular appearance and is frequently used, but it involves connections passing through four thicknesses which tend to bunch up because the sheets are of uniform profile throughout and cannot nest together in full contact, especially in the stouter gauges. The other layouts shown overcome this disadvantage, although in some cases it means splitting sheets lengthwise at the ends of alternate strakes, or increasing the first side lap.

Accessories

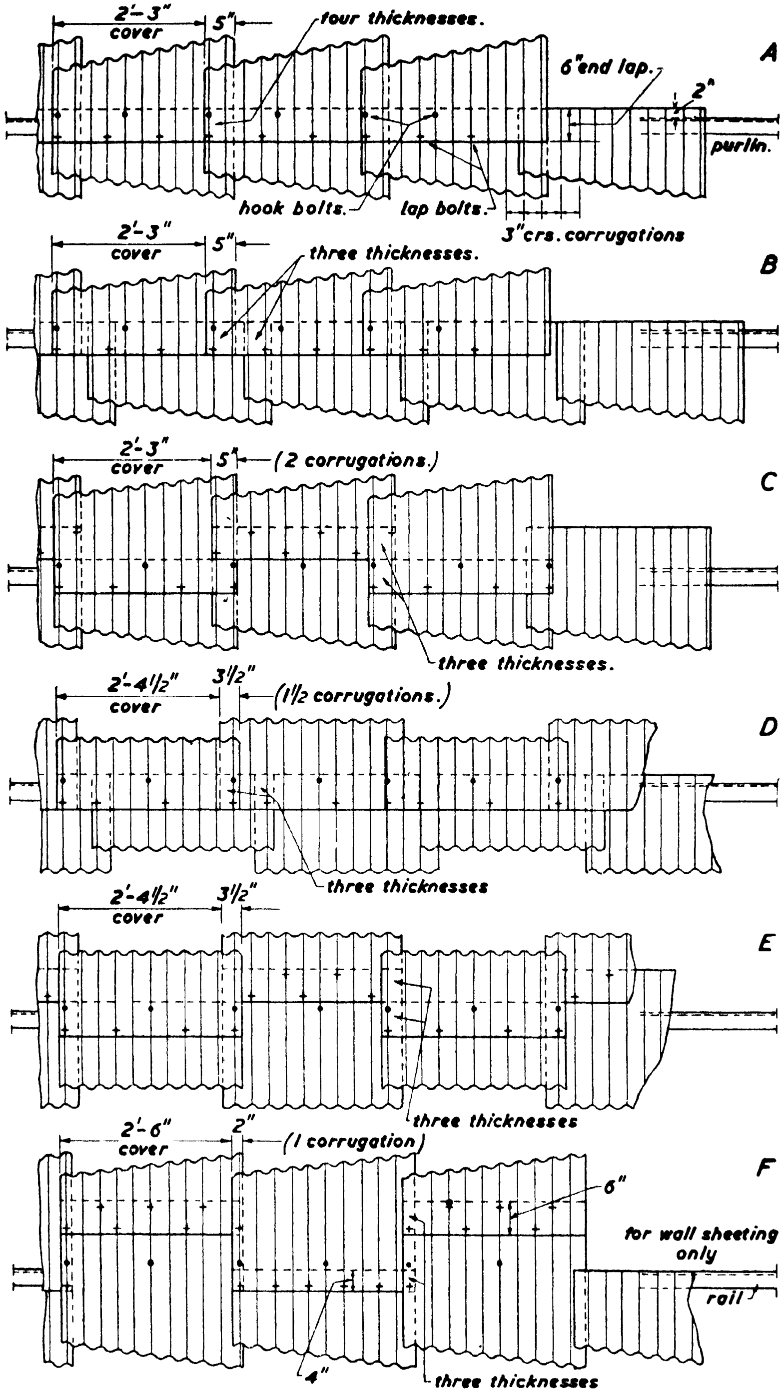
Ridgecaps, corner and eaves closures, aprons, louvres and flashings are usually of plain flat galvanised sheet, bent at the works to any angle required. The girth should be 12 in., 15 in. or 18 in., for cutting from a standard width sheet and the gauge the same as the corrugated sheeting or 20-gauge minimum. A lap of 3 in. is generally sufficient, and ridgecap lengths should be laid so that the prevailing wind does not tend to blow rain into the lap.

Corrosion

Whilst a steel sheet will provide all the strength and covering required it will corrode if not protected. Galvanising in accordance with B.S. 798 with a minimum of $1\frac{3}{4}$ oz. per sq. ft. provides a very good long-term protection for all but very special cases of acid pollution. Most makers are prepared to quote for sheets with a heavier weight of spelter if required, up to $2\frac{1}{2}$ oz. per sq. ft.

The life of galvanised corrugated steel sheeting can be extended indefinitely by regular painting. The adhesion of paint to new sheets is likely to be poor and in general it is good practice to defer painting for some time to allow the surface to weather. In cases where immediate painting is required, bituminous paints are recommended as less likely to peel. The alternative is to treat the surface to give a key to the paint. A number of proprietary materials are available for this purpose, many based on phosphoric acid. The Iron & Steel Institute recommend a compound of (by volume) industrial spirit 60 per cent, toluol 30 per cent, carbon tetrachloride 5 per cent, hydrochloric acid 5 per cent. Where such mordants are used they must be well washed off after application and the surface allowed to dry before being painted.

If the first painting is not done for some years and the zinc coating has deteriorated, the surface should be wire brushed to remove all rust and



TYPICAL LAYOUTS AND END LAPS OF 10/3" CORRUGATED STEEL SHEETING.

FIG. 520

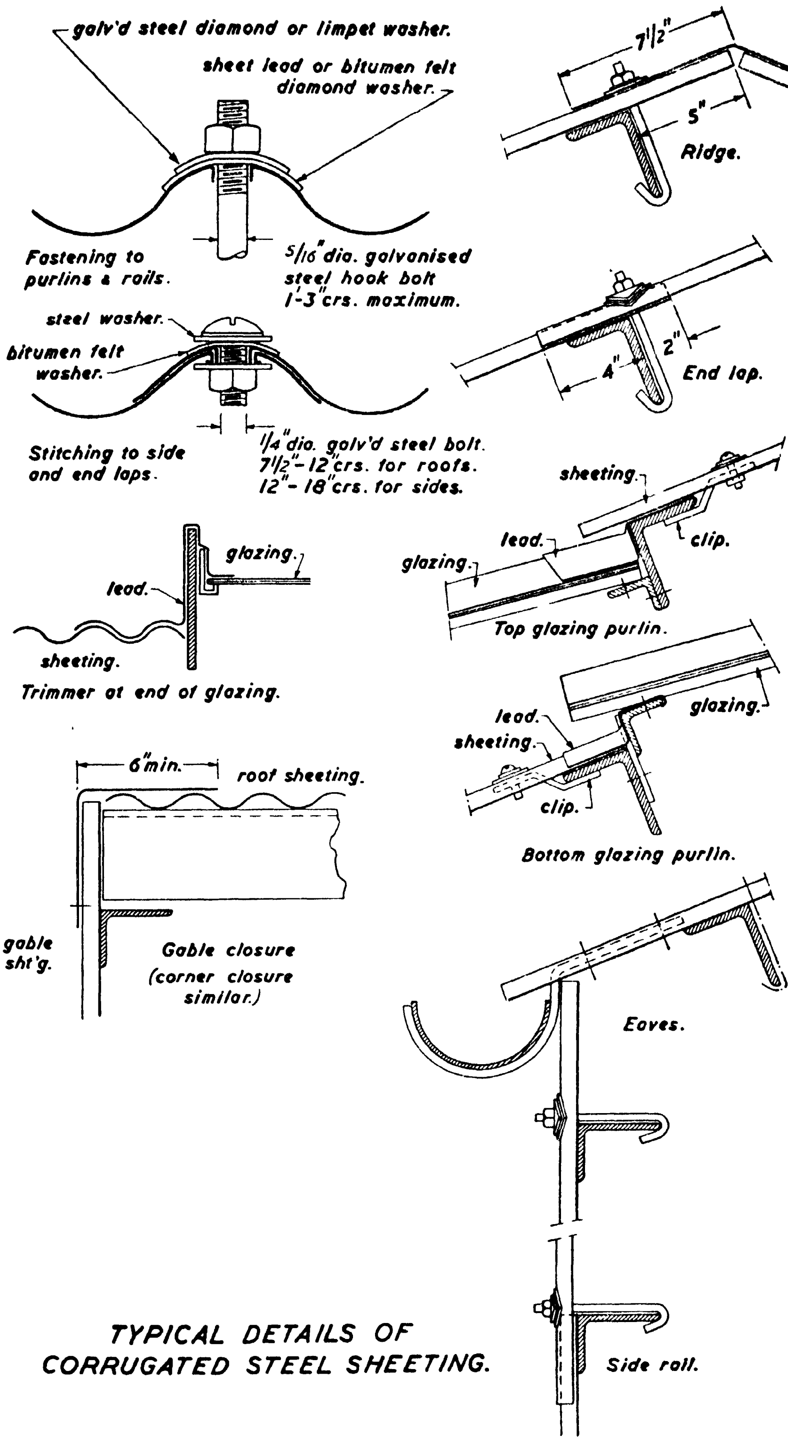

golv'd steel diamond or limpet washer.
sheet lead or bitumen felt diamond washer.
Fastening to purlins & rails.
5/16" dia. galvanised steel hook bolt 1'-3" crs. maximum.
steel washer.
bitumen felt washer.
Stitching to side and end laps.
1/4" dia. galv'd steel bolt. 7½"-12" crs. for roofs. 12"-18" crs. for sides.
7½"
5"
Ridge.
2"
4"
End lap.
glazing.
lead.
sheeting.
Trimmer at end of glazing.
sheeting.
lead.
clip.
glazing.
Top glazing purlin.
lead.
sheeting.
glazing.
clip.
Bottom glazing purlin.
6" min.
roof sheeting.
gable sht'g.
Gable closure (corner closure similar.)
Eaves.
Side roll.
TYPICAL DETAILS OF CORRUGATED STEEL SHEETING.

FIG. 521

similarly when repainting all loose paint should be removed by wire brushing. Where the zinc is sound no priming coat is necessary.

The following paints are recommended for galvanised sheeting:

(1) A non-glossy rust-inhibiting oil undercoat followed by a finishing coat of linseed oil gloss.
(2) A solution of bitumen in drying oil or varnish.
(3) Bituminous emulsion.
(4) Tar paint.

Thermal Insulation

The thermal resistance of steel sheeting is very low but can be readily improved to any required standard and at reasonable cost by the use of insulation board which is laid between the purlins and the sheeting, or under the purlins, supported in each case by strip steel tees. A number of proprietary arrangements are available and particulars can be obtained from the suppliers, who will usually also carry out the erection.

41. STEEL WINDOWS AND PATENT GLAZING

Steel Windows

Three British Standards have been issued relating to steel windows, namely:

B.S. 990—Metal Casement Windows and Casement Doors.

B.S. 1787—Steel Windows for Industrial Buildings.

B.S. 2503—Agricultural Windows.

The windows described in the first specification are of the domestic variety, and, although they are used in the administrative blocks of industrial buildings, they are normally set in brickwork or masonry. Consequently, they are not considered here. The windows in the second specification are those which are used in the production blocks, boiler-houses, etc., where they are frequently attached directly to steel frames.

Although the methods shown are by no means exhaustive, the diagrams in Fig. 522 demonstrate some of the methods of fixing the windows and show how the lead flashings may be arranged. For bolt spacings, see B.S. 1787.

This information has been supplied by The Metal Window Association Ltd., of Burwood House, Caxton Street, London, S.W.1, to whom all inquiries should be addressed.

Patent Glazing

General

The following information on patent glazing has been supplied by the Patent Glazing Conference, Burwood House, Caxton Street, London, S.W.1., to whom grateful acknowledgement is made and to whom all enquiries relating to manufacturers and glazing systems should be addressed.

While the sheets of glass normally used for patent glazing are 24 in. wide, the spacing of the glazing bars and the method of fixing vary slightly with individual manufacturers. It may be accepted, however, that each system conforms with CP 145.101 which is the relevant British Standard Code of Practice for Patent Glazing.

Minimum Slopes

Patent glazing can be fixed with complete success at any slope between the vertical and the recommended minimum pitch of 20°. The lower limit is dictated by two separate factors which are common to glass disposed at shallow pitches:

(*a*) the tendency of condensation to collect more readily into large drops and thus fall directly from the glass.

(*b*) in exposed positions rainwater sluggishly drained away on the outside surface tends to be blown under the top flashings.

Shallower slopes of patent glazing have been successfully installed, but the most careful consideration in the light of local conditions should be given to these problems before a pitch lower than 20° is decided upon.

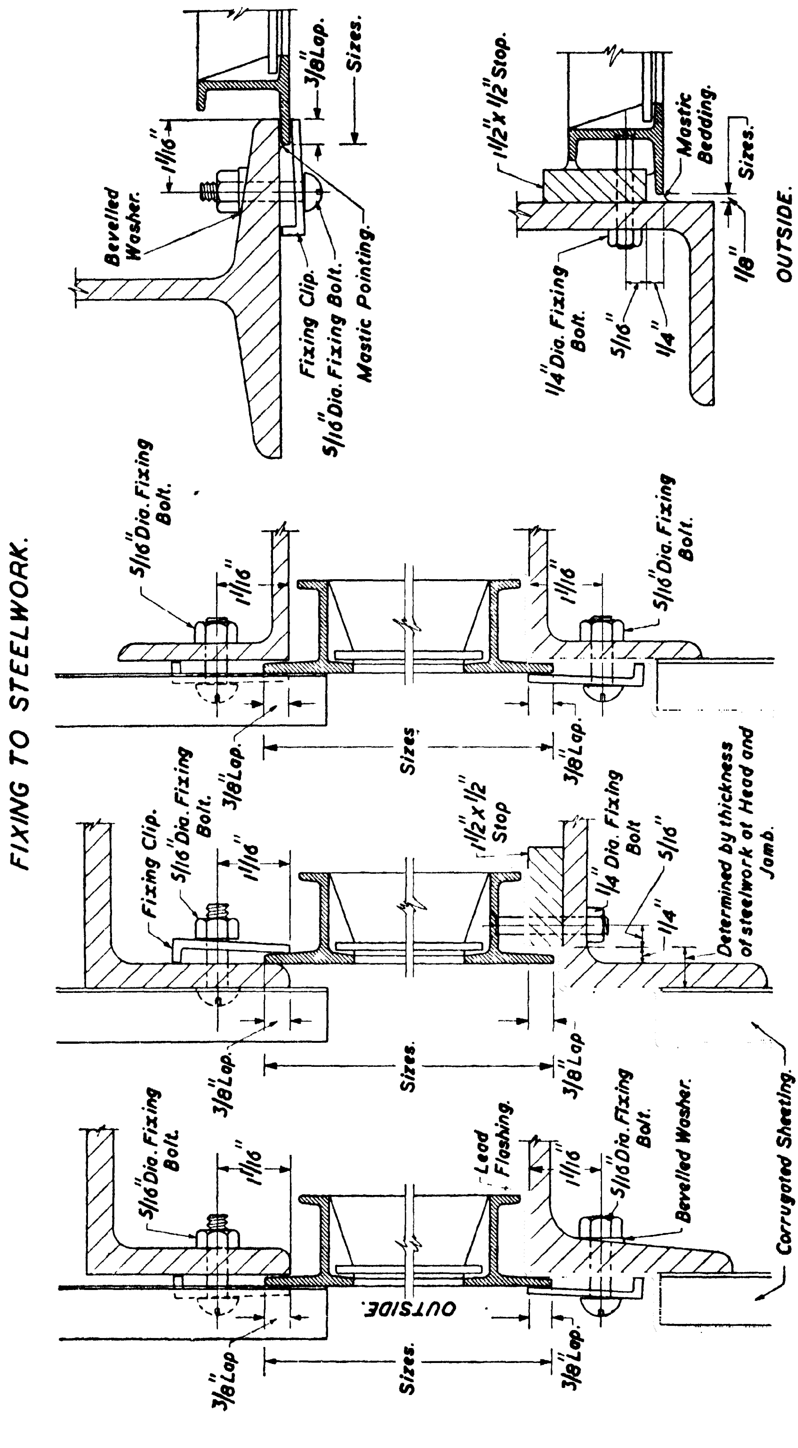

FIXING TO STEELWORK.
Bevelled Washer.
1 1/16"
3/8" Lap.
Sizes.
Fixing Clip.
5/16" Dia. Fixing Bolt.
Mastic Pointing.
1 1/2" x 1 1/2" Stop.
Mastic Bedding.
1/4" Dia. Fixing Bolt.
5/16"
1/4"
1/8"
OUTSIDE.
5/16" Dia. Fixing Bolt.
1 1/2" x 1/2" Stop
1/4" Dia. Fixing Bolt
Determined by thickness of steelwork at Head and Jamb.
Lead Flashing.
Bevelled Washer.
Corrugated Sheeting.

FIG. 522

Mid-Roof and North-Light Glazing

Methods of fixing mid-roof and north-light glazing are shown in Figs. 523 and 524.

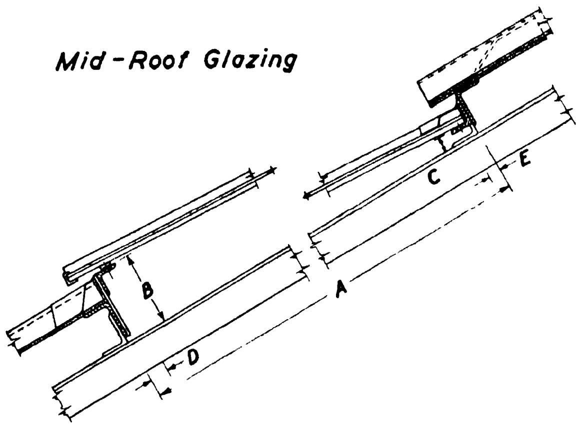


FIG. 523

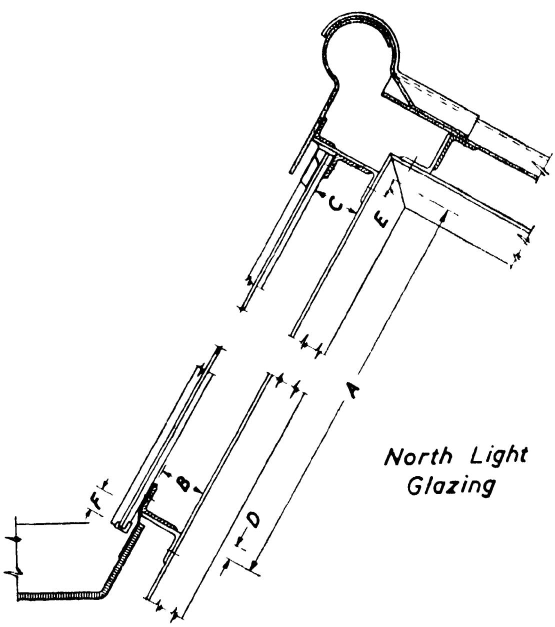


FIG. 524

The dimensions to specify are as follows:

	Mid-roof Glazing	*North-light Glazing*
Pitch	Roof pitch on rafter back.	Roof pitch on rafter back.
A	Back to back of purlin cleats.	Back to back of purlin cleats.
B	Height—lower purlin above rafter back.	Height—lower purlin above rafter back.
C	Height—upper purlin above rafter back.	Height—Upper purlin above rafter back.
D	Back of purlin cleat to lower fixing hole.	Back of purlin cleat to lower fixing hole.
E	Back of purlin cleat to upper fixing hole.	Back of purlin cleat to upper fixing hole.
F		Overhang required.
Run	Total run of glazing between end rails.	Total run of glazing between end rails.
Height	Height of glazing above ground level.	Height of glazing above ground level.

Ends of patent glazing runs may be finished in various ways, a typical example being shown in Fig. 525. The spacing for the fixing bolts should not exceed 18 ins.

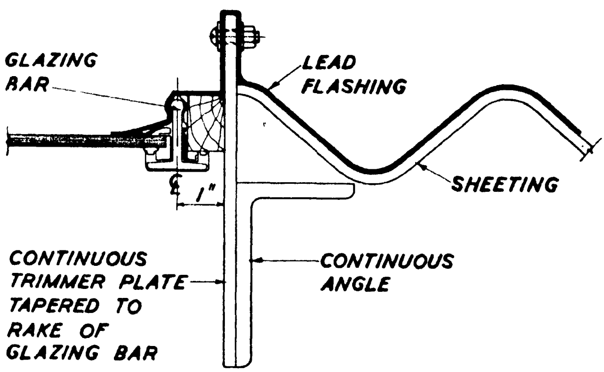


FIG. 525

Vertical Patent Glazing

Recent years have seen an increasing emphasis on glazed wall construction, and vertical glazing has achieved popularity in meeting this demand. Typical methods of fixing vertical glazing are shown in Fig. 526.

Patent Double Glazing

The modern need for fuel economy has increased the demand for double glazing owing to its greater thermal insulating qualities and a number of proprietary systems are available. The use of patent double glazing reduces heat losses by approximately 50 per cent as compared with single glazing.

TYPICAL DETAILS FOR VERTICAL GLAZING

Scale: 3 ins. = 1 ft.

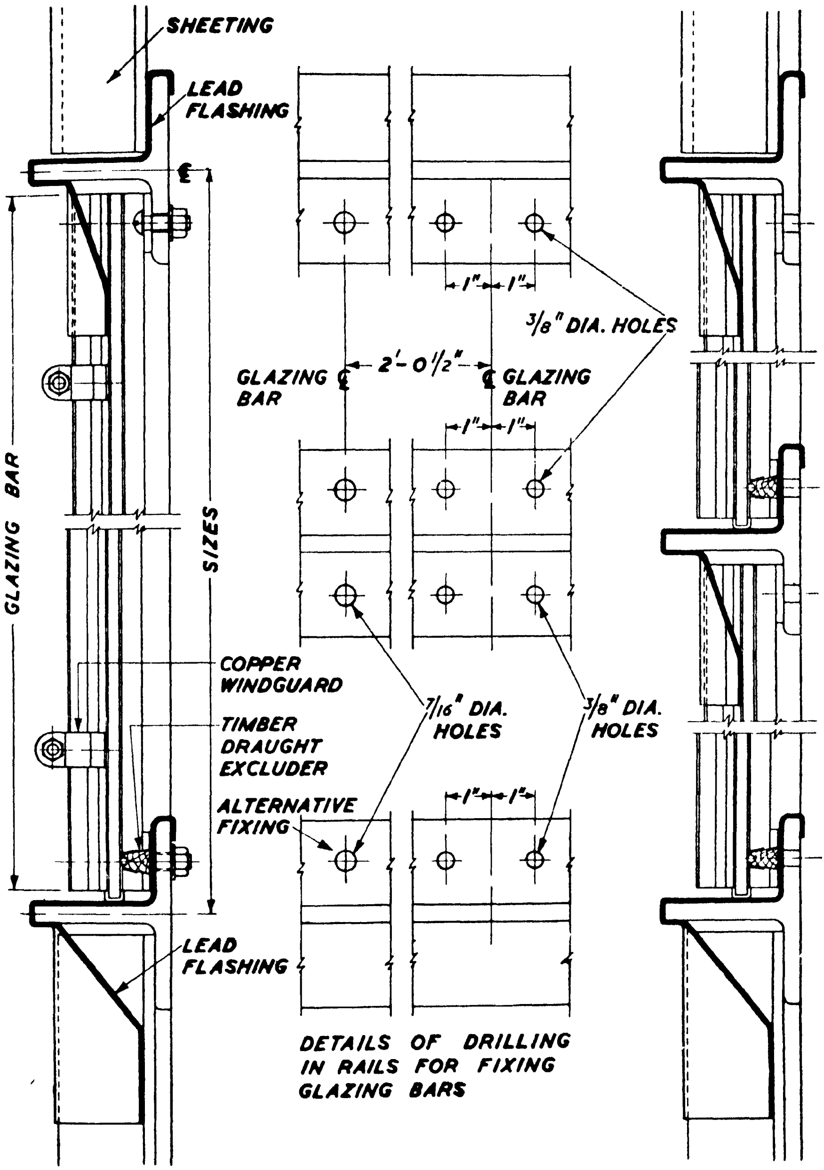


FIG. 526

Weight per Square Foot

Patent glazing is usually reckoned in weight at 6 lb. per sq. ft. for the lead-clothed bar system. This figure is inclusive of the weight of ¼-in thick glass and envisages the bars being spaced at approximately 24-in. centres.

Ventilation

Ventilation, either controllable or permanent, is often required in patent glazing, and, in the former type, the 'top hung' method has become general. Most makers are prepared to supply ventilators either in separate units or continuous ranges. They can be constructed in steel, galvanised or lead covered.

Since these ventilators are situated in remote positions, mechanical opening gear is frequently necessary to operate them and a variety of methods, manually, electrically or hydraulically controlled, is available.

Details of construction both for the ventilators and the operating gear differ considerably, and early consultation with the makers is recommended to ensure that satisfactory preparations are made for fixing them.

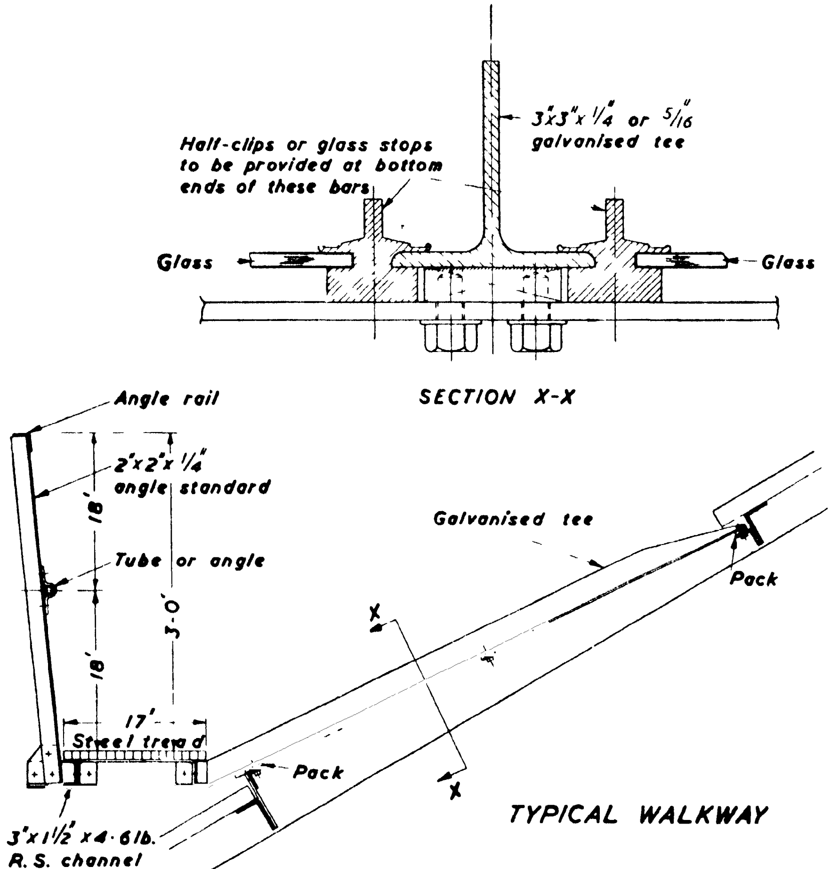


FIG. 527

Walkways

The Code of Practice for patent glazing, already mentioned, recommends that walkways should not be attached to ordinary glazing bars, but difficulties often arise in attaching them to, or through, other forms of roof covering. An illustration is given in Fig. 527 which shows a method by which both these objections are avoided.

This consists of introducing 'T'-bearers fixed direct to the purlins between pairs of closely spaced glazing bars, so that additional loads are not imposed on the bars. The bearers extend beyond the lower edge of the glazing and are cranked to form a horizontal cantilever support for the walkway. To simplify the design of purlins and to minimise the number of bearers required, the latter are intended to be fixed immediately over the roof trusses.

The major dimensions have been interpreted in accordance with the Factories Act and Building Regulations. It should, however, be borne in mind that such walkways are intended as working platforms only, and not as gangways for the passage or stacking of materials which would, under the Building Regulations, necessitate heavier and wider construction.

Glass

The varieties of glass which are used in patent glazing are:

(*a*) ¼-in. wired cast (hexagon mesh);
(*b*) ¼-in. Georgian wired cast (square mesh);
(*c*) ¼-in. rough cast;
(*d*) ¼-in. polished plate;
(*e*) ¼-in. heat-resisting glass;
(*f*) the heavier types of sheet glass.

It may perhaps be mentioned here that wire-reinforced glass has achieved considerable popularity because of its ability to delay the spread of fire and the additional safety factor which it confers. This is not, as is sometimes supposed, due to any greater resistance to breakage but rather to the effect of the wire mesh in restraining the glass from falling after it has once been broken. Glazing bars are usually spaced about 24 in. from centre to centre and, although wider spacing is occasionaly employed, it should only be specified with due caution and recognition given to the increased difficulties of obtaining and handling the wider squares of glass involved. Glazing bars should not normally exceed 10 ft. in length. Here, again, convenience of handling the glass is one of the criteria, but it is also important to remember that the cost of glass replacements in these sizes is proportionately high.

Lead Flashings

Lead flashings are a necessary adjunct to patent glazing and generally 4 lb. (per sq. ft.) lead flashings are used, but in exposed situations the weight should be increased to 5 lb.

42. SHEET STEEL PILING

Introduction

The following tables provide the designer with the dimensions and properties of the steel sheet piling sections which are rolled in this country and which are available for use at home or abroad.

The applications of steel sheet piling cover the construction of docks, harbours, wharves, jetties, river and canal training walls, cut-off walls under dams and embankments, sea-defence works, and so on. Steel sheet piling is also used for temporary work in the construction of cofferdams for bridge piers, deep sewers, river and canal crossings, intakes and outfalls, etc.

All steel piling is manufactured by the open-hearth process and is available to the following specifications:

B.S. 15 : 1948. (Ultimate tensile stress 28/33 tons per sq. in. Minimum guaranteed yield point 15¼ tons per sq. in.)

B.S. 24 : part 6 : 1942, Specification 18. (Ultimate tensile stress 28/33 tons per sq. in.)

Medium-tensile Steel. (Ultimate tensile stress 33/38 tons per sq. in.)

High-tensile Structural Steel. (Ultimate tensile stress 37/43 tons per sq. in.)

All these qualities can be supplied with copper content with the object of increasing resistance to corrosion. Special rust-resisting high-tensile steel is also available. In addition, the South Durham Steel & Iron Company Limited supply piles shot-blasted and painted with anti-corrosive compound.

There are four specific types, each having its own range of sections, known respectively as Frodingham, D.L., Larssen and Rendhex. The first is of the 'Z' type, the second and third are trough-shaped and the fourth hexagonal. The Frodingham sections are rolled by the Appleby Frodingham Steel Company, Scunthorpe, Lincolnshire, the D.L. sections by Dorman Long & Company Limited, Middlesbrough, and the Larssen and Rendhex sections by the South Durham Steel & Iron Company Limited, Middlesbrough.

Various types of standard or special bent and riveted corners, junctions, tapers and cruciform piles can be supplied to suit requirements.

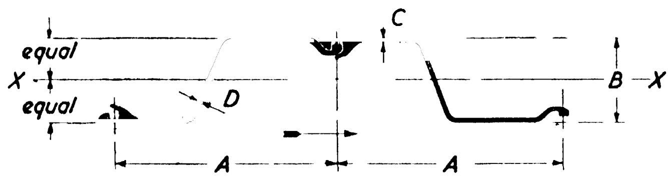
The box piles shown in the following diagrams are fabricated from sheet piles and, when used alone as bearing piles or interlocked as a continuous retaining wall, are capable of withstanding very heavy loads. The Rendhex products, however, are fabricated by welding together pairs of semi-hexagonal rolled sections designed for the specific purpose of producing foundation columns. Full details and information are available on application to the individual makers.

The working stresses used for design must obviously depend on the degree of permanence of the structure. For permanent structures, using mild-steel

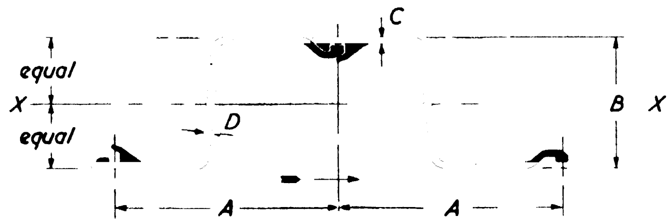
piles, it is normal to use a working stress of 8 tons per sq. in., but if the work is of a temporary nature the stress can be increased up to 10 tons per sq. in. as the designer desires. The makers concerned are prepared to submit, free on request, designs for any work involving the prospective use of their products.

APPLEBY-FRODINGHAM STEEL COMPANY
(BRANCH OF THE UNITED STEEL COMPANIES LTD)
FRODINGHAM SHEET PILING

DIMENSIONS AND PROPERTIES



Profile of No 1A Section



Profile of Nos 2, 3 and 4 Sections

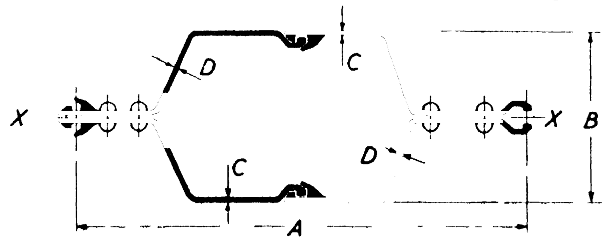
Direction of drive is shown thus ➡—➤

Section No	Size in inches				Cross Sectional Area per foot of piling wall in square inches	Weight		Moment of Inertia about X-X per foot of piling wall in ins⁴	Section Modulus about X-X per foot of piling wall in ins³	Radius of gyration about X-X in inches
	A	B	C	D		of section in lb per foot run	of piling wall in lb per sq foot			
1A	15¾	5¾	27	27	5 37	23 95	18 25	29 02	10 09	2 32
2	15¾	7 9/32	32	30	7 12	31 72	24 17	67 46	18 53	3 08
3	15¾	9	42	40	9 28	41 34	31 50	118 90	26 42	3 58
4	15¾	10¾	55	45	12 07	53 79	40 98	235 18	43 75	4 41

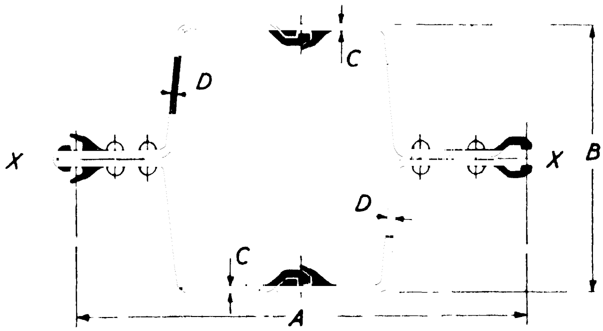
NOTE—The upright figures are for a single pile and not for the double pile as shown in the diagrams

APPLEBY-FRODINGHAM STEEL COMPANY
(BRANCH OF THE UNITED STEEL COMPANIES LTD.)
FRODINGHAM SHEET PILING
DIMENSIONS AND PROPERTIES FOR BOX PILING

For forming a continuous interlocking wall of box piles or for use as king piles driven intermittently in a Single Wall of Piling.



Profile of No. 1A. Section.



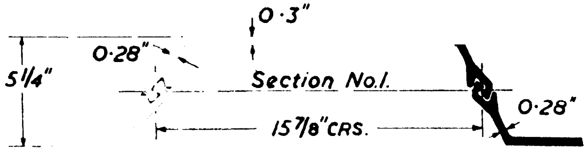
Profile for Nos 2, 3 and 4 Sections.

Section No	Size in inches A	B	C	D	Cross Sectional Area per foot of piling wall in square inches.	Weight of Box Pile in lb. per foot run	Weight of piling wall in lb. per sq foot.	Moment of Inertia about X–X per foot of piling wall in ins.⁴	Section Modulus about X–X per foot of piling wall in ins ³	Radius of gyration about X–X in inches	Dia. of Rivets in ins.
1A	31½	11½	·27	·27	10·74	95·80	36·50	146·84	25·54	3·70	¾
2	31½	14 9/16	·32	·30	14·23	126·88	48·34	323·55	44·44	4·77	¾
3	31½	18	·42	·40	18·56	165·36	63·00	613·55	68·17	5·75	⅞
4	31½	21½	·55	·45	24·14	215·16	81 96	1167·60	108·61	6·96	⅞

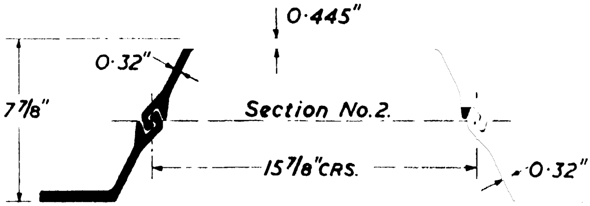
The weight per foot does not include rivet heads

For 3/4 in. dia. rivets at 3 in. staggered pitch 6 in. on line add 2 25 lb.
·· 3/4 in. ·· ·· 4 in. ·· ·· 8 in. ·· 1·69 lb.
·· 3/4 in. ·· ·· 6 in. ·· ·· 12 in ·· 1·13 lb.
·· 7/8 in. ·· ·· 3 in. ·· ·· 6 in ·· 3·58 lb.
·· 7/8 in. ·· ·· 4 in. ·· ·· 8 in ·· 2·68 lb.
·· 7/8 in. ·· ·· 6 in. ·· ·· 12 in. ·· 1·79 lb.

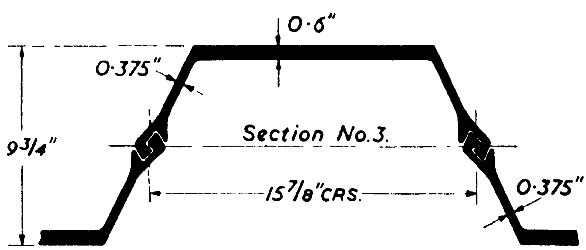
DORMAN LONG.
DORMAN LONG SHEET PILING
DIMENSIONS AND PROPERTIES



Sectional Area square inches	Weight		Moment of Inertia		Section Modulus		Modulus weight ratio	Radius of gyration inches
	Section lb./ft.	Wall lb./ft.²	Section ins.⁴	Wall ins.⁴/ft.	Section ins.³	Wall ins.³/ft.		
5·40 ins²/ft.	24·05	18·35	28·44	21·67	10·84	8·27	·450	2·00



Sectional Area square inches	Weight		Moment of Inertia		Section Modulus		Modulus weight ratio	Radius of gyration inches
	Section lb./ft.	Wall lb./ft.²	Section ins.⁴	Wall ins.⁴/ft.	Section ins.³	Wall ins.³/ft.		
7·09 ins²/ft.	31·68	24·12	87·21	66·45	22·14	16·87	·699	3·06

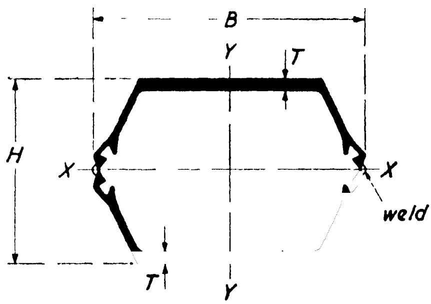


Sectional Area square inches	Weight		Moment of Inertia		Section Modulus		Modulus weight ratio	Radius of gyration inches
	Section lb./ft.	Wall lb./ft.²	Section ins.⁴	Wall ins.⁴/ft.	Section ins.³	Wall ins.³/ft.		
9·17 ins²/ft.	40·95	31·20	170·54	130·0	35·00	26·70	·856	3·77

DORMAN LONG

DORMAN LONG SHEET PILING.

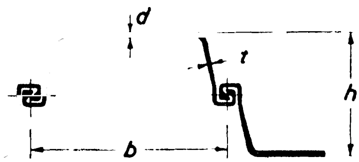
DIMENSIONS AND PROPERTIES FOR BOX PILING.



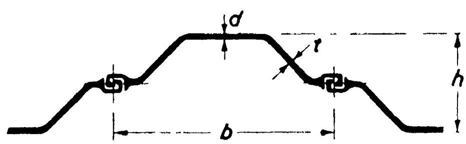
Section No.	B ins.	H ins.	T ins.	Weight lb./ft.	Sectional Area sq. ins.	Section Modulus ins.^{3}		Moment of Inertia ins.^{4}		Radius of Gyration inches	
						Axis X–X	Axis Y–Y	Axis X–X	Axis Y–Y	Axis X–X	Axis Y–Y
2	17	9 1/2	0·445	63·36	18·64	54·4	67·4	247·5	574·9	3·64	5·55
3	17 1/4	11 11/16	0·600	81·90	24·10	81·6	81·8	451·4	704·1	4·33	5·41

SOUTH DURHAM STEEL & IRON CO. LTD.
(INCORPORATING CARGO FLEET IRON COMPANY LTD.)

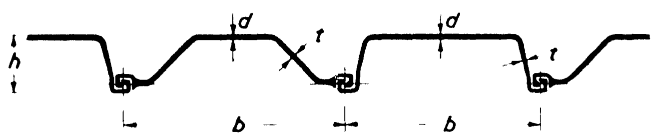
LARSSEN SHEET PILING
DIMENSIONS AND PROPERTIES



Sections 1GB, 1U, 2, 3, 4B, 5 and 6



Section 10A

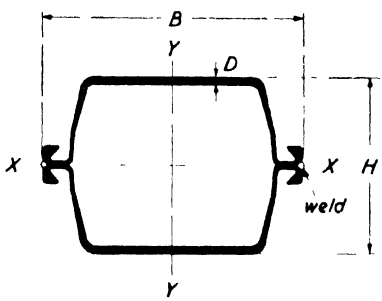


Section 2/10A

Section	b ins	h ins	d ins	t ins.	Sectional Area sq ins per ft of wall	Weight lbs per lineal ft	Weight lbs per sq ft	Section Modulus ins³ per ft
1GB	15¾	5⅛	0 32	0 23	5·44	24 30	18 50	7 8
1U	15¾	5⅛	0 375	0 375	6·38	28 50	21 70	9 1
2	15¾	7⅞	0·40	0·31	7·35	32·79	24·98	15·8
3	15¾	9¾	0·55	0 35	9·33	41·66	31 74	25·3
4B	16 9/16	13½	0·61	0·43	12·07	56·75	41·17	42·5
5	16 9/16	13½	0·87	0·47	14·34	67·19	48·74	55·1
6	16 17/32	17 5/16	0·87	0·55	17·47	81·84	59·39	78·12
10A	17¾	7⅞	0·50	0·50	8·03	40·40	27·30	11·7
2/10A	15¾/17¾	4 13/16	0·40/0·50	0·31/0·50	7·73	32·79/40·4	26·30	6·9

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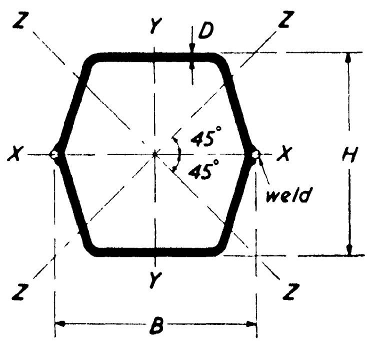
LARSSEN SHEET PILING



DIMENSIONS AND PROPERTIES FOR BOX PILING.

Section	Dimensions ins.			Weight lb. per linear foot	Section Modulus ins^3		Moment of Inertia ins^4		Least Radius of Gyration ins	Sectional Area ins^2		Perimeter inches
	B	H	D		Axis X–X	Axis Y–Y	Axis X–X	Axis Y–Y		Steel only	whole pile	
BP1	17	6½	0·31	48·60	27·8	40·5	90·4	344·3	2·52	14·28	96	44
BP1U	17	6½	0·375	57·00	32·8	50·1	106·6	425·9	2·52	16·78	96	44
BP2	17¼	9 7/16	0·41	65·58	52·0	71·8	245·2	619·3	3·57	19·28	133	49
BP3	17¼	11⅜	0·55	83·32	81·5	82·9	463·5	715·0	4·35	24·50	158	52
BP4	18	15½	0·63	113·50	141·2	126·8	1094·3	1141·2	5·74	33·20	222	61
BP5	18	15½	0·87	134·38	180·0	137·4	1395·0	1236·6	5·60	39·52	222	61
BP6	18¼	19¾	0·87	163·68	257·3	177·7	2542·3	1623·2	8·21	48·14	278	82

SOUTH DURHAM STEEL & IRON CO. LTD.
(INCORPORATING CARGO FLEET IRON COMPANY LTD.)
DIMENSIONS AND PROPERTIES FOR
RENDHEX FOUNDATION COLUMNS.



Section	Dimensions ins. B	H	D	Weight lb. per linear foot (including weld)	Section Modulus ins.³	Moment of Inertia ins.⁴	Radius of Gyration ins.	Axis
3	13	13	1/2	72·50	72·2	469·3	4·69	X–X
					58·7	381·3	4·23	Y–Y
					57·3	425·3	4·47	Z–Z
4	16 1/2	16	5/8	117·75	151·75	1214·0	5·95	X–X
					125·82	1038·0	5·50	Y–Y
					115·60	1126·0	5·70	Z–Z

43. FLOORS

In this section it is proposed to give some notes on reinforced concrete floors, which may be cast in situ or supplied in precast units. Those in the former group consist of reinforced concrete slabs and hollow tile or hollow concrete floors. Many different types of precast units are obtainable, most of which are patented.

Particulars of manufacturers and other information may be obtained from the Association of Constructional Floor Specialists, Leicester Square, London, W.C.2.

The design of reinforced concrete slabs is adequately treated in most textbooks on reinforced concrete. Other information may be obtained from the Code of Practice, C.P. 114.100–114.105 (1950) Suspended Concrete Floors and Roofs (including stairs).

The reinforcement must comply with one of the following British Standards:

B.S. 785—*Rolled Steel Bars and Hard Drawn Steel Wire for Concrete Reinforcement.*
B.S. 1144—*Cold Twisted Steel Bars for Concrete Reinforcement.*
B.S. 1221—*Steel Fabric for Concrete Reinforcement.*

Hollow floors are much lighter in weight than reinforced concrete slabs, as will be seen from Fig. 528.

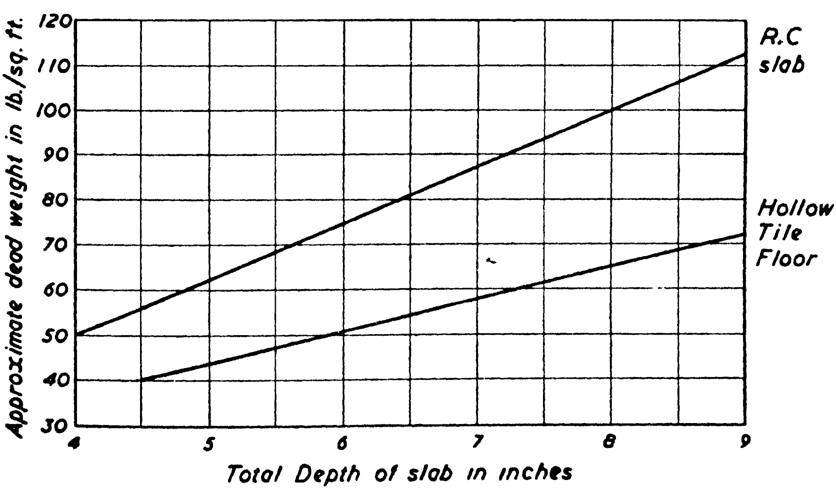


Fig. 528

The standard sizes of hollow tiles are specified in B.S. 1190. The tiles may be laid end to end in rows when the main reinforcement runs in one direction,

as shown in Fig. 529, or in two directions at right angles, when the tiles are isolated and provision is made to prevent concrete entering the open ends.

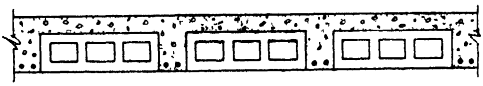


FIG. 529

Hollow tile floors are designed as 'T'-beams, the breadth of the ribs being regulated by the amount of reinforcement to be housed and the depth by the span and loads to be carried. Hollow floors are usually continuous over at least two spans and, consequently, the maximum B.M. induced by U.D.L.s is assumed to be WL/10. This value has been assumed in preparing the tables which follow.

A typical method of carrying hollow-tile floors over joist supports is shown in Fig. 530.

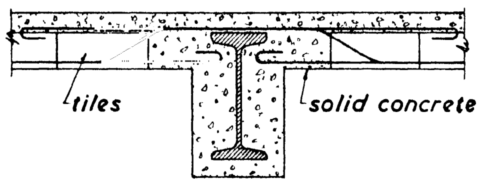


FIG. 530

Precast units can be of the types shown in Fig. 531, which may require an integral concrete topping to complete the floor, or they can be little more than permanent shuttering on which the concrete floor has to be poured.

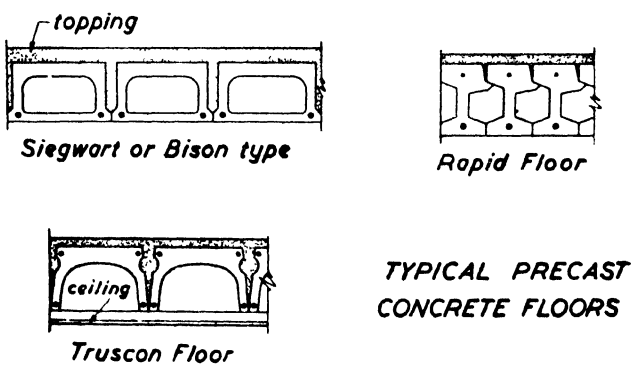


FIG. 531

Precast units are designed as simply supported beams. Continuity bars are required when the units are designed as continuous over more than one span, as shown in Fig. 532.

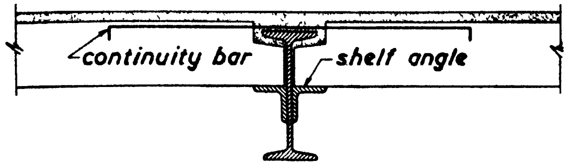


FIG. 532

HOLLOW TILE FLOORS WITH SINGLE-WAY REINFORCEMENT
SCHEDULE OF FLOOR THICKNESSES IN INCHES

Super-load in lb. per sq. ft.	*Span in Feet*										
	10	11	12	13	14	15	16	17	18	19	20
30	4½	5	5	5	5½	5½	6	6½	7	7½	7½
50	5	5	5	5½	5½	6	6½	7	7½	7½	8
80	5	5	5½	5½	6	6½	7	7½	8	8	9
100	5	5½	5½	6	6½	7	7½	8	8	9	9

Note.—In addition to the superload, 20 lb. per sq. ft. has been allowed for floor finishes.

HOLLOW TILE FLOORS WITH TWO-WAY REINFORCEMENT
SCHEDULE OF FLOOR THICKNESSES IN INCHES

Super-load in lb. per sq. ft.	*Span in Feet*										
	10	11	12	13	14	15	16	17	18	19	20
30	4½	4½	4½	4½	5	5	5	5½	5½	6	6
50	5	5	5	5	5½	5½	5½	6	6	6½	7
80	5	5	5	5	5½	5½	6	6½	6½	7	7
100	5	5	5	5½	5½	6	6½	6½	7	7	7

Notes.—The bays are assumed to be square. In addition to the superload, 20 lb. per sq. ft. has been allowed for floor finishes.

44. WALLS

THE following by-law has been reproduced from the London Building (Constructional) By-Laws 1952, by kind permission of the London County Council.

5.24. Panel walling in framed buildings

(1) If an external wall is constructed of materials used in combination with a framework of metal or reinforced concrete, any part of that wall which does not sustain and transmit any load other than that due to its own weight and to wind pressure on its own surface may be deemed to be a separate panel wall and thereupon shall

(a) comply with the provisions of paragraphs (2), (3), (4) or (5) (as the case may be) of this by-law; and

(b) be provided with such support and restraint as shall, to the satisfaction of the district surveyor, give that part of the wall an adequate degree of stability.

(2) In every panel wall constructed (otherwise than as a cavity wall) of bricks or blocks,

(a) the thickness shall be not less than 8½ in. throughout;

(b) the height shall not exceed 25 ft.;

(c) either the height or the length (whichever is the less) shall not exceed 18 times the thickness; and

(d) the base shall not overhang the beam upon which it is supported, to a greater extent than one-third of the thickness of the panel.

(3) In every panel wall constructed as a cavity wall,

(a) the provisions of by-law 5.16 (Cavity Walls) shall be complied with except that the inner leaf may be constructed of solid or hollow bricks or blocks not less than 3 in. thick;

(b) the height shall not exceed 25 ft.;

(c) either the height or the length (whichever is the less) shall not exceed 13 ft.;

(d) the area shall not exceed 200 sq. ft.; and

(e) the base shall not overhang the beam upon which it is supported, to a greater extent than one-third of the thickness of the overhanging leaf: provided that, if the bottom courses are built solid for the full thickness of the panel wall to a height above its base at least equal to that full thickness, the base may overhang the beam to an extent not exceeding one-third of that full thickness.

(4) Every panel wall constructed of reinforced concrete shall be not less than 4 in. thick in every part.

(5) Every panel wall constructed of concrete shall be not less than 4 ir thick in every part and shall be provided with shrinkage reinforcement i accordance with by-law 5.21 (3).

(6) For the purpose of this by-law, the expression 'beam' includes th solid concrete casing of any beam, where that casing is reinforced to the satisfaction of the district surveyor.

45. FIRE-RESISTING CONSTRUCTION

PART IX of the London Building (Constructional) By-Laws 1952 relates to fire-resisting construction and contains, *inter alia*, the following by-laws, which are reproduced by kind permission of the London County Council:

9.01. Application

Without prejudice to the generality of by-law 9.03, this Part of these by-laws shall apply to those buildings or parts or divisions of buildings and to those purposes which are specified in by-law 9.02.

9.02. Fire resistance period of elements of construction

(1) Each element of construction of a building, or of a part or of a division of a building, shall be capable of resisting the action of fire for a period of not less than that specified in the following Classification for the use, and the cubical extent or floor area on any one floor (as the case may be), of the building or part or division of the building.

CLASSIFICATION

Class No.	*Use*	*Cubical Extent or Floor Area on any one Floor (as the case may be)*	*Fire Resistance Period*
1	A building or division of a building used for bulk storage or warehouse purposes	(a) Exceeding 25,000 cu. ft., but not exceeding 50,000 cu. ft. in cubical extent	½ hour
		(b) Exceeding 50,000 cu. ft., but not exceeding 125,000 cu. ft. in cubical extent	1 hour
		(c) Exceeding 125,000 cu. ft., but not exceeding 250,000 cu. ft. in cubical extent	2 hours
2	A building or division of a building used for the purposes of trade or manufacture	(a) Exceeding 50,000 cu. ft., but not exceeding 125,000 cu. ft. in cubical extent	½ hour
		(b) Exceeding 125,000 cu. ft., but not exceeding 250,000 cu. ft. in cubical extent and	
		(i) not exceeding 7500 sq. ft. in floor area on any one floor; or	1 hour
		(ii) exceeding 7500 sq. ft. in floor area on any one floor	2 hours
3	A building used for office or dwelling purposes	(a) (i) Exceeding 50,000 cu. ft., but not exceeding 125,000 cu. ft. in cubical extent, or (ii) Exceeding 1000 sq. ft., but not exceeding 2500 sq. ft. in floor area on any one floor	½ hour
		(b) (i) Exceeding 125,000 cu. ft. in cubical extent, or (ii) Exceeding 2500 sq. ft. in floor area on any one floor	1 hour

Class No.	*Use*	*Cubical Extent or Floor Area on any one Floor (as the case may be)*	*Fire Resistance Period*
4	In a building used partly for office purposes and partly for the purposes of trade or manufacture, any part used for the purposes of trade or manufacture	(a) (i) Not exceeding 64,000 cu. ft. in cubical extent	½ hour
		(ii) Exceeding 64,000 cu. ft., but not exceeding 125,000 cu. ft. in cubical extent	1 hour
		(iii) Exceeding 125,000 cu. ft., but not exceeding 250,000 cu. ft. in cubical extent	2 hours
		(b) (i) Not exceeding 2500 sq. ft. in floor area on any one floor	½ hour
		(ii) Exceeding 2500 sq. ft., but not exceeding 5000 sq. ft. in floor area on any one floor	1 hour
		(iii) Exceeding 5000 sq. ft. in floor area on any one floor	2 hours
5	In a building used partly for dwelling purposes and partly for the purposes of trade or manufacture, any part used for the purposes of trade or manufacture	(a) (i) Not exceeding 32,000 cu. ft. in cubical extent	½ hour
		(ii) Exceeding 32,000 cu. ft., but not exceeding 64,000 cu ft. in cubical extent	1 hour
		(iii) Exceeding 64,000 cu. ft., but not exceeding 250,000 cu. ft. in cubical extent	2 hours
		(b) (i) Not exceeding 1000 sq. ft. in floor area on any one floor	½ hour
		(ii) Exceeding 1000 sq. ft., but not exceeding 2500 sq. ft. in floor area on any one floor	1 hour
		(iii) Exceeding 2500 sq. ft. in floor area on any one floor	2 hours
6	In a building used for any of the above-mentioned purposes, any part used for		
	(a) A transformer chamber or for any purpose involving a similar fire risk		2 hours
	(b) Garage purposes	(i) Not exceeding 500 sq. ft. in floor area	½ hour
		(ii) Exceeding 500 sq. ft., but not exceeding 1000 sq. ft. in floor area	1 hour
		(iii) Exceeding 1000 sq. ft. in floor area	2 hours

For the purposes of this Classification, 'floor area' shall not include the area of any underground cellar outside the external walls of the building above, when that cellar is entirely separated from the main building or is connected therewith only by an opening not exceeding 5 ft. in width.

(2) Where walls are used to divide parts of buildings used for the same purpose, those walls shall be capable of resisting the action of fire for a period of not less than 2 hours.

(3) Where an element of construction is required to be capable of resisting the action of fire for a period of not less than 1 hour, that element of construction shall be constructed of incombustible materials.

(4) The floor area on any one floor of a building of class No. 1 (c) shall not, without the approval of the Council, exceed 7,500 sq. ft.

(5) Where, in any building, the level of the surface of any floor is more than 42 ft. above the level of the footway immediately in front of the centre of the face of the building, or if there is no footway, above the level of the ground before excavation, the elements of construction of that building shall be capable of resisting the action of fire for a period of not less than one hour.

(6) Where in a building of class No. 3 (a) (ii), the floors are constructed of incombustible materials, the limitations specified with respect to floor area shall not apply.

(7) In a building of class Nos. 4 or 5, the elements of construction of the part used for office purposes, or for dwelling purposes, (as the case may be), shall be capable of resisting the action of fire for the same period as that required if the whole building were used for office or dwelling purposes.

(8) Where, in the same building or part or division of a building, a different period of resistance to the action of fire is required by this by-law according to whether regard is had to the cubical extent, or to the floor area on any one floor, or to the level of the surface of any floor of the building or part or division of the building, the longer period shall be taken.

(9) Where a single-storey building or a division of a single-storey building does not exceed 250,000 cu. ft. in cubical extent or 25 ft. in height, the steelwork in that building or division may, subject to the provisions of by-law 6.02, be unprotected.

(10) Each element of construction of a basement storey (which for the purpose of this paragraph of this by-law shall be deemed to include the floor over the basement storey and any separations between the basement storey and any underground cellar outside the external walls of the building above) shall be capable of resisting the action of fire for a period of not less than twice that required for the elements of construction of the building or part or division of the building in which that basement storey is situated:

provided that in no case need that period exceed two hours.

(11) All joints of and in elements of construction, separations and enclosures shall be tight and proof against the passage of smoke or flame and shall be constructed to the satisfaction of the district surveyor.

9.03. Separation of buildings

Notwithstanding the provisions of Part V (Walls and Piers) of these by-laws, every building shall be separated from an adjoining building by such an external wall, party wall or other form of separation as shall be capable of resisting the action of fire for a period of not less than four hours.

Schedule VI

Furthermore, Schedule VI of the By-Laws contains the minimum requirements for construction and materials to be capable of resisting the action of fire for specified periods, among which are the following:

TABLE A.—WALLS AND PARTITIONS

Construction and Materials		*Minimum Thickness in inches (excluding plaster) for period of* 4 *hours*	2 *hours*	1 *hour*	½ *hour*
SOLID CONSTRUCTION					
Bricks of clay, concrete or sand lime	No plaster * Where plastered at least ½ in. thick on each side and where the wall does not exceed 10 ft. in either height or length, the thickness for this period may be 4 in.	8½	8½*	4	4
Concrete blocks	*Class* 1 *Aggregate*				
	No plaster		4	3	2½
	Plastered at least ½ in. thick on each side		4	2½	2
	Class 2 *Aggregate*				
	No plaster			4	3
	Plastered at least ½ in. thick on each side		4	3	2
Gypsum blocks	No plaster		4	3	2
	Plastered at least ½ in. thick on each side		3	2	2
Wood wool slabs	Plastered at least ½ in. thick on each side		3	2	2
Reinforced concrete	Aggregate in accordance with by-law 3.02, with minimum concrete cover to reinforcement of 1 in.	7	4	3	3
Plaster board	Supported at top and bottom edges in steel channels and plastered on each side at least ⅝ in. thickness with gypsum plaster			¾	
HOLLOW BLOCK CONSTRUCTION					
Clay blocks	Plastered at least ½ in. thick on each side and shells not less than ¾ in. thick:				
	1 cell in each block and each block not less than 50 per cent solid			4	3
	1 cell in each block and each block not less than 30 per cent solid			6	
	2 cells in each block and each block not less than 50 per cent solid		8½	4	
	2 cells in each block and each block not less than 30 per cent solid			6	
Concrete blocks	Plastered at least ½ in. thick on each side and 1 cell in wall thickness:				
	Class 1 *Aggregate*	8¾	4½	3	2½
	Class 2 *Aggregate*			8¾	3
Gypsum blocks	Not less than 70 per cent solid:				
	No plaster		4	3	2
	Plastered at least ½ in. thick on each side		3	2	2

TABLE A.—WALLS AND PARTITIONS—*continued*

Construction and Materials		Minimum Thickness in inches (excluding plaster) for period of 4 hours	2 hours	1 hour	½ hour
CAVITY WALL CONSTRUCTION	Walls constructed in accordance with by-law 5.16:				
	(a) of solid bricks or blocks of clay, concrete or sand-lime: thickness of each leaf	4			
	(b) with outer leaf, not less than 4 in. thick of solid bricks or blocks of clay, concrete or sand-lime:				
	(i) Thickness of inner leaf of solid or hollow concrete blocks of Class I Aggregate	4	3		
	(ii) Thickness of inner leaf of hollow clay or solid or hollow concrete blocks of Class 2 Aggregate		3		

In this table:

Class 1 *Aggregate* means foamed slag, pumice, blast furnace slag, crushed brick and burnt clay products, well burned clinker, crushed limestone.

Class 2 *Aggregate* means flint, gravel, granite and all crushed natural stones other than limestone.

TABLE B.—FLOORS AND LANDINGS

Construction and Materials	Minimum Thickness in inches for period of 4 hours	2 hours	1 hour	½ hour
FILLER JOIST CONSTRUCTION				
Thickness of concrete	6	5	4	3½
Concrete cover on bottom of joist	1	1	½	½
SOLID REINFORCED CONCRETE CONSTRUCTION (including flat slab construction and floors constructed of precast inverted 'U'-, channel or 'T'-sections, without a ceiling or soffit)				
Thickness of concrete	6	5	4	3½
Concrete cover to reinforcement	1	½	½	½
HOLLOW BLOCK FLOOR CONSTRUCTION (including floors constructed of precast concrete units of box-section or 'I'-section)				
Aggregate thickness of incombustible material [excluding ceiling finishes (if any)]	5	3½	3	2½
Concrete cover to reinforcement	1	½	½	

TABLE B.—FLOORS AND LANDINGS—*continued*

Construction and Materials	*Mininium Thickness in inches for period of*			
	4 *hours*	2 *hours*	1 *hour*	½ *hour*
STRUCTURAL TIMBER CONSTRUCTION				
Boards ⅞ in. thick on timber joists not less than 7 in. deep by 2 in. wide:				
(a) Tongued and grooved boards with ceiling of:				
(i) timber lath and plaster. Thickness of plaster				⅝
(ii) metal lath and plaster. Thickness of plaster				⅝
(iii) two layers of plaster-board, each of minimum thickness				⅜
(iv) one layer of plaster-board of minimum thickness				¾
(v) one layer of plaster-board of minimum thickness of ⅜ in. covered with gypsum plaster of minimum thickness				⅝
(b) Plain edge boarding with ceiling of:				
(i) metal lath and plaster. Thickness of plaster				⅝
(ii) timber lath and plaster with plaster of minimum thickness of ⅝ in., covered with sheets of plaster-board of minimum thickness				⅜

TABLF C.—STEEL COLUMNS AND BEAMS

Construction and Materials	*Minimum Thickness of Protection in inches for period of*			
	4 *hours*	2 *hours*	1 *hour*	½ *hour*
SOLID PROTECTION				
Columns				
Concrete not inferior to that designated Grade III in by-law 3.07 and reinforced	2½*	2*	1	1
* The thickness of protection on any projecting cleat, projecting rivet head and the like need not exceed 1 in.				
Solid bricks of burnt clay or sand lime	3	2	2	2
Solid blocks: reinforced in every horizontal joint:				
(i) Foamed slag or pumice concrete	2½	2	2	2
(ii) Gypsum blocks	2	2	2	2
Sprayed asbestos	2	1	½	¼
Beams				
Concrete not inferior to that designated Grade III in by-law 3.07 and reinforced	2½*	2*	1	1
* The thickness of protection on the upper surface of the upper flange of an internal beam, and on any projecting cleat, projecting rivet head and the like need not exceed 1 in.				
Sprayed asbestos	2	1	½	¼

TABLE C.—STEEL COLUMNS AND BEAMS—*continued*

Construction and Materials	*Minimum Thickness of Protection in inches for period of*			
	4 hours	*2 hours*	*1 hour*	*½ hour*
HOLLOW PROTECTION				
Columns				
Solid bricks of burnt clay or sand lime: reinforced in every horizontal joint	4½	3	2	2
Solid bricks of foamed slag or pumice concrete or gypsum: reinforced in every horizontal joint	3	2	2	2
Moulded asbestos bound in position with nicrome wire not less than No. 16 S.W.G. in thickness, the wires to be sunk in grooves not less than ⅛ in. deep in the outer surface of the asbestos and the grooves and all joints in the asbestos to be filled with refractory cement	2½	1½	1	1
Cement plaster or cement-lime plaster on metal lathing				¾
Cement plaster or cement-lime plaster on metal lathing with reinforcement over rendering coat			1	
Gypsum plaster on metal lathing			⅞	⅝
Gypsum plaster on ⅜-in. gypsum plaster board with No. 16 S.W.G. wire binding at 4 in. pitch			½	
Gypsum plaster on ¾-in. gypsum plaster board with No. 16 S.W.G. wire binding at 4 in. pitch.		½		
Two layers of metal lathing plastered with gypsum plaster on each layer, each	¾			
Precast concrete consisting of 4 volumes of vermiculite to 1 volume of Portland Cement, reinforced with expanded metal, wire mesh or with No. 16 S.W.G. wire binding at 4 in. pitch			1	
Beams				
Moulded asbestos bound in position with nicrome wire not less than No. 16 S.W.G. in thickness, the wires to be sunk in grooves not less than ⅛ in. deep in the outer surface of the asbestos and the grooves and all joints in the asbestos to be filled with refractory cement	2½	1½	1	1
Cement plaster or cement-lime plaster on metal lathing				¾
Cement plaster or cement-lime plaster on metal lathing with reinforcement over the rendering coat			1	
Gypsum plaster on metal lathing		⅞	⅝	
Gypsum plaster on ⅜-in. gypsum plaster board with No. 16 S.W.G. wire binding at 4 in. pitch			½	
Gypsum plaster on ⅜-in. gypsum plaster board supported on wood battens				3/16 (neat single coat)
Gypsum plaster on ¾-in. gypsum plaster board with No. 16 S.W.G. wire binding at 4 in. pitch		½		
Precast concrete consisting of 4 volumes of vermiculite to 1 volume of Portland Cement, reinforced with expanded metal, wire mesh or with No. 16 S.W.G. wire binding at 4 in. pitch			1	

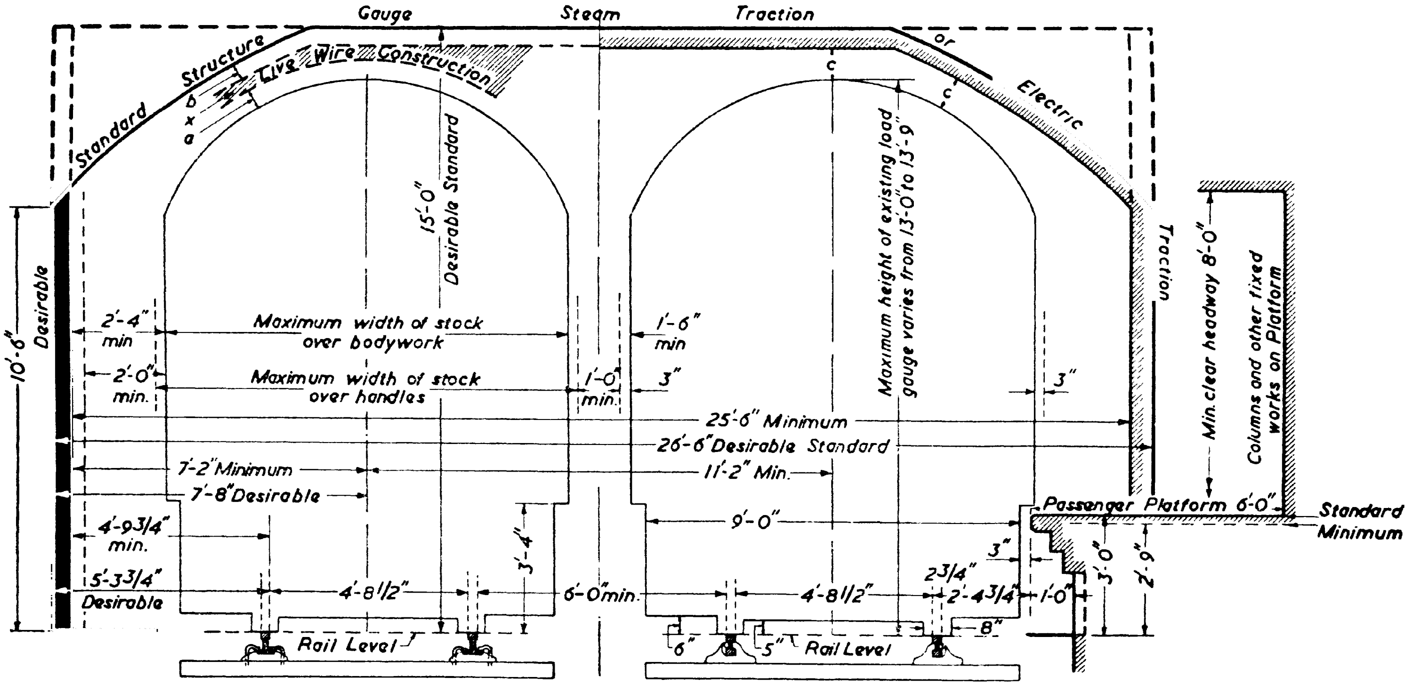
In this table:

SOLID PROTECTION means casing which is bedded close up to the steel without any intervening cavities and with all joints in that casing made full and solid.

HOLLOW PROTECTION means that there is a void between the protective material and the steel. All hollow protection to columns shall be effectively sealed at each floor level.

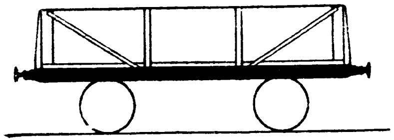
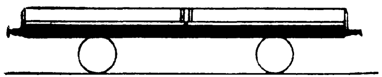
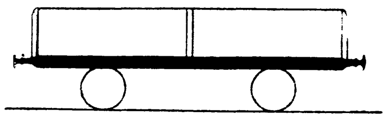
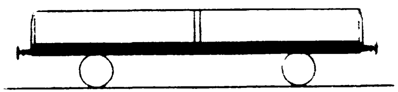
REINFORCEMENT. Where reinforcement is required in this table, that reinforcement shall consist of steel binding wire not less than No. 13 S.W.G. in thickness, or a steel mesh weighing not less than 1 lb. per sq. yd. In the case of concrete protection, the spacing of that reinforcement shall not exceed 12 in. in any direction.

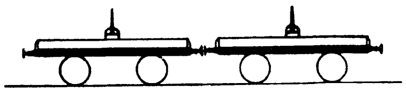
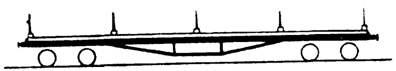
Diagram illustrating lateral and overhead clearances on straight double track in accordance with the Requirements, 1950.



Further particulars can be found in *Requirements for Passenger Lines and Recommendations for Goods Lines of the Minister of Transport in regard to Railway Construction and Operation*, published by H.M.S.O.

a = clearance between load gauge and underside of conductor = 10 in. desirable standard = 4 in. absolute minimum.
b = clearance between structure and upper side of live equipment = 6 in. desirable standard = 4 in. absolute minimum.
c = overhead clearance between load gauge and structure for steam traction or electric traction without overhead equipment = 12 in. desirable standard = 6 in. absolute minimum.
x = depth of live-wire construction and flexibility.

Type of Wagon		Capacity in Tons	Internal Dimensions			Height of Floor from Rail
			Length	Width	Height	
	Open Goods Steel Body Wooden Body	 13 13	 17 ft. $5\frac{1}{2}$ in. 17 ft. $1\frac{1}{4}$ in.	 7 ft. $11\frac{3}{8}$ in. 7 ft. $7\frac{1}{4}$ in.	 3 ft. $2\frac{13}{16}$ in. 3 ft. $1\frac{3}{4}$ in.	 4 ft. $0\frac{1}{8}$ in. 4 ft. $0\frac{1}{4}$ in.
	Plate	22	27 ft. 0 in.	8 ft. $4\frac{3}{8}$ in.	1 ft. $2\frac{3}{8}$ in.	4 ft. $2\frac{3}{4}$ in.
	Tube	22	31 ft. $7\frac{1}{4}$ in.	7 ft. $7\frac{1}{4}$ in.	3 ft. $1\frac{3}{4}$ in.	4 ft. $0\frac{13}{16}$ in.
	Pipe	12	21 ft. $1\frac{1}{4}$ in.	7 ft. $11\frac{3}{4}$ in.	3 ft. $1\frac{3}{4}$ in.	4 ft. $0\frac{1}{2}$ in.
	Low Machine (Well Wagon)	25	30 ft. 0 in.	8 ft. $6\frac{3}{4}$ in.	16 ft. 0 in.	2 ft. $3\frac{3}{4}$ in.

Type of Wagon		Capacity in Tons	Length	Width between Stanchions	Height of Bolster from Rail Level
	Single Bolster	13	15 ft. 5⅜ in.	6 ft. 8¼ in.	5 ft. 3¼ in.
	Bogie Bolster C	30	45 ft. 0 in.	7 ft. 7¾ in.	4 ft. 11 in.
	Bogie Bolster D	42	52 ft. 0 in.	7 ft. 3 in.	5 ft. 8¼ in.

47. COLD-ROLLED SECTIONS

Structural Uses

Cold-rolled or cold-formed sections are made from steel strip, the raw material being supplied in coils of long length and uniform thickness. The main prerequisite of the material for cold forming is ductility, both mild and alloy steels being suitable media.

Light steel structures have been developed from cold-formed sections, used either alone or in conjunction with the heavier hot-rolled B.S. sections, the most usual components being roof trusses, purlins, bracings and floors units. For structural work the sections are cold formed from hot-rolled, pickled and oiled strip which complies with B.S. 1449 (including amendments 1 and 2 for EN. 2C/2, 24-ton steel).

The sections are rolled or formed by feeding strip into one end of a rolling machine where it passes through a series of specially shaped rollers which form it in easy stages from a flat strip to a finished section. There is virtually no limit to the variety of shapes that can be rolled, a consideration which affords the designer a wide choice, resulting in increased efficiency and substantial overall economies in the weight of steel used. Basic profiles are angles, channels, box channels, top hats, tees and zeds. Some guidance on the optimum proportions of the basic types to resist bending, tension and compression is given in the references listed in the bibliography.

Tables giving the geometrical properties of certain standard profiles appear on pages 830 to 837.

The composite sections, such as channels placed back to back, lend themselves readily to axial loads, while the box sections have considerable torsional strength. All types of resistance welding (including spot welding), pop riveting and self-tapping screws have been employed in their assembly.

One advantage of cold-rolled sections from the fabrication aspect is that sections can be formed to push fit inside one another. For simple girders or trusses top-hat boom or rafter sections can be used in conjunction with internal members which fit inside them, enabling node connections to be made without introducing gusset plates, thus eliminating the chances of eccentric loading and secondary bending.

Protective Treatment

Protective treatments for cold-rolled sections are described in PD 420/1953, *Methods of Protection against Corrosion for Light-Gauge Steel used in Buildings*, published by the British Standards Institution.

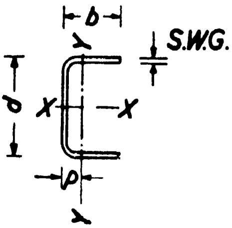
Cold-rolled Sections Association

The foregoing information and the tables appearing at the end of this section are reproduced by courtesy of the Cold Rolled Sections Association,

c/o Frank Impey & Co., Great Charles St., Birmingham, 3, to whom all inquiries should be addressed.

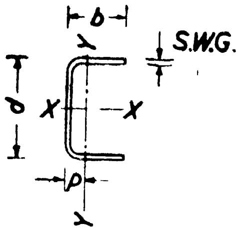
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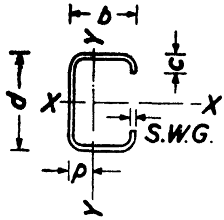
COLD-ROLLED SECTIONS
PLAIN CHANNELS
DIMENSIONS AND PROPERTIES
IN INCH UNITS

Size		S.W.G. No.	Weight per ft. in lb.	Area	Moments of Inertia		Section Moduli		Radii of Gyration		Value of p
d × b					About XX	About YY	About XX	About YY	About XX	About YY	
6	3	8 10	6·386 5·033	1·881 1·482	10·696 8·457	1·659 1·339	3·565 2·819	0·787 0·611	2·385 2·389	0·939 0·951	0·808 0·809
5	3	8 10	5·843 4·599	1·721 1·354	7·030 5·568	1·567 1·261	2·812 2·227	0·738 0·595	2·021 2·028	0·954 0 965	0·876 0·879
5	2½	8 10 12	5·300 4·164 3·411	1·561 1·226 1·004	6·093 4·809 3·985	0·944 0·764 0·631	2·437 1·923 1·594	0·520 0·420 0·345	1·976 1·980 1·992	0·778 0·789 0·792	0·684 0·684 0·673
4	2½	10 12	3·729 3·058	1·098 0·900	2·877 2·392	0·709 0·586	1·439 1·196	0·406 0·334	1·619 1·630	0·803 0·807	0·756 0·744
4	2	10 12 14	3·295 2·705 2·102	0·970 0·796 0·619	2·397 1·997 1·575	0·383 0·317 0·249	1·199 0·999 0·787	0·265 0·218 0·170	1·572 1·583 1·595	0·628 0·631 0·634	0·559 0·548 0·537
3	2	12 14	2·352 1·830	0·692 0·539	1·031 0·817	0·288 0·226	0·687 0·545	0·209 0·163	1·220 1·231	0·644 0·648	0·622 0·610
3	1½	12 14 16	1·998 1·559 1·258	0·588 0·459 0·371	0·813 0·646 0·529	0·130 0·103 0·084	0·542 0·431 0·352	0·120 0·094 0·076	1·175 1·187 1·194	0·470 0·473 0·475	0·423 0·412 0·404
2	1½	14 16	1·287 1·041	0·379 0·307	0·255 0·210	0·089 0·073	0·255 0·210	0·088 0·072	0·820 0·827	0·485 0·488	0·490 0·482
2	1	14 16 18	1·015 0·824 0·626	0·299 0·243 0·184	0·181 0·150 0·116	0·029 0·024 0·018	0·181 0·150 0·116	0·041 0·033 0·025	0·778 0·786 0·794	0·312 0·314 0·314	0·287 0·279 0·271
1½	1	16 18	0·716 0·545	0·211 0·160	0·077 0·060	0·022 0·016	0·103 0·080	0·032 0·024	0·605 0·612	0·321 0·320	0·317 0·308



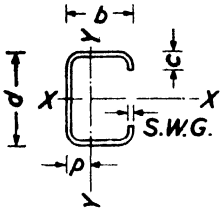
COLD-ROLLED SECTIONS
PLAIN CHANNELS
DIMENSIONS AND PROPERTIES
IN INCH UNITS

Size		S.W.G. No.	Weight per ft. in lb.	Area	Moments of Inertia		Section Moduli		Radii of Gyration		Value of p
d × b					About XX	About YY	About XX	About YY	About XX	About YY	
4	4	8	6·235	1·836	5·256	3·131	2·628	1·213	1·692	1·306	1·419
		10	5·033	1·482	4·317	2·550	2·159	0·982	1·707	1·312	1·402
3½	3½	8	5·419	1·596	3·456	2·071	1·975	0·922	1·472	1·139	1·253
		10	4·381	1·290	2·850	1·692	1·628	0·747	1·486	1·145	1·236
3	3	8	4·604	1·356	2·122	1·282	1·414	0·670	1·251	0·972	1·086
		10	3·729	1·098	1·759	1·051	1·173	0·544	1·266	0·978	1·069
		12	3·058	0·900	1·467	0·869	0·978	0·447	1·276	0·982	1·056
2½	2½	8	3·789	1·116	1·185	0·724	0·948	0·458	1·030	0·805	0·920
		10	3·077	0·906	0·990	0·597	0·792	0·374	1·045	0·812	0·903
		12	2·528	0·744	0·830	0·495	0·664	0·307	1·056	0·816	0·889
		14	1·966	0·579	0·659	0·389	0·527	0·240	1·067	0·820	0·876
2	2	8	2·974	0·876	0·574	0·357	0·574	0·286	0·810	0·638	0·754
		10	2·425	0·714	0·486	0·297	0·486	0·235	0·825	0·645	0·736
		12	1·998	0·588	0·411	0·248	0·411	0·194	0·836	0·649	0·723
		14	1·559	0·459	0·329	0·196	0·329	0·152	0·846	0·653	0·710
		16	1·258	0·371	0·270	0·159	0·270	0·123	0·853	0·656	0·701
1½	1½	10	1·773	0·522	0·202	0·119	0·269	0·128	0·622	0·478	0·570
		12	1·468	0·432	0·164	0·100	0·218	0·106	0·615	0·482	0·556
		14	1·151	0·339	0·133	0·080	0·177	0·084	0·626	0·486	0·543
		16	0·932	0·275	0·110	0·066	0·147	0·068	0·633	0·489	0·535
		18	0·708	0·208	0·085	0·050	0·114	0·051	0·640	0·490	0·525
1	1	14	0·744	0·219	0·036	0·022	0·072	0·036	0·405	0·320	0·377
		16	0·606	0·179	0·030	0·019	0·061	0·029	0·412	0·322	0·368
		18	0·463	0·136	0·024	0·014	0·048	0·022	0·420	0·322	0·358



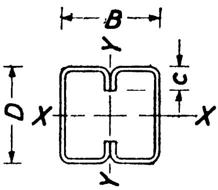
COLD-ROLLED SECTIONS
LIPPED CHANNELS
DIMENSIONS AND PROPERTIES
IN INCH UNITS

Size			S.W.G. No.	Weight per ft. in lb.	Area	Moments of Inertia		Section Moduli		Radii of Gyration		Value of p
d	b	c				About XX	About YY	About XX	About YY	About XX	About YY	
6	4	½	8	7·883	2·321	14·268	4·484	4·756	1·685	2·479	1·390	1·338
		½	10	6·154	1·812	11·165	3·468	3·722	1·292	2·482	1·383	1·316
		½	12	5·056	1·489	9·269	2·910	3·090	1·086	2·495	1·398	1·320
6	3	½	8	6·796	2·001	11·539	2·142	3·846	1·036	2·401	1·035	0·932
		½	10	5·285	1·556	8·958	1·656	2·986	0·792	2·399	1·031	0·909
		½	12	4·350	1·281	7·461	1·401	2·487	0·671	2·413	1·046	0·912
5	3	½	8	6·253	1·841	7·585	2·016	3·034	1·011	2·030	1·046	1·006
		½	10	4·880	1·428	5·895	1·556	2·358	0·772	2·032	1·044	0·985
		½	12	3·997	1·177	4·922	1·317	1·969	0·655	2·045	1·058	0·988
5	2½	½	10	4·416	1·300	5·136	0·976	2·054	0·568	1·987	0·866	0·783
		½	12	3·644	1·073	4·298	0·831	1·719	0·485	2·002	0·880	0·785
4	2½	½	10	3·655	1·076	3·067	0·887	1·536	0·530	1·688	0·908	0·827
		½	12	3·290	0·969	2·576	0·769	1·288	0·470	1·631	0·891	0·864
		⅜	14	2·506	0·738	1·998	0·568	0·999	0·339	1·646	0·878	0·824
4	2	½	12	2·937	0·865	2·181	0·440	1·091	0·328	1·588	0·713	0·658
		⅜	14	2·234	0·658	1·691	0·323	0·845	0·234	1·603	0·701	0·621
3	2	½	12	2·584	0·761	1·120	0·396	0·746	0·315	1·213	0·722	0·741
		⅜	14	1·963	0·578	0·876	0·292	0·584	0·225	1·231	0·711	0·701
		⅜	16	1·593	0·469	0·720	0·243	0·480	0·188	1·239	0·720	0·703
3	1½	⅜	14	1·691	0·498	0·705	0·142	0·470	0·141	1·190	0·534	0·493
		⅜	16	1·376	0·405	0·582	0·119	0·388	0·119	1·199	0·543	0·495
2	1½	⅜	14	1·419	0·418	0·275	0·122	0·275	0·133	0·812	0·541	0·580
		⅜	16	1·158	0·341	0·229	0·103	0·229	0·112	0·820	0·549	0·582
		¼	18	0·845	0·249	0·173	0·071	0·173	0·074	0·834	0·535	0·540
2	1	⅜	16	0·941	0·277	0·169	0·038	0·169	0·060	0·782	0·371	0·365
		¼	18	0·682	0·201	0·127	0·026	0·127	0·039	0·796	0·359	0·329



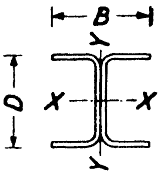
COLD-ROLLED SECTIONS
LIPPED CHANNELS
DIMENSIONS AND PROPERTIES
IN INCH UNITS

Size			S.W.G. No.	Weight per ft. in lb.	Area	Moments of Inertia		Section Moduli		Radii of Gyration		Value of p
d	b	c				About XX	About YY	About XX	About YY	About XX	About YY	
4	4	½	8	6·492	1·912	5·434	3·580	2·717	1·442	1·686	1·368	1·518
		½	10	5·285	1·556	4·507	3·000	2·253	1·211	1·702	1·388	1·522
		½	12	4·350	1·281	3·760	2·521	1·880	1·019	1·713	1·403	1·526
		⅜	14	3·321	0·978	2·920	1·885	1·460	0·748	1·728	1·388	1·479
3½	3½	½	8	5·677	1·672	3·579	2·405	2·045	1·119	1·463	1·199	1·350
		½	10	4·633	1·364	2·985	2·028	1·706	0·945	1·479	1·219	1·355
		½	12	3·820	1·125	2·499	1·712	1·428	0·799	1·491	1·234	1·358
		⅜	14	2·913	0·858	1·946	1·277	1·112	0·583	1·506	1·220	1·311
		⅜	16	2·354	0·693	1·589	1·048	0·908	0·479	1·514	1·230	1·314
3	3	½	8	4·862	1·432	2·201	1·518	1·467	0·835	1·240	1·030	1·183
		½	10	3·981	1·172	1·849	1·291	1·233	0·712	1·256	1·049	1·186
		½	12	3·290	0·969	1·556	1·095	1·037	0·605	1·267	1·063	1·189
		⅜	14	2·506	0·738	1·217	0·815	0·811	0·439	1·284	1·051	1·144
		⅜	16	2·028	0·597	0·996	0·672	0·644	0·362	1·292	1·061	1·146
		¼	18	1·497	0·441	0·748	0·478	0·498	0·251	1·302	1·041	1·098
2½	2½	½	10	3·329	0·980	1·043	0·756	0·835	0·510	1·032	0·878	1·018
		½	12	2·761	0·813	0·884	0·647	0·707	0·437	1·043	0·892	1·020
		⅜	14	2·098	0·618	0·696	0·480	0·557	0·315	1·061	0·882	0·976
		⅜	16	1·702	0·501	0·572	0·398	0·458	0·261	1·069	0·891	0·978
		¼	18	1·253	0·369	0·431	0·281	0·345	0·179	1·081	0·873	0·931
2	2	⅜	14	1·691	0·498	0·349	0·252	0·349	0·211	0·837	0·711	0·807
		⅜	16	1·376	0·405	0·289	0·210	0·289	0·176	0·845	0·720	0·809
		¼	18	1·008	0·297	0·219	0·147	0·219	0·119	0·859	0·704	0·763
1½	1½	⅜	16	1·050	0·309	0·119	0·092	0·158	0·107	0·620	0·546	0·638
		¼	18	0·764	0·225	0·091	0·064	0·121	0·071	0·636	0·534	0·595



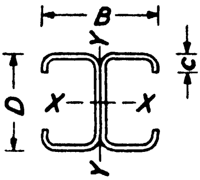
COLD-ROLLED-SECTIONS
LIPPED CHANNELS TOE TO TOE
DIMENSIONS AND PROPERTIES IN INCH UNITS

Size			SWG No.	Weight per ft. in lb.	Area	Moments of Inertia		Section Moduli		Radii of Gyration	
D	B	c				About XX	About YY	About XX	About YY	About XX	About YY
9	4	½	8	14·679	4·323	46·950	11·993	10·433	5·996	3·296	1·666
			10	11·440	3·369	36·064	9·596	8·014	4·798	3·272	1·688
			12	9·406	2·770	30·029	7·942	6·673	3·971	3·293	1·693
7	4	½	8	12·506	3·683	25·503	9·633	7·287	4·817	2·632	1·617
			10	9·701	2·857	19·552	7·676	5·586	3·838	2·616	1·640
			12	7·993	2·354	16·335	6·363	4·667	3·182	2·634	1·644
6	6	½	8	13·593	4·003	23·079	21·395	7·693	7·131	2·401	2·312
			10	10·570	3·113	17·916	16·914	5·972	5·638	2·399	2·331
			12	8·700	2·562	14·922	13·969	4·974	4·656	2·417	2·334
6	4	½	8	11·419	3·363	17·622	8·454	5 874	4·227	2·289	1·586
			10	8·831	2·601	13·502	6·716	4·501	3·358	2·279	1·607
			14	5·691	1·676	8·959	4·379	2·986	2·189	2·312	1·616
5	5	½	10	8·831	2·601	10·271	9·618	4·109	3·847	1·987	1·923
			12	7·287	2·146	8·597	7·970	3·439	3·188	2·002	1·927
			14	5·691	1·676	6·809	6·252	2·723	2·501	2·016	1·931
5	3	½	10	7·093	2·089	7·232	2·993	2·893	1·995	1·861	1·197
			12	5·874	1·730	6·103	2·499	2·441	1·666	1·878	1·202
			14	4·604	1·356	4·872	1·975	1·949	1·317	1·896	1·207
4	4	½	12	5·874	1·730	4·362	3·995	2·181	1·997	1·588	1·520
			14	4·604	1·356	3·479	3·149	1·740	1·574	1·602	1·524
4	3	½	12	5·168	1·522	3·573	2·062	1·786	1·375	1·532	1·164
			14	4·061	1·196	2·865	1·634	1·432	1·089	1·548	1·169
3	3	⅜	14	3·382	0·996	1·410	1·293	0·940	0 862	1·190	1·139
			16	2·751	0·810	1·165	1·057	0·777	0·705	1·199	1·142
2	1	⅜	16	1·447	0·426	0·219	0 0609	0·219	0·122	0·716	0·378
		¼	18	1·038	0·306	0·163	0·0481	0 163	0·096	0·731	0·397



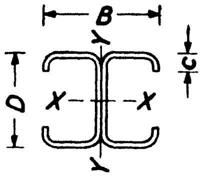
COLD-ROLLED SECTIONS
PLAIN CHANNELS BACK TO BACK
DIMENSIONS AND PROPERTIES
IN INCH UNITS

Size		S.W.G. No.	Weight per ft. in lb.	Area	Moments of Inertia		Section Moduli		Radii of Gyration	
D × B					About XX	About YY	About XX	About YY	About XX	About YY
10	5	8	15·729	4·632	61·295	3·358	12·259	1·343	3·638	0·851
		10	12·675	3·732	49·881	2·681	9·976	1·072	3·656	0·847
		12	10·354	3·049	41·046	2·174	8·209	0·870	3·669	0·844
	4	10	11·805	3·476	43·643	1·379	8·729	0·690	3·543	0·630
		12	9·648	2·841	35·954	1·117	7·191	0·558	3·558	0·627
		14	7·464	2·198	28·040	0·857	5·608	0·428	3·572	0·624
9	5	8	14·642	4·312	55·008	3·355	12·224	1·342	3·572	0·882
		10	11·805	3·476	38·607	2·679	8·579	1·072	3·333	0·878
		12	9·648	2·841	31·798	2·173	7·066	0·869	3·346	0·857
8	4	10	10·067	2·964	25·106	1·376	6·277	0·688	2·910	0·681
		12	8·235	2·425	20·728	1·115	5·182	0·558	2·924	0·678
		14	6·377	1·878	16·201	0·856	4·050	0·428	2·937	0·675
7	4	10	9·197	2·708	18·126	1·375	5·179	0·688	2·587	0·713
		12	7·529	2·217	14·986	1·114	4·282	0·557	2·600	0·709
		14	5·834	1·718	11·729	0·856	3·351	0·428	2·613	0·706
6	4	10	8·328	2·452	12·499	1·374	4·166	0·687	2·258	0·748
		12	6·822	2·009	10·352	1·114	3·451	0·557	2·270	0·745
		14	5·291	1·558	8·117	0·855	2·706	0·428	2·283	0·741
	3	12	6·116	1·801	8·544	0·472	2·848	0·315	2·178	0·512
		14	4·747	1·398	6·715	0·362	2·238	0·241	2·192	0·509
		16	3·821	1·126	5·448	0·289	1·816	0·193	2·201	0·507
5	3	12	5·409	1·593	5·476	0·472	2·191	0·314	1·854	0·544
		14	4·204	1·238	4·315	0·362	1·726	0·241	1·867	0·540
		16	3·386	0·997	3·507	0·289	1·403	0·193	1·876	0·538
	2	14	3·661	1·078	3·347	0·108	1·339	0·108	1·762	0·317
		16	2·951	0·869	2·728	0·086	1·091	0·086	1·772	0·315
		18	2·231	0·657	2·084	0·064	0·834	0·064	1·781	0·313
4	3	12	4·703	1·385	3·205	0·471	1·602	0·314	1·521	0·583
		14	3·661	1·078	2·534	0·361	1·267	0·241	1·533	0·579
		16	2·951	0·869	2·065	0·289	1·032	0·192	1·541	0·576
	2	14	3·117	0·918	1·920	0·108	0·960	0·108	1·446	0·343
		16	2·517	0·741	1·569	0·086	0·785	0·086	1·455	0·341
		18	1·905	0·561	1·202	0·064	0·601	0·064	1·464	0·339



COLD-ROLLED SECTIONS
LIPPED CHANNELS BACK TO BACK
DIMENSIONS AND PROPERTIES
IN INCH UNITS

Size			S.W.G. No.	Weight per ft. in lb.	Area	Moments of Inertia		Section Moduli		Radii of Gyration	
D	B	c				About XX	About YY	About XX	About YY	About XX	About YY
10	6	1	8	18·418	5·423	82·910	11·443	16·582	3·814	3·910	1·453
		¾	12	11·878	3·498	51·078	5·840	10·216	1·947	3·821	1·292
		¾	16	7·424	2·186	32·404	3·759	6·481	1·253	3·850	1·311
	5	¾	8	16·788	4·943	72·441	6·293	14·488	2·517	3·828	1·128
		¾	12	11·172	3·290	45·985	3·614	9·197	1·445	3·739	1·048
		¾	16	6·989	2·058	29·245	2·340	5·849	0·936	3·770	1·066
	4	½	10	12·309	3·625	46·784	1·929	9·357	0·965	3·593	0·730
		½	14	7·865	2·316	30·614	1·308	6·123	0·654	3·636	0·752
		½	18	4·787	1·410	18·924	0·828	3·785	0·414	3·664	0·766
9	6	1	8	17·331	5·103	64·618	11·441	14·360	3·814	3·558	1·497
		¾	12	11·172	3·290	39·822	5·839	8·849	1·946	3·479	1·332
		¾	16	6·989	2·058	25·306	3·759	5·623	1·253	3·507	1·351
	5	¾	8	15·701	4·623	56·263	6·290	12·503	2·516	3·488	1·166
		¾	12	10·466	3·082	35·707	3·613	7·935	1·445	3·404	1·083
		¾	16	6·555	1·930	22·750	2·339	5·056	0·936	3·433	1·101
	4	½	10	11·440	3·369	36·064	1·928	8·014	0·964	3·272	0·757
		½	14	7·321	2·156	23·663	1·308	5·258	0·654	3·313	0·779
		½	18	4·461	1·314	14·653	0·828	3·256	0·414	3·340	0·794
8	6	¾	8	15·701	4·623	47·417	10·074	11·854	3·358	3·203	1·476
		¾	12	10·466	3·082	30·211	5·838	7·553	1·946	3·131	1·376
		¾	16	6·555	1·930	19·234	3·758	4·809	1·253	3·157	1·395
	5	¾	8	14·613	4·303	42·500	6·287	10·625	2·515	3·143	1·209
		¾	12	9·759	2·874	26·969	3·612	6·742	1·445	3·063	1·121
		¾	16	6·120	1·082	17·219	2·339	4·305	0·936	3·091	1·139
	4	½	10	10·570	3·113	27·030	1·926	6·758	0·963	2·947	0·787
		½	14	6·778	1·996	17·791	1·308	4·448	0·654	2·986	0·810
		½	18	4·135	1·218	11·038	0·828	2·760	0·414	3·011	0·825
7	5	¾	8	13·527	3·983	30·991	6·284	8·854	2·514	2·789	1·256
		¾	12	9·053	2·666	19·668	3·611	5·620	1·445	2·716	1·164
		¾	16	5·685	1·674	12·589	2·339	3·597	0·936	2·742	1·182
	4	½	10	9·701	2·857	19·552	1·925	5·586	0·963	2·616	0·821
		½	14	6·235	1·836	12·916	1·307	3·690	0·654	2·652	0·844
		½	18	3·809	1·122	8·032	0·828	2·295	0·414	2·676	0·859
	3	½	10	8·831	2·601	16·529	0·887	4·723	0·592	2·521	0·584
		½	14	5·691	1·676	11·000	0·613	3·143	0·408	2·562	0·605
		½	18	3·483	1·026	6·873	0·392	1·964	0·262	2·589	0·618



COLD-ROLLED SECTIONS
LIPPED CHANNELS BACK TO BACK
DIMENSIONS AND PROPERTIES
IN INCH UNITS

Size D	Size B	Size c	S.W.G. No.	Weight per ft. in lb.	Area	Moments of Inertia About XX	Moments of Inertia About YY	Section Moduli About XX	Section Moduli About YY	Radii of Gyration About XX	Radii of Gyration About YY
6	5	¾	8	12·441	3·663	21·575	6·282	7·192	2·513	2·427	1·309
		¾	12	8·347	2·458	13·701	3·611	4·567	1·444	2·361	1·212
		¾	16	5·251	1·546	8·797	2·339	2·932	0·936	2·385	1·230
	4	½	10	8·831	2·601	13·502	1·924	4·501	0·962	2·279	0·860
		½	14	5·691	1·676	8·959	1·307	2·986	0·653	2·312	0·883
		½	18	3·483	1·026	5·588	0·828	1·863	0·414	2·334	0·898
	3	½	10	7·692	2·345	11·295	0·886	3·765	0·591	2·195	0·615
		½	14	5·148	1·516	7·557	0·612	2·519	0·408	2·233	0·635
		½	18	3·157	0·930	4·737	0·392	1·579	0·262	2·257	0·649
5	4	½	10	7·962	2·344	8·752	1·922	3·501	0·961	1·932	0·905
		½	14	5·148	1·516	5·840	1·307	2·336	0·653	1·963	0·929
		½	18	3·157	0·930	3·656	0·828	1·462	0·414	1·983	0·944
	3	½	10	7·093	2·089	7·232	0·885	2·893	0·590	1·861	0·651
		½	14	4·604	1·356	4·872	0·612	1·949	0·408	1·896	0·672
		½	18	2·831	0·834	3·067	0·392	1·227	0·262	1·918	0·686
	2	⅜	10	6·006	1·769	5·441	0·249	2·176	0·249	1·754	0·375
		¼	14	3·934	1·116	3·542	0·143	1·417	0·143	1·782	0·358
		¼	18	2·342	0·690	2·261	0·096	0·905	0·096	1·811	0·373
4	4	½	10	7·093	2·089	5·174	1·921	2·587	0·960	1·574	0·959
		½	14	4·604	1·356	3·479	1·306	1·740	0·653	1·602	0·981
		½	18	2·831	0·834	2·188	0·828	1·094	0·414	1·620	0·997
	3	½	10	6·223	1·833	4·215	0·883	2·107	0·589	1·517	0·694
		½	14	4·061	1·196	2·865	0·612	1·432	0·408	1·548	0·715
		½	18	2·505	0·738	1·814	0·392	0·907	0·261	1·568	0·729
	2	⅜	10	5·137	1·513	3·098	0·248	1·549	0·248	1·431	0·405
		¼	14	3·246	0·956	2·038	0·142	1·019	0·142	1·460	0·385
		¼	18	2·016	0·594	1·312	0·095	0·656	0·095	1·487	0·400
3	3	½	10	5·354	1·577	2·113	0·882	1·409	0·588	1·158	0·748
		½	14	3·518	1·036	1·455	0·611	0·970	0·407	1·185	0·768
		½	18	2·179	0·642	0·929	0·392	0·619	0·261	1·203	0·782
	2	⅜	10	4·267	1·257	1·512	0·246	1·008	0·246	1·097	0·442
		¼	14	2·703	0·796	1·012	0·142	0·675	0·142	1·128	0·422
		¼	18	1·690	0·498	0·659	0·095	0·439	0·095	1·151	0·437
2	2	⅜	10	3·398	1·001	0·555	0·245	0·555	0·245	0·745	0·495
		¼	14	2·159	0·636	0·384	0·142	0·384	0·142	0·777	0·473
		¼	18	1·364	0·402	0·255	0·095	0·255	0·095	0·797	0·486

The above Tables of Properties of Cold-Rolled Sections for Structural Purposes have been supplied by the Cold Rolled Sections Association, Birmingham, 3.

48. MISCELLANEOUS TABLES

NATURAL SINES

	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	*Mean Differences*				
											1′	2′	3′	4′	5′
0°	0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	0349	0366	0384	0401	0419	0436	0454	0471	0488	0506	3	6	9	12	15
3	0523	0541	0558	0576	0593	0610	0628	0645	0663	0680	3	6	9	12	15
4	0698	0715	0732	0750	0767	0785	0802	0819	0837	0854	3	6	9	12	14
5	0872	0889	0906	0924	0941	0958	0976	0993	1011	1028	3	6	9	12	14
6	1045	1063	1080	1097	1115	1132	1149	1167	1184	1201	3	6	9	12	14
7	1219	1236	1253	1271	1288	1305	1323	1340	1357	1374	3	6	9	12	14
8	1392	1409	1426	1444	1461	1478	1495	1513	1530	1547	3	6	9	12	14
9	1564	1582	1599	1616	1633	1650	1668	1685	1702	1719	3	6	9	12	14
10	1736	1754	1771	1788	1805	1822	1840	1857	1874	1891	3	6	9	12	14
11	1908	1925	1942	1959	1977	1994	2011	2028	2045	2062	3	6	9	11	14
12	2079	2096	2113	2130	2147	2164	2181	2198	2215	2233	3	6	9	11	14
13	2250	2267	2284	2300	2317	2334	2351	2368	2385	2402	3	6	8	11	14
14	2419	2436	2453	2470	2487	2504	2521	2538	2554	2571	3	6	8	11	14
15	2588	2605	2622	2639	2656	2672	2689	2706	2723	2740	3	6	8	11	14
16	2756	2773	2790	2807	2823	2840	2857	2874	2890	2907	3	6	8	11	14
17	2924	2940	2957	2974	2990	3007	3024	3040	3057	3074	3	6	8	11	14
18	3090	3107	3123	3140	3156	3173	3190	3206	3223	3239	3	6	8	11	14
19	3256	3272	3289	3305	3322	3338	3355	3371	3387	3404	3	5	8	11	14
20	3420	3437	3453	3469	3486	3502	3518	3535	3551	3567	3	5	8	11	14
21	3584	3600	3616	3633	3649	3665	3681	3697	3714	3730	3	5	8	11	14
22	3746	3762	3778	3795	3811	3827	3843	3859	3875	3891	3	5	8	11	14
23	3907	3923	3939	3955	3971	3987	4003	4019	4035	4051	3	5	8	11	14
24	4067	4083	4099	4115	4131	4147	4163	4179	4195	4210	3	5	8	11	13
25	4226	4242	4258	4274	4289	4305	4321	4337	4352	4368	3	5	8	11	13
26	4384	4399	4415	4431	4446	4462	4478	4493	4509	4524	3	5	8	10	13
27	4540	4555	4571	4586	4602	4617	4633	4648	4664	4679	3	5	8	10	13
28	4695	4710	4726	4741	4756	4772	4787	4802	4818	4833	3	5	8	10	13
29	4848	4863	4879	4894	4909	4924	4939	4955	4970	4985	3	5	8	10	13
30	5000	5015	5030	5045	5060	5075	5090	5105	5120	5135	3	5	8	10	13
31	5150	5165	5180	5195	5210	5225	5240	5255	5270	5284	2	5	7	10	12
32	5299	5314	5329	5344	5358	5373	5388	5402	5417	5432	2	5	7	10	12
33	5446	5461	5476	5490	5505	5519	5534	5548	5563	5577	2	5	7	10	12
34	5592	5606	5621	5635	5650	5664	5678	5693	5707	5721	2	5	7	10	12
35	5736	5750	5764	5779	5793	5807	5821	5835	5850	5864	2	5	7	10	12
36	5878	5892	5906	5920	5934	5948	5962	5976	5990	6004	2	5	7	9	12
37	6018	6032	6046	6060	6074	6088	6101	6115	6129	6143	2	5	7	9	12
38	6157	6170	6184	6198	6211	6225	6239	6252	6266	6280	2	5	7	9	11
39	6293	6307	6320	6334	6347	6361	6374	6388	6401	6414	2	4	7	9	11
40	6428	6441	6455	6468	6481	6494	6508	6521	6534	6547	2	4	7	9	11
41	6561	6574	6587	6600	6613	6626	6639	6652	6665	6678	2	4	7	9	11
42	6691	6704	6717	6730	6743	6756	6769	6782	6794	6807	2	4	6	9	11
43	6820	6833	6845	6858	6871	6884	6896	6909	6921	6934	2	4	6	8	11
44	6947	6959	6972	6984	6997	7009	7022	7034	7046	7059	2	4	6	8	10

NATURAL SINES

	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	Mean Differences 1′	2′	3′	4′	5′
45°	7071	7083	7096	7108	7120	7133	7145	7157	7169	7181	2	4	6	8	10
46	7193	7206	7218	7230	7242	7254	7266	7278	7290	7302	2	4	6	8	10
47	7314	7325	7337	7349	7361	7373	7385	7396	7408	7420	2	4	6	8	10
48	7431	7443	7455	7466	7478	7490	7501	7513	7524	7536	2	4	6	8	10
49	7547	7559	7570	7581	7593	7604	7615	7627	7638	7649	2	4	6	8	9
50	7660	7672	7683	7694	7705	7716	7727	7738	7749	7760	2	4	6	7	9
51	7771	7782	7793	7804	7815	7826	7837	7848	7859	7869	2	4	5	7	9
52	7880	7891	7902	7912	7923	7934	7944	7955	7965	7976	2	4	5	7	9
53	7986	7997	8007	8018	8028	8039	8049	8059	8070	8080	2	3	5	7	9
54	8090	8100	8111	8121	8131	8141	8151	8161	8171	8181	2	3	5	7	8
55	8192	8202	8211	8221	8231	8241	8251	8261	8271	8281	2	3	5	7	8
56	8290	8300	8310	8320	8329	8339	8348	8358	8368	8377	2	3	5	6	8
57	8387	8396	8406	8415	8425	8434	8443	8453	8462	8471	2	3	5	6	8
58	8480	8490	8499	8508	8517	8526	8536	8545	8554	8563	2	3	5	6	8
59	8572	8581	8590	8599	8607	8616	8625	8634	8643	8652	1	3	4	6	7
60	8660	8669	8678	8686	8695	8704	8712	8721	8729	8738	1	3	4	6	7
61	8746	8755	8763	8771	8780	8788	8796	8805	8813	8821	1	3	4	6	7
62	8829	8838	8846	8854	8862	8870	8878	8886	8894	8902	1	3	4	5	7
63	8910	8918	8926	8934	8942	8949	8957	8965	8973	8980	1	3	4	5	6
64	8988	8996	9003	9011	9018	9026	9033	9041	9048	9056	1	3	4	5	6
65	9063	9070	9078	9085	9092	9100	9107	9114	9121	9128	1	2	4	5	6
66	9135	9143	9150	9157	9164	9171	9178	9184	9191	9198	1	2	3	5	6
67	9205	9212	9219	9225	9232	9239	9245	9252	9259	9265	1	2	3	4	6
68	9272	9278	9285	9291	9298	9304	9311	9317	9323	9330	1	2	3	4	5
69	9336	9342	9348	9354	9361	9367	9373	9379	9385	9391	1	2	3	4	5
70	9397	9403	9409	9415	9421	9426	9432	9438	9444	9449	1	2	3	4	5
71	9455	9461	9466	9472	9478	9483	9489	9494	9500	9505	1	2	3	4	5
72	9511	9516	9521	9527	9532	9537	9542	9548	9553	9558	1	2	3	4	4
73	9563	9568	9573	9578	9583	9588	9593	9598	9603	9608	1	2	2	3	4
74	9613	9617	9622	9627	9632	9636	9641	9646	9650	9655	1	2	2	3	4
75	9659	9664	9668	9673	9677	9681	9686	9690	9694	9699	1	1	2	3	4
76	9703	9707	9711	9715	9720	9724	9728	9732	9736	9740	1	1	2	3	3
77	9744	9748	9751	9755	9759	9763	9767	9770	9774	9778	1	1	2	3	3
78	9781	9785	9789	9792	9796	9799	9803	9806	9810	9813	1	1	2	2	3
79	9816	9820	9823	9826	9829	9833	9836	9839	9842	9845	1	1	2	2	3
80	9848	9851	9854	9857	9860	9863	9866	9869	9871	9874	0	1	1	2	2
81	9877	9880	9882	9885	9888	9890	9893	9895	9898	9900	0	1	1	2	2
82	9903	9905	9907	9910	9912	9914	9917	9919	9921	9923	0	1	1	2	2
83	9925	9928	9930	9932	9934	9936	9938	9940	9942	9943	0	1	1	1	2
84	9945	9947	9949	9951	9952	9954	9956	9957	9959	9960	0	1	1	1	1
85	9962	9963	9965	9966	9968	9969	9971	9972	9973	9974	0	0	1	1	1
86	9976	9977	9978	9979	9980	9981	9982	9983	9984	9985	0	0	1	1	1
87	9986	9987	9988	9989	9990	9990	9991	9992	9993	9993	0	0	0	1	1
88	9994	9995	9995	9996	9996	9997	9997	9997	9998	9998	0	0	0	0	0
89	9998	9999	9999	9999	9999	1·000	1·000	1·000	1·000	1·000	0	0	0	0	0

NATURAL COSINES

N.B.—Subtract mean differences

	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	*Mean Differences*				
											1′	2′	3′	4′	5′
0°	1·000	1·000	1·000	1·000	1·000	1·000	9999	9999	9999	9999	0	0	0	0	0
1	9998	9998	9998	9997	9997	9997	9996	9996	9995	9995	0	0	0	0	0
2	9994	9993	9993	9992	9991	9990	9990	9989	9988	9987	0	0	0	1	1
3	9986	9985	9984	9983	9982	9981	9980	9979	9978	9977	0	0	1	1	1
4	9976	9974	9973	9972	9971	9969	9968	9966	9965	9963	0	0	1	1	1
5	9962	9960	9959	9957	9956	9954	9952	9951	9949	9947	0	1	1	1	2
6	9945	9943	9942	9940	9938	9936	9934	9932	9930	9928	0	1	1	1	2
7	9925	9923	9921	9919	9917	9914	9912	9910	9907	9905	0	1	1	2	2
8	9903	9900	9898	9895	9893	9890	9888	9885	9882	9880	0	1	1	2	2
9	9877	9874	9871	9869	9866	9863	9860	9857	9854	9851	0	1	1	2	2
10	9848	9845	9842	9839	9836	9833	9829	9826	9823	9820	1	1	2	2	3
11	9816	9813	9810	9806	9803	9799	9796	9792	9789	9785	1	1	2	2	3
12	9781	9778	9774	9770	9767	9763	9759	9755	9751	9748	1	1	2	3	3
13	9744	9740	9736	9732	9728	9724	9720	9715	9711	9707	1	1	2	3	3
14	9703	9699	9694	9690	9686	9681	9677	9673	9668	9664	1	1	2	3	4
15	9659	9655	9650	9646	9641	9636	9632	9627	9622	9617	1	2	2	3	4
16	9613	9608	9603	9598	9593	9588	9583	9578	9573	9568	1	2	2	3	4
17	9563	9558	9553	9548	9542	9537	9532	9527	9521	9516	1	2	3	4	4
18	9511	9505	9500	9494	9489	9483	9478	9472	9466	9461	1	2	3	4	5
19	9455	9449	9444	9438	9432	9426	9421	9415	9409	9403	1	2	3	4	5
20	9397	9391	9385	9379	9373	9367	9361	9354	9348	9342	1	2	3	4	5
21	9336	9330	9323	9317	9311	9304	9298	9291	9285	9278	1	2	3	4	5
22	9272	9265	9259	9252	9245	9239	9232	9225	9219	9212	1	2	3	4	6
23	9205	9198	9191	9184	9178	9171	9164	9157	9150	9143	1	2	3	5	6
24	9135	9128	9121	9114	9107	9100	9092	9085	9078	9070	1	2	4	5	6
25	9063	9056	9048	9041	9033	9026	9018	9011	9003	8996	1	3	4	5	6
26	8988	8980	8973	8965	8957	8949	8942	8934	8926	8918	1	3	4	5	6
27	8910	8902	8894	8886	8878	8870	8862	8854	8846	8838	1	3	4	5	7
28	8829	8821	8813	8805	8796	8788	8780	8771	8763	8755	1	3	4	6	7
29	8746	8738	8729	8721	8712	8704	8695	8686	8678	8669	1	3	4	6	7
30	8660	8652	8643	8634	8625	8616	8607	8599	8590	8581	1	3	4	6	7
31	8572	8563	8554	8545	8536	8526	8517	8508	8499	8490	2	3	5	6	8
32	8480	8471	8462	8453	8443	8434	8425	8415	8406	8396	2	3	5	6	8
33	8387	8377	8368	8358	8348	8339	8329	8320	8310	8300	2	3	5	6	8
34	8290	8281	8271	8261	8251	8241	8231	8221	8211	8202	2	3	5	7	8
35	8192	8181	8171	8161	8141	8151	8131	8121	8111	8100	2	3	5	7	8
36	8090	8080	8070	8059	8049	8039	8028	8018	8007	7997	2	3	5	7	9
37	7986	7976	7965	7955	7944	7934	7923	7912	7902	7891	2	4	5	7	9
38	7880	7869	7859	7848	7837	7826	7815	7804	7793	7782	2	4	5	7	9
39	7771	7760	7749	7738	7727	7716	7705	7694	7683	7672	2	4	6	7	9
40	7660	7649	7638	7627	7615	7604	7593	7581	7570	7559	2	4	6	8	9
41	7547	7536	7524	7513	7501	7490	7478	7466	7455	7443	2	4	6	8	10
42	7431	7420	7408	7396	7385	7373	7361	7349	7337	7325	2	4	6	8	10
43	7314	7302	7290	7278	7266	7254	7242	7230	7218	7206	2	4	6	8	10
44	7193	7181	7169	7157	7145	7133	7120	7108	7096	7083	2	4	6	8	10

NATURAL COSINES

N.B.—Subtract mean differences

	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	*Mean Differences*				
											1′	2′	3′	4′	5′
45°	7071	7059	7046	7034	7022	7009	6997	6984	6972	6959	2	4	6	8	10
46	6947	6934	6921	6909	6896	6884	6871	6858	6845	6833	2	4	6	8	11
47	6820	6807	6794	6782	6769	6756	6743	6730	6717	6704	2	4	6	9	11
48	6691	6678	6665	6652	6639	6626	6613	6600	6587	6574	2	4	7	9	11
49	6561	6547	6534	6521	6508	6494	6481	6468	6455	6441	2	4	7	9	11
50	6428	6414	6401	6388	6374	6361	6347	6334	6320	6307	2	4	7	9	11
51	6293	6280	6266	6252	6239	6225	6211	6198	6184	6170	2	5	7	9	11
52	6157	6143	6129	6115	6101	6088	6074	6060	6046	6032	2	5	7	9	12
53	6018	6004	5990	5976	5962	5948	5934	5920	5906	5892	2	5	7	9	12
54	5878	5964	5850	5835	5821	5807	5793	5779	5764	5750	2	5	7	9	12
55	5736	5721	5707	5693	5678	5664	5650	5635	5621	5606	2	5	7	10	12
56	5592	5577	5563	5548	5534	5519	5505	5490	5476	5461	2	5	7	10	12
57	5446	5432	5417	5402	5388	5373	5358	5344	5329	5314	2	5	7	10	12
58	5299	5284	5270	5255	5240	5225	5210	5195	5180	5165	2	5	7	10	12
59	5150	5135	5120	5105	5090	5075	5060	5045	5030	5015	3	5	8	10	13
60	5000	4985	4970	4955	4939	4924	4909	4894	4879	4863	3	5	8	10	13
61	4848	4833	4818	4802	4787	4772	4756	4741	4726	4710	3	5	8	10	13
62	4695	4679	4664	4648	4633	4617	4602	4586	4571	4555	3	5	8	10	13
63	4540	4524	4509	4493	4478	4462	4446	4431	4415	4399	3	5	8	10	13
64	4384	4368	4352	4337	4321	4305	4289	4274	4258	4242	3	5	8	11	13
65	4226	4210	4195	4179	4163	4147	4131	4115	4099	4083	3	5	8	11	13
66	4067	4051	4035	4019	4003	3987	3971	3955	3939	3923	3	5	8	11	14
67	3907	3891	3875	3859	3843	3827	3811	3795	3778	3762	3	5	8	11	14
68	3746	3730	3714	3697	3681	3665	3649	3633	3616	3600	3	5	8	11	14
69	3584	3567	3551	3535	3518	3502	3486	3469	3453	3437	3	5	8	11	14
70	3420	3404	3387	3371	3355	3338	3322	3305	3289	3272	3	5	8	11	14
71	3256	3239	3223	3206	3190	3173	3156	3140	3123	3107	3	6	8	11	14
72	3090	3074	3057	3040	3024	3007	2990	2974	2957	2940	3	6	8	11	14
73	2924	2907	2890	2874	2857	2840	2823	2807	2790	2773	3	6	8	11	14
74	2756	2740	2723	2706	2689	2672	2656	2639	2622	2605	3	6	8	11	14
75	2588	2571	2554	2538	2521	2504	2487	2470	2453	2436	3	6	8	11	14
76	2419	2402	2385	2368	2351	2334	2317	2300	2284	2267	3	6	8	11	14
77	2250	2233	2215	2198	2181	2164	2147	2130	2113	2096	3	6	9	11	14
78	2079	2062	2045	2028	2011	1994	1977	1959	1942	1925	3	6	9	11	14
79	1908	1891	1874	1857	1840	1822	1805	1788	1771	1754	3	6	9	12	14
80	1736	1719	1702	1685	1668	1650	1633	1616	1599	1582	3	6	9	12	14
81	1564	1547	1530	1513	1495	1478	1461	1444	1426	1409	3	6	9	12	14
82	1392	1374	1357	1340	1323	1305	1288	1271	1253	1236	3	6	9	12	14
83	1219	1201	1184	1167	1149	1132	1115	1097	1080	1063	3	6	9	12	14
84	1045	1028	1011	0993	0976	0958	0941	0924	0906	0889	3	6	9	12	14
85	0872	0854	0837	0819	0802	0785	0767	0750	0732	0715	3	6	9	12	14
86	0698	0680	0663	0645	0628	0610	0593	0576	0558	0541	3	6	9	12	15
87	0523	0506	0488	0471	0454	0436	0419	0401	0384	0366	3	6	9	12	15
88	0349	0332	0314	0297	0279	0262	0244	0227	0209	0192	3	6	9	12	15
89	0175	0157	0140	0122	0105	0087	0070	0052	0035	0017	3	6	9	12	15

NATURAL TANGENTS

	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	Mean Differences				
											1′	2′	3′	4′	5′
0°	·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	·0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	·0349	0367	0384	0402	0419	0437	0454	0472	0489	0507	3	6	9	12	15
3	·0524	0542	0559	0577	0594	0612	0629	0647	0664	0682	3	6	9	12	15
4	·0699	0717	0734	0752	0769	0787	0805	0822	0840	0857	3	6	9	12	15
5	·0875	0892	0910	0928	0945	0963	0981	0998	1016	1033	3	6	9	12	15
6	·1051	1069	1086	1104	1122	1139	1157	1175	1192	1210	3	6	9	12	15
7	·1228	1246	1263	1281	1299	1317	1334	1352	1370	1388	3	6	9	12	15
8	·1405	1423	1441	1459	1477	1495	1512	1530	1548	1566	3	6	9	13	15
9	·1584	1602	1620	1638	1655	1673	1691	1709	1727	1745	3	6	9	12	15
10	·1763	1781	1799	1817	1835	1853	1871	1890	1908	1926	3	6	9	12	15
11	·1944	1962	1980	1998	2016	2035	2053	2071	2089	2107	3	6	9	12	15
12	·2126	2144	2162	2180	2199	2217	2235	2254	2272	2290	3	6	9	12	15
13	·2309	2327	2345	2364	2382	2401	2419	2438	2456	2475	3	6	9	12	15
14	·2493	2512	2530	2549	2568	2586	2605	2623	2642	2661	3	6	9	12	16
15	·2679	2698	2717	2736	2754	2773	2792	2811	2830	2849	3	6	9	13	16
16	·2867	2886	2905	2924	2943	2962	2981	3000	3019	3038	3	6	9	13	16
17	·3057	3076	3096	3115	3134	3153	3172	3191	3211	3230	3	6	10	13	16
18	·3249	3269	3288	3307	3327	3346	3365	3385	3404	3424	3	6	10	13	16
19	·3443	3463	3482	3502	3522	3541	3561	3581	3600	3620	3	7	10	13	16
20	·3640	3659	3679	3699	3719	3739	3759	3779	3799	3819	3	7	10	13	17
21	·3839	3859	3879	3899	3919	3939	3959	3979	4000	4020	3	7	10	13	17
22	·4040	4061	4081	4101	4122	4142	4163	4183	4204	4224	3	7	10	14	17
23	·4245	4265	4286	4307	4327	4348	4369	4390	4411	4431	3	7	10	14	17
24	·4452	4473	4494	4515	4536	4557	4578	4599	4621	4642	4	7	11	14	18
25	·4663	4684	4706	4727	4748	4770	4791	4813	4834	4856	4	7	11	14	18
26	·4877	4899	4921	4942	4964	4986	5008	5029	5051	5073	4	7	11	15	18
27	·5095	5117	5139	5161	5184	5206	5228	5250	5272	5295	4	7	11	15	18
28	·5317	5340	5362	5384	5407	5430	5452	5475	5498	5520	4	8	11	15	19
29	·5543	5566	5589	5612	5635	5658	5681	5704	5727	5750	4	8	12	15	19
30	·5774	5797	5820	5844	5867	5890	5914	5938	5961	5895	4	8	12	16	20
31	·6009	6032	6056	6080	6104	6128	6152	6176	6200	6224	4	8	12	16	20
32	·6249	6273	6297	6322	6346	6371	6395	6420	6445	6496	4	8	12	16	20
33	·6494	6519	6544	6569	6594	6619	6644	6669	6694	6720	4	8	13	17	21
34	·6745	6771	6796	6822	6847	6873	6899	6924	6950	6976	4	9	13	17	21
35	·7002	7028	7054	7080	7107	7133	7159	7186	7212	7239	4	9	13	18	22
36	·7265	7292	7319	7346	7373	7400	7427	7454	7481	7508	5	9	14	18	23
37	·7536	7563	7590	7618	7646	7673	7701	7729	7757	7785	5	9	14	18	23
38	·7813	7841	7869	7898	7926	7954	7983	8012	8040	8069	5	9	14	19	24
39	·8098	8127	8156	8185	8214	8243	8273	8302	8332	8361	5	10	15	20	24
40	·8391	8421	8451	8481	8511	8541	8571	8601	8632	8662	5	10	15	20	25
41	·8693	8724	8754	8785	8816	8847	8878	8910	8941	8972	5	10	16	21	26
42	·9004	9036	9067	9099	9131	9163	9195	9228	9260	9293	5	11	16	21	27
43	·9325	9358	9391	9424	9457	9490	9523	9556	9590	9623	6	11	17	22	28
44	·9657	9691	9725	9759	9793	9827	9861	9896	9930	9965	6	11	17	23	29

NATURAL TANGENTS

	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	Mean Differences 1′	2′	3′	4′	5′
45°	1·0000	0035	0070	0105	0141	0176	0212	0247	0283	0319	6	12	18	24	30
46	1·0355	0392	0428	0464	0501	0538	0575	0612	0649	0686	6	12	18	25	31
47	1·0724	0761	0799	0837	0875	0913	0951	0990	1028	1067	6	13	19	25	32
48	1·1106	1145	1184	1224	1263	1303	1343	1383	1423	1463	7	13	20	27	33
49	1·1504	1544	1585	1626	1667	1708	1750	1792	1833	1875	7	14	21	28	34
50	1·1918	1960	2002	2045	2088	2131	2174	2218	2261	2305	7	14	22	29	36
51	1·2349	2393	2437	2482	2527	2572	2617	2662	2708	2753	8	15	23	30	38
52	1·2799	2846	2892	2938	2985	3032	3079	3127	3175	3222	8	16	24	31	39
53	1·3270	3319	3367	3416	3465	3514	3564	3613	3663	3713	8	16	25	33	41
54	1·3764	3814	3865	3916	3968	4019	4071	4124	4176	4229	9	17	26	34	43
55	1·4281	4335	4388	4442	4496	4550	4605	4659	4715	4770	9	18	27	36	45
56	1·4826	4882	4938	4994	5051	5108	5166	5224	5282	5340	10	19	29	38	48
57	1·5399	5458	5517	5577	5637	5697	5757	5818	5880	5941	10	20	30	40	50
58	1·6003	6066	6128	6191	6255	6319	6383	6447	6512	6577	11	21	32	43	53
59	1·6643	6709	6775	6842	6909	6977	7045	7113	7182	7251	11	23	34	45	56
60	1·7321	7391	7461	7532	7603	7675	7747	7820	7893	7966	12	24	36	48	60
61	1·8040	8115	8190	8265	8341	8418	8495	8572	8650	8728	13	26	38	51	64
62	1·8807	8887	8967	9047	9128	9210	9292	9375	9458	9542	14	27	41	55	68
63	1·9626	9711	9797	9883	9970	*0057*	*0145*	*0233*	*0323*	*0413*	15	29	44	58	73
64	2·0503	0594	0686	0778	0872	0965	1060	1155	1251	1348	16	31	47	63	78
65	2·1445	1543	1642	1742	1842	1943	2045	2148	2251	2355	17	34	51	68	85
66	2·2460	2566	2673	2781	2889	2998	3109	3220	3332	3445	18	37	55	73	92
67	2·3559	3673	3789	3906	4023	4142	4262	4383	4504	4627	20	40	60	79	99
68	2·4751	4876	5002	5129	5257	5386	5517	5649	5782	5916	22	43	65	87	108
69	2·6051	6187	6325	6464	6605	6746	6889	7034	7179	7326	24	47	71	95	119
70	2·7475	7625	7776	7929	8083	8239	8397	8556	8716	8878	26	52	78	104	131
71	2·9042	9208	9375	9544	9714	9887	*0061*	*0237*	*0415*	*0595*	29	58	87	116	14[illegible]
72	3·0777	0961	1146	1334	1524	1716	1910	2106	2305	2506	32	64	96	129	161
73	3·2709	2914	3122	3332	3544	3759	3977	4197	4420	4646	36	72	108	144	180
74	3·4874	5105	5339	5576	5816	6059	6305	6554	6806	7062	41	81	122	163	204
75	3·7321	7583	7848	8118	8391	8667	8947	9232	9520	9812					
76	4·0108	0408	0713	1022	1335	1653	1976	2303	2635	2972					
77	4·3315	3662	4015	4374	4737	5107	5483	5864	6252	6646					
78	4·7046	7453	7867	8288	8716	9152	9594	*0045*	*0504*	*0970*					
79	5·1446	1929	2422	2924	3435	3955	4486	5026	5578	6140					
80	5·6713	7297	7894	8502	9124	9758	*0405*	*1066*	*1742*	*2432*					
81	6·3138	3859	4596	5350	6122	6912	7720	8548	9395	*0264*					
82	7·1154	2066	3002	3962	4947	5958	6996	8062	9158	*0285*					
83	8·1443	2636	3863	5126	6427	7769	9152	*0579*	*2052*	*3572*					
84	9·5144	9·677	9·845	10·02	10·20	10·39	10·58	10·78	10·99	11·20					
85	11·43	11·66	11·91	12·16	12·43	12·71	13·00	13·30	13·62	13·95					
86	14·30	14·67	15·06	15·46	15·89	16·35	16·83	17·34	17·89	18·46					
87	19·08	19·74	20·45	21·20	22·02	22·90	23·86	24·90	26·03	27·27					
88	28·64	30·14	31·82	33·69	35·80	38·19	40·92	44·07	47·74	52·08					
89	57·29	63·66	71·62	81·85	95·49	114·6	143·2	191·0	286·5	573·0					

Owing to the rapidity with which the tangent changes, mean differences cease to be useful.

Note that the integer changes where the values are italicised.

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9968	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9812	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

ANTILOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
·09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
·22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

ANTILOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
·51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
·52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
·53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
·54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
·55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
·56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
·57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
·58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
·59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
·61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
·62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
·67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
·68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
·69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
·81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
·82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
·83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
·84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	4	5	7	9	10	12	14	16
·88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
·99	9772	9795	9817	9804	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

CONVERSION TABLES

MOMENTS OF INERTIA

Inches⁴ Units to Centimetres⁴ Units

Based on 1 inch = 25 4 millimetres

in^4	0	1	2	3	4	5	6	7	8	9
	cm^4	cm^4	cm^4	cm^4	cm^4	cm^4	cm^4	cm^4	cm^4	cm^4
1	41 62314256	45 785	49 948	54 110	58 272	62 435	66 597	70 759	74 922	79 084
2	83 24628	87 409	91 571	95 733	99 895	104 058	108 220	112 382	116 545	120 707
3	124 86942	129 032	133 194	137 356	141 519	145 681	149 843	154 006	158 168	162 330
4	166 49257	170 655	174 817	178 979	183 142	187 304	191 466	195 629	199 791	203 953
5	208 11571	212 278	216 440	220 603	224 765	228 927	233 089	237 252	241 414	245 576
6	249 73886	253 901	258 063	262 226	266 388	270 550	274 713	278 875	283 037	287 200
7	291 36200	295 524	299 687	303 849	308 011	312 173	316 336	320 498	324 660	328 883
8	332 98514	337 147	341 310	345 472	349 634	353 797	357 959	362 121	366 284	370 446
9	374 60829	378 771	382 933	387 095	391 257	395 420	399 582	403 744	407 907	412 069
10	416 23143									

Centimetres⁴ Units to Inches⁴ Units

Based on 1 inch = 25 4 millimetres

cm^4	0	1	2	3	4	5	6	7	8	9
	in^4	in^4	in^4	in^4	in^4	in^4	in^4	in^4	in^4	in^4
1	0 02402510	0 02643	0 02883	0 03123	0 03363	0 03604	0 03844	0 04084	0 04324	0 04565
2	0 04805019	0 05045	0 05285	0 05526	0 05766	0 06006	0 06246	0 06487	0 06727	0 06967
3	0 07207529	0 07448	0 07688	0 07928	0 08168	0 08409	0 08649	0 08889	0 09129	0 09370
4	0 09610038	0 09850	0 10091	0 10331	0 10571	0 10811	0 11052	0 11292	0 11532	0 11772
5	0 12012548	0 12253	0 12493	0 12733	0 12974	0 13214	0 13454	0 13694	0 13935	0 14175
6	0 14415058	0 14655	0 14895	0 15136	0 15376	0 15616	0 15856	0 16097	0 16337	0 16577
7	0 16817567	0 17058	0 17298	0 17538	0 17779	0 18019	0 18259	0 18499	0 18740	0 18980
8	0 19220077	0 19460	0 19701	0 19941	0 20181	0 20421	0 20662	0 20902	0 21142	0 21382
9	0 21622586	0 21863	0 22103	0 22343	0 22584	0 22824	0 23064	0 23304	0 23545	0 23785
10	0 24025096									

MODULI OF SECTION

Inches³ Units to Centimetres³ Units

Based on 1 inch = 25 4 millimetres

in^3	0	1	2	3	4	5	6	7	8	9
	cm^3	cm^3	cm^3	cm^3	cm^3	cm^3	cm^3	cm^3	cm^3	cm^3
1	16 387064	18 026	19 664	21 303	22 942	24 580	26 219	27 858	29 497	31 135
2	32 77413	34 413	36 051	37 690	39 329	40 967	42 606	44 245	45 884	47 522
3	49 16119	50 800	52 438	54 077	55 716	57 355	58 993	60 632	62 271	63 909
4	65 54826	67 187	68 826	70 464	72 103	73 742	75 380	77 019	78 658	80 297
5	81 93532	83 574	85 213	86 851	88 490	90 129	91 767	93 406	95 045	96 684
6	98 32238	99 961	101 600	103 238	104 877	106 516	108 154	109 793	111 432	113 071
7	114 70945	116 348	117 987	119 625	121 624	122 903	124 542	126 180	127 819	129 458
8	131 09651	132 735	134 374	136 012	137 651	139 290	140 929	142 567	144 206	145 845
9	147 48358	149 122	150 761	152 400	154 038	155 677	157 316	158 954	160 593	162 232
10	163 87064									

Centimetres³ Units to Inches³ Units

Based on 1 inch = 25 4 millimetres

cm^3	0	1	2	3	4	5	6	7	8	9
	in^3	in^3	in^3	in^3	in^3	in^3	in^3	in^3	in^3	in^3
1	0 061024	0 06713	0 07323	0 07933	0 08543	0 09154	0 09764	0 10374	0 10984	0 11594
2	0 122047	0 12815	0 13425	0 14035	0 14646	0 15256	0 15866	0 16476	0 17087	0 17697
3	0 183071	0 18917	0 19527	0 20138	0 20748	0 21358	0 21968	0 22579	0 23189	0 23799
4	0 244095	0 25020	0 25630	0 26240	0 26850	0 27461	0 28071	0 28681	0 29291	0 29902
5	0 305119	0 31122	0 31732	0 32342	0 32953	0 33563	0 34173	0 34783	0 35394	0 36004
6	0 366142	0 37224	0 37835	0 38445	0 39055	0 39665	0 40276	0 40886	0 41496	0 42106
7	0 427166	0 43327	0 43937	0 44547	0 45158	0 45768	0 46378	0 46988	0 47599	0 48209
8	0 488190	0 49429	0 50040	0 50650	0 51260	0 51870	0 52480	0 53091	0 53701	0 54311
9	0 549213	0 55532	0 56142	0 56752	0 57362	0 57973	0 58583	0 59193	0 59803	0 60414
10	0 610237									

CONVERSION TABLES

Extract from B.S. 350 : 1944

WEIGHTS PER UNIT LENGTH

POUNDS PER FOOT TO KILOGRAMS PER METRE

Based on 1 inch = 25·4 millimetres ; 1 pound = 0·45359243 kilograms

Pounds per ft.	0	1	2	3	4	5	6	7	8	9
	kg./m.	kg./m.	kg./m.	kg./m.	kg./m.	kg./m.	kg./m.	kg./m.	kg./m.	kg./m.
—	—	1·4882	2·9763	4·4645	5·9527	7·4408	8·9290	10·417	11·905	13·393
10	14·882	16·370	17·858	19·346	20·834	22·322	23·811	25·299	26·787	28·275
20	29·763	31·251	32·740	34·228	35·716	37·204	38·692	40·180	41·669	43·157
30	44·645	46·133	47·621	49·109	50·598	52·086	53·574	55·062	56·550	58·038
40	59·527	61·015	62·503	63·991	65·479	66·967	68·456	69·944	71·432	72·920
50	74·408	75·896	77·385	78·873	80·361	81·849	83·337	84·825	86·314	87·802
60	89·290	90·778	92·266	93·754	95·242	96·731	98·219	99·707	101·195	102·683
70	104·171	105·660	107·148	108·636	110·124	111·612	113·100	114·589	116·077	117·565
80	119·053	120·541	122·029	123·518	125·006	126·494	127·982	129·470	130·958	132·447
90	133·935	135·423	136·911	138·399	139·887	141·376	142·864	144·352	145·840	147·328
100	148·816									

KILOGRAMS PER METRE TO POUNDS PER FOOT

Based on 1 inch = 25·4 millimetres ; 1 pound 0·45359243 kilograms

Kg. per m.	0	1	2	3	4	5	6	7	8	9
	lb./ft.	lb./ft.	lb./ft.	lb./ft.	lb./ft.	lb./ft.	lb./ft.	lb./ft.	lb./ft.	lb./ft.
—	—	0·67197	1·3439	2·0159	2·6879	3·3598	4·0318	4·7038	5·3758	6·0477
10	6·7197	7·3917	8·0636	8·7356	9·4076	10·0795	10·7515	11·4235	12·0954	12·7674
20	13·4394	14·1113	14·7833	15·4553	16·1273	16·7992	17·4712	18·1432	18·8151	19·4871
30	20·1591	20·8310	21·5030	22·1750	22·8469	23·5189	24·1909	24·8629	25·5348	26·2068
40	26·8788	27·5507	28·2227	28·8947	29·5666	30·2386	30·9106	31·5825	32·2545	32·9265
50	33·5984	34·2704	34·9424	35·6144	36·2863	36·9583	37·6303	38·3022	38·9742	39·6462
60	40·3181	40·9901	41·6621	42·3340	43·0060	43·6780	44·3500	45·0219	45·6939	46·3659
70	47·0378	47·7098	48·3818	49·0537	49·7257	50·398	51·070	51·742	52·414	53·086
80	53·758	54·429	55·101	55·773	56·445	57·117	57·789	58·461	59·133	59·805
90	60·477	61·149	61·821	62·493	63·165	63·837	64·509	65·181	65·853	66·525
100	67·197	67·869	68·541	69·213	69·885	70·557	71·229	71·901	72·573	73·245
110	73·917	74·589	75·261	75·932	76·604	77·276	77·948	78·620	79·292	79·964
120	80·636	81·308	81·980	82·652	83·324	83·996	84·668	85·340	86·012	86·684
130	87·356	88·028	88·700	89·372	90·044	90·716	91·388	92·060	92·732	93·404
140	94·076	94·748	95·420	96·092	96·764	97·436	98·107	98·779	99·451	100·123

CONVERSION TABLES

Extract from B S 350 1944

WEIGHTS

POUNDS TO KILOGRAMS

Based on 1 pound = 0 45359243 kilograms

lb.	0	1	2	3	4	5	6	7	8	9
	kg	kg	kg	kg	kg	kg	kg	kg	kg	kg
—	—	0 45359	0 90718	1 36078	1 81437	2 26796	2 72155	3 17515	3 62874	4 08233
10	4 53592	4 98952	5 4431	5 8967	6 3503	6 8039	7 2575	7 7111	8 1647	8 6183
20	9 0718	9 5254	9 9790	10 4326	10 8862	11 3398	11 7934	12 2470	12 7006	13 1542
30	13 6078	14 0614	14 5150	14 9686	15 4221	15 8757	16 3293	16 7829	17 2365	17 6901
40	18 1437	18 5973	19 0509	19 5045	19 9581	20 4117	20 8653	21 3188	21 7724	22 2260
50	22 6796	23 1332	23 5868	24 0404	24 4940	24 9476	25 4012	25 8548	26 3084	26 7620
60	27 2155	27 6691	28 1227	28 5763	29 0299	29 4835	29 9371	30 3907	30 8443	31 2979
70	31 7515	32 2051	32 6587	33 1122	33 5658	34 0194	34 4730	34 9266	35 3802	35 8338
80	36 2874	36 7410	37 1946	37 6482	38 1018	38 5554	39 0089	39 4625	39 9161	40 3697
90	40 8233	41 2769	41 7305	42 1841	42 6377	43 0913	43 5449	43 9985	44 4521	44 9057
100	45 3592	45 8128	46 2664	46 7200	47 1736	47 6272	48 0808	48 5344	48 9880	49 4416
10	49 8952	50 349	50 802	51 256	51 710	52 163	52 617	53 070	53 524	53 977
20	54 431	54 885	55 338	55 792	56 245	56 699	57 153	57 606	58 060	58 513
30	58 967	59 421	59 874	60 328	60 781	61 235	61 689	62 142	62 596	63 049
40	63 503	63 957	64 410	64 864	65 317	65 771	66 224	66 678	67 132	67 585
50	68 039	68 492	68 946	69 400	69 853	70 307	70 760	71 214	71 668	72 121
60	72 575	73 028	73 482	73 936	74 389	74 843	75 296	75 750	76 204	76 657
70	77 111	77 564	78 018	78 471	78 925	79 379	79 832	80 286	80 739	81 193
80	81 647	82 100	82 554	83 007	83 461	83 915	84 368	84 822	85 275	85 729
90	86 183	86 636	87 090	87 543	87 997	88 451	88 904	89 358	89 811	90 265
200	90 718	91 172	91 626	92 079	92 533	92 986	93 440	93 894	94 347	94 801
10	95 254	95 708	96 162	96 615	97 069	97 522	97 976	98 430	98 883	99 337
20	99 790	100 244	100 698	101 151	101 605	102 058	102 512	102 965	103 419	103 873
30	104 326	104 780	105 233	105 687	106 141	106 594	107 048	107 501	107 955	108 409
40	108 862	109 316	109 769	110 223	110 677	111 130	111 584	112 037	112 491	112 945
50	113 398	113 852	114 305	114 759	115 212	115 666	116 120	116 573	117 027	117 480
60	117 934	118 388	118 841	119 295	119 748	120 202	120 656	121 109	121 563	122 016
70	122 470	122 924	123 377	123 831	124 284	124 738	125 192	125 645	126 099	126 552
80	127 006	127 459	127 913	128 367	128 820	129 274	129 727	130 181	130 635	131 088
90	131 542	131 995	132 449	132 903	133 356	133 810	134 263	134 717	135 171	135 624
300	136 078	136 531	136 985	137 439	137 892	138 346	138 799	139 253	139 706	140 160
10	140 614	141 067	141 521	141 974	142 428	142 882	143 335	143 789	144 242	144 696
20	145 150	145 603	146 057	146 510	146 964	147 418	147 871	148 325	148 778	149 232
30	149 686	150 139	150 593	151 046	151 500	151 953	152 407	152 861	153 314	153 768
40	154 221	154 675	155 129	155 582	156 036	156 489	156 943	157 397	157 850	158 304
50	158 757	159 211	159 665	160 118	160 572	161 025	161 479	161 932	162 386	162 840
60	163 293	163 747	164 200	164 654	165 108	165 561	166 015	166 468	166 922	167 376
70	167 829	168 283	168 736	169 190	169 644	170 097	170 551	171 004	171 458	171 912
80	172 365	172 819	173 272	173 726	174 179	174 633	175 087	175 540	175 994	176 447
90	176 901	177 355	177 808	178 262	178 715	179 169	179 623	180 076	180 530	180 983
400	181 437	181 891	182 344	182 798	183 251	183 705	184 159	184 612	185 066	185 519
10	185 973	186 426	186 880	187 334	187 787	188 241	188 694	189 148	189 602	190 055
20	190 509	190 962	191 416	191 870	192 323	192 777	193 230	193 684	194 138	194 591
30	195 045	195 498	195 952	196 406	196 859	197 313	197 766	198 220	198 673	199 127
40	199 581	200 034	200 488	200 941	201 395	201 849	202 302	202 756	203 209	203 663
50	204 117	204 570	205 024	205 477	205 931	206 385	206 838	207 292	207 745	208 199
60	208 653	209 106	209 560	210 013	210 467	210 920	211 374	211 828	212 281	212 735
70	213 188	213 642	214 096	214 549	215 003	215 456	215 910	216 364	216 817	217 271
80	217 724	218 178	218 632	219 085	219 539	219 992	220 446	220 900	221 353	221 807
90	222 260	222 714	223 167	223 621	224 075	224 528	224 982	225 435	225 889	226 343

CONVERSION TABLES

Extract from B S. 350: 1944

WEIGHTS

POUNDS TO KILOGRAMS (continued)

Based on 1 pound = 0 45359243 kilograms

lb.	0	1	2	3	4	5	6	7	8	9
	kg	kg	kg	kg	kg	kg	kg	kg	kg	kg
500	226 796	227 250	227 703	228 157	228 611	229 064	229 518	229 971	230 425	230 879
10	231 332	231 786	232 239	232 693	233 147	233 600	234 054	234 507	234 961	235 414
20	235 868	236 322	236 775	237 229	237 682	238 136	238 590	239 043	239 497	239 950
30	240 404	240 858	241 311	241 765	242 218	242 672	243 126	243 579	244 033	244 486
40	244 940	245 394	245 847	246 301	246 754	247 208	247 661	248 115	248 569	249 022
50	249 476	249 929	250 383	250 837	251 290	251 744	252 197	252 651	253 105	253 558
60	254 012	254 465	254 919	255 373	255 826	256 280	256 733	257 187	257 640	258 094
70	258 548	259 001	259 455	259 908	260 362	260 816	261 269	261 723	262 176	262 630
80	263 084	263 537	263 991	264 444	264 898	265 352	265 805	266 259	266 712	267 166
90	267 620	268 073	268 527	268 980	269 434	269 887	270 341	270 795	271 248	271 702
600	272 155	272 609	273 063	273 516	273 970	274 423	274 877	275 331	275 784	276 238
10	276 691	277 145	277 599	278 052	278 506	278 959	279 413	279 867	280 320	280 774
20	281 227	281 681	282 134	282 588	283 042	283 495	283 949	284 402	284 856	285 310
30	285 763	286 217	286 670	287 124	287 578	288 031	288 485	288 938	289 392	289 846
40	290 299	290 753	291 206	291 660	292 114	292 567	293 021	293 474	293 928	294 381
50	294 835	295 289	295 742	296 196	296 649	297 103	297 557	298 010	298 464	298 917
60	299 371	299 825	300 278	300 732	301 185	301 639	302 093	302 546	303 000	303 453
70	303 907	304 361	304 814	305 268	305 721	306 175	306 628	307 082	307 536	307 989
80	308 443	308 896	309 350	309 804	310 257	310 711	311 164	311 618	312 072	312 525
90	312 979	313 432	313 886	314 340	314 793	315 247	315 700	316 154	316 608	317 061
700	317 515	317 968	318 422	318 875	319 329	319 783	320 236	320 690	321 143	321 597
10	322 051	322 504	322 958	323 411	323 865	324 319	324 772	325 226	325 679	326 133
20	326 587	327 040	327 494	327 947	328 401	328 855	329 308	329 762	330 215	330 669
30	331 122	331 576	332 030	322 483	332 937	333 390	333 844	334 298	334 751	335 205
40	335 658	336 112	336 566	337 019	337 473	337 926	338 380	338 834	339 287	339 741
50	340 194	340 648	341 101	341 555	342 009	342 462	342 916	343 369	343 823	344 277
60	344 730	345 184	345 637	346 091	346 545	346 998	347 452	347 905	348 359	348 813
70	349 266	349 720	350 173	350 627	351 081	351 534	351 988	352 441	352 895	353 349
80	353 802	354 256	354 709	355 163	355 616	356 070	356 524	356 977	357 431	357 884
90	358 338	358 792	359 245	359 699	360 152	360 606	361 060	361 513	361 967	362 420
800	362 874	363 328	363 781	364 235	364 688	365 142	365 595	366 049	366 503	366 956
10	367 410	367 863	368 317	368 771	369 224	369 678	370 131	370 585	371 039	371 492
20	371 946	372 399	372 853	373 307	373 760	374 214	374 667	375 121	375 575	376 028
30	276 482	376 935	377 389	377 842	378 296	378 750	379 203	379 657	380 110	380 564
40	381 018	381 471	381 925	382 378	382 832	383 286	383 739	384·193	384 646	385 100
50	385 554	386 007	386 461	386 914	387 368	387 822	388 275	388 729	389 182	389 636
60	390 089	390 543	390 997	391 450	391 904	392 357	392 811	393 265	393 718	394 172
70	394 625	395 079	395 533	395 986	396 440	396 893	397 347	397 801	398 254	398 708
80	399 161	399 615	400 069	400 522	400 976	401 429	401 883	402 336	402 790	403 244
90	403 697	404 151	404 604	405 058	405 512	405 965	406 419	406 872	407 326	407 780
900	408 233	408 687	409 140	409 594	410 048	410 501	410 955	411 408	411 862	412 316
10	412 769	413 223	413 676	414 130	414 583	415 037	415 491	415 944	416 398	416 851
20	417 305	417 759	418 212	418 666	419 119	419 573	420 027	420 481	420 934	421 387
30	421 841	422 295	422 748	423 202	423 655	424 109	424 563	425 016	425 470	425 923
40	426 377	426 830	427 284	427 738	428 191	428 645	429 098	429 552	430 006	430 459
50	430 913	431 366	431 820	432 274	432 727	433 181	433 634	434 088	434 542	434 995
60	435 449	435 902	436 356	436 810	437 263	437 717	438 170	438 624	439 077	439 531
70	439 985	440 438	440 892	441 345	441 799	442 253	442 706	443 160	443 613	444 067
80	444 521	444 974	445 428	445 881	446 335	446 789	447 242	447 696	448 149	448 603
90	449 057	449 510	449 964	450 417	450 871	451 324	451 778	452 232	452 685	453 139
1000	453 592									

CONVERSION TABLES

Extract from B.S. 350 : 1944

KILOGRAMS TO POUNDS

Based on 1 pound = 0·45359243 kilograms

Kg.	0	1	2	3	4	5	6	7	8	9
	lb	lb	lb	lb	lb	lb	lb	lb	lb	lb.
—	—	2 20462	4 40924	6 6139	8 8185	11 0231	13 2277	15 4324	17 6370	19 8416
10	22 0462	24 2508	26 4555	28 6601	30 8647	33 0693	35 2740	37 4786	39 6832	41 8878
20	44 0924	46 2971	48 5017	50 706	52 911	55 116	57 320	59 525	61 729	63 934
30	66 139	68 343	70 548	72 753	74 957	77 162	79 366	81 571	83 776	85 980
40	88 185	90 390	92 594	94 799	97 003	99 208	101 413	103 617	105 822	108 026
50	110 231	112 436	114 640	116 845	119 050	121 254	123 459	125 663	127 868	130 073
60	132 277	134 482	136 687	138 891	141 096	143 300	145 505	147 710	149 914	152 119
70	154 324	156 528	158 733	160 937	163 142	165 347	167 551	169 756	171 961	174 165
80	176 370	178 574	180 779	182 984	185 188	187 393	189 598	191 802	194 007	196 211
90	198 416	200 621	202 825	205 030	207 235	209 439	211 644	213 848	216 053	218 258
100	220 462	222 667	224 871	227 076	229 281	231 485	233 690	235 895	238 099	240 304
10	242 508	244 713	246 918	249 122	251 327	253 532	255 736	257 941	260 145	262 350
20	264 555	266 759	268 964	271 169	273 373	275 578	277 782	279 987	282 192	284 396
30	286 601	288 806	291 010	293 215	295 419	297 624	299 829	302 033	304 238	306 442
40	308 647	310 852	313 056	315 261	317 466	319 670	321 875	324 079	326 284	328 489
50	330 693	332 898	335 103	337 307	339 512	341 716	343 921	346 126	348 330	350 535
60	352 740	354 944	357 149	359 353	361 558	363 763	365 967	368 172	370 377	372 581
70	374 786	376 990	379 195	381 400	383 604	385 809	388 014	390 218	392 423	394 627
80	396 832	399 037	401 241	403 446	405 651	407 855	410 060	412 264	414 469	416 674
90	418 878	421 083	423 288	425 492	427 697	429 901	432 106	434 311	436 515	438 720
200	440 924	443 129	445 334	447 538	449 743	451 948	454 152	456 357	458 561	460 766
10	462 971	465 175	467 380	469 585	471 789	473 994	476 198	478 403	480 608	482 812
20	485 017	487 222	489 426	491 631	493 835	496 040	498 245	500 45	502 65	504 86
30	507 06	509 27	511 47	513 68	515 88	518 09	520 29	522 50	524 70	526 90
40	529 11	531 31	533 52	535 72	537 93	540 13	542 34	544 54	546 75	548 95
50	551 16	553 36	555 56	557 77	559 97	562 18	564 38	566 59	568 79	571 00
60	573 20	575 41	577 61	579 82	582 02	584 22	586 43	588 63	590 84	593 04
70	595 25	597 45	599 66	601 86	604 07	606 27	608 48	610 68	612 89	615 09
80	617 29	619 50	621 70	623 91	626 11	628 32	630 52	632 73	634 93	637 14
90	639 34	641 55	643 75	645 95	648 16	650 36	652 57	654 77	656 98	659 18
300	661 39	663 59	665 80	668 00	670 21	672 41	674 61	676 82	679 02	681 23
10	683 43	685 64	687 84	690 05	692 25	694 46	696 66	698 87	701 07	703 27
20	705 48	707 68	709 89	712 09	714 30	716 50	718 71	720 91	723 12	725 32
30	727 53	729 73	731 93	734 14	736 34	738 55	740 75	742 96	745 16	747 37
40	749 57	751 78	753 98	756 19	758 39	760 59	762 80	765 00	767 21	769 41
50	771 62	773 82	776 03	778 23	780 44	782 64	784 85	787 05	789 25	791 46
60	793 66	795 87	798 07	800 28	802 48	804 69	806 89	809 10	811 30	813 51
70	815 71	817 91	820 12	822 32	824 53	826 73	828 94	831 14	833 35	835 55
80	837 76	839 96	842 17	844 37	846 57	848 78	850 98	853 19	855 39	857 60
90	859 80	862 01	864 21	866 42	868 62	870 83	873 03	875 24	877 44	879 64
400	881 85	884 05	886 26	888 46	890 67	892 87	895 08	897 28	899 49	901 69
10	903 90	906 10	908 30	910 51	912 71	914 92	917 12	919 33	921 53	923 74
20	925 94	928 15	930 35	932 56	934 76	936 96	939 17	941 37	943 58	945 78
30	947 99	950 19	952 40	954 60	956 81	959 01	961 22	963 42	965 62	967 83
40	970 03	972 24	974 44	976 65	978 85	981 06	983 26	985 47	987 67	989 88
50	992 08	994 28	996 49	998 69	1000 90	1003 10	1005 31	1007 51	1009 72	1011 92
60	1014 13	1016 33	1018 54	1020 74	1022 94	1025 15	1027 35	1029 56	1031 76	1033 97
70	1036 17	1038 38	1040 58	1042 79	1044 99	1047 20	1049 40	1051 60	1053 81	1056 01
80	1058 22	1060 42	1062 63	1064 83	1067 04	1069 24	1071 45	1073 65	1075 86	1078 06
90	1080 26	1082 47	1084 67	1086 88	1089 08	1091 29	1093 49	1095 70	1097 90	1100 11

CONVERSION TABLES

Extract from B.S. 350 : 1944

KILOGRAMS TO POUNDS (continued)

Based on 1 pound = 0·45359243 kilograms

Kg.	0	1	2	3	4	5	6	7	8	9
	lb	lb	lb	lb	lb	lb	lb	lb	lb	lb
500	1102 31	1104 52	1106 72	1108 93	1111 13	1113 33	1115 54	1117 74	1119 95	1122 15
10	1124 36	1126 56	1128 77	1130 97	1133 18	1135 38	1137 59	1139 79	1141 99	1144 20
20	1146 40	1148 61	1150 81	1153 02	1155 22	1157 43	1159 63	1161 84	1164 04	1166 25
30	1168 45	1170 65	1172 86	1175 06	1177 27	1179 47	1181 68	1183 88	1186 09	1188 29
40	1190 50	1192 70	1194 91	1197 11	1199 31	1201 52	1203 72	1205 93	1208 13	1210 34
50	1212 54	1214 75	1216 95	1219 16	1221 36	1223 57	1225 77	1227 97	1230 18	1232 38
60	1234 59	1236 79	1239 00	1241 20	1243 41	1245 61	1247 82	1250 02	1252 23	1254 43
70	1256 63	1258 84	1261 04	1263 25	1265 45	1267 66	1269 86	1272 07	1274 27	1276 48
80	1278 68	1280 89	1283 09	1285 29	1287 50	1289 70	1291 91	1294 11	1296 32	1298 52
90	1300 73	1302 93	1305 14	1307 34	1309 55	1311 75	1313 95	1316 16	1318 36	1320 57
600	1322 77	1324 98	1327 18	1329 39	1331 59	1333 80	1336 00	1338 21	1340 41	1342 62
10	1344 82	1347 02	1349 23	1351 43	1353 64	1355 84	1358 05	1360 25	1362 46	1364 66
20	1366 87	1369 07	1371 28	1373 48	1375 68	1377 89	1380 09	1382 30	1384 50	1386 71
30	1388 91	1391 12	1393 32	1395 53	1397 73	1399 94	1402 14	1404 34	1406 55	1408 75
40	1410 96	1413 16	1415 37	1417 57	1419 78	1421 98	1424 19	1426 39	1428 60	1430 80
50	1433 00	1435 21	1437 41	1439 62	1441 82	1444 03	1446 23	1448 44	1450 64	1452 85
60	1455 05	1457 26	1459 46	1461 66	1463 87	1466 07	1468 28	1470 48	1472 69	1474 89
70	1477 10	1479 30	1481 51	1483 71	1485 92	1488 12	1490 32	1492 53	1494 73	1496 94
80	1499 14	1501 35	1503 55	1505 76	1507 96	1510 17	1512 37	1514 58	1516 78	1518 98
90	1521 19	1523 39	1525 60	1527 80	1530 01	1532 21	1534 42	1536 62	1538 83	1541 03
700	1543 24	1545 44	1547 64	1549 85	1552 05	1554 26	1556 46	1558 67	1560 87	1563 08
10	1565 28	1567 49	1569 69	1571 90	1574 10	1576 30	1578 51	1580 71	1582 92	1585 12
20	1587 33	1589 53	1591 74	1593 94	1596 15	1598 35	1600 56	1602 76	1604 97	1607 17
30	1609 37	1611 58	1613 78	1615 99	1618 19	1620 40	1622 60	1624 81	1627 01	1629 22
40	1631 42	1633 63	1635 83	1638 03	1640 24	1642 44	1644 65	1646 85	1649 06	1651 26
50	1653 47	1655 67	1657 88	1660 08	1662 29	1664 49	1666 69	1668 90	1671 10	1673 31
60	1675 51	1677 72	1679 92	1682 13	1684 33	1686 54	1688 74	1690 95	1693 15	1695 35
70	1697 56	1699 76	1701 97	1704 17	1706 38	1708 58	1710 79	1712 99	1715 20	1717 40
80	1719 61	1721 81	1724 01	1726 22	1728 42	1730 63	1732 83	1735 04	1737 24	1739 45
90	1741 65	1743 86	1746 06	1748 27	1750 47	1752 67	1754 88	1757 08	1759 29	1761 49
800	1763 70	1765 90	1768 11	1770 31	1772 52	1774 72	1776 93	1779 13	1781 33	1783 54
10	1785 74	1787 95	1790 15	1792 36	1794 56	1796 77	1798 97	1801 18	1803 38	1805 59
20	1807 79	1809 99	1812 20	1814 40	1816 61	1818 81	1821 02	1823 22	1825 43	1827 63
30	1829 84	1832 04	1834 25	1836 45	1838 65	1840 86	1843 06	1845 27	1847 47	1849 68
40	1851 88	1854 09	1856 29	1858 50	1860 70	1862 91	1865 11	1867 32	1869 52	1871 72
50	1873 93	1876 13	1878 34	1880 54	1882 75	1884 95	1887 16	1889 36	1891 57	1893 77
60	1895 98	1898 18	1900 38	1902 59	1904 79	1907 00	1909 20	1911 41	1913 61	1915 82
70	1918 02	1920 23	1922 43	1924 64	1926 84	1929 04	1931 25	1933 45	1935 66	1937 86
80	1940 07	1942 27	1944 48	1946 68	1948 89	1951 09	1953 30	1955 50	1957 70	1959 91
90	1962 11	1964 32	1966 52	1968 73	1970 93	1973 14	1975 34	1977 55	1979 75	1981 96
900	1984 16	1986 36	1988 57	1990 77	1992 98	1995 18	1997 39	1999 59	2001 80	2004 00
10	2006 21	2008 41	2010 62	2012 82	2015 02	2017 23	2019 43	2021 64	2023 84	2026 05
20	2028 25	2030 46	2032 66	2034 87	2037 07	2039 28	2041 48	2043 68	2045 89	2048 09
30	2050 30	2052 50	2054 71	2056 91	2059 12	2061 32	2063 53	2065 73	2067 94	2070 14
40	2072 35	2074 55	2076 75	2078 96	2081 16	2083 37	2085 57	2087 78	2089 98	2092 19
50	2094 39	2096 60	2098 80	2101 01	2103 21	2105 41	2107 62	2109 82	2112 03	2114 23
60	2116 44	2118 64	2120 85	2123 05	2125 26	2127 46	2129 67	2131 87	2134 07	2136 28
70	2138 48	2140 69	2142 89	2145 10	2147 30	2149 51	2151 71	2153 92	2156 12	2158 33
80	2160 53	2162 73	2164 94	2167 14	2169 35	2171 55	2173 76	2175 96	2178 17	2180 37
90	2182 58	2184 78	2186 99	2189 19	2191 39	2193 60	2195 80	2198 01	2200 21	2202 42
1000	2204 62									

CONVERSION TABLES

Extract from B.S. 350 : 1944

FEET AND INCHES TO METRES

Based on 1 inch = 25·4 millimetres

Uncontracted values

Feet	Inches 0	1	2	3	4	5	Differences for sixteenths of an inch	
	m	m	m	m	m	m		m
0	—	0 0254	0 0508	0 0762	0 1016	0 1270		
1	0 3048	0 3302	0 3556	0 3810	0 4064	0 4318		
2	0 6096	0 6350	0 6604	0 6858	0 7112	0 7366	1	0 0016
3	0 9144	0 9398	0 9652	0 9906	1 0160	1 0414		
4	1 2192	1 2446	1 2700	1 2954	1 3208	1 3462		
5	1 5240	1 5494	1 5748	1 6002	1 6256	1 6510	2	0 0032
6	1 8288	1 8542	1 8796	1 9050	1 9304	1 9558		
7	2 1336	2 1590	2 1844	2 2098	2 2352	2 2606		
8	2 4384	2 4638	2 4892	2 5146	2 5400	2 5654	3	0 0048
9	2 7432	2 7686	2 7940	2 8194	2 8448	2 8702		
10	3 0480	3 0734	3 0988	3 1242	3 1496	3 1750		
11	3 3528	3 3782	3 4036	3 4290	3 4544	3 4798	4	0 0064
12	3 6576	3 6830	3 7084	3 7338	3 7592	3 7846		
13	3 9624	3 9878	4 0132	4 0386	4 0640	4 0894		
14	4 2672	4 2926	4 3180	4 3434	4 3688	4 3942	5	0·0079
15	4 5720	4 5974	4 6228	4 6482	4 6736	4 6990		
16	4 8768	4 9022	4 9276	4 9530	4 9784	5 0038		
17	5 1816	5 2070	5 2324	5 2578	5 2832	5 3086	6	0 0095
18	5 4864	5 5118	5 5372	5 5626	5 5880	5 6134		
19	5 7912	5 8166	5 8420	5 8674	5 8928	5 9182		
20	6 0960	6 1214	6 1468	6 1722	6 1976	6 2230	7	0 0111
21	6 4008	6 4262	6 4516	6 4770	6 5024	6 5278		
22	6 7056	6 7310	6 7564	6 7818	6 8072	6 8326		
23	7 0104	7 0358	7 0612	7 0866	7 1120	7 1374	8	0 0127
24	7 3152	7 3406	7 3660	7 3914	7 4168	7 4422		
25	7 6200	7 6454	7 6708	7 6962	7 7216	7 7470		
26	7 9248	7 9502	7 9756	8 0010	8 0264	8 0518	9	0 0143
27	8 2296	8 2550	8 2804	8 3058	8 3312	8 3566		
28	8 5344	8 5598	8 5852	8 6106	8 6360	8 6614		
29	8 8392	8 8646	8 8900	8 9154	8 9408	8 9662	10	0 0159
30	9 1440	9 1694	9 1948	9 2202	9 2456	9 2710		
31	9 4488	9 4742	9 4996	9 5250	9 5504	9 5758		
32	9 7536	9 7790	9 8044	9 8298	9 8552	9 8806	11	0 0175
33	10 0584	10 0838	10 1092	10 1346	10 1600	10 1854		
34	10 3632	10 3886	10 4140	10 4394	10 4648	10 4902		
35	10 6680	10 6934	10 7188	10 7442	10 7696	10 7950	12	0 0190
36	10 9728	10 9982	11 0236	11 0490	11 0744	11 0998		
37	11 2776	11 3030	11 3284	11 3538	11 3792	11 4046		
38	11 5824	11 6078	11 6332	11 6586	11 6840	11 7094	13	0 0206
39	11 8872	11 9126	11 9380	11 9634	11 9888	12 0142		
40	12 1920	12 2174	12 2428	12 2682	12 2936	12 3190		
41	12 4968	12 5222	12 5476	12 5730	12 5984	12 6238	14	0 0222
42	12 8016	12 8270	12 8524	12 8778	12 9032	12 9286		
43	13 1064	13 1318	13 1572	13 1826	13 2080	13 2334		
44	13 4112	13 4366	13 4620	13 4874	13 5128	13 5382	15	0·0238
45	13 7160	13 7414	13 7668	13 7922	13 8176	13 8430		
46	14 0208	14 0462	14 0716	14 0970	14 1224	14 1478		
47	14 3256	14 3510	14 3764	14 4018	14 4272	14 4526		
48	14 6304	14 6558	14 6812	14 7066	14 7320	14 7574		
49	14 9352	14 9606	14 9860	15 0114	15 0368	15 0622		

CONVERSION TABLES

Extract from B.S. 350: 1944

FEET AND INCHES TO METRES

Based on 1 inch = 25·4 millimetres

Uncontracted values

Feet	Inches 6	Inches 7	Inches 8	Inches 9	Inches 10	Inches 11	Differences for sixteenths of an inch	
	m.	m.	m.	m.	m.	m.		m.
0	0·1524	0·1778	0·2032	0·2286	0 2540	0·2794		
1	0·4572	0·4826	0·5080	0·5334	0·5588	0·5842		
2	0·7620	0·7874	0·8128	0·8382	0·8636	0·8890	1	0·0016
3	1·0668	1·0922	1·1176	1·1430	1·1684	1·1938		
4	1·3716	1·3970	1·4224	1·4478	1·4732	1·4986		
5	1·6764	1·7018	2·7272	1·7526	1·7780	1·8034	2	0·0032
6	1·9812	2·0066	2·0320	2·0574	2·0828	2·1082		
7	2·2860	2·3114	2·3368	2·3622	2·3876	2·4130		
8	2·5908	2·6162	2·6416	2·6670	2·6924	2·7178	3	0·0048
9	2·8956	2·9210	2·9464	2·9718	2·9972	3·0226		
10	3·2004	3·2258	3·2512	3·2766	3·3020	3·3274		
11	3·5052	3·5306	3·5560	3·5814	3·6068	3·6322	4	0·0064
12	3·8100	3·8354	3·8608	3·8862	3·9116	3 9370		
13	4·1148	4·1402	4·1656	4·1910	4·2164	4·2418		
14	4·4196	4·4450	4·4704	4·4958	4·5212	4·5466	5	0·0079
15	4·7244	4·7498	4·7752	4·8006	4·8260	4·8514		
16	5·0292	5·0546	5 0800	5·1054	5·1308	5·1562		
17	5·3340	5·3594	5 3848	5·4102	5·4356	5·4610	6	0·0095
18	5·6388	5·6642	5 6896	5·7150	5·7404	5·7658		
19	5·9436	5·9690	5·9944	6·0198	6·0452	6·0706		
20	6·2484	6·2738	6·2992	6·3246	6·3500	6·3754	7	0·0111
21	6·5532	6·5786	6·6040	6·6294	6·6548	6·6802		
22	6·8580	6·8834	6·9088	6·9342	6·9596	6·9850		
23	7·1628	7·1882	7·2136	7·2390	7·2644	7·2898	8	0·0127
24	7·4676	7·4930	7·5184	7·5438	7·5692	7·5946		
25	7·7724	7·7978	7·8232	7·8486	7·8740	7·8994		
26	8·0772	8·1026	8·1280	8·1534	8·1788	8 2042	9	0·0143
27	8 3820	8·4074	8·4328	8·4582	8·4836	8·5090		
28	8·6868	8·7122	8·7376	8·7630	8·7884	8·8138		
29	8·9916	9·0170	9·0424	9 0678	9·0932	9·1186	10	0·0159
30	9·2964	9·3218	9·3472	9 3726	9·3980	9·4234		
31	9·6012	9·6266	9 6520	9 6774	9·7028	9·7282		
32	9·9060	9·9314	9·9568	9·9822	10 0076	10·0330	11	0·0175
33	10·2108	10·2362	10·2616	10·2870	10 3124	10·3378		
34	10·5156	10·5410	10·5664	10·5918	10 6172	10·6426		
35	10·8204	10·8458	10·8712	10 8966	10 9220	10·9474	12	0·0190
36	11·1252	11·1506	11·1760	11·2014	11·2268	11·2522		
37	11·4300	11·4554	11·4808	11 5062	11·5316	11·5570		
38	11·7348	11·7602	11·7856	11·8110	11·8364	11 8618	13	0·0206
39	12·0396	12·0650	12·0904	12·1158	12·1412	12·1666		
40	12·3444	12·3698	12·3952	12·4206	12·4460	12·4714		
41	12·6492	12·6746	12·7000	12·7254	12·7508	12·7762	14	0·0222
42	12·9540	12·9794	13·0048	13·0302	13·0556	13·0810		
43	13·2588	13·2842	13·3096	13·3350	13·3604	13·3858		
44	13·5636	13·5890	13·6144	13·6398	13·6652	13·6906	15	0·0238
45	13·8684	13·8938	13·9192	13·9446	13·9700	13·9954		
46	14·1732	14·1986	14·2240	14·2494	14·2748	14·3002		
47	14·4780	14·5034	14·5288	14·5542	14·5796	14·6050		
48	14·7828	14·8082	14·8336	14·8590	14·8844	14·9098		
49	15·0876	15·1130	15·1384	15·1638	15·1892	15·2146		

CONVERSION TABLES

Extract from B.S. 350 : 1944

FEET AND INCHES TO METRES

Based on 1 inch = 25·4 millimetres

Uncontracted values

Feet	Inches 0	Inches 1	Inches 2	Inches 3	Inches 4	Inches 5	Differences for sixteenths of an inch	
	m.	m.	m.	m.	m.	m.		m.
50	15·2400	15·2654	15·2908	15·3162	15·3416	15·3670		
51	15·5448	15·5702	15·5956	15·6210	15·6464	15·6718		
52	15·8496	15·8750	15·9004	15·9258	15·9512	15·9766	**1**	0·0016
53	16·1544	16·1798	16·2052	16·2306	16·2560	16·2814		
54	16·4592	16·4846	16·5100	16·5354	16·5608	16·5862		
55	16·7640	16·7894	16·8148	16·8402	16·8656	16·8910	2	0·0032
56	17·0688	17·0942	17·1196	17·1450	17·1704	17·1958		
57	17·3736	17·3990	17·4244	17·4498	17·4752	17·5006		
58	17·6784	17·7038	17·7292	17·7546	17·7800	17·8054	3	0·0048
59	17·9832	18·0086	18·0340	18·0594	18·0848	18·1102		
60	18·2880	18·3134	18·3388	18·3642	18·3896	18·4150		
61	18·5928	18·6182	18·6436	18·6690	18·6944	18·7198	4	0·0064
62	18·8976	18·9230	18·9484	18·9738	18·9992	19·0246		
63	19·2024	19·2278	19·2532	19·2786	19·3040	19·3294		
64	19·5072	19·5326	19·5580	19·5834	19·6088	19·6342	5	0·0079
65	19·8120	19·8374	19·8628	19·8882	19·9136	19·9390		
66	20·1168	20·1422	20·1676	20·1930	20·2184	20·2438		
67	20·4216	20·4470	20·4724	20·4978	20·5232	20·5486	6	0·0095
68	20·7264	20·7518	20·7772	20·8026	20·8280	20·8534		
69	21·0312	21·0566	21·0820	21·1074	21·1328	21·1582		
70	21·3360	21·3614	21·3868	21·4122	21·4376	21·4630	7	0·0111
71	21·6408	21·6662	21·6916	21·7170	21·7424	21·7678		
72	21·9456	21·9710	21·9964	22·0218	22·0472	22·0726		
73	22·2504	22·2758	22·3012	22·3266	22·3520	22·3774	8	0·0127
74	22·5552	22·5806	22·6060	22·6314	22·6568	22·6822		
75	22·8600	22·8854	22·9108	22·9362	22·9616	22·9870		
76	23·1648	23·1902	23·2156	23·2410	23·2664	23·2918	9	0·0143
77	23·4696	23·4950	23·5204	23·5458	23·5712	23·5966		
78	23·7744	23·7998	23·8252	23·8506	23·8760	23·9014		
79	24·0792	24·1046	24·1300	24·1554	24·1808	24·2062	10	0·0159
80	24·3840	24·4094	24·4348	24·4602	24·4856	24·5110		
81	24·6888	24·7142	24·7396	24·7650	24·7904	24·8158		
82	24·9936	25·0190	25·0444	25·0698	25·0952	25·1206	11	0·0175
83	25·2984	25·3238	25·3492	25·3746	25·4000	25·4254		
84	25·6032	25·6286	25·6540	25·6794	25·7048	25·7302		
85	25·9080	25·9334	25·9588	25·9842	26·0096	26·0350	12	0·0190
86	26·2128	26·2382	26·2636	26·2890	26·3144	26·3398		
87	26·5176	26·5430	26·5684	26·5938	26·6192	26·6446		
88	26·8224	26·8478	26·8732	26·8986	26·9240	26·9494	13	0·0206
89	27·1272	27·1526	27·1780	27·2034	27·2288	27·2542		
90	27·4320	27·4574	27·4828	27·5082	27·5336	27·5590		
91	27·7368	27·7622	27·7876	27·8130	27·8384	27·8638	14	0·0222
92	28·0416	28·0670	28·0924	28·1178	28·1432	28·1686		
93	28·3464	28·3718	28·3972	28·4226	28·4480	28·4734		
94	28·6512	28·6766	28·7020	28·7274	28·7528	28·7782	15	0·0238
95	28·9560	28·9814	29·0068	29·0322	29·0576	29·0830		
96	29·2608	29·2862	29·3116	29·3370	29·3624	29·3878		
97	29·5656	29·5910	29·6164	29·6418	29·6672	29·6926		
98	29·8704	29·8958	29·9212	29·9466	29·9720	29·9974		
99	30·1752	30·2006	30·2260	30·2514	30·2768	30·3022		
100	30·4800							

CONVERSION TABLES

Extract from B.S. 350 : 1944

FEET AND INCHES TO METRES

Based on 1 inch = 25 4 millimetres

Uncontracted values

Feet	Inches 6	7	8	9	10	11	Differences for sixteenths of an inch	
	m	m	m	m	m	m		m
50	15 3924	15 4178	15 4432	15 4686	15 4940	15 5194		
51	15 6972	15 7226	15 7480	15 7734	15 7988	15 8242		
52	16 0020	16 0274	16 0528	16 0782	16 1036	16 1290	1	0 0016
53	16 3068	16 3322	16 3576	16 3830	16 4084	16 4338		
54	16 6116	16 6370	16 6624	16 6878	16 7132	16 7386		
55	16 9164	16 9418	16 9672	16 9926	17 0180	17 0434	2	0 0032
56	17 2212	17 2466	17 2720	17 2974	17 3228	17 3482		
57	17 5260	17 5514	17 5768	17 6022	17 6276	17 6530		
58	17 8304	17 8562	17 8816	17 9070	17 9324	17 9578	3	0 0048
59	18 1356	18 1610	18 1864	18 2118	18 2372	18 2626		
60	18 4404	18 4658	18 4912	18 5166	18 5420	18 5674		
61	18 7452	18 7706	18 7960	18 8214	18 8468	18 8722	4	0 0064
62	19 0500	19 0754	19 1008	19 1262	19 1516	19 1770		
63	19 3548	19 3802	19 4056	19 4310	19 4564	19 4818		
64	19 6596	19 6850	19 7104	19 7358	19 7612	19 7866	5	0 0079
65	19 9644	19 9898	20 0152	20 0406	20 0660	20 0914		
66	20 2692	20 2946	20 3200	20 3454	20 3708	20 3962		
67	20 5740	20 5994	20 6248	20 6502	20 6756	20 7010	6	0 0095
68	20 8788	20 9042	20 9296	20 9550	20 9804	21 0058		
69	21 1836	21 2090	21 2344	21 2598	21 2852	21 3106		
70	21 4884	21 5138	21 5392	21 5646	21 5900	21 6154	7	0 0111
71	21 7932	21 8186	21 8440	21 8694	21 8948	21 9202		
72	22 0980	22 1234	22 1488	22 1742	22 1996	22 2250		
73	22 4028	22 4282	22 4536	22 4790	22 5044	22 5298	8	0 0127
74	22 7076	22 7330	22 7584	22 7838	22 8092	22 8346		
75	23 0124	23 0378	23 0632	23 0886	23 1140	23 1394		
76	23 3172	23 3426	23 3680	23 3934	23 4188	23 4442	9	0 0143
77	23 6220	23 6474	23 6728	23 6982	23 7236	23 7490		
78	23 9268	23 9522	23 9776	24 0030	24 0284	24 0538		
79	24 2316	24 2570	24 2824	24 3078	24 3332	24 3586	10	0 0159
80	24 5364	24 5618	24 5872	24 6126	24 6380	24 6634		
81	24 8412	24 8666	24 8920	24 9174	24 9428	24 9682		
82	25 1460	25 1714	25 1968	25 2222	25 2476	25 2730	11	0 0175
83	25 4508	25 4762	25 5016	25 5270	25 5524	25 5778		
84	25 7556	25 7810	25 8064	25 8318	25 8572	25 8826		
85	26 0604	26 0858	26 1112	26 1366	26 1620	26 1874	12	0 0190
86	26 3652	26 3906	26 4160	26 4414	26 4668	26 4922		
87	26 6700	26 6954	26 7208	26 7462	26 7716	26 7970		
88	26 9748	27 0002	27 0256	27 0510	27 0764	27 1018	13	0 0206
89	27 2796	27 3050	27 3304	27 3558	27 3812	27 4066		
90	27 5844	27 6098	27 6352	27 6606	27 6860	27 7114		
91	27 8892	27 9146	27 9400	27 9654	27 9908	28 0162	14	0 0222
92	28 1940	28 2194	28 2448	28 2702	28 2956	28 3210		
93	28 4988	28 5242	28 5496	28 5750	28 6004	28 6258		
94	28 8036	28 8290	28 8544	28 8798	28 9052	28 9306	15	0·0238
95	29 1084	29 1338	29 1592	29 1846	29 2100	29 2354		
96	29 4132	29 4386	29 4640	29 4894	29 5148	29 5402		
97	29 7180	29 7434	29 7688	29 7942	29 8196	29 8450		
98	30 0228	30 0482	30 0736	30 0990	30 1244	30 1498		
99	30 3276	30 3530	30 3784	30 4038	30 4292	30 4546		

CONVERSION TABLES

Extract from B.S. 350 : 1944

METRES TO FEET

Based on 1 inch = 25·4 millimetres

m.	0	1	2	3	4	5	6	7	8	9
	ft.	ft.	ft.	ft.	ft.	ft.	ft.	ft.	ft.	ft.
—	—	3·28084	6·5617	9·8425	13·1234	16·4042	19·6850	22·9659	26·2467	29·5276
10	32·8084	36·0892	39·3701	42·6509	45·9317	49·2126	52·493	55·774	59·055	62·336
20	65·617	68·898	72·178	75·459	78·740	82·021	85·302	88·583	91·863	95·144
30	98·425	101·706	104·987	108·268	111·549	114·829	118·110	121·391	124·672	127·953
40	131·234	134·514	137·795	141·076	144·357	147·638	150·919	154·199	157·480	160·761
50	164·042	167·323	170·604	173·884	177·165	180·446	183·727	187·008	190·289	193·570
60	196·850	200·131	203·412	206·693	209·974	213·255	216·535	219·816	223·097	226·378
70	229·659	232·940	236·220	239·501	242·782	246·063	249·344	252·625	255·905	259·186
80	262·467	265·748	269·029	272·310	275·590	278·871	282·152	285·433	288·714	291·995
90	295·276	298·556	301·837	305·118	308·399	311·680	314·961	318·241	321·522	324·803
100	328·084	331·365	334·646	337·926	341·207	344·488	347·769	351·050	354·331	357·611
10	360·892	364·173	367·454	370·735	374·016	377·296	380·577	383·858	387·139	390·420
20	393·701	396·982	400·262	403·543	406·824	410·105	413·386	416·667	419·947	423·228
30	426·509	429·790	433·071	436·352	439·632	442·913	446·194	449·475	452·756	456·037
40	459·317	462 598	465 879	469 160	472·441	475·722	479·002	482·283	485·564	488·845
50	492·126	495·407	498·688	501·97	505·25	508·53	511·81	515·09	518·37	521·65
60	524·93	528·22	531·50	534·78	538·06	541·34	544·62	547·90	551·18	554·46
70	557 74	561·02	564·30	567·59	570·87	574·15	577·43	580·71	583·99	587·27
80	590·55	593·83	597·11	600·39	603·67	606·96	610 24	613·52	616·80	620·08
90	623·36	626·64	629·92	633·20	636·48	639·76	643·04	646·33	649·61	652·89
200	656·17	659·45	662·73	666·01	669·29	672·57	675·85	679·13	682·41	685·70
10	688 98	692·26	695·54	698·82	702·10	705·38	708·66	711·94	715·22	718·50
20	721·78	725·07	728·35	731·63	734·91	738·19	741·47	744·75	748·03	751·31
30	754·59	757·87	761·15	764·44	767·72	771·00	774·28	777·56	780·84	784·12
40	787·40	790·68	793·96	797·24	800·52	803·81	807·09	810·37	813·65	816·93
50	820·21	823·49	826·77	830·05	833·33	836·61	839·89	843·18	846·46	849·74
60	853·02	856·30	859·58	862·86	866·14	869·42	872·70	875·98	879·26	882·55
70	885·83	889·11	892·39	895·67	898·95	902·23	905·51	908·79	912·07	915·35
80	918·63	921·92	925·20	928·48	931·76	935·04	938·32	941·60	944·88	948·16
90	951·44	954·72	958 00	961·29	964·57	967·85	971·13	974·41	977·69	980·97
300	984·25	987·53	990·81	994·09	997·38	1000·66	1003·94	1007·22	1010·50	1013·78
10	1017·06	1020 34	1023·62	1026·90	1030·18	1033·46	1036·75	1040·03	1043·31	1046·59
20	1049·87	1053·15	1056·43	1059·71	1062·99	1066·27	1069·55	1072·83	1076·12	1079·40
30	1082·68	1085·96	1089·24	1092·52	1095·80	1099·08	1102·36	1105·64	1108·92	1112·20
40	1115·49	1118·77	1122·05	1125·33	1128·61	1131·89	1135·17	1138·45	1141·73	1145·01
50	1148·29	1151·57	1154·86	1158·14	1161·42	1164·70	1167·98	1171·26	1174·54	1177·82
60	1181 10	1184·38	1187·66	1190·94	1194 23	1197·51	1200·79	1204·07	1207·35	1210·63
70	1213·91	1217·19	1220·47	1223·75	1227·03	1230·31	1233·60	1236·88	1240·16	1243·44
80	1246·72	1250·00	1253·28	1256·56	1259·84	1263·12	1266·40	1269·68	1272·97	1276·25
90	1279·53	1282·81	1286·09	1289·37	1292·65	1295·93	1299·21	1302·49	1305·77	1309·05
400	1312·34	1315·62	1318·90	1322·18	1325·46	1328·74	1332·02	1335·30	1338·58	1341·86
10	1345·14	1348·42	1351·71	1354·99	1358·27	1361·55	1364·83	1368·11	1371·39	1374·67
20	1377·95	1381·23	1384·51	1387·79	1391·08	1394·36	1397·64	1400·92	1404·20	1407·48
30	1410·76	1414·04	1417·32	1420·60	1423·88	1427·16	1430·45	1433·73	1437·01	1440·29
40	1443·57	1446·85	1450·13	1453·41	1456·69	1459·97	1463·25	1466·54	1469·82	1473·10
50	1476·38	1479·66	1482·94	1486·22	1489·50	1492·78	1496·06	1499·34	1502·62	1505·91
60	1509·19	1512·47	1515·75	1519·03	1522·31	1525·59	1528·87	1532·15	1535·43	1538·71
70	1541·99	1545·28	1548·56	1551·84	1555·12	1558·40	1561·68	1564·96	1568·24	1571·52
80	1574·80	1578·08	1581·36	1584·65	1587·93	1591·21	1594·49	1597·77	1601·05	1604·33
90	1607·61	1610·89	1614·17	1617·45	1620·73	1624·02	1627·30	1630·58	1633·86	1637·14

CONVERSION TABLES

Extract from B.S. 350 : 1944

METRES TO FEET

Based on 1 inch = 25·4 millimetres

m.	0	1	2	3	4	5	6	7	8	9
	ft.	ft.	ft.	ft.	ft.	ft.	ft.	ft.	ft.	ft.
500	1640·42	1643·70	1646·98	1650·26	1653·54	1656·82	1660·10	1663·39	1666·67	1669·95
10	1673·23	1676·51	1679·79	1683·07	1686·35	1689·63	1692·91	1696·19	1699·48	1702·76
20	1706·04	1709·32	1712·60	1715·88	1719·16	1722·44	1725·72	1729·00	1732·28	1735·56
30	1738·84	1742·13	1745·41	1748·69	1751·97	1755·25	1758·53	1761·81	1765·09	1768·37
40	1771·65	1774·93	1778·22	1781·50	1784·78	1788·06	1791·34	1794·62	1797·90	1801·18
50	1804·46	1807·74	1811·02	1814·30	1817·59	1820·87	1824·15	1827·43	1830·71	1833·99
60	1837·27	1840·55	1843·83	1847·11	1850·39	1853·67	1856·96	1860·24	1863·52	1866·80
70	1870·08	1873·36	1876·64	1879·92	1883·20	1886·48	1889·76	1893·04	1896·33	1899·61
80	1902·89	1906·17	1909·45	1912·73	1916·01	1919·29	1922·57	1925·85	1929·13	1932·41
90	1935·70	1938·98	1942·26	1945·54	1948·82	1952·10	1955·38	1958·66	1961·94	1965·22
600	1968·50	1971·78	1975·07	1978·35	1981·63	1984·91	1988·19	1991·47	1994·75	1998·03
10	2001·31	2004·59	2007·87	2011·15	2014·44	2017·72	2021·00	2024·28	2027·56	2030·84
20	2034·12	2037·40	2040·68	2043·96	2047·24	2050·52	2053·81	2057·09	2060·37	2063·65
30	2066·93	2070·21	2073·49	2076·77	2080·05	2083·33	2086·61	2089·90	2093·18	2096·46
40	2099·74	2103·02	2106·30	2109·58	2112·86	2116·14	2119·42	2122·70	2125·98	2129·27
50	2132·55	2135·83	2139·11	2142·39	2145·67	2148·95	2152·23	2155·51	2158·79	2162·07
60	2165·35	2168·63	2171·92	2175·20	2178·48	2181·76	2185·04	2188·32	2191·60	2194·88
70	2198·16	2201·44	2204·72	2208·01	2211·29	2214·57	2217·85	2221·13	2224·41	2227·69
80	2230·97	2234·25	2237·53	2240·81	2244·09	2247·38	2250·66	2253·94	2257·22	2260·50
90	2263·78	2267·06	2270·34	2273·62	2276·90	2280·18	2283·46	2286·75	2290·03	2293·31
700	2296·59	2299·87	2303·15	2306·43	2309·71	2312·99	2316·27	2319·55	2322·83	2326·12
10	2329·40	2332·68	2335·96	2339·24	2342·52	2345·80	2349·08	2352·36	2355·64	2358·92
20	2362·20	2365·49	2368·77	2372·05	2375·33	2378·61	2381·89	2385·17	2388·45	2391·73
30	2395·01	2398·29	2401·57	2404·86	2408·14	2411·42	2414·70	2417·98	2421·26	2424·54
40	2427·82	2431·10	2434·38	2437·66	2440·94	2444·23	2447·51	2450·79	2454·07	2457·35
50	2460·63	2463·91	2467·19	2470·47	2473·75	2477·03	2480·31	2483·60	2486·88	2490·16
60	2493·44	2496·72	2500·00	2503·28	2506·56	2509·84	2513·12	2516·40	2519·69	2522·97
70	2526·25	2529·53	2532·81	2536·09	2539·37	2542·65	2545·93	2549·21	2552·49	2555·77
80	2559·06	2562·34	2565·62	2568·90	2572·18	2575·46	2578·74	2582·02	2585·30	2588·58
90	2591·86	2595·14	2598·43	2601·71	2604·99	2608·27	2611·55	2614·83	2618·11	2621·39
800	2624·67	2627·95	2631·23	2634·51	2637·80	2641·08	2644·36	2647·64	2650·92	2654·20
10	2657·48	2660·76	2664·04	2667·32	2670·60	2673·88	2677·17	2680·45	2683·73	2687·01
20	2690·29	2693·57	2696·85	2700·13	2703·41	2706·69	2709·97	2713·25	2716·54	2719·82
30	2723·10	2726·38	2729·66	2732·94	2736·22	2739·50	2742·78	2746·06	2749·34	2752·62
40	2755·91	2759·19	2762·47	2765·75	2769·03	2772·31	2775·59	2778·87	2782·15	2785·43
50	2788·71	2791·99	2795·28	2798·56	2801·84	2805·12	2808·40	2811·68	2814·96	2818·24
60	2821·52	2824·80	2828·08	2831·36	2834·65	2837·93	2841·21	2844·49	2847·77	2851·05
70	2854·33	2857·61	2860·89	2864·17	2867·45	2870·73	2874·02	2877·30	2880·58	2883·86
80	2887·14	2890·42	2893·70	2896·98	2900·26	2903·54	2906·82	2910·10	2913·39	2916·67
90	2919·95	2923·23	2926·51	2929·79	2933·07	2936·35	2939·63	2942·91	2946·19	2949·48
900	2952·76	2956·04	2959·32	2962·60	2965·88	2969·16	2972·44	2975·72	2979·00	2982·28
10	2985·56	2988·85	2992·13	2995·41	2998·69	3001·97	3005·25	3008·53	3011·81	3015·09
20	3018·37	3021·65	3024·93	3028·22	3031·50	3034·78	3038·06	3041·34	3044·62	3047·90
30	3051·18	3054·46	3057·74	3061·02	3064·30	3067·59	3070·87	3074·15	3077·43	3080·71
40	3083·99	3087·27	3090·55	3093·83	3097·11	3100·39	3103·67	3106·96	3110·24	3113·52
50	3116·80	3120·08	3123·36	3126·64	3129·92	3133·20	3136·48	3139·76	3143·04	3146·33
60	3149·61	3152·89	3156·17	3159·45	3162·73	3166·01	3169·29	3172·57	3175·85	3179·13
70	3182·41	3185·70	3188·98	3192·26	3195·54	3198·82	3202·10	3205·38	3208·66	3211·94
80	3215·22	3218·50	3221·78	3225·07	3228·35	3231·63	3234·91	3238·19	3241·47	3244·75
90	3248·03	3251·31	3254·59	3257·87	3261·15	3264·44	3267·72	3271·00	3274·28	3277·56
1000	3280·84									

CONVERSION TABLES

Extract from B.S. 350 : 1944

SQUARE FEET TO SQUARE METRES

Based on 1 inch - 25 4 millimetres

Sq. ft.	0	1	2	3	4	5	6	7	8	9
	m^2	m^2	m^2	m^2	m^2	m^2	m^2	m^2	m^2	m^2
—	—	0 09290	0 18581	0 27871	0 37161	0 46452	0 55742	0 65032	0 74322	0 83613
10	0 92903	1 02193	1 11484	1 20774	1 30064	1 39355	1 48645	1 57935	1 67225	1 76516
20	1 85806	1 95096	2 04387	2 13677	2 22967	2 32258	2 41548	2 50838	2 60129	2 69419
30	2 78709	2 87999	2 97290	3 06580	3 15870	3 25161	3 34451	3 43741	3 53032	3 62322
40	3 71612	3 80902	3 90193	3 99483	4 08773	4 18064	4 27354	4 36644	4 45935	4 55225
50	4 64515	4 73806	4 83096	4 92386	5 0168	5 1097	5 2026	5 2955	5 3884	5 4813
60	5 5742	5 6671	5 7600	5 8529	5 9458	6 0387	6 1316	6 2245	6 3174	6 4103
70	6 5032	6 5961	6 6890	6 7819	6 8748	6 9677	7 0606	7 1535	7 2464	7 3393
80	7 4322	7 5251	7 6180	7 7110	7 8039	7 8968	7 9897	8 0826	8 1755	8 2684
90	8 3613	8 4542	8 5471	8 6400	8 7329	8 8258	8 9187	9 0116	9 1045	9 1974
100	9 2903	9 3832	9 4761	9 5690	9 6619	9 7548	9 8477	9 9406	10 0335	10 1264
10	10 2193	10 3122	10 4051	10 4980	10 5909	10 6838	10 7768	10 8697	10 9626	11 0555
20	11 1484	11 2413	11 3342	11 4271	11 5200	11 6129	11 7058	11 7987	11 8916	11 9845
30	12 0774	12 1703	12 2632	12 3561	12 4490	12 5419	12 6348	12 7277	12 8206	12 9135
40	13 0064	13 0993	13 1922	13 2851	13 3780	13 4709	13 5638	13 6567	13 7496	13 8426
50	13 9355	14 0284	14 1213	14 2142	14 3071	14 4000	14 4929	14 5858	14 6787	14 7716
60	14 8645	14 9574	15 0503	15 1432	15 2361	15 3290	15 4219	15 5148	15 6077	15 7006
70	15 7935	15 8864	15 9793	16 0722	16 1651	16 2580	16 3509	16 4438	16 5367	16 6296
80	16 7225	16 8155	16 9084	17 0013	17 0942	17 1871	17 2800	17 3729	17 4658	17 5587
90	17 6516	17 7445	17 8374	17 9303	18 0232	18 1161	18 2090	18 3019	18 3948	18 4877
200	18 5806	18 6735	18 7664	18 8593	18 9522	19 0451	19 1380	19 2309	19 3238	19 4167
10	19 5096	19 6025	19 6954	19 7883	19 8813	19 9742	20 0671	20 1600	20 2529	20 3458
20	20 4387	20 5316	20 6245	20 7174	20 8103	20 9032	20 9961	21 0890	21 1819	21 2748
30	21 3677	21 4606	21 5535	21 6464	21 7393	21 8322	21 9251	22 0180	22 1109	22 2038
40	22 2967	22 3896	22 4825	22 5754	22 6683	22 7612	22 8541	22 9471	23 0400	23 1329
50	23 2258	23 3187	23 4116	23 5045	23 5974	23 6903	23 7832	23 8761	23 9690	24 0619
60	24 1548	24 2477	24 3406	24 4335	24 5264	24 6193	24 7122	24 8051	24 8980	24 9909
70	25 0838	25 1767	25 2696	25 3625	25 4554	25 5483	25 6412	25 7341	25 8270	25 9199
80	26 0129	26 1058	26 1987	26 2916	26 3845	26 4774	26 5703	26 6632	26 7561	26 8490
90	26 9419	27 0348	27 1277	27 2206	27 3135	27 4064	27 4993	27 5922	27 6851	27 7780
300	27 8709	27 9638	28 0567	28 1496	28 2425	28 3354	28 4283	28 5212	28 6141	28 7070
10	28 7999	28 8928	28 9857	29 0787	29 1716	29 2645	29 3574	29 4503	29 5432	29 6361
20	29 7290	29 8219	29 9148	30 0077	30 1006	30 1935	30 2864	30 3793	30 4722	30 5651
30	30 6580	30 7509	30 8438	30 9367	31 0296	31 1225	31 2154	31 3083	31 4012	31 4941
40	31 5870	31 6799	31 7728	31 8657	31 9586	32 0515	32 1445	32 2374	32 3303	32 4232
50	32 5161	32 6090	32 7019	32 7948	32 8877	32 9806	33 0735	33 1664	33 2593	33 3522
60	33 4451	33 5380	33 6309	33 7238	33 8167	33 9096	34 0025	34 0954	34 1883	34 2812
70	34 3741	34 4670	34 5599	34 6528	34 7457	34 8386	34 9315	35 0244	35 1173	35 2103
80	35 3032	35 3961	35 4890	35 5819	35 6748	35 7677	35 8606	35 9535	36 0464	36 1393
90	36 2322	36 3251	36 4180	36 5109	36 6038	36 6967	36 7896	36 8825	36 9754	37 0683
400	37 1612	37 2541	37 3470	37 4399	37 5328	37 6257	37 7186	37 8115	37 9044	37 9973
10	38 0902	38 1831	38 2761	38 3690	38 4619	38 5548	38 6477	38 7406	38 8335	38 9264
20	39 0193	39 1122	39 2051	39 2980	39 3909	39 4838	39 5767	39 6696	39 7625	39 8554
30	39 9483	40 0412	40 1341	40 2270	40 3199	40 4128	40 5057	40 5986	40 6915	40 7844
40	40 8773	40 9702	41 0631	41 1560	41 2489	41 3419	41 4348	41 5277	41 6206	41 7135
50	41 8064	41 8993	41 9922	42 0851	42 1780	42 2709	42 3638	42 4567	42 5496	42 6425
60	42 7354	42 8283	42 9212	43 0141	43 1070	43 1999	43 2928	43 3857	43 4786	43 5715
70	43 6644	43 7573	43 8502	43 9431	44 0360	44 1289	44 2218	44 3148	44 4077	44 5006
80	44 5935	44 6864	44 7793	44 8722	44 9651	45 0580	45 1509	45 2438	45 3367	45 4296
90	45 5225	45 6154	45 7083	45 8012	45 8941	45 9870	46 0799	46 1728	46 2657	46 3586

CONVERSION TABLES

Extract from B.S. 350: 1944

SQUARE FEET TO SQUARE METRES (continued)

Based on 1 inch = 25·4 millimetres

Sq. ft.	0	1	2	3	4	5	6	7	8	9
	m^2	m^2	m^2	m^2	m^2	m^2	m^2	m^2	m^2	m^2
500	46 4515	46 5444	46 6373	46 7302	46 8231	46 9160	47 0089	47 1018	47 1947	47 2876
10	47 3806	47 4735	47 5664	47 6593	47 7522	47 8451	47 9380	48 0309	48 1238	48 2167
20	48 3096	48 4025	48 4954	48 5883	48·6812	48 7741	48 8670	48 9599	49 0528	49 1457
30	49 2386	49 3315	49 4244	49 5173	49 6102	49 7031	49 7960	49 8889	49 9818	50 075
40	50 168	50 261	50 353	50 446	50 539	50 632	50 725	50 818	50 911	51 004
50	51 097	51 190	51 282	51 375	51 468	51 561	51 654	51 747	51 840	51 933
60	52 026	52 119	52 212	52 304	52 397	52 490	52 583	52 676	52 769	52 862
70	52 955	53 048	53 141	53 233	53 326	53 419	53 512	53 605	53 698	53 791
80	53 884	53 977	54 070	54 162	54 255	54 348	54 441	54 534	54 627	54 720
90	54 813	54 906	54 999	55 092	55 184	55 277	55 370	55 463	55 556	55 649
600	55 742	55 835	55 928	56 021	56 113	56 206	56 299	56 392	56 485	56 578
10	56 671	56 764	56 857	56 950	57 042	57 135	57 228	57 321	57 414	57 507
20	57 600	57 693	57 786	57 879	57 971	58 064	58 157	58 250	58 343	58 436
30	58 529	58 622	58 715	58 808	58 901	58 993	59 086	59 179	59 272	59 365
40	59 458	59 551	59 644	59 737	59 830	59 922	60 015	60 108	60 201	60 294
50	60 387	60 480	60 573	60 666	60 759	60 851	60 944	61 037	61 130	61 223
60	61 316	61 409	61 502	61 595	61 688	61 781	61 873	61 966	62 059	62 152
70	62 245	62 338	62 431	62 524	62 617	62 710	62 802	62 895	62 988	63 081
80	63 174	63 267	63 360	63 453	63 546	63 639	63 731	63 824	63 917	64 010
90	64 103	64 196	64 289	64 382	64 475	64 568	64 661	64 753	64 846	64 939
700	65 032	65 125	65 218	65 311	65 404	65 497	65 590	65 682	65 775	65 868
10	65 961	66 054	66 147	66 240	66 333	66 426	66 519	66 611	66 704	66 797
20	66 890	66 983	67 076	67 169	67 262	67 355	67 448	67 541	67 633	67 726
30	67 819	67 912	68 005	68 098	68 191	68 284	68 377	68 470	68 562	68 655
40	68 748	68 841	68 934	69 027	69 120	69 213	69 306	69 399	69 491	69 584
50	69 677	69 770	69 863	69 956	70 049	70 142	70 235	70 328	70 421	70 513
60	70 606	70 699	70 792	70 885	70 978	71 071	71 164	71 257	71 350	71 442
70	71 535	71 628	71 721	71 814	71 907	72 000	72 093	72 186	72 279	72 371
80	72 464	72 557	72 650	72 743	72 836	72 929	73 022	73 115	73 208	73 300
90	73 393	73 486	73 579	73 672	73 765	73 858	73 951	74 044	74 137	74 230
800	74 322	74 415	74 508	74 601	74 694	74 787	74 880	74 973	75 066	75 159
10	75 251	75 344	75 437	75 530	75 623	75 716	75 809	75 902	75 995	76 088
20	76 180	76 273	76 366	76 459	76 552	76 645	76 738	76 831	76 924	77 017
30	77 110	77 202	77 295	77 388	77 481	77 574	77 667	77 760	77 853	77 946
40	78 039	78 131	78 224	78 317	78 410	78 503	78 596	78 689	78 782	78 875
50	78 968	79 060	79 153	79 246	79 339	79 432	79 525	79 618	79 711	79 804
60	79 897	79 990	80 082	80 175	80 268	80 361	80 454	80 547	80 640	80 733
70	80 826	80 919	81 011	81 104	81 197	81 290	81 383	81 476	81 569	81 662
80	81 755	81 848	81 940	82 033	82 126	82 219	82 312	82 405	82 498	82 591
90	82 684	82 777	82 870	82 962	83 055	83 148	83 241	83 334	83 427	83 520
900	83 613	83 706	83 799	83 891	83 984	84 077	84 170	84 263	84 356	84 449
10	84 542	84 635	84 728	84 820	84 913	85 006	85 099	85 192	85 285	85 378
20	85 471	85 564	85 657	85 750	85 842	85 935	86 028	86 121	86 214	86 307
30	86 400	86 493	86 586	86 679	86 771	86 864	86 957	87 050	87 143	87 236
40	87 329	87 422	87 515	87 608	87 700	87 793	87 886	87 979	88 072	88 165
50	88 258	88 351	88 444	88 537	88 630	88 722	88 815	88 908	89 001	89 094
60	89 187	89 280	89 373	89 466	89 559	89 651	89 744	89 837	89 930	90 023
70	90 116	90 209	90 302	90 395	90 488	90 580	90 673	90 766	90 859	90 952
80	91 045	91 138	91 231	91 324	91 417	91 509	91 602	91 695	91 788	91 881
90	91 974	92 067	92 160	92 253	92 346	92 439	92 531	92 624	92 717	92 810
1000	92 903									

CONVERSION TABLES

Extract from B.S. 350 : 1944

SQUARE METRES TO SQUARE FEET

Based on 1 inch = 25 4 millimetres

m².	0	1	2	3	4	5	6	7	8	9
	sq ft	sq ft	sq ft	sq ft	sq ft	sq ft	sq ft	sq ft	sq ft	sq ft
—	—	10 7639	21 5278	32 2917	43 0556	53 820	64 583	75 347	86 111	96 875
10	107 639	118 403	129 167	139 931	150 695	161 459	172 223	182 986	193 750	204 514
20	215 278	226 042	236 806	247 570	258 334	269 098	279 862	290 626	301 389	312·153
30	322 917	333 681	344 445	355 209	365 973	376 737	387 501	398 265	409 029	419 792
40	430 556	441 320	452 084	462 848	473 612	484 376	495 140	505 90	516 67	527 43
50	538 20	548 96	559 72	570 49	581 25	592 01	602 78	613 54	624 31	635 07
60	645 83	656 60	667 36	678 13	688 89	699 65	710 42	721 18	731 95	742 71
70	753 47	764 24	775 00	785 77	796 53	807 29	818 06	828 82	839 58	850 35
80	861 11	871 88	882 64	893 40	904 17	914 93	925 70	936 46	947 22	957 99
90	968 75	979 52	990 28	1001 04	1011 81	1022 57	1033 34	1044 10	1054 86	1065 63
100	1076 39	1087 15	1097 92	1108 68	1119 45	1130 21	1140 97	1151 74	1162 50	1173 27
10	1184 03	1194 79	1205 56	1216 32	1227 09	1237 85	1248 61	1259 38	1270 14	1280 91
20	1291 67	1302 43	1313 20	1323 96	1334 72	1345 49	1356 25	1367 02	1377 78	1388 54
30	1399 31	1410 07	1420 84	1431 60	1442 36	1453 13	1463 89	1474 66	1485 42	1496 18
40	1506 95	1517 71	1528 48	1539 24	1550 00	1560 77	1571 53	1582 29	1593 06	1603 82
50	1614 59	1625 35	1636 11	1646 88	1657 64	1668 41	1679 17	1689 93	1700 70	1711 46
60	1722 23	1732 99	1743 75	1754 52	1765 28	1776 04	1786 81	1797 57	1808 34	1819 10
70	1829 86	1840 63	1851 39	1862 16	1872 92	1883 68	1894 45	1905 21	1915 98	1926 74
80	1937 50	1948 27	1959 03	1969 80	1980 56	1991 32	2002 09	2012 85	2023 61	2034 38
90	2045 14	2055 91	2066 67	2077 43	2088 20	2098 96	2109 73	2120 49	2131 25	2142 02
200	2152 78	2163 55	2174 31	2185 07	2195 84	2206 60	2217 37	2228 13	2238 89	2249 66
10	2260 42	2271 18	2281 95	2292 71	2303 48	2314 24	2325 00	2335 77	2346 53	2357 30
20	2368 06	2378 82	2389 59	2400 35	2411 12	2421 88	2432 64	2443 41	2454 17	2464 94
30	2475 70	2486 46	2497 23	2507 99	2518 75	2529 52	2540 28	2551 05	2561 81	2572 57
40	2583 34	2594 10	2604 87	2615 63	2626 39	2637 16	2647 92	2658 69	2669 45	2680 21
50	2690 98	2701 74	2712 51	2723 27	2734 03	2744 80	2755 56	2766 32	2777 09	2787 85
60	2798 62	2809 38	2820 14	2830 91	2841 67	2852 44	2863 20	2873 96	2884 73	2895 49
70	2906 26	2917 02	2927 78	2938 55	2949 31	2960 07	2970 84	2981 60	2992 37	3003 13
80	3013 89	3024 66	3035 42	3046 19	3056 95	3067 71	3078 48	3089 24	3100 01	3110 77
90	3121 53	3132 30	3143 06	3153 83	3164 59	3175 35	3186 12	3196 88	3207 64	3218 41
300	3229 17	3239 94	3250 70	3261 46	3272 23	3282 99	3293 76	3304 52	3315 28	3326 05
10	3336 81	3347 58	3358 34	3369 10	3379 87	3390 63	3401 40	3412 16	3422 92	3433 69
20	3444 45	3455 21	3465 98	3476 74	3487 51	3498 27	3509 03	3519 80	3530 56	3541 33
30	3552 09	3562 85	3573 62	3584 38	3595 15	3605 91	3616 67	3627 44	3638 20	3648 97
40	3659 73	3670 49	3681 26	3692 02	3702 78	3713 55	3724 31	3735 08	3745 84	3756 60
50	3767 37	3778 13	3788 90	3799 66	3810 42	3821 19	3831 95	3842 72	3853 48	3864 24
60	3875 01	3885 77	3896 53	3907 30	3918 06	3928 83	3939 59	3950 35	3961 12	3971 88
70	3982 65	3993 41	4004 17	4014 94	4025 70	4036 47	4047 23	4057 99	4068 76	4079 52
80	4090 29	4101 05	4111 81	4122 58	4133 34	4144 10	4154 87	4165 63	4176 40	4187 16
90	4197 92	4208 69	4219 45	4230 22	4240 98	4251 74	4262 51	4273 27	4284 04	4294 80
400	4305 56	4316 33	4327 09	4337 86	4348 62	4359 38	4370 15	4380 91	4391 67	4402 44
10	4413 20	4423 97	4434 73	4445 49	4456 26	4467 02	4477 79	4488 55	4499 31	4510 08
20	4520 84	4531 61	4542 37	4553 13	4563 90	4574 66	4585 43	4596 19	4606 95	4617 72
30	4628 48	4639 24	4650 01	4660 77	4671 54	4682 30	4693 06	4703 83	4714 59	4725 36
40	4736 12	4746 88	4757 65	4768 41	4779 18	4789 94	4800 70	4811 47	4822 23	4833 00
50	4843 76	4854 52	4865 29	4876 05	4886 81	4897 58	4908 34	4919 11	4929 87	4940 63
60	4951 40	4962 16	4972 93	4983 69	4994 45	5005 2	5016 0	5026 7	5037 5	5048 3
70	5059 0	5069 8	5080 6	5091 3	5102 1	5112 9	5123 6	5134 4	5145 1	5155 9
80	5166 7	5177 4	5188 2	5199 0	5209 7	5220 5	5231 3	5242 0	5252 8	5263 6
90	5274 3	5285 1	5295 8	5306 6	5317 4	5328·1	5338 9	5349 7	5360 4	5371 2

CONVERSION TABLES

Extract from B.S. 350 : 1944

SQUARE METRES TO SQUARE FEET (continued)

Based on 1 inch = 25·4 millimetres

m².	0	1	2	3	4	5	6	7	8	9
	sq ft	sq ft	sq ft	sq ft	sq ft	sq ft	sq ft	sq ft	sq ft	sq ft
500	5382 0	5392 7	5403 5	5414 2	5425 0	5435 8	5446 5	5457 3	5468 1	5478 8
10	5489 6	5500 4	5511 1	5521 9	5532 6	5543 4	5554 2	5564 9	5575 7	5586 5
20	5597 2	5608 0	5618 8	5629 5	5640 3	5651 1	5661 8	5672 6	5683 3	5694 1
30	5704 9	5715 6	5726 4	5737 2	5747 9	5758 7	5769 5	5780 2	5791 0	5801 7
40	5812 5	5823 3	5834 0	5844 8	5855 6	5866 3	5877 1	5887 9	5898 6	5909 4
50	5920 1	5930 9	5941 7	5952 4	5963 2	5974 0	5984 7	5995 5	6006 3	6017 0
60	6027 8	6038 6	6049 3	6060 1	6070 8	6081 6	6092 4	6103 1	6113 9	6124 7
70	6135 4	6146 2	6157 0	6167 7	6178 5	6189 2	6200 0	6210 8	6221 5	6232 3
80	6243 1	6253 8	6264 6	6275 4	6286 1	6296 9	6307 7	6318 4	6329 2	6339 9
90	6350 7	6361 5	6372 2	6383 0	6393 8	6404 5	6415 3	6426 1	6436 8	6447 6
600	6458 3	6469 1	6479 9	6490 6	6501 4	6512 2	6522 9	6533 7	6544 5	6555 2
10	6566 0	6576 7	6587 5	6598 3	6609 0	6619 8	6630 6	6641 3	6652 1	6662 9
20	6673 6	6684 4	6695 2	6705 9	6716 7	6727 4	6738 2	6749 0	6759 7	6770 5
30	6781 3	6792 0	6802 8	6813 6	6824 3	6835 1	6845 8	6856 6	6867 4	6878 1
40	6888 9	6899 7	6910 4	6921 2	6932 0	6942 7	6953 5	6964 2	6975 0	6985 8
50	6996 5	7007 3	7018 1	7028 8	7039 6	7050 4	7061 1	7071 9	7082 7	7093 4
60	7104 2	7114 9	7125 7	7136 5	7147 2	7158 0	7168 8	7179 5	7190 3	7201 1
70	7211 8	7222 6	7233 3	7244 1	7254 9	7265 6	7276 4	7287 2	7297 9	7308 7
80	7319 5	7330 2	7341 0	7351 7	7362 5	7373 3	7384 0	7394 8	7405 6	7416 3
90	7427 1	7437 9	7448 6	7459 4	7470 2	7480 9	7491 7	7502 4	7513 2	7524 0
700	7534 7	7545 5	7556 3	7567 0	7577 8	7588 6	7599 3	7610 1	7620 8	7631 6
10	7642 4	7653 1	7663 9	7674 7	7685 4	7696 2	7707 0	7717 7	7728 5	7739 3
20	7750 0	7760 8	7771 5	7782 3	7793 1	7803 8	7814 6	7825 4	7836 1	7846 9
30	7857 7	7868 4	7879 2	7889 9	7900 7	7911 5	7922 2	7933 0	7943 8	7954 5
40	7965 3	7976 1	7986 8	7997 6	8008 3	8019 1	8029 9	8040 6	8051 4	8062 2
50	8072 9	8083 7	8094 5	8105 2	8116 0	8126 8	8137 5	8148 3	8159 0	8169 8
60	8180 6	8191 3	8202 1	8212 9	8223 6	8234 4	8245 2	8255 9	8266 7	8277 4
70	8288 2	8299 0	8309 7	8320 5	8331 3	8342 0	8352 8	8363 6	8374 3	8385 1
80	8395 8	8406 6	8417 4	8428 1	8438 9	8449 7	8460 4	8471 2	8482 0	8492 7
90	8503 5	8514 3	8525 0	8535 8	8546 5	8557 3	8568 1	8578 8	8589 6	8600 4
800	8611 1	8621 9	8632 7	8643 4	8654 2	8664 9	8675 7	8686 5	8697 2	8708 0
10	8718 8	8729 5	8740 3	8751 1	8761 8	8772 6	8783 3	8794 1	8804 9	8815 6
20	8826 4	8837 2	8847 9	8858 7	8869 5	8880 2	8891 0	8901 8	8912 5	8923 3
30	8934 0	8944 8	8955 6	8966 3	8977 1	8987 9	8998 6	9009 4	9020 2	9030 9
40	9041 7	9052 4	9063 2	9074 0	9084 7	9095 5	9106 3	9117 0	9127 8	9138 6
50	9149 3	9160 1	9170 9	9181 6	9192 4	9203 1	9213 9	9224 7	9235 4	9246 2
60	9257 0	9267 7	9278 5	9289 3	9300 0	9310 8	9321 5	9332 3	9343 1	9353 8
70	9364 6	9375 4	9386 1	9396 9	9407 7	9418 4	9429 2	9439 9	9450 7	9461 5
80	9472 2	9483 0	9493 8	9504 5	9515 3	9526 1	9536 8	9547 6	9558 4	9569 1
90	9579 9	9590 6	9601 4	9612 2	9622 9	9633 7	9644 5	9655 2	9666 0	9676 8
900	9687 5	9698 3	9709 0	9719 8	9730 6	9741 3	9752 1	9762 9	9773 6	9784 4
10	9795 2	9805 9	9816 7	9827 4	9838 2	9849 0	9859 7	9870 5	9881 3	9892 0
20	9902 8	9913 6	9924 3	9935 1	9945 9	9956 6	9967 4	9978 1	9988 9	9999 7
30	10010 4	10021 2	10032 0	10042 7	10053 5	10064 3	10075 0	10085 8	10096 5	10107 3
40	10118 1	10128 8	10139 6	10150 4	10161 1	10171 9	10182 7	10193 4	10204 2	10214 9
50	10225 7	10236 5	10247 2	10258 0	10268 8	10279 5	10290 3	10301 1	10311 8	10322 6
60	10333 4	10344 1	10354 9	10365 6	10376 4	10387 2	10397 9	10408 7	10419 5	10430 2
70	10441 0	10451 8	10462 5	10473 3	10484 0	10494 8	10505 6	10516 3	10527 1	10537 9
80	10548 6	10559 4	10570 2	10580 9	10591 7	10602 5	10613 2	10624 0	10634 7	10645 5
90	10656 3	10667 0	10677 8	10688 6	10699 3	10710 1	10720 9	10731 6	10742 4	10753 1
1000	10763 9									

CONVERSION TABLES

Extract from B S. 350: 1944

TONS PER SQUARE INCH TO KILOGRAMS PER SQUARE MILLIMETRE

Based on 1 inch — 25 4 millimetres, 1 pound — 0 45359243 kilograms

Tons/ sq. in	0	0 1	0 2	0 3	0 4	0 5	0 6	0 7	0 8	0 9
	kg / mm²	kg / mm²	kg / mm²	kg / mm²	kg / mm²	kg / mm²	kg / mm²	kg / mm²	kg / mm²	kg / mm²
--	—	0 15749	0 31498	0 47246	0 62995	0 78744	0 94493	1 10241	1 25990	1 41739
1	1 57488	1 73236	1 88985	2 04734	2 20483	2 36231	2 51980	2 67729	2 83478	2 99226
2	3 14975	3 30724	3 46473	3 62222	3 77970	3 93719	4 09468	4 25217	4 40965	4 56714
3	4 72463	4 88212	5 0396	5 1971	5 3546	5 5121	5 6696	5 8270	5 9845	6 1420
4	6 2995	6 4570	6 6145	6 7720	6 9295	7 0869	7 2444	7 4019	7 5594	7 7169
5	7 8744	8 0319	8 1894	8 3468	8 5043	8 6618	8 8193	8 9768	9 1343	9 2918
6	9 4493	9 6067	9 7642	9 9217	10 0792	10 2367	10 3942	10 5517	10 7092	10 8666
7	11 0241	11 1816	11 3391	11 4966	11 6541	11 8116	11 9691	12 1265	12 2840	12 4415
8	12 5990	12 7565	12 9140	13 0715	13 2290	13 3864	13 5439	13 7014	13 8589	14 0164
9	14 1739	14 3314	14 4889	14 6463	14 8038	14 9613	15 1188	15 2763	15 4338	15 5913
10	15 7488	15 9062	16 0637	16 2212	16 3787	16 5362	16 6937	16 8512	17 0087	17 1661
11	17 3236	17 4811	17 6386	17 7961	17 9536	18 1111	18 2686	18 4261	18 5835	18 7410
12	18 8985	19 0560	19 2135	19 3710	19 5285	19 6860	19 8434	20 0009	20 1584	20 3159
13	20 4734	20 6309	20 7884	20 9459	21 1033	21 2608	21 4183	21 5758	21 7333	21 8908
14	22 0483	22 2058	22 3632	22 5207	22 6782	22 8357	22 9932	23 1507	23 3082	23 4657
15	23 6231	23 7806	23 9381	24 0956	24 2531	24 4106	24 5681	24 7256	24 8830	25 0405
16	25 1980	25 3555	25 5130	25 6705	25 8280	25 9855	26 1429	26 3004	26 4579	26 6154
17	26 7729	26 9304	27 0879	27 2454	27 4028	27 5603	27 7178	27 8753	28 0328	28 1903
18	28 3478	28 5053	28 6627	28 8202	28 9777	29 1352	29 2927	29 4502	29 6077	29 7652
19	29 9226	30 0801	30 2376	30 3951	30 5526	30 7101	30 8676	31 0251	31 1825	31 3400
20	31 4975	31 6550	31 8125	31 9700	32 1275	32 2850	32 4424	32 5999	32 7574	32 9149
21	33 0724	33 2299	33 3874	33 5449	33 7023	33 8598	34 0173	34 1748	34 3323	34 4898
22	34 6473	34 8048	34 9622	35 1197	35 2772	35 4347	35 5922	35 7497	35 9072	36 0647
23	36 2222	36 3796	36 5371	36 6946	36 8521	37 0096	37 1671	37 3246	37 4821	37 6395
24	37 7970	37 9545	38 1120	38 2695	38 4270	38 5845	38 7420	38 8994	39 0569	39 2144
25	39 3719	39 5294	39 6869	39 8444	40 0019	40 1593	40 3168	40 4743	40 6318	40 7893
26	40 9468	41 1043	41 2618	41 4192	41·5767	41 7342	41 8917	42 0492	42 2067	42 3642
27	42 5217	42 6791	42 8366	42 9941	43 1516	43 3091	43 4666	43 6241	43 7816	43 9390
28	44 0965	44 2540	44 4115	44 5690	44 7265	44 8840	45 0415	45 1989	45 3564	45 5139
29	45 6714	45 8289	45 9864	46 1439	46 3014	46 4588	46 6163	46 7738	46 9313	47 0888
30	47 2463	47 4038	47 5613	47 7187	47 8762	48 0337	48 1912	48 3487	48 5062	48 6637
31	48 8212	48 9786	49 1361	49 2936	49 4511	49 6086	49 7661	49 9236	50 081	50 239
32	50 396	50 554	50 711	50 868	51 026	51 183	51 341	51 498	51 656	51 813
33	51 971	52 128	52 286	52 443	52 601	52 758	52 916	53 073	53 231	53 388
34	53 546	53 703	53 861	54 018	54 176	54 333	54 491	54 648	54 806	54 963
35	55 121	55 278	55 436	55 593	55 751	55 908	56 066	56 223	56 381	56 538
36	56 696	56 853	57 011	57 168	57 325	57 483	57 640	57 798	57 955	58 113
37	58 270	58 428	58 585	58 743	58 900	59 058	59 215	59 373	59 530	59 688
38	59 845	60 003	60 160	60 318	60 475	60 633	60 790	60 948	61 105	61 263
39	61 420	61 578	61 735	61 893	62 050	62 208	62 365	62 523	62 680	62 838
40	62 995	63 153	63 310	63 468	63 625	63 782	63 940	64 097	64 255	64 412
41	64 570	64 727	64 885	65 042	65 200	65 357	65 515	65 672	65 830	65 987
42	66 145	66 302	66 460	66 617	66 775	66 932	67 090	67 247	67 405	67 562
43	67 720	67 877	68 035	68 192	68 350	68 507	68 665	68 822	68 980	69 137
44	69 295	69 452	69 610	69 767	69 924	70 082	70 239	70 397	70 554	70 712
45	70 869	71 027	71 184	71 342	71 499	71 657	71 814	71 972	72 129	72 287
46	72 444	72 602	72 759	72 917	73 074	73 232	73 389	73 547	73 704	73 862
47	74 019	74 177	74 334	74 492	74 649	74 807	74 964	75 122	75 279	75 437
48	75 594	75 752	75 909	76 067	76 224	76 381	76 539	76 696	76 854	77 011
49	77 169	77 326	77 484	77 641	77 799	77·956	78 114	78 271	78 429	78 586

CONVERSION TABLES

Extract from B.S. 350 : 1944

TONS PER SQUARE INCH TO KILOGRAMS PER SQUARE MILLIMETRE (continued)

Based on 1 inch 25·4 millimetres , 1 pound 0 45359243 kilograms

Tons/ sq. in	0	0 1	0 2	0 3	0 4	0 5	0 6	0 7	0 8	0 9
	kg / mm²	kg / mm²	kg / mm²	kg / mm²	kg / mm²	kg / mm²	kg / mm²	kg / mm²	kg / mm²	kg / mm²
50	78 744	78 901	79 059	79 216	79 374	79 531	79 689	79 846	80 004	80 161
51	80 319	80 476	80 634	80 791	80 949	81 106	81 264	81 421	81 579	81 736
52	81 894	82 051	82 209	82 366	82 524	82 681	82 838	82 996	83 153	83 311
53	83 468	83 626	83 783	83 941	84 098	84 256	84 413	84 571	84 728	84 886
54	85 043	85 201	85 358	85 516	85 673	85 831	85 988	86 146	86 303	86 461
55	86 618	86 776	86 933	87 091	87 248	87 406	87 563	87 721	87 878	88 036
56	88 193	88 351	88 508	88 666	88 823	88 980	89 138	89 295	89 453	89 610
57	89 768	89 925	90 083	90 240	90 398	90 555	90 713	90 870	91 028	91 185
58	91 343	91 500	91 658	91 815	91 973	92 130	92 288	92 445	92 603	92 760
59	92 918	93 075	93 233	93 390	93 548	93 705	93 863	94 020	94 178	94 335
60	94 493	94 650	94 808	94 965	95 123	95 280	95 437	95 595	95 752	95 910
61	96 067	96 225	96 382	96 540	96 697	96 855	97 012	97 170	97 327	97 485
62	97 642	97 800	97 957	98 115	98 272	98 430	98 587	98 745	98 902	99 060
63	99 217	99 375	99 532	99 690	99 847	100 005	100 162	100 320	100 477	100 635
64	100 792	100 950	101 107	101 265	101 422	101 580	101 737	101 894	102 052	102 209
65	102 367	102 524	102 682	102 839	102 997	103 154	103 312	103 469	103 627	103 784
66	103 942	104 099	104 257	104 414	104 572	104 729	104 887	105 044	105 202	105 359
67	105 517	105 674	105 832	105 989	106 147	106 304	106 462	106 619	106 777	106 934
68	107 092	107 249	107 407	107 564	107 722	107 879	108 036	108 194	108 351	108 509
69	108 666	108 824	108 981	109 139	109 296	109 454	109 611	109 769	109 926	110 084
70	110 241	110 399	110 556	110 714	110 871	111 029	111 186	111 344	111 501	111 659
71	111 816	111 974	112 131	112 289	112 446	112 604	112 761	112 919	113 076	113 234
72	113 391	113 549	113 706	113 864	114 021	114 179	114 336	114 493	114 651	114 808
73	114 966	115 123	115 281	115 438	115 596	115 753	115 911	116 068	116 226	116 383
74	116 541	116 698	116 856	117 013	117 171	117 328	117 486	117 643	117 801	117 958
75	118 116	118 273	118 431	118 588	118 746	118 903	119 061	119 218	119 376	119 533
76	119 691	119 848	120 006	120 163	120 321	120 478	120 636	120 793	120 950	121 108
77	121 265	121 423	121 580	121 738	121 895	122 053	122 210	122 368	122 525	122 683
78	122 840	122 998	123 155	123 313	123 470	123 628	123 785	123 943	124 100	124 258
79	124 415	124 573	124 730	124 888	125 045	125 203	125 360	125 518	125 675	125 833
80	125 990	126 148	126 305	126 463	126 620	126 778	126 935	127 092	127 250	127 407
81	127 565	127 722	127 880	128 037	128 195	128 352	128 510	128 667	128 825	128 982
82	129 140	129 297	129 455	129 612	129 770	129 927	130 085	130 242	130 400	130 557
83	130 715	130 872	131 030	131 187	131 345	131 502	131 660	131 817	131 975	132 132
84	132 290	132 447	132 605	132 762	132 920	133 077	133 235	133 392	133 549	133 707
85	133 864	134 022	134 179	134 337	134 494	134 652	134 809	134 967	135 124	135 282
86	135 439	135 597	135 754	135 912	136 069	136 227	136 384	136 542	136 699	136 857
87	137 014	137 172	137 329	137 487	137 644	137 802	137 959	138 117	138 274	138 432
88	138 589	138 747	138 904	139 062	139 219	139 377	139 534	139 692	139 849	140 006
89	140 164	140 321	140 479	140 636	140 794	140 951	141 109	141 266	141 424	141 581
90	141 739	141 896	142 054	142 211	142 369	142 526	142 684	142 841	142 999	143 156
91	143 314	143 471	143 629	143 786	143 944	144 101	144 259	144 416	144 574	144 731
92	144 889	145 046	145 204	145 361	145 519	145 676	145 834	145 991	146 148	146 306
93	146 463	146 621	146 778	146 936	147 093	147 251	147 408	147 566	147 723	147 881
94	148 038	148 196	148 353	148 511	148 668	148 826	148 983	149 141	149 298	149 456
95	149 613	149 771	149 928	150 086	150 243	150 401	150 558	150 716	150 873	151 031
96	151 188	151 346	151 503	151 661	151 818	151 976	152 133	152 291	152 448	152 605
97	152 763	152 920	153 078	153 235	153 393	153 550	153 708	153 865	154 023	154 180
98	154 338	154 495	154 653	154 810	154 968	155 125	155 283	155 440	155 598	155 755
99	155 913	156 070	156 228	156 385	156 543	156 700	156 858	157 015	157 173	157 330
100	157 488									

CONVERSION TABLES

Extract from B S 350 1944

KILOGRAMS PER SQUARE MILLIMETRE TO TONS PER SQUARE INCH

Based on 1 inch = 25 4 millimetres , 1 pound – 0 45359243 kilograms

Kg/mm²	0	0 1	0 2	0 3	0 4	0 5	0 6	0 7	0 8	0 9
	tons/ sq in	tons/ sq in	tons/ sq in	tons/ sq in	tons/ sq in	tons/ sq in	tons/ sq in	tons/ sq in	tons/ sq in	tons/ sq in
—	—	0 06350	0 12699	0 19049	0 25399	0 31749	0 38098	0 44448	0 50798	0 57147
1	0 63497	0 69847	0 76196	0 82546	0 88896	0 95246	1 01595	1 07945	1 14295	1 20644
2	1 26994	1 33344	1 39694	1 46043	1 52393	1 58743	1 65092	1 71442	1 77792	1 84141
3	1 90491	1 96841	2 03191	2 09540	2 15890	2 22240	2 28589	2 34939	2 41289	2 47639
4	2 53988	2 60338	2 66688	2 73037	2 79387	2 85737	2 92086	2 98436	3 04786	3 11136
5	3 17485	3 23835	3 30185	3 36534	3 42884	3 49234	3 55584	3 61933	3 68283	3 74633
6	3 80982	3 87332	3 93682	4 00031	4 06381	4 12731	4 19081	4 25430	4 31780	4 38130
7	4 44479	4 50829	4 57179	4 63529	4 69878	4 76228	4 8 578	4 88927	4 95277	5 0163
8	5 0798	5 1433	5 2068	5 2703	5 3338	5 3973	5 4607	5 5242	5 5877	5 6512
9	5 7147	5 7782	5 8417	5 9052	5 9687	6 0322	6 0957	6 1592	6 2227	6 2862
10	6 3497	6 4132	6 4767	6 5402	6 6037	6 6672	6 7307	6 7942	6 8577	6 9212
11	6 9847	7 0482	7 1117	7 1752	7 2387	7 3022	7 3657	7 4292	7 4927	7 5562
12	7 6196	7 6831	7 7466	7 8101	7 8736	7 9371	8 0006	8 0641	8 1276	8 1911
13	8 2546	8 3181	8 3816	8 4451	8 5086	8 5721	8 6356	8 6991	8 7626	8 8261
14	8 8896	8 9531	9 0166	9 0801	9 1436	9 2071	9 2706	9 3341	9 3976	9 4611
15	9 5246	9 5881	9 6516	9 7151	9 7785	9 8420	9 9055	9 9690	10 0325	10 0960
16	10 1595	10 2230	10 2865	10 3500	10 4135	10 4770	10 5405	10 6040	10 6675	10 7310
17	10 7945	10 8580	10 9215	10 9850	11 0485	11 1120	11 1755	11 2390	11 3025	11 3660
18	11 4295	11 4930	11 5565	11 6200	11 6835	11 7470	11 8105	11 8740	11 9374	12 0009
19	12 0644	12 1279	12 1914	12 2549	12 3184	12 3819	12 4454	12 5089	12 5724	12 6359
20	12 6994	12 7629	12 8264	12 8899	12 9534	13 0169	13 0804	13 1439	13 2074	13 2709
21	13 3344	13 3979	13 4614	13 5249	1 5884	13 6519	13 7154	13 7789	13 8424	13 9059
22	13 9694	14 0329	14 0963	14 1598	14 2233	14 2868	14 3503	14 4138	14 4773	14 5408
23	14 6043	14 6678	14 7313	14 7948	14 8583	14 9218	14 985	15 0488	15 1123	15 1758
24	15 2393	15 3028	15 3663	15 4298	15 4933	15 5568	15 6203	15 6838	15 7473	15 8108
25	15 8743	15 9378	16 0013	16 0648	16 1283	16 1918	16 2552	16 3187	16 3822	16 4457
26	16 5092	16 5727	16 6362	16 6997	16 7632	16 8267	16 8902	16 9537	17 0172	17 0807
27	17 1442	17 2077	17 2712	17 3347	17 3982	17 4617	17 5252	17 5887	17 6522	17 7157
28	17 7792	17 8427	17 9062	17 9697	18 0332	18 0967	18 1602	18 2237	18 2872	18 3507
29	18 4141	18 4776	18 5411	18 6046	18 6681	18 7316	18 7951	18 8586	18 9221	18 9856
30	19 0491	19 1126	19 1761	19 2396	19 3031	19 3666	19 4301	19 4936	19 5571	19 6206
31	19 6841	19 7476	19 8111	19 8746	19 9381	20 0016	20 0651	20 1286	20 1921	30 2556
32	20 3191	20 3826	20 4461	20 5096	20 5730	20 6365	20 7000	20 7635	20 8270	20 8905
33	20 9540	21 0175	21 0810	21 1445	21 2080	21 2715	21 3350	21 3985	21 4620	21 5255
34	21 5890	21 6525	21 7160	21 7795	21 8430	21 9065	21 9700	22 0335	22 0970	22 1605
35	22 2240	22 2875	22 3510	22 4145	22 4780	22 5415	22 6050	22 6685	22 7319	22 7954
36	22 8589	22 9224	22 9859	23 0494	23 1129	23 1764	23 2399	23 3034	23 3669	23 4304
37	23 4939	23 5574	23 6209	23 6844	23 7479	23 8114	23 8749	23 9384	24 0019	24 0654
38	24 1289	24 1924	24 2559	24 3194	24 3829	24 4464	24 5099	24 5734	24 6369	24 7004
39	24 7639	24 8274	24 8908	24 9543	25 0178	25 0813	25 1448	25 2083	25 2718	25 3353
40	25 3988	25 4623	25 5258	25 5893	25 6528	25 7163	25 7798	25 8433	25 9068	25 9703
41	26 0338	26 0973	26 1608	26 2243	26 2878	26 3513	26 4148	26 4783	26 5418	26 6053
42	26 6688	26 7323	26 7958	26 8593	26 9228	26 9863	27 0497	27 1132	27 1767	27 2402
43	27 3037	27 3672	27 4307	27 4942	27 5577	27 6212	27 6847	27 7482	27 8117	27 8752
44	27 9387	28 0022	28 0657	28 1292	28 1927	28 2562	28 3197	28 3832	28 4467	28 5102
45	28 5737	28 6372	28 7007	28 7642	28 8277	28 8912	28 9547	29 0182	29 0817	29 1452
46	29 2086	29 2721	29 3356	29 3991	29 4626	29 5261	29 5896	29 6531	29 7166	29 7801
47	29 8436	29 9071	29 9706	30 0341	30 0976	30 1611	30 2246	30 2881	30 3516	30 4151
48	30 4786	30 5421	30 6056	30 6691	30 7326	30 7961	30 8596	30 9231	30 9866	31 0501
49	31 1136	31 1771	31 2406	31 3041	31 3675	31 4310	31 4945	31 5580	31 6215	31 6850

CONVERSION TABLES

Extract from B.S. 350: 1944

KILOGRAMS PER SQUARE MILLIMETRE TO TONS PER SQUARE INCH (continued)

Based on 1 inch = 25·4 millimetres ; 1 pound = 0·45359243 kilograms

Kg./mm.²	0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9
	tons/ sq. in.	tons/ sq. in.	tons/ sq. in.	tons/ sq. in.	tons/ sq. in.	tons/ sq. in.	tons/ sq. in.	tons/ sq. in.	tons/ sq. in.	tons/ sq. in.
50	31·7485	31·8120	31·8755	31·9390	32·0025	32·0660	32·1295	32·1930	32·2565	32·3200
51	32·3835	32·4470	32·5105	32·5740	32·6375	32·7010	32·7645	32·8280	32·8915	32·9550
52	33·0185	33 0820	33·1455	33 2090	33·2725	33·3360	33·3995	33 4630	33·5264	33·5899
53	33·6534	33·7169	33·7804	33·8439	33·9074	33·9709	34·0344	34·0979	34·1614	34·2249
54	34·2884	34·3519	34·4154	34·4789	34·5424	34·6059	34·6694	34·7329	34·7964	34·8599
55	34·9234	34·9869	35·0504	35·1139	35·1774	35·2409	35·3044	35·3679	35·4314	35·4949
56	35·5584	35·6219	35·6853	35·7488	35·8123	35·8758	35 9393	36·0028	36·0663	36·1298
57	36·1933	36·2568	36·3203	36·3838	36·4473	36·5108	36·5743	36·6378	36·7013	36·7648
58	36·8283	36·8918	36 9553	37·0188	37·0823	37·1458	37·2093	37·2728	37·3363	37·3998
59	37·4633	37·5268	37·5903	37·6538	37·7173	37·7808	37 8442	37 9077	37·9712	38·0347
60	38·0982	38·1617	38·2252	38·2887	38·3522	38·4157	38·4792	38·5427	38·6062	38·6697
61	38·7332	38·7967	38·8602	38 9237	38·9872	39·0507	39·1142	39·1777	39·2412	39·3047
62	39·3682	39·4317	39·4952	39·5587	39·6222	39·6857	39·7492	39·8127	39·8762	39·9397
63	40·0031	40·0666	40·1301	40·1936	40·2571	40·3206	40·3841	40·4476	40·5111	40·5746
64	40·6381	40·7016	40·7651	40·8286	40·8921	40·9556	41·0191	41·0826	41·1461	41·2096
65	41·2731	41·3366	41·4001	41·4636	41·5271	41·5906	41·6541	41 7176	41·7811	41·8446
66	41·9081	41·9716	42·0351	42·0986	42·1620	42·2255	42 2890	42·3525	42·4160	42·4795
67	42·5430	42·6065	42·6700	42·7335	42·7970	42·8605	42·9240	42·9875	43·0510	43·1145
68	43·1780	43·2415	43·3050	43·3685	43·4320	43·4955	43·5590	43·6225	43·6860	43·7495
69	43·8130	43·8765	43·9400	44·0035	44·0670	44·1305	44·1940	44·2575	44·3209	44·3844
70	44·4479	44·5114	44·5749	44·6384	44·7019	44·7654	44·8289	44·8924	44·9559	45·0194
71	45·0829	45 1464	45·2099	45 2734	45·3369	45·4004	45·4639	45·5274	45·5909	45·6544
72	45·7179	45·7814	45·8449	45·9084	45 9719	46·0354	46 0989	46 1624	46·2259	46·2894
73	46·3529	46·4164	46·4798	46·5433	46·6068	46·6703	46·7338	46 7973	46·8608	46·9243
74	46·9878	47·0513	47·1148	47·1783	47·2418	47·3053	47·3688	47·4323	47·4958	47·5593
75	47·6228	47·6863	47·7498	47·8133	47·8768	47·9403	48·0038	48 0673	48 1308	48·1943
76	48·2578	48·3213	48·3848	48·4483	48·5118	48·5753	48·6387	48·7022	48·7657	48·8292
77	48·8927	48·9562	49 0197	49 0832	49·1467	49 2102	49·2737	49·3372	49·4007	49·4642
78	49·5277	49·5912	49·6547	49·7182	49·7817	49·8452	49·9087	49·9722	50·036	50·099
79	50·163	50·226	50·290	50·353	50·417	50·480	50 544	50·607	50·671	50·734
80	50·798	50·861	50·925	50·988	51·052	51·115	51·179	51·242	51·306	51·369
81	51·433	51·496	51·560	51·623	51·687	51·750	51·814	51·877	51·941	52·004
82	52·068	52·131	52·195	52·258	52·322	52·385	52·449	52·512	52·576	52·639
83	52·703	52·766	52·830	52·893	52·957	53·020	53 084	53·147	53·211	53·274
84	53·338	53·401	53·465	53·528	53 592	53·655	53 719	53·782	53 846	53·909
85	53·973	54·036	54·099	54·163	54·226	54·290	54·353	54·417	54·480	54·544
86	54·607	54·671	54·734	54·798	54·861	54·925	54·988	55·052	55·115	55·179
87	55·242	55·306	55·369	55 433	55·496	55·560	55 623	55·687	55·750	55·814
88	55·877	55·941	56·004	56·068	56·131	56·195	56·258	56 322	56·385	56·449
89	56·512	56·576	56·639	56·703	56·766	56·830	56·893	56·957	57·020	57·084
90	57·147	57·211	57·274	57·338	57·401	57·465	57·528	57·592	57·655	57·719
91	57·782	57·846	57·909	57·973	58·036	58·100	58·163	58·227	58·290	58·354
92	58·417	58·481	58·544	58·608	58·671	58·735	58·798	58·862	58·925	58·989
93	59·052	59·116	59·179	59·243	59 306	59·370	59·433	59·497	59·560	59·624
94	59·687	59·751	59·814	59·878	59·941	60·005	60·068	60·132	60·195	60·259
95	60·322	60·386	60·449	60·513	60·576	60·640	60·703	60·767	60·830	60·894
96	60 957	16·021	61·084	61·148	61·211	61·275	61·338	61·402	61·465	61·529
97	61·592	61·656	61·719	61·783	61·846	61·910	61·973	62·037	62·100	62·164
98	62·227	62·291	62·354	62·418	62·481	62·545	62·608	62·672	62·735	62·799
99	62·862	62·926	62·989	63·053	63·116	63·180	63·243	63·307	63·370	63·434
100	63·497									

WIRE AND SHEET METAL GAUGES

Extract from B.S. 350 : 1944

DIAMETER OR THICKNESS IN INCHES AND MILLIMETRES

No.	Standard wire gauge		Birmingham gauge		American Brown & Sharpe's gauge		U.S. Government Standard	
	in.	mm.	in.	mm.	in.	mm.	in.	mm.
15/0	—	—	1·000	25·4	—	—	—	—
14/0	—	—	0·958 3	24·34	—	—	—	—
13/0	—	—	0·916 7	23·28	—	—	—	—
12/0	—	—	0·875 0	22·22	—	—	—	—
11/0	—	—	0·833 3	21·17	—	—	—	—
10/0	—	—	0·791 7	20·11	—	—	—	—
9/0	—	—	0·750	19·05	—	—	—	—
8/0	—	—	0·708 3	17·99	—	—	—	—
7/0	0·500	12·700	0·666 6	16·93	—	—	0·500 0	12·700
6/0	0·464	11·786	0·625	15·88	—	—	0·468 75	11·906
5/0	0·432	10·973	0·588 3	14·94	—	—	0·437 5	11·112
4/0	0·400	10·160	0·541 6	13·76	0·460 0	11·68	0·406 25	10·319
3/0	0·372	9·449	0·500	12·70	0·409 6	10·40	0·375	9·525
2/0	0·348	8·839	0·445 2	11·31	0·364 8	9·27	0·343 7	8·730
0	0·324	8·230	0·396 4	10·07	0·324 9	8·25	0·312 5	7·938
1	0·300	7·620	0·353 2	8·971	0·289 3	7·35	0·281 2	7·144
2	0·276	7·010	0·314 7	7·993	0·257 6	6·54	0·265 6	6·747
3	0·252	6·401	0·280 4	7·122	0·229 4	5·83	0·250 0	6·35
4	0·232	5·893	0·250	6·350	0·204 3	5·19	0·234 4	5·953
5	0·212	5·385	0·222 5	5·652	0·181 9	4·62	0·218 7	5·556
6	0·192	4·877	0·198 1	5·032	0·162 0	4·11	0·203 1	5·159
7	0·176	4·470	0·176 4	4·481	0·144 3	3·67	0·187 5	4·762
8	0·160	4·064	0·157 0	3·988	0·128 5	3·26	0·171 9	4·366
9	0·144	3·658	0·139 8	3·551	0·114 4	2·91	0·156 2	3·969
10	0·128	3·251	0·125 0	3·175	0·101 9	2·59	0·140 6	3·572
11	0·116	2·946	0·111 3	2·827	0·090 74	2·30	0·125 0	3·175
12	0·104	2·642	0·099 1	2·517	0·080 81	2·05	0·109 4	2·778
13	0·092	2·337	0·088 2	2·240	0·071 96	1·83	0·093 7	2·381
14	0·080	2·032	0·078 5	1·994	0·064 08	1·63	0·078 1	1·984
15	0·072	1·829	0·069 9	1·775	0·057 07	1·45	0·070 3	1·786
16	0·064	1·626	0·062 5	1·588	0·050 82	1·29	0·062 5	1·588
17	0·056	1·422	0·055 6	1·412	0·045 26	1·15	0·056 2	1·429
18	0·048	1·219	0·049 5	1·257	0·040 30	1·02	0·05	1·270
19	0·040	1·016	0·044 0	1·118	0·035 89	0·91	0·043 7	1·111

WIRE AND SHEET METAL GAUGES (continued)

Extract from B.S. 350 : 1944

DIAMETER OR THICKNESS IN INCHES AND MILLIMETRES

No.	Standard wire gauge		Birmingham gauge		American Brown & Sharpe's gauge		U.S. Government Standard	
	in.	mm.	in.	mm.	in.	mm.	in.	mm.
20	0·036	0·914	0·039 2	0·995 7	0·031 96	0·812	0·037 5	0·952
21	0·032	0·813	0·034 9	0·886 5	0·028 46	0·723	0·034 4	0·873
22	0·028	0·711	0·031 25	0·793 8	0·025 35	0·644	0·031 25	0·794
23	0·024	0·610	0·027 82	0·706 6	0·022 57	0·573	0·028 1	0·714
24	0·022	0·559	0·024 76	0·628 9	0·020 10	0·511	0·025	0·635
25	0·020	0·508	0·022 04	0·559 8	0·017 90	0·455	0·021 9	0·556
26	0·018	0·457	0·019 61	0·498 1	0·015 94	0·405	0·018 8	0·476
27	0·016 4	0·416 6	0·017 45	0·443 2	0·014 20	0·361	0·017 2	0·437
28	0·014 8	0·375 9	0·015 625	0·396 9	0·012 64	0·321	0·015 6	0·397
29	0·013 6	0·345 4	0·013 9	0·353 1	0·011 26	0·286	0·014 1	0·357
30	0·012 4	0·315 0	0·012 3	0·312 4	0·010 03	0·255	0·012 5	0·318
31	0·011 6	0·294 6	0·011 0	0·279 4	0·008 928	0·227	0·010 9	0·278
32	0·010 8	0·274 3	0·009 8	0·248 9	0·007 950	0·202	0·010 2	0·258
33	0·010 0	0·254 0	0·008 7	0·221 0	0·007 080	0·180	0·009 4	0·238
34	0·009 2	0·233 7	0·007 7	0·195 6	0·006 304	0·160	0·008 6	0·218
35	0·008 4	0·213 4	0·006 9	0·175 3	0·005 615	0·143	0·007 8	0·198
36	0·007 6	0·193 0	0·006 1	0·154 9	0·005 000	0·127	0·007	0·179
37	0·006 8	0·172 7	0·005 4	0·137 2	0·004 453	0·113	0·006 64	0·169
38	0·006 0	0·152 4	0·004 8	0·121 9	0·003 965	0·101	0·006 25	0·159
39	0·005 2	0·132 1	0·004 3	0·109 2	0·003 531	0·090	—	—
40	0·004 8	0·121 9	0·003 86	0·098 04	0·003 145	0·080	—	—
41	0·004 4	0·111 8	0·003 43	0·087 12	—	—	—	—
42	0·004 0	0·101 6	0·003 06	0·077 72	—	—	—	—
43	0·003 6	0·091 4	0·002 72	0·069 09	—	—	—	—
44	0·003 2	0·081 3	0·002 42	0·061 47	—	—	—	—
45	0·002 8	0·071 1	0·002 15	0·054 61	—	—	—	—
46	0·002 4	0·061 0	0·001 92	0·048 77	—	—	—	—
47	0·002 0	0·050 8	0·001 70	0·043 18	—	—	—	—
48	0·001 6	0·040 6	0·001 52	0·038 61	—	—	—	—
49	0·001 2	0·030 5	0·001 35	0·034 29	—	—	—	—
50	0·001 0	0·025 4	0·001 20	0·030 48	—	—	—	—
51	—	—	0·001 07	0·027 18	—	—	—	—
52	—	—	0·000 95	0·024 13	—	—	—	—

CONVERSION TABLES

Extract from B.S. 350 : 1944

MOMENTS

Based on: 1 inch = 25·4 millimetres
1 pound = 0·45359243 kilograms

Inch pounds	*Foot pounds*	*Inch tons*	*Foot tons*	*Kilogram metres*	*Tonne metres*
1	0·083333	0·00044643	0·000037202	0·0115212	0·0000115212
12	1	0·0053571	0·00044643	0·138255	0·000138255
2,240	186·667	1	0·083333	25·8076	0·0258076
26,880	2240	12	1	309·691	0·309691
86·796	7·2330	0·0387483	0·0032290	1	0·001
86,796	7233·0	38·7483	3·22902	1000	1

This table can also be used for the conversion of units of work and energy.

THE GREEK ALPHABET

Name	*Capital Letter*	*Small Letter*	*English Equivalent*	*Name*	*Capital Letter*	*Small Letter*	*English Equivalent*
Alpha	A	α	a	Nu	N	ν	n
Beta	B	β	b	Xi	Ξ	ξ	x
Gamma	Γ	γ	g	Omicron	O	o	short o
Delta	Δ	δ	d	Pi	Π	π	p
Epsilon	E	ϵ	short e	Rho	P	ρ	rh
Zeta	Z	ζ	z	Sigma	Σ	σ	s
Eta	H	η	long e	Tau	T	τ	t
Theta	Θ	θ	th	Upsilon	Υ	υ	u
Iota	I	ι	i	Phi	Φ	ϕ	ph
Kappa	K	κ	k	Chi	X	χ	ch
Lambda	Λ	λ	l	Psi	Ψ	ψ	ps
Mu	M	μ	m	Omega	Ω	ω	long o

BRITISH STANDARD CONVERSIONS OF DEGREES, MINUTES AND SECONDS TO RADIANS

Based on 180° – 3·14159265 radians

Degrees	0	1°	2°	3°	4°	5°	6°	7°	8°	9°
—	—	0·017453	0·034907	0·052360	0·069813	0·087266	0·104720	0·122173	0·139626	0·157080
10	0·174533	0·191986	0·209440	0·226893	0·244346	0·261799	0·279253	0·296706	0·314159	0·331613
20	0·349066	0·366519	0·383972	0·401426	0·418879	0·436332	0·453786	0·471239	0·488692	0·50615
30	0·52360	0·54105	0·55851	0·57596	0·59341	0·61087	0·62832	0·64577	0·66323	0·68068
40	0·69813	0·71558	0·73304	0·75049	0·76794	0·78540	0·80285	0·82030	0·83776	0·85521
50	0·87266	0·89012	0·90757	0·92502	0·94248	0·95993	0·97738	0·99484	1·01229	1·02974
60	1·04720	1·06465	1·08210	1·09956	1·11701	1·13446	1·15192	1·16937	1·18682	1·20428
70	1·22173	1·23918	1·25664	1·27409	1·29154	1·30900	1·32645	1·34390	1·36136	1·37881
80	1·39626	1·41372	1·43117	1·44862	1·46608	1·48353	1·50098	1·51844	1·53589	1·55334
90	1·57080	1·58825	1·60570	1·62316	1·64061	1·65806	1·67552	1·69297	1·71042	1·72788
100	1·74533	1·76278	1·78024	1·79769	1·81514	1·83260	1·85005	1·86750	1·88496	1·90241

Minutes	0	1′	2′	3′	4′	5′	6′	7′	8′	9′
—	—	0·000291	0·000582	0·000873	0·001164	0·001454	0·001745	0·002036	0·002327	0·002618
10	0·002909	0·003200	0·003491	0·003782	0·004072	0·004363	0·004654	0·004945	0·005236	0·005527
20	0·005818	0·006109	0·006400	0·006690	0·006981	0·007272	0·007563	0·007854	0·008145	0·008436
30	0·008727	0·009018	0·009308	0·009599	0·009890	0·010181	0·010472	0·010763	0·011054	0·011345
40	0·011636	0·011926	0·012217	0·012508	0·012799	0·013090	0·013381	0·013672	0·013963	0·014254
50	0·014544	0·014835	0·015126	0·015417	0·015708	0·015999	0·016290	0·016581	0·016872	0·017162

Seconds	0	10″	20″	30″	40″	50″				
—	—	0·000048	0·000097	0·000145	0·000194	0·000242				

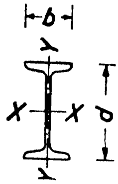
RADIANS TO DEGREES, MINUTES AND SECONDS

Based on 1 Radian – 57°·2957795 (Diff. for 0·001 radn. 3′ 26¼″)

R	0	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09
	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″
—	—	34 23	1 8 45	1 43 8	2 17 31	2 51 53	3 26 16	4 0 39	4 35 1	5 9 24
0·1	5 43 46	6 18 9	6 52 32	7 26 54	8 1 17	8 35 40	9 10 2	9 44 25	10 18 48	10 53 10
0·2	11 27 33	12 1 56	12 36 18	13 10 41	13 45 4	14 19 26	14 53 49	15 28 11	16 2 34	16 36 57
0·3	17 11 19	17 45 42	18 20 5	18 54 27	19 28 50	20 3 13	20 37 35	21 11 58	21 46 21	22 20 43
0·4	22 55 6	23 29 29	24 3 51	24 38 14	25 12 37	25 46 59	26 21 22	26 55 44	27 30 7	28 4 30
0·5	28 38 52	29 13 15	29 47 38	30 22 0	30 56 23	31 30 46	32 5 8	32 39 31	33 13 54	33 48 16
0·6	34 22 39	34 57 2	35 31 24	36 5 47	36 40 9	37 14 32	37 48 55	38 23 17	38 57 40	39 32 3
0·7	40 6 25	40 40 48	41 15 11	41 49 33	42 23 56	42 58 19	43 32 41	44 7 4	44 41 27	45 15 49
0·8	45 50 12	46 24 34	46 58 57	47 33 20	48 7 42	48 42 5	49 16 28	49 50 50	50 25 13	50 59 36
0·9	51 33 58	52 8 21	52 42 44	53 17 6	53 51 29	54 25 52	55 0 14	55 34 37	56 9 0	56 43 22
1·0	57 17 45	57 52 7	58 26 30	59 0 53	59 35 15	60 9 38	60 44 1	61 18 23	61 52 46	62 27 9
1·1	63 1 31	63 35 54	64 10 17	64 44 39	65 19 2	65 53 25	66 27 47	67 2 10	67 36 32	68 10 55
1·2	68 45 18	69 19 40	69 54 3	70 28 26	71 2 48	71 37 11	72 11 34	72 45 56	73 20 19	73 54 42
1·3	74 29 4	75 3 27	75 37 50	76 12 12	76 46 35	77 20 57	77 55 20	78 29 43	79 4 5	79 38 28
1·4	80 12 51	80 47 13	81 21 36	81 55 59	82 30 21	83 4 44	83 39 7	84 13 29	84 47 52	85 22 15
1·5	85 56 37	86 31 0	87 5 22	87 39 45	88 14 8	88 48 30	89 22 53	89 57 16	90 31 38	91 6 1

49. PROPERTIES OF STEEL SECTIONS

BRITISH STANDARD SECTIONS



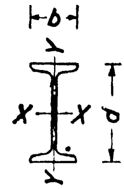
JOISTS
DIMENSIONS AND PROPERTIES IN INCH UNITS

Reference Mark	Size d×b	Weight per ft in lb	Area	Standard Thicknesses		Moments of Inertia			Section Moduli		
						About XX		About YY	About XX		About YY
				Web	Flange	Gross	Net*	Gross	Gross	Net*	Gross
BSB 140	24 × 7½	95	27 94	0 57	1 011	2533 04	2289 84	62 54	211 09	190 82	16 68
BSB 139	22 × 7	75	22 06	0 50	0 834	1676 80	1504 98	41 07	152 44	136 81	11 73
BSB 138	20 × 7½	89	26 19	0 60	1 010	1672 85	1506 77	62 54	167 29	150 68	16 68
BSB 137	20 × 6½	65	19 12	0 45	0 820	1226 17	1087 74	32 56	122 62	108 77	10 02
BSB 136	18 × 8	80	23 53	0 50	0 950	1292 07	1166 77	69 43	143 56	129 64	17 36
BSB 135	18 × 7	75	22 09	0 55	0 928	1151 18	1026 21	46 56	127 91	114 02	13 30
BSB 134	18 × 6	55	16 18	0 42	0 757	841 76	752 52	23 64	93 53	83 61	7 88
BSB 133	16 × 8	75	22 06	0 48	0 938	973 91	877 40	68 30	121 74	109 67	17 08
BSB 132	16 × 6 H	62	18 21	0 55	0 847	725 05	647 28	27 14	90 63	80 91	9 05
BSB 131	16 × 6 L	50	14 71	0 40	0 726	618 09	551 05	22 47	77 26	68 88	7 49
BSB 130	15 × 6	45	13 24	0 38	0 655	491 91	438 78	19 87	65 59	58 50	6 62
BSB 129	15 × 5	42	12 36	0 42	0 647	428 49	374 52	11 81	57 13	49 94	4 72
BSB 128	14 × 8	70	20 59	0 46	0 920	705 58	634 23	66 67	100 80	90 60	16 67
BSB 127	14 × 6 H	57	16 78	0 50	0 873	533 34	473 21	27 94	76 19	67 60	9 31
BSB 126	14 × 6 L	46	13 59	0 40	0 698	442 57	393 71	21 45	63 22	56 24	7 15
BSB 125	13 × 5	35	10 30	0 35	0 604	283 51	246 07	10 82	43 62	37 86	4 33
BSB 124	12 × 8	65	19 12	0 43	0 904	487 77	437 33	65 18	81 30	72 89	16·30
BSB 123	12 × 6 H	54	15 89	0 50	0 883	375 77	332 09	28 28	62 63	55 35	9 43
BSB 122	12 × 6 L	44	13 00	0 40	0 717	316 76	280 59	22 12	52 79	46 76	7 37
BSB 121	12 × 5	32	9 45	0 35	0 550	221 07	192 01	9 69	36 84	32 00	3 88

* The net moments of inertia and section moduli allow for one hole per flange, $\frac{1}{16}$ in. greater in diameter than the appropriate bolt size given on the next page

BRITISH STANDARD SECTIONS

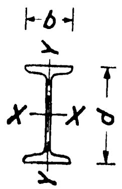
JOISTS
DIMENSIONS AND PROPERTIES IN INCH UNITS



Radii of Gyration		*Web*					*Bolts*		*Size d×b*	*Reference Mark*
		Web Depth		*Buckling Values*		*Shear Value in Tons*				
About XX	*About YY*	*Clear**	*Net**	*Joist*	*1-in. Bearing*		*Cross Centres*	*Diam.*		
9·52	1·50	20·22	21·41	39·94	3·32	88·92	$4\frac{1}{2}$	$\frac{7}{8}$	$24\times7\frac{1}{2}$	BSB 140
8·72	1·36	18·68	19·81	31·18	2·83	71·50	4	$\frac{7}{8}$	22×7	BSB 139
7·99	1·55	16·22	17·41	39·35	3·93	78·00	$4\frac{1}{2}$	$\frac{7}{8}$	$20\times7\frac{1}{2}$	BSB 138
8·01	1·31	16·81	17·87	25·47	2·54	58·50	$3\frac{3}{4}$	$\frac{7}{8}$	$20\times6\frac{1}{2}$	BSB 137
7·41	1·72	14·23	15·50	28·77	3·19	58·50	$4\frac{3}{4}$	$\frac{7}{8}$	18×8	BSB 136
7·22	1·45	14·50	15·61	32·72	3·63	64·35	4	$\frac{7}{8}$	18×7	BSB 135
7·21	1·21	15·03	16·03	21·89	2·43	49·14	$3\frac{1}{2}$	$\frac{3}{4}$	18×6	BSB 134
6·64	1·76	12·26	13·53	25·46	3·18	49·92	$4\frac{3}{4}$	$\frac{7}{8}$	16×8	BSB 133
6·31	1·22	12·86	13·85	30·28	3·78	57·20	$3\frac{1}{2}$	$\frac{3}{4}$	16×6 H	BSB 132
6·48	1·24	13·09	14·10	19·32	2·41	41·60	$3\frac{1}{2}$	$\frac{3}{4}$	16×6 L	BSB 131
6·10	1·23	12·23	13·24	17·29	2·30	37·05	$3\frac{1}{2}$	$\frac{3}{4}$	15×6	BSB 130
5·89	0·98	12·46	13·33	19·94	2·65	40·95	$2\frac{3}{4}$	$\frac{3}{4}$	15×5	BSB 129
5·85	1·80	10·29	11·57	22·18	3·16	41·86	$4\frac{3}{4}$	$\frac{7}{8}$	14×8	BSB 128
5·64	1·29	10·80	11·80	24·55	3·50	45·50	$3\frac{1}{2}$	$\frac{3}{4}$	14×6 H	BSB 127
5·71	1·26	11·14	12·15	18·05	2·57	36·40	$3\frac{1}{2}$	$\frac{3}{4}$	14×6 L	BSB 126
5·25	1·03	10·54	11·42	14·23	2·19	29·57	$2\frac{3}{4}$	$\frac{3}{4}$	13×5	BSB 125
5·05	1·85	8·32	9·60	18·37	3·06	33·54	$4\frac{3}{4}$	$\frac{7}{8}$	12×8	BSB 124
4·86	1·33	8·78	9·78	22·06	3·67	39·00	$3\frac{1}{2}$	$\frac{3}{4}$	12×6 H	BSB 123
4·94	1·30	9·12	10·12	16·49	2·74	31·20	$3\frac{1}{2}$	$\frac{3}{4}$	12×6 L	BSB 122
4·84	1·01	9·66	10·52	13·58	2·26	27·30	$2\frac{3}{4}$	$\frac{3}{4}$	12×5	BSB 121

* Clear web depth is the depth of the parallel portion of the web. Net depth is the depth ignoring root fillets.

BRITISH STANDARD SECTIONS



JOISTS
DIMENSIONS AND PROPERTIES IN INCH UNITS

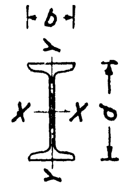
Reference Mark	Size $d \times b$	Weight per ft. in lb.	Area	Standard Thicknesses		Moments of Inertia About XX		Moments of Inertia About YY	Section Moduli About XX		Section Moduli About YY
				Web	Flange	Gross	Net*	Gross	Gross	Net*	Gross
BSB 120	10 ×8	55	16·18	0·40	0·783	288·69	258·74	54·74	57·74	51·75	13·69
BSB 119	10 ×6	40	11·77	0·36	0·709	204·80	180·56	21·76	40·96	36·11	7·25
BSB 118	10 ×5	30	8·85	0·36	0·552	146·23	126·35	9·73	29·25	25·27	3·89
BSB 117	10 ×4½	25	7·35	0·30	0·505	122·34	104·06	6·49	24·47	20·81	2·88
BSB 116	9 ×7	50	14·71	0·40	0·825	208·13	182·73	40·17	46·25	40·61	11·48
BSB 115	9 ×4	21	6·18	0·30	0·457	81·13	69·81	4·15	18·03	15·51	2·07
BSB 114	8 ×6	35	10·30	0·35	0·648	115·06	101·21	19·54	28·76	25·30	6·51
BSB 113	8 ×5	28	8·28	0·35	0·575	89·69	76·89	10·19	22·42	19·22	4·08
BSB 112	8 ×4	18	5·30	0·28	0·398	55·63	47·85	3·51	13·91	11·96	1·75
BSB 111	7 ×4	16	4·75	0·25	0·387	39·51	33·80	3·37	11·29	9·66	1·69
BSB 110	6 ×5	25	7·37	0·41	0·520	43·69	37·36	9·10	14·56	12·45	3·64
BSB 109	6 ×4½	20	5·89	0·37	0·431	34·71	29·32	5·40	11·57	9·77	2·40
BSB 108	6 ×3	12	3·53	0·23	0·377	20·99	17·57	1·46	7·00	5·86	0·97
BSB 107	5 ×4½	20	5·88	0·29	0·513	25·03	20·86	6·59	10·01	8·34	2·93
BSB 106	5 ×3	11	3·26	0·22	0·376	13·68	11·37	1·45	5·47	4·54	0·97
BSB 105	4¾×1¾	6·5	1·91	0·18	0·325	6·73	5·72	0·26	2·83	2·41	0·30
BSB 104	4 ×3	10	2·94	0·24	0·347	7·79	6·46	1·33	3·89	3·23	0·88
BSB 103	4 ×1¾	5	1·47	0·17	0·239	3·66	3·12	0·19	1·83	1·56	0·21
BSB 102	3 ×3	8·5	2·52	0·20	0·332	3·81	3·13	1·25	2·54	2·09	0·83
BSB 101	3 ×1½	4	1·18	0·16	0·249	1·66	1·36	0·13	1·11	0·91	0·17

* The net moments of inertia and section moduli allow for one hole per flange, $\frac{1}{16}$ in. greater in diameter than the appropriate bolt size given on the next page.

BRITISH STANDARD SECTIONS

JOISTS

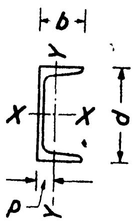
DIMENSIONS AND PROPERTIES IN INCH UNITS



Radii of Gyration		*Web*					*Bolts*		*Size d×b*	*Reference Mark*
		Web Depth		*Buckling Values*		*Shear Value in Tons*				
About XX	*About YY*	*Clear**	*Net**	*Joist*	*1-in. Bearing*		*Cross Centres*	*Diam.*		
4·22	1·84	6·56	7·84	14·69	2·93	26·00	4¾	⅞	10 × 8	BSB 120
4·17	1·36	7·13	8·14	12·78	2·55	23·40	3½	¾	10 × 6	BSB 119
4·06	1·05	7·64	8·52	12·61	2·52	23·40	2¾	¾	10 × 5	BSB 118
4·08	0·94	7·84	8·65	9·85	1·97	19·50	2½	¾	10 × 4½	BSB 117
3·76	1·65	5·69	6·83	13·61	3·02	23·40	4	⅞	9 × 7	BSB 116
3·62	0·82	7·04	7·78	9·20	2·04	17·55	2¼	⅝	9 × 4	BSB 115
3·34	1·38	5·25	6·26	10·49	2·62	18·20	3½	¾	8 × 6	BSB 114
3·29	1·11	5·60	6·47	10·42	2·60	18 20	2¾	¾	8 × 5	BSB 113
3·24	0·81	6·16	6·90	7·75	1·93	14·56	2¼	⅝	8 × 4	BSB 112
2·89	0·84	5·18	5·93	6·12	1·75	11·37	2¼	⅝	7 × 4	BSB 111
2·44	1·11	3·72	4·58	9·91	3·30	15·99	2¾	¾	6 × 5	BSB 110
2·43	0·96	4·00	4·80	8·77	2·92	14·43	2½	¾	6 × 4½	BSB 109
2·44	0·64	4·41	5·02	4·94	1·64	8·97	1½	½	6 × 3	BSB 108
2·06	1·06	2·83	3·64	5·76	2·30	9·42	2½	¾	5 × 4½	BSB 107
2·05	0·67	3·40	4·02	4·10	1·64	7·15	1½	½	5 × 3	BSB 106
1·88	0·37	3·52	3·96	3·05	1·28	5·55	⅞	¼	4¾ × 1¾	BSB 105
1·63	0·67	2·47	3·08	3·80	1·90	6·24	1½	½	4 × 3	BSB 104
1·58	0·36	2·94	3·39	2·49	1·24	4·42	⅞	¼	4 × 1¾	BSB 103
1·23	0·70	1·50	2·11	2·43	1·62	3·90	1½	½	3 × 3	BSB 102
1·19	0·33	1·97	2·39	1·85	1·23	3·12	¾	¼	3 × 1½	BSB 101

* Clear web depth is the depth of the parallel portion of the web. Net depth is the depth ignoring root fillets.

BRITISH STANDARD SECTIONS

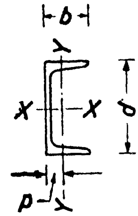


CHANNELS
DIMENSIONS AND PROPERTIES IN INCH UNITS

Reference Mark	*Size d×b*	*Weight per ft. in lb.*	*Area*	*p*	*Standard Thicknesses*		*Moments of Inertia*			*Section Moduli*		
							About XX		*About YY*	*About XX*		*About YY*
					Web	*Flange*	*Gross*	*Net**	*Gross*	*Gross*	*Net**	*Gross*
BSC 120A	17×4	51·28	15·08	0·91	0·60	0·68	569·31	494·73	16·96	66·98	58·20	5·28
BSC 120	17×4	44·34	13·04	0·92	0·48	0·68	520·18	446·65	15·26	61·20	52·55	4·96
BSC 119A	15×4	42·49	12·50	0·94	0·53	0·62	382·85	330·23	14·97	51·05	44·03	4·71
BSC 119	15×4	36·37	10·70	0·97	0·41	0·62	349·10	297·29	13·34	46·55	39·64	4·40
BSC 118A	13×4	38·92	11·45	1·01	0·53	0·62	270·66	231·63	14·51	41·64	35·63	4·64
BSC 118	13×4	33·18	9·76	1·04	0·40	0·62	246·86	208·47	12·76	37·98	32·07	4·31
BSC 117A	12×4	36·63	10·77	1·02	0·53	0·60	218·81	186·77	13·80	36·47	31·13	4·44
BSC 117	12×4	31·33	9·21	1·06	0·40	0·60	200·09	168·59	12·12	33·35	28·10	4·12
BSC 116A	12×3½	30·45	8·96	0·81	0·48	0·50	174·13	147·08	7·96	29·02	24·51	2·86
BSC 116	12×3½	26·37	7·76	0·83	0·38	0·50	159·73	133·11	7·15	26·62	22·18	2·68
BSC 115A	11×3½	30·52	8·98	0·91	0·48	0·58	152·96	127·21	8·86	27·81	23·13	3·30
BSC 115	11×3½	26·78	7·88	0·93	0·38	0·58	141·87	116·47	7·93	25·80	21·17	3·09
BSC 114A	10×3½	28·54	8·39	0·94	0·48	0·56	119·52	99·08	8·50	23·90	19·82	3·17
BSC 114	10×3½	24·46	7·19	0·97	0·36	0·56	109·52	89·42	7·42	21·90	17·88	2·93
BSC 113A	10×3	21·33	6·27	0·73	0·38	0·45	87·66	71·06	4·31	17·53	14·21	1·85
BSC 113	10×3	19·28	5·67	0·74	0·32	0·45	82·66	66·24	3·98	16·53	13·25	1·76
BSC 112B	9×3½	25·63	7·54	0·97	0·45	0·54	89·30	73·51	7·86	19·84	16·34	2·98
BSC 112A	9×3½	23·49	6·91	0·99	0·38	0·54	85·05	69·42	7·26	18·90	15·43	2·85
BSC 112	9×3½	22·27	6·55	1·00	0·34	0·54	82·62	67·08	6·90	18·36	14·91	2·76
BSC 111A	9×3	19·91	5·86	0·76	0·38	0·44	67·38	54·32	4·18	14·97	12·07	1·80
BSC 111	9×3	17·46	5·14	0·78	0·30	0·44	62·52	49·65	3·75	13·89	11·03	1·69

* The net moments of inertia and section moduli allow for one hole per flange, $\frac{1}{16}$ in. greater in diameter than the appropriate bolt size given on the next page.

BRITISH STANDARD SECTIONS



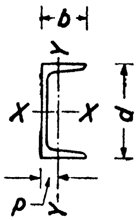
CHANNELS

DIMENSIONS AND PROPERTIES
IN INCH UNITS

Radii of Gyration		*Web*					*Holes*		*Size d×b*	*Reference Mark*
		Web Depth		*Buckling Values*		*Shear Value in Tons*				
About XX	*About YY*	*Clear**	*Net**	*Channel*	*1-in. Bearing*		*Back Mark*	*Diam.*		
6·14	1·06	14·24	15·23	35·00	4·11	66·30	2¼	¾	17×4	BSC 120A
6·32	1·08	14·23	15·25	25·82	3·03	53·04	2¼	¾	17×4	BSC 120
5·54	1·09	12·36	13·35	27·36	3·64	51·67	2¼	¾	15×4	BSC 119A
5·71	1·12	12·35	13·37	19·24	2·56	39·97	2¼	¾	15×4	BSC 119
4·86	1·13	10·36	11·35	24·80	3·81	44·78	2¼	¾	13×4	BSC 118A
5·03	1·14	10·35	11·37	17·19	2·64	33·80	2¼	¾	13×4	BSC 118
4·51	1·13	9·40	10·39	23·37	3·89	41·34	2¼	¾	12×4	BSC 117A
4·66	1·15	9·39	10·42	16·34	2·72	31·20	2¼	¾	12×4	BSC 117
4·41	0·94	9·75	10·64	20·55	3·42	37·44	2	¾	12×3½	BSC 116A
4·54	0·96	9·74	10·66	15·14	2·52	29·64	2	¾	12×3½	BSC 116
4·13	0·99	8·59	9·48	19·37	3·52	34·32	2	¾	11×3½	BSC 115A
4·24	1·00	8·58	9·50	14·41	2·62	27·17	2	¾	11×3½	BSC 115
3·77	1·01	7·63	8·52	18·02	3·60	31·20	2	¾	10×3½	BSC 114A
3·90	1·02	7·62	8·54	12·60	2·52	23·40	2	¾	10×3½	BSC 114
3·74	0·83	7·99	8·80	13·40	2·68	24·70	1¾	¾	10×3	BSC 113A
3·82	0·84	7·99	8·81	10·69	2·13	20·80	1¾	¾	10×3	BSC 113
3·44	1·02	6·66	7·56	15·36	3·41	26·32	2	¾	9×3½	BSC 112B
3·51	1·03	6·66	7·58	12·51	2·78	22·23	2	¾	9×3½	BSC 112A
3·55	1·03	6·65	7·58	10·90	2·42	19·89	2	¾	9×3½	BSC 112
3·39	0·85	7·01	7·82	12·43	2·76	22·23	1¾	¾	9×3	BSC 111A
3·49	0·86	7·00	7·83	9·18	2·04	17·55	1¾	¾	9×3	BSC 111

* Clear web depth is the depth of the parallel portion of the web. Net depth is the depth ignoring root fillets.

BRITISH STANDARD SECTIONS



CHANNELS
DIMENSIONS AND PROPERTIES IN INCH UNITS

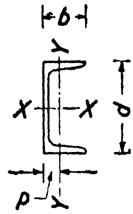
Reference Mark	Size $d \times b$	Weight per ft. in lb.	Area	p	Standard Thicknesses		Moments of Inertia			Section Moduli		
					Web	Flange	About XX Gross	About XX Net*	About YY Gross	About XX Gross	About XX Net*	About YY Gross
BSC 110A	8 × 3½	23·20	6·82	1·01	0·43	0·52	65·27	53·40	7·30	16·32	13·35	2·81
BSC 110	8 × 3½	20·21	5·94	1·05	0·32	0·52	60·57	48·89	6·37	15·14	12·22	2·60
BSC 109A	8 × 3	18·68	5·49	0·81	0·38	0·44	50·99	40·78	4·11	12·75	10·20	1·79
BSC 109	8 × 3	15·96	4·69	0·83	0·28	0·44	46·72	36·69	3·58	11·68	9·17	1·65
BSC 108A	7 × 3½	20·18	5·94	1·07	0·38	0·50	45·12	36·55	6·48	12·89	10·44	2·58
BSC 108	7 × 3½	18·28	5·38	1·09	0·30	0·50	42·83	34·36	5·83	12·24	9·82	2·42
BSC 107A	7 × 3	17·07	5·02	0·84	0·38	0·42	36·18	28·78	3·87	10·34	8·22	1·70
BSC 107	7 × 3	14·22	4·18	0·88	0·26	0·42	32·75	25·51	3·26	9·36	7·29	1·53
BSC 106A	6 × 3½	18·52	5·45	1·11	0·38	0·48	30·68	24·73	6·05	10·23	8·24	2·43
BSC 106	6 × 3½	16·48	4·85	1·14	0·28	0·48	28·88	23·02	5·29	9·63	7·67	2·25
BSC 105A	6 × 3	17·53	5·16	0·90	0·43	0·48	27·18	21·23	3·95	9·06	7·08	1·84
BSC 105	6 × 3	16·51	4·86	0·91	0·38	0·48	26·28	20·38	3·70	8·76	6·79	1·77
BSC 104A	6 × 3	13·64	4·01	0·87	0·31	0·38	22·35	17·53	3·10	7·45	5·84	1·42
BSC 104	6 × 3	12·41	3·65	0·89	0·25	0·38	21·27	16·51	2·83	7·09	5·50	1·34
BSC 103A	5 × 2½	11·24	3·31	0·76	0·31	0·38	12·50	9·16	1·82	5·00	3·66	1·01
BSC 103	5 × 2½	10·22	3·01	0·77	0·25	0·38	11·87	8·57	1·64	4·75	3·43	0·95
BSC 102A	4 × 2	7·91	2·33	0·59	0·30	0·31	5·38	3·91	0·79	2·69	1·96	0·54
BSC 102	4 × 2	7·09	2·09	0·60	0·24	0·31	5·06	3·61	0·70	2·53	1·80	0·50
BSC 101A	3 × 1½	5·11	1·50	0·48	0·25	0·28	1·94	1·35	0·30	1·29	0·90	0·28
BSC 101	3 × 1½	4·60	1·35	0·48	0·20	0·28	1·82	1·24	0·26	1·22	0·83	0·26

*** The net moments of inertia and net section moduli allow for one hole per flange, $\frac{1}{16}$ in. greater in diameter than the appropriate bolt size given on the next page.**

BRITISH STANDARD SECTIONS

CHANNELS

DIMENSIONS AND PROPERTIES IN INCH UNITS

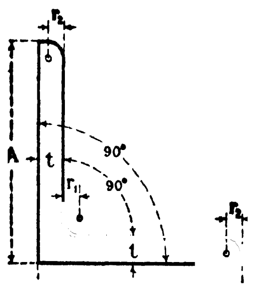


Radii of Gyration		*Web*					*Holes*		*Size d×b*	*Reference Mark*
		Web Depth		*Buckling Values*		*Shear Value in Tons*				
About XX	*About YY*	*Clear**	*Net**	*Channel*	*1-in. Bearing*		*Back Mark*	*Diam.*		
3·09	1·03	5·70	6·61	13·25	3·31	22·36	2	¾	8×3½	BSC 110A
3·19	1·04	5·69	6·63	9·28	2·32	16·64	2	¾	8×3½	BSC 110
3·05	0·87	6·01	6·82	11·38	2·84	19·76	1¾	¾	8×3	BSC 109A
3·16	0·87	6·00	6·83	7·78	1·94	14·56	1¾	¾	8×3	BSC 109
2·76	1·05	4·74	5·65	10·30	2·94	17·29	2	¾	7×3½	BSC 108A
2·82	1·04	4·73	5·67	7·78	2·22	13·65	2	¾	7×3½	BSC 108
2·68	0·88	5·05	5·85	10·24	2·92	17·29	1¾	¾	7×3	BSC 107A
2·80	0·88	5·04	5·87	6·46	1·84	11·83	1¾	¾	7×3	BSC 107
2·37	1·05	3·78	4·69	9·07	3·02	14·82	2	¾	6×3½	BSC 106A
2·44	1·05	3·77	4·71	6·37	2·12	10·92	2	¾	6×3½	BSC 106
2·30	0·88	3·94	4·74	10·41	3·47	16·77	1¾	¾	6×3	BSC 105A
2·33	0·87	3·93	4·74	9·06	3·02	14·82	1¾	¾	6×3	BSC 105
2·36	0·88	4·13	4·95	7·11	2·37	12·09	1¾	¾	6×3	BSC 104A
2·41	0·88	4·12	4·96	5·49	1·83	9·75	1¾	¾	6×3	BSC 104
1·94	0·74	3·28	3·99	6·13	2·45	10·07	1⅜	¾	5×2½	BSC 103A
1·99	0·74	3·27	4·00	4·78	1·91	8·12	1⅜	¾	5×2½	BSC 103
1·52	0·58	2·57	3·17	4·86	2·43	7·80	1⅛	⅝	4×2	BSC 102A
1·56	0·58	2·57	3·18	3·78	1·89	6·24	1⅛	⅝	4×2	BSC 102
1·14	0·44	1·78	2·28	3·08	2·05	4·87	⅞	½	3×1½	BSC 101A
1·16	0·44	1·78	2·29	2·41	1·60	3·90	⅞	½	3×1½	BSC 101

* Clear web depth is the depth of the parallel portion of the web. Net depth is the depth ignoring root fillets.

BRITISH STANDARD SECTIONS

*EQUAL ANGLES



a = Sectional area in sq. in.

w = Weight in lb. per ft. = $3{\cdot}4a$.

$r_1 = 0{\cdot}03\ (A+B) + 0{\cdot}12$.

$r_2 = 0{\cdot}7r_1$.

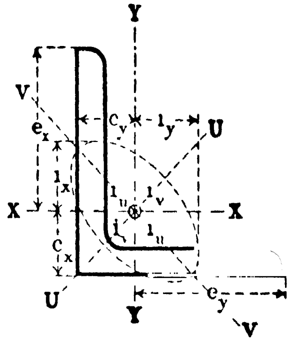
Note.—The thicknesses printed in heavy type are the Standards. For note on other thicknesses see Remarks, pages 884 and 885.

1	2	3	4	5	6	7	8
				Radii			
Reference No.	*Size* $A \times B$	*Thickness* t	*Minimum Thickness Rolled*	*Root* r_1	*Toe* r_2	*Calculated Weight per foot* w	*Sectional Area* a
	in.	in.	in.	in.		lb.	in.^{2}
BSEA 101	1 × 1	·125 ·1875 *·25*	·125	·18	·13	·80 1·15 *1·49*	·23 ·34 *·44*
BSEA 102	1¼ × 1¼	·125 ·1875 *·25*	·125	·20	·14	1·01 1·47 *1·91*	·30 ·43 *·56*
BSEA 103	1½ × 1½	·1875 ·25 *·3125*	*·125*	·21	·15	1·79 2·34 *2·85*	·53 ·69 *·84*
BSEA 104	1¾ × 1¾	·1875 ·25 *·3125*	*·175*	·23	·16	2·11 2·76 *3·39*	·62 ·81 *1·00*
BSEA 105	2 × 2	·1875 ·25 ·3125 *·375*	*·175*	·24	·17	2·43 3·19 3·92 *4·62*	·71 ·94 1·15 *1·36*
BSEA 106	2¼ × 2¼	·1875 ·25 ·3125 *·375*	*·175*	·26	·18	2·75 3·61 4·45 *5·26*	·81 1·06 1·31 *1·55*
BSEA 107	2½ × 2½	·25 ·3125 ·375 *·4375* *·5*	*·2*	·27	·19	4·04 4·98 5·90 *6·79* *7·65*	1·19 1·46 1·73 *2·00* *2·25*

* See remarks at end of Table.

BRITISH STANDARD SECTIONS

EQUAL ANGLES



c_x, c_y Distance of centre of gravity from back lines of angle.

$J = ai^2$ Moment of inertia.

$i = \sqrt{(J/a)}$ Radius of gyration.

e_x, e_y Distance of outer fibres from X and Y axes.

$Z = J/e$ Modulus of section.

9	10	11	12	13	14	15	16	17	18	19	20	21
Centre of Gravity		*Moments of Inertia*				*Radii of Gyration*				*Moduli of Section*		*Reference No.*
				Max.	*Min.*			*Max.*	*Min.*			
c_x	c_y	J_x	J_y	J_u	J_v	i_x	i_y	i_u	i_v	Z_x	Z_y	
in.		in.[4]		in.[4]		in.		in.		in.[3]		
·29	·29	·02	·02	·03	·01	·29	·29	·37	·19	·03	·03	
·31	31	·03	·03	·04	·01	·29	·29	·36	·19	·04	04	
·33	*·33*	*·04*	*·04*	*·05*	*·02*	*·28*	*·28*	*·35*	*·19*	*·05*	*·05*	BSEA 101
·34	·34	·04	·04	·06	·02	·37	·37	·47	·24	·05	·05	
·37	·37	·06	·06	·09	·02	·37	·37	·46	·24	·07	·07	BSEA 102
·40	*·40*	*·07*	*·07*	*·12*	*·03*	*·36*	*·36*	*·45*	*·24*	*09*	*09*	
·43	·43	·10	·10	·17	·04	·45	·45	·56	·29	·10	·10	
·46	·46	·13	·13	·21	·06	·44	·44	·55	·29	·13	·13	BSEA 103
·48	*·48*	*·16*	*·16*	*·25*	*·07*	*·44*	*·44*	*·55*	*·29*	*·16*	*·16*	
·49	·49	·17	·17	·27	·07	·52	·52	·66	·34	·14	·14	
·52	·52	·22	·22	·35	·09	·52	·52	·65	·34	·18	·18	BSEA 104
·54	*·54*	*·26*	*·26*	*·41*	*·11*	*·51*	*·51*	*·64*	*·34*	*·22*	*·22*	
·56	·56	·26	·26	·41	·11	·60	·60	·76	·39	·18	·18	
·58	·58	·34	·34	·53	·14	·60	·60	·75	·39	·24	·24	BSEA 105
·61	·61	·40	·40	·64	·17	·59	·59	·75	·38	·29	·29	
·63	*·63*	*·47*	*·47*	*·74*	*·20*	*·59*	*·59*	*·74*	*·38*	*·34*	*·34*	
·62	·62	·38	·38	·60	·16	·68	·68	·86	·44	·23	·23	
·64	·64	·49	·49	·77	·20	·68	·68	·85	·44	·30	·30	BSEA 106
·67	·67	·59	·59	·94	·25	·67	·67	·85	·43	·37	·37	
·69	*·69*	*·69*	*·69*	*1·08*	*·29*	*·67*	*·67*	*·84*	*·43*	*·44*	*·44*	
·70	·70	·68	·68	1·08	·28	·76	·76	·95	·49	·38	·38	
·73	·73	·83	·83	1·31	·34	·75	·75	·95	·48	·47	·47	
·75	·75	·96	·96	1·52	·40	·74	·74	·94	·48	·55	·55	BSEA 107
·78	*·78*	*1·09*	*1·09*	*1·71*	*·46*	*·74*	*·74*	*·93*	*·48*	*·63*	*·63*	
·80	*·80*	*1·21*	*1·21*	*1·89*	*·52*	*·73*	*·73*	*·92*	*·48*	*·71*	*·71*	

BRITISH STANDARD SECTIONS

*EQUAL ANGLES

(continued)

1	2	3	4	5	6	7	8
				Radii			
Reference No.	*Size* $A \times B$	*Thickness* t	*Minimum Thickness Rolled*	*Root* r_1	*Toe* r_2	*Calculated Weight per foot* w	*Sectional Area* a
	in.	in.	in.	in.		lb.	in.^{2}
		·25				**4·89**	**1·44**
		·3125				*6·04*	*1·78*
BSEA 108	3 × 3	**·375**	**·25**	**·30**	**·21**	**7·17**	**2·11**
		·4375				*8·28*	*2·43*
		·5				**9·35**	**2·75**
		·5625				*10·40*	*3·06*
		·25				**5·74**	**1·69**
		·3125				*7·11*	*2·09*
		·375				**8·45**	**2·49**
BSEA 109	3½ × 3½	*·4375*	**·25**	·33	·23	*9·76*	*2·87*
		·5				**11·05**	**3·25**
		·5625				*12·31*	*3·62*
		·625				*13·55*	*3·99*
		·3125				*8·17*	*2·40*
		·375				**9·73**	**2·86**
		·4375				*11·25*	*3·31*
BSEA 110	4 × 4	**·5**	*·3*	·36	·25	**12·75**	**3·75**
		·5625				*14·23*	*4·18*
		·625				**15·68**	**4·61**
		·6875				*17·10*	*5·03*
		·75				*18·49*	*5·44*
		·3125				*9·24*	*2·72*
		·375				**11·00**	**3·24**
		·4375				*12·74*	*3·75*
BSEA 111	4½ × 4½	**·5**	*·3*	**·39**	**·27**	**14·45**	**4·25**
		·5625				*16·14*	*4·75*
		·625				**17·80**	**5·24**
		·6875				*19·44*	*5·72*
		·75				*21·04*	*6·19*
		·375				**12·28**	**3·61**
		·4375				*14·23*	*4·19*
		·5				**16·16**	**4·75**
BSEA 112	5 × 5	*·5625*	*·35*	**·42**	·29	*18·06*	*5·31*
		·625				**19·93**	**5·86**
		·6875				*21·77*	*6·40*
		·75				*23·59*	*6·94*
		·375				**14·82**	**4·36**
		·4375				*17·20*	*5·06*
		·5				**19·55**	**5·75**
		·5625				*21·87*	*6·43*
BSEA 113	6 × 6	**·625**	·375	**·48**	·34	**24·17**	**7·11**
		·6875				*26·44*	*7·78*
		·75				**28·69**	**8·44**
		·8125				*30·90*	*9·09*
		·875				*33·10*	*9·73*

* See remarks at end of Table.

BRITISH STANDARD SECTIONS

EQUAL ANGLES

(continued)

9	10	11	12	13	14	15	16	17	18	19	20	21
Centre of Gravity		Moments of Inertia				Radii of Gyration				Moduli of Section		Reference No.
				Max.	Min.			Max.	Min.			
c_x	c_y	J_x	J_y	J_u	J_v	i_x	i_y	i_u	i_v	Z_x	Z_y	
in.		in.^{4}		in.^{4}		in.		in.		in.^{3}		
·83	·83	1·20	1·20	1·91	·49	·91	·91	1·15	·59	·55	·55	
·85	·85	1·47	1·47	2·33	·60	·91	·91	1·15	·58	·68	·68	
·88	·88	1·72	1·72	2·73	·71	·90	·90	1·14	·58	·81	·81	BSEA 108
·90	·90	1·96	1·96	3·09	·82	·90	·90	1·13	·58	·93	·93	
·92	·92	2·18	2·18	3·44	·92	·89	·89	1·12	·58	1·05	1·05	
·95	·95	2·39	2·39	3·75	1·02	·88	·88	1·11	·58	1·16	1·16	
·95	·95	1·94	1·94	3·09	·80	1·07	1·07	1·35	·69	·76	·76	
·97	·97	2·38	2·38	3·79	·98	1·07	1·07	1·35	·68	·94	·94	
1·00	1·00	2·80	2·80	4·45	1·15	1·06	1·06	1·34	·68	1·12	1·12	
1·02	1·02	3·20	3·20	5·07	1·32	1·05	1·05	1·33	·68	1·29	1·29	BSEA 109
1·05	1·05	3·57	3·57	5·65	1·49	1·05	1·05	1·32	·68	1·46	1·46	
1·07	1·07	3·93	3·93	6·20	1·65	1·04	1·04	1·31	·68	1·62	1·62	
1·09	1·09	4·27	4·27	6·72	1·82	1·03	1·03	1·30	·68	1·77	1·77	
1·10	1·10	3·61	3·61	5·74	1·48	1·23	1·23	1·55	·78	1·24	1·24	
1·12	1·12	4·26	4·26	6·77	1·75	1·22	1·22	1·54	·78	1·48	1·48	
1·15	1·15	4·87	4·87	7·74	2·00	1·21	1·21	1·53	·78	1·71	1·71	
1·17	1·17	5·46	5·46	8·66	2·26	1·21	1·21	1·52	·78	1·93	1·93	BSEA 110
1·20	1·20	6·02	6·02	9·54	2·51	1·20	1·20	1·51	·77	2·15	2·15	
1·22	1·22	6·56	6·56	10·37	2·76	1·19	1·19	1·50	·77	2·36	2·36	
1·24	1·24	7·08	7·08	11·15	3·00	1·19	1·19	1·49	·77	2·57	2·57	
1·26	1·26	7·57	7·57	11·89	3·25	1·18	1·18	1·48	·77	2·77	2·77	
1·22	1·22	5·21	5·21	8·27	2·14	1·38	1·38	1·75	·89	1·59	1·59	
1·24	1·24	6·15	6·15	9·78	2·52	1·38	1·38	1·74	·88	1·89	1·89	
1·27	1·27	7·05	7·05	11·22	2·89	1·37	1·37	1·37	·88	2·18	2·18	
1·29	1·29	7·92	7·92	12.59	3·26	1·37	1·37	1·72	·88	2·47	2·47	BSEA 111
1·32	1·32	8·76	8·76	13·90	3·62	1·36	1·36	1·71	·87	2·75	2·75	
1·34	1·34	9·56	9·56	15·15	3·98	1·35	1·35	1·70	·87	3·03	3·03	
1·37	1·37	10·34	10·34	16·34	4·33	1·34	1·34	1·69	·87	3·30	3·30	
1·39	1·39	11·08	11·08	17·47	4·68	1·34	1·34	1·68	·87	3·56	3·56	
1·37	1·37	8·53	8·53	13·57	3·49	1·54	1·54	1·94	·98	2·35	2·35	
1·39	1·39	9·81	9·81	15·60	4·01	1·53	1·53	1·93	·98	2·72	2·72	
1·42	1·42	11·04	11·04	17·55	4·53	1·52	1·52	1·92	·98	3·08	3·08	
1·44	1·44	12·22	12·22	19·42	5·03	1·52	1·52	1·91	·97	3·44	3·44	BSEA 112
1·47	1·47	13·37	13·37	21·21	5·53	1·51	1·51	1·90	·97	3·78	3·78	
1·49	1·49	14·47	14·47	22·93	6·02	1·50	1·50	1·89	·97	4·12	4·12	
1·51	1·51	15·54	15·54	24·57	6·50	1·50	1·50	1·88	·97	4·46	4·46	
1·61	1·61	14·95	14·95	23·79	6·11	1·85	1·85	2·34	1·18	3·40	3·40	
1·64	1·64	17·25	17·25	27·45	7·04	1·85	1·85	2·33	1·18	3·95	3·95	
1·66	1·66	19·48	19·48	30·99	7·96	1·84	1·84	2·32	1·18	4·49	4·49	
1·69	1·69	21·64	21·64	34·42	8·86	1·83	1·83	2·31	1·17	5·02	5·02	
1·71	1·71	23·73	23·73	37·73	9·74	1·83	1·83	2·30	1·17	5·54	5·54	BSEA 113
1·74	1·74	25·77	25·77	40·92	10·61	1·82	1·82	2·29	1·17	6·04	6·04	
1·76	1·76	27·74	27·74	44·01	11·47	1·81	1·81	2·28	1·17	6·54	6·54	
1·78	1·78	29·65	29·65	46·99	12·31	1·81	1·81	2·27	1·16	7·03	7·03	
1·81	1·81	31·51	31·51	49·86	13·16	1·80	1·80	2·26	1·16	7·51	7·51	

BRITISH STANDARD SECTIONS

EQUAL ANGLES

(continued)

1	2	3	4	5	6	7	8
				Radii			
Reference No.	*Size* $A \times B$	*Thickness* t	*Minimum Thickness Rolled*	*Root* r_1	*Toe* r_2	*Calculated Weight per foot* w	*Sectional Area* a
	in.	in.	in.	in.		lb.	in.^{2}
		·5				**22·95**	**6·75**
		·5625				*25·70*	*7·56*
		·625				**28·42**	**8·36**
		·6875				*31·12*	*9·15*
BSEA 114	7 × 7	**·75**	**·5**	**·54**	**·38**	**33·79**	**9·94**
		·8125				*36·43*	*10·72*
		·875				*39·05*	*11·48*
		·9375				*41·64*	*12·25*
		1·00				*44·20*	*13·00*
		·625				**32·68**	**9·61**
		·6875				*35·80*	*10·53*
BSEA 115	8 × 8	**·75**				**38·89**	**11·44**
		·8125	*·55*	**·60**	**·42**	*41·96*	*12·34*
		·875				**45·00**	**13·24**
		·9375				*48·02*	*14·12*
		1·00				*51·01*	*15·00*

Remarks

1. The thicknesses printed in heavy type are the Standards. For the information of designers of structural steelwork the properties are given of sections differing from the standard thicknesses in steps of 0·0625 inch, but other intermediate thicknesses can be obtained.

2. The dimensions, thickness and profile of Standard Equal Angles shall be in accordance with the accompanying list and sketch, but finished sections in which the angle between the flanges is not less than 89° nor more than 91° shall be accepted as conforming to the Standard.

BRITISH STANDARD SECTIONS

EQUAL ANGLES

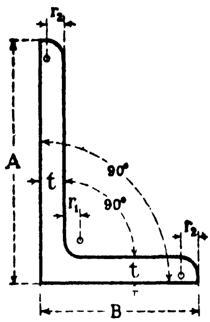
(*continued*)

9	10	11	12	13	14	15	16	17	18	19	20	21
Centre of Gravity		*Moments of Inertia*				*Radii of Gyration*				*Moduli of Section*		*Reference No.*
				Max.	*Min.*			*Max.*	*Min.*			
c_x	c_y	J_x	J_y	J_u	J_v	i_x	i_y	i_u	i_v	Z_x	Z_y	
in.		in.⁴		in.⁴		in.		in.		in.³		
1·91	**1·91**	**31·42**	**31·42**	**50·02**	**12·82**	**2·16**	**2·16**	**2·72**	**1·38**	**6·17**	**6·17**	
1·93	*1·93*	*34·98*	*34·98*	*55·68*	*14·29*	*2·15*	*2·15*	*2·71*	*1·37*	*6·90*	*6·90*	
1·96	**1·96**	**38·45**	**38·45**	**61·19**	**15·72**	**2·14**	**2·14**	**2·71**	**1·37**	**7·63**	**7·63**	
1·98	*1·98*	*41·83*	*41·83*	*66·53*	*17·14*	*2·14*	*2·14*	*2·70*	*1·37*	*8·34*	*8·34*	
2·01	**2·01**	**45·12**	**45·12**	**71·72**	**18·53**	**2·13**	**2·13**	**2·69**	**1·37**	**9·04**	**9·04**	BSEA 114
2·03	*2·03*	*48·33*	*48·33*	*76·75*	*19·91*	*2·12*	*2·12*	*2·68*	*1·36*	*9·73*	*9·73*	
2·06	*2·06*	*51·45*	*51·45*	*81·64*	*21·27*	*2·12*	*2·12*	*2·67*	*1·36*	*10·41*	*10·41*	
2·08	*2·08*	*54·50*	*54·50*	*86·37*	*22·62*	*2·11*	*2·11*	*2·66*	*1·36*	*11·07*	*11·07*	
2·10	*2·10*	*57·46*	*57·46*	*90·97*	*23·96*	*2·10*	*2·10*	*2·65*	*1·36*	*11·73*	*11·73*	
2·20	**2·20**	**58·26**	**58·26**	**92·75**	**23·78**	**2·46**	**2·46**	**3·11**	**1·57**	**10·05**	**10·05**	
2·23	*2·23*	*63·48*	*63·48*	*101·03*	*25·93*	*2·46*	*2·46*	*3·10*	*1·57*	*11·00*	*11·00*	
2·25	**2·25**	**68·58**	**68·58**	**109·11**	**28·06**	**2·45**	**2·45**	**3·09**	**1·57**	**11·94**	**11·94**	
2·28	*2·28*	*73·57*	*73·57*	*116·98*	*30·16*	*2·44*	*2·44*	*3·08*	*1·56*	*12·86*	*12·86*	BSEA 115
2·30	**2·30**	**78·44**	**78·44**	**124·65**	**32·23**	**2·43**	**2·43**	**3·07**	**1·56**	**13·77**	**13·77**	
2·33	*2·33*	*83·20*	*83·20*	*132·12*	*34·28*	*2·43*	*2·43*	*3·06*	*1·56*	*14·66*	*14·66*	
2·35	*2·35*	*87·85*	*87·85*	*139·40*	*36·31*	*2·42*	*2·42*	*3·05*	*1·56*	*15·55*	*15·55*	

3. Angles ordered to the standard thickness shall be practically accurate in profile; but if the thickness is above the Standards, the flanges will be longer than the Standards. The profile at the back of the toe will be slightly rounded when above the Standards, instead of square; but the radii at the root and toe will remain unchanged. In Equal Sided Angles the thickness of the flanges will be the same.

4. Angles may be ordered by width of flanges and thickness, or by width of flanges and weight per foot, but not by both thickness and weight per foot.

BRITISH STANDARD SECTIONS



* UNEQUAL ANGLES

a = Sectional area in sq. in.

w = Weight in lb. ft. = $3{\cdot}4a$.

$r_1 = 0{\cdot}03\ (A+B) + 0{\cdot}12$.

$r_2 = 0{\cdot}7r_1$.

Note.—The thicknesses printed in heavy type are the Standards. For note on other thicknesses see Remarks, pages 892 and 893.

1	2	3	4	5	6	7	8
Reference No.	*Size* $A \times B$	*Thickness* t	*Minimum Thickness Rolled*	*Radii* Root r_1	*Radii* Toe r_2	*Calculated Weight per foot* w	*Sectional Area* a
	in.	in.	in.	in.		lb.	in.²
BSUA 101	2 × 1½	**·1875** **·25** *·3125*	*·15*	**·23**	**·16**	**2·11** **2·76** *3·39*	**·62** **·81** *1·00*
BSUA 102	2½ × 1½	**·1875** **·25** *·3125*	*·15*	**·24**	**·17**	**2·43** **3·19** *3·92*	**·71** **·94** *1·15*
BSUA 103	2½ × 2	**·1875** **·25** **·3125** *·375*	*·175*	**·26**	**·18**	**2·75** **3·61** **4·45** *5·26*	**·81** **1·06** **1·31** *1·55*
BSUA 104	3 × 2	*·1875* **·25** **·3125** **·375** *·4375*	*·1875*	**·27**	**·19**	*3·07* **4·04** **4·98** **5·90** *6·79*	*·90* **1·19** **1·46** **1·73** *2·00*
BSUA 105	3 × 2½	**·25** **·3125** **·375** *·4375*	*·225*	**·29**	**·20**	**4·47** **5·51** **6·54** *7·53*	**1·31** **1·62** **1·92** *2·22*
BSUA 106	3½ × 2½	**·25** **·3125** **·375** *·4375*	**·25**	**·30**	**·21**	**4·89** **6·04** **7·17** *8·28*	**1·44** **1·78** **2·11** *2·43*

* See remarks at end of Table.

BRITISH STANDARD SECTIONS

UNEQUAL ANGLES

c_x, c_y Distance of centre of gravity from back lines of angle.

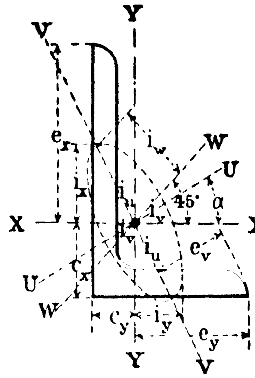
$J = ai^2$ Moment of inertia.

$i = \sqrt{(J/a)}$ Radius of gyration.

$$\text{Tan } 2a = \frac{J_x + J_y - 2J_w}{J_y - J_x}.$$

e_x, e_y, e_v Distance of outer fibres from X, Y and V axes.

$Z_v = J_v/e_v$ Minimum modulus of section.



9	10	11	12	13	14	15	16	17	18	19	20	21	22
Centre of Gravity		Moments of Inertia				Radii of Gyration				Tan. of Angle	Moduli of Section		Reference No.
				Max.	Min.			Max.	Min.				
c_x	c_y	J_x	J_y	J_u	J_v	i_x	i_y	i_u	i_v	a	Z_x	Z_y	
in.		in.⁴		in.⁴		in.		in.			in.³		
·63	·38	·24	·11	·29	·06	·62	·43	·68	·32	·54	·17	·10	
·65	·41	·31	·15	·37	·08	·61	·42	·68	·31	·54	·23	·13	BSUA 101
·68	*·43*	*·37*	*·17*	*·44*	*·10*	*·61*	*·42*	*·67*	*·31*	*·53*	*·28*	*·16*	
·83	·34	·45	·12	·49	·07	·79	·41	·83	·32	·35	·27	·10	
·86	·37	·58	·15	·64	·09	·78	·40	·82	·32	·35	·35	·14	BSUA 102
·89	*·39*	*·70*	*·18*	*·77*	*·11*	*·78*	*·40*	*·82*	*·32*	*·35*	*·43*	*·17*	
·75	·50	·49	·28	·62	·14	·78	·58	·88	·42	·62	·28	·18	
·77	·53	·63	·36	·81	·19	·77	·58	·87	·42	·62	·37	·24	BSUA 103
·80	·55	·77	·43	·98	·23	·77	·57	·86	·42	·62	·45	·30	
·82	*·57*	*·89*	*·50*	*1·13*	*·27*	*·76*	*·57*	*·85*	*·41*	*·61*	*·53*	*·35*	
·95	*·46*	*·81*	*·29*	*·94*	*·17*	*·95*	*·57*	*1·02*	*·43*	*·43*	*·40*	*·19*	
·98	·48	1·06	·38	1·22	·22	·94	·56	1·01	·43	·43	·52	·25	
1·00	·51	1·29	·45	1·48	·26	·94	·56	1·00	·43	·43	·65	·30	BSUA 104
1·03	·53	1·50	·53	1·72	·31	·93	·55	1·00	·42	·42	·76	·36	
1·05	*·56*	*1·71*	*·59*	*1·94*	*·36*	*·92*	*·55*	*·99*	*·42*	*·42*	*·88*	*·41*	
·89	·65	1·14	·72	1·50	·36	·93	·74	1·07	·52	·68	·54	·39	
·92	·67	1·39	·87	1·82	·44	·93	·73	1·06	·52	·68	·67	·48	BSUA 105
·94	·70	1·62	1·02	2·13	·51	·92	·73	1·05	·52	·67	·79	·56	
·97	*·72*	*1·84*	*1·15*	*2·41*	*·59*	*·91*	*·72*	*1·04*	*·52*	*·67*	*·91*	*·65*	
1·09	·60	1·75	·74	2·08	·41	1·10	·72	1·20	·54	·50	·73	·39	
1·12	·63	2·14	·91	2·55	·51	1·10	·71	1·20	·53	·50	·90	·48	BSUA 106
1·15	·65	2·51	1·06	2·98	·59	1·09	·71	1·19	·53	·49	1·07	·57	
1·17	*·68*	*2·86*	*1·20*	*3·38*	*·68*	*1·08*	*·70*	*1·18*	*·53*	*·49*	*1·23*	*·66*	

BRITISH STANDARD SECTIONS

* UNEQUAL ANGLES
(*continued*)

1	2	3	4	5	6	7	8
Reference No.	*Size* $A \times B$	*Thickness* t	*Minimum Thickness Rolled*	*Radii* *Root* r_1	*Toe* r_2	*Calculated Weight per foot* w	*Sectional Area* a
	in.	in.	in.	in.		lb.	in.^{2}
BSUA 107	3½ × 3	·25	·25	·32	·22	5·32	1·56
		·3125				6·58	1·93
		·375				7·81	2·30
		·4375				9·02	2·65
		·5				10·20	3·00
		·5625				11·36	3·34
BSUA 108	4 × 2½	·25	·25	·32	·22	5·32	1·56
		·3125				6·58	1·93
		·375				7·81	2·30
		·4375				9·02	2·65
BSUA 109	4 × 3	·3125	·275	·33	·23	7·11	2·09
		·375				8·45	2·49
		·4375				9·76	2·87
		·5				11·05	3·25
		·5625				12·31	3·62
BSUA 110	4 × 3½	·3125	·275	·35	·24	7·64	2·25
		·375				9·09	2·67
		·4375				10·51	3·09
		·5				11·91	3·50
		·5625				13·27	3·90
		·625				14·61	4·30
BSUA 111	4½ × 3	·3125	·3	·35	·24	7·64	2·25
		·375				9·09	2·67
		·4375				10·51	3·09
		·5				11·91	3·50
		·5625				13·27	3·90
BSUA 112	5 × 3	·3125	·3	·36	·25	8·17	2·40
		·375				9·73	2·86
		·4375				11·25	3·31
		·5				12·75	3·75
		·5625				14·23	4·18
BSUA 113	5 × 3½	·3125	·3125	·38	·26	8·71	2·56
		·375				10·37	3·05
		·4375				12·00	3·53
		·5				13·61	4·00
		·5625				15·19	4·47
		·625				16·74	4·92
BSUA 114	5 × 4	·375	·3125	·39	·27	11·00	3·24
		·4375				12·74	3·75
		·5				14·45	4·25
		·5625				16·14	4·75
		·625				17·80	5·24

* See remarks at end of Table.

BRITISH STANDARD SECTIONS

UNEQUAL ANGLES

(continued)

9	10	11	12	13	14	15	16	17	18	19	20	21	22
Centre of Gravity		Moments of Inertia				Radii of Gyration				Angle	Moduli of Section		Reference No.
				Max.	Min.			Max.	Min.				
c_x	c_y	J_x	J_y	J_u	J_v	i_x	i_y	i_u	i_v	a	Z_x	Z_y	
in.		in.[4]		in.[4]		in.		in.			in.[3]		
1·01	·77	1·86	1·26	2·50	·61	1·09	·90	1·27	·62	·72	·75	·56	
1·04	·79	2·27	1·54	3·06	·75	1·08	·89	1·26	·62	·72	·92	·70	
1·07	·82	2·67	1·80	3·59	·88	1·08	·88	1·25	·62	·72	1·10	·83	BSUA 107
1·09	·84	3·04	2·05	4·08	1·01	1·07	·88	1·24	·62	·72	1·26	·95	
1·12	·87	3·40	2·28	4·55	1·33	1·06	·87	1·23	·61	·71	1·43	1·07	
1·14	·89	3·74	2·51	4·98	1·26	1·06	·87	1·22	·61	·71	1·58	1·19	
1·30	·56	2·54	·77	2·85	·46	1·27	·70	1·35	·54	·39	94	·40	
1·33	·59	3·11	·94	3·49	·56	1·27	·70	1·34	·54	·39	1·17	·49	BSUA 108
1·36	·61	3·66	1·10	4·10	·66	1·26	·69	1·34	·53	·38	1·38	·58	
1·38	·64	4·18	1·24	4·67	·75	1·25	·68	1·33	·53	·38	1·59	·67	
1·24	·75	3·30	1·59	4·04	·86	1·26	·87	1·39	·64	·55	1·20	·71	
1·27	·77	3·89	1·87	4·75	1·01	1·25	·87	1·38	·64	·55	1·42	·84	
1·29	·80	4·44	2·13	5·41	1·16	1·24	·86	1·37	·63	·54	1·64	·96	BSUA 109
1·32	·82	4·97	2·37	6·04	1·30	1·24	·85	1·36	·63	·54	1·85	1·09	
1·34	·84	5·48	2·60	6·64	1·44	1·23	·85	1·35	·63	·54	2·06	1·21	
1·16	·92	3·47	2·47	4·77	1·18	1·24	1·05	1·46	·72	·75	1·22	·96	
1·19	·94	4·09	2·91	5·61	1·39	1·24	1·04	1·45	·72	·75	1·45	1·14	
1·21	·97	4·68	3·32	6·41	1·59	1·23	1·04	1·44	·72	·75	1·68	1·31	BSUA 110
1·24	·99	5·24	3·72	7·16	1·79	1·22	1·03	1·43	·72	·75	1·90	1·48	
1·26	1·01	5·77	4·09	7·87	1·99	1·22	1·02	1·42	·71	·75	2·11	1·65	
1·29	1·04	6·28	4·45	8·55	2·19	1·21	1·02	1·41	·71	·74	2·31	1·81	
1·44	·70	4·59	1·64	5·28	·94	1·43	·85	1·53	·65	·44	1·50	·71	
1·47	·73	5·41	1·92	6·22	1·11	1·42	·85	1·53	·64	·44	1·79	·85	
1·50	·75	6·19	2·19	7·11	1·27	1·42	·84	1·52	·64	·43	2·06	·98	
1·52	·78	6·94	2·45	7·96	1·43	1·41	·84	1·51	·64	·43	2·33	1·10	BSUA 111
1·55	·80	7·66	2·69	8·76	1·59	1·40	·83	1·50	·64	·43	2·59	1·22	
1·66	·67	6·14	1·68	6·80	1·01	1·60	·84	1·68	·65	·36	1·84	·72	
1·68	·69	7·25	1·97	8·03	1·19	1·59	·83	1·68	·65	·36	2·18	·85	
1·71	·72	8·31	2·25	9·19	1·37	1·58	·82	1·67	·64	·36	2·53	·99	BSUA 112
1·73	·74	9·33	2·51	10·31	1·54	1·58	·82	1·66	·64	·35	2·86	1·11	
1·76	·77	10·31	2·76	11·37	1·71	1·57	·81	1·65	·64	·35	3·18	1·24	
1·56	·82	6·46	2·62	7·61	1·47	1·59	1·01	1·72	·76	·48	1·88	·98	
1·59	·85	7·63	3·09	8·99	1·73	1·58	1·01	1·72	·75	·48	2·24	1·16	
1·62	·87	8·76	3·53	10·30	1·99	1·58	1·00	1·71	·75	·48	2·59	1·34	
1·64	·90	9·84	3·95	11·56	2·24	1·57	·99	1·70	·75	·48	2·93	1·52	BSUA 113
1·66	·92	10·88	4·36	12·76	2·48	1·56	·99	1·69	·75	·47	3·26	1·69	
1·69	·94	11·89	4·74	13·91	2·72	1·55	·98	1·68	·74	·47	3·59	1·86	
1·51	1·01	7·97	4·53	10·18	2·32	1·57	1·18	1·77	·85	·63	2·28	1·52	
1·53	1·04	9·15	5·19	11·68	2·67	1·56	1·18	1·77	·84	·62	2·64	1·75	
1·56	1·06	10·29	5·83	13·11	3·01	1·56	1·17	1·76	·84	·62	2·99	1·98	BSUA 114
1·58	1·09	11·39	6·44	14·48	3·34	1·55	1·16	1·75	·84	·62	3·33	2·21	
1·61	1·11	12·44	7·02	15·79	3·67	1·54	1·16	1·74	·84	·62	3·67	2·43	

BRITISH STANDARD SECTIONS

* UNEQUAL ANGLES

(continued)

1	2	3	4	5	6	7	8
Reference No.	Size $A \times B$	Thickness t	Minimum Thickness Rolled	Radii Root r_1	Radii Toe r_2	Calculated Weight per foot w	Sectional Area a
	in.	in.	in.	in.		lb.	in.^{2}
BSUA 115	6 × 3	·3125	·3125	**·39**	**·27**	9·24	2·72
		·375				**11·00**	**3·24**
		·4375				12·74	3·75
		·5				**14·45**	**4·25**
		·5625				16·14	4·75
		·625				17·80	5·24
BSUA 116	6 × 3½	·3125	·3125	**·41**	**·29**	9·76	2·87
		·375				**11·63**	**3·42**
		·4375				13·48	3·96
		·5				**15·30**	**4·50**
		·5625				17·09	5·03
		·625				18·86	5·55
BSUA 117	6 × 4	**·375**	·35	**·42**	**·29**	**12·28**	**3·61**
		·4375				14·23	4·19
		·5				**16·16**	**4·75**
		·5625				18·06	5·31
		·625				**19·93**	**5·86**
		·6875				21·77	6·40
		·75				23·59	6·94
BSUA 118	7 × 3½	·375	·375	**·44**	**·31**	12·91	3·80
		·4375				**14·97**	**4·40**
		·5				**17·00**	**5·00**
		·5625				19·01	5·59
		·625				20·99	6·17
BSUA 119	7 × 4	**·5**	·425	**·45**	**·32**	**17·85**	**5·25**
		·5625				19·96	5·87
		·625				**22·05**	**6·48**
		·6875				24·10	7·09
		·75				26·14	7·69
BSUA 120	8 × 3½	·375	·375	**·47**	**·33**	14·19	4·17
		·4375				**16·46**	**4·84**
		·5				**18·70**	**5·50**
		·5625				20·92	6·15
		·625				23·11	6·80
BSUA 121	8 × 4	**·5**	·425	**·48**	**·34**	**19·55**	**5·75**
		·5625				21·87	6·43
		·625				**24·17**	**7·11**
		·6875				26·44	7·78
		·75				28·69	8·44

* See remarks at end of Table.

BRITISH STANDARD SECTIONS

UNEQUAL ANGLES

(continued)

9	10	11	12	13	14	15	16	17	18	19	20	21	22
Centre of Gravity		Moments of Inertia				Radii of Gyration				Tan. of Angle	Moduli of Section		Reference No.
				Max.	Min.			Max.	Min.				
c_x	c_y	J_x	J_y	J_u	J_v	i_x	i_y	i_u	i_v	α	Z_x	Z_y	
in.		in.[4]		in.[4]		in.		in.			in.[3]		
2·09	·61	10·13	1·74	10·76	1·12	1·93	·80	1·99	·64	·26	2·59	·73	
2·12	·63	11·99	2·05	12·72	1·32	1·93	·80	1·98	·64	·26	3·09	·87	
2·15	·66	13·78	2·34	14·61	1·51	1·92	·79	1·97	·64	·26	3·58	1·00	
2·17	·68	15·51	2·62	16·43	1·70	1·91	·78	1·97	·63	·26	4·05	1·13	BSUA 115
2·20	·71	17·18	2·88	18·18	1·89	1·90	·78	1·96	·63	·26	4·52	1·26	
2·22	·73	18·80	3·14	19·86	2·07	1·89	·77	1·95	·63	·25	4·98	1·38	
1·98	·75	10·65	2·72	11·72	1·65	1·93	·97	2·02	·76	·34	2·65	·99	
2·01	·77	12·62	3·21	13·87	1·96	1·92	·97	2·01	·76	·34	3·16	1·18	
2·04	·80	14·52	3·68	15·95	2·25	1·91	·96	2·01	·75	·34	3·66	1·36	
2·06	·82	16·36	4·13	17·95	2·53	1·91	·96	2·00	·75	·34	4·15	1·54	BSUA 116
2·09	·85	18·13	4·55	19·87	2·81	1·90	·95	1·99	·75	·34	4·63	1·72	
2·11	·87	19·85	4·96	21·73	3·08	1·89	·95	1·98	·75	·33	5·11	1·89	
1·91	·92	13·21	4·74	15·24	2·72	1·91	1·15	2·05	·87	·44	3·23	1·54	
1·94	·95	15·21	5·44	17·53	3·13	1·91	1·14	2·05	·86	·44	3·75	1·78	
1·97	·97	17·14	6·11	19·73	3·52	1·90	1·13	2·04	·86	·44	4·25	2·02	
1·99	1·00	19·01	6·76	21·85	3·91	1·89	1·13	2·03	·86	·43	4·74	2·25	BSUA 117
2·02	1·02	20·82	7·37	23·90	4·29	1·88	1·12	2·02	·86	·43	5·23	2·48	
2·04	1·05	22·57	7·96	25·86	4·67	1·88	1·12	2·01	·85	·43	5·70	2·70	
2·06	1·07	24·26	8·53	27·75	5·04	1·87	1·11	2·00	·85	·43	6·16	2·91	
2·44	·71	19·28	3·31	20·47	2·12	2·25	·93	2·32	·75	·26	4·23	1·19	
2·47	·74	22·22	3·80	23·58	2·44	2·25	·93	2·31	·74	·26	4·91	1·38	
2·50	·76	25·07	4·27	26·59	2·75	2·24	·92	2·31	·74	·26	5·57	1·56	BSUA 118
2·53	·79	27·84	4·71	29·50	3·06	2·23	·92	2·30	·74	·26	6·22	1·74	
2·55	·81	30·53	5·14	32·31	3·35	2·22	·91	2·29	·74	·26	6·86	1·91	
2·39	·90	26·26	6·33	28·69	3·89	2·24	1·10	2·34	·86	·33	5·70	2·04	
2·42	·93	29·17	7·00	31·85	4·32	2·23	1·09	2·33	·86	·33	6·37	2·28	
2·44	·95	32·00	7·64	34·90	4·74	2·22	1·09	2·32	·86	·33	7·02	2·51	BSUA 119
2·47	·98	34·75	8·26	37·86	5·16	2·21	1·08	2·31	·85	·32	7·67	2·73	
2·49	1·00	37·42	8·86	40·72	5·57	2·21	1·07	2·30	·85	·32	8·30	2·95	
2·89	·66	27·79	3·39	28·93	2·25	2·58	·90	2·63	·73	·21	5·44	1·20	
2·92	·69	32·08	3·89	33·38	2·59	2·57	·90	2·63	·73	·21	6·31	1·39	
2·95	·72	36·24	4·38	37·69	2·93	2·57	·89	2·62	·73	·21	7·17	1·57	BSUA 120
2·97	·74	40·30	4·84	41·88	3·25	2·56	·89	2·61	·73	·21	8·02	1·75	
3·00	·77	44·24	5·28	45·95	3·57	2·55	·88	2·60	·72	·21	8·85	1·93	
2·83	·85	37·95	6·50	40·28	4·18	2·57	1·06	2·65	·85	·26	7·34	2·06	
2·86	·87	42·22	7·20	44·78	4·64	2·56	1·06	2·64	·85	·26	8·21	2·30	
2·88	·90	46·37	7·87	49·15	5·10	2·55	1·05	2·63	·85	·26	9·06	2·54	BSUA 121
2·91	·92	50·42	8·52	53·40	5·54	2·55	1·05	2·62	·84	·26	9·90	2·77	
2·93	·94	54·36	9·14	57·52	5·98	2·54	1·04	2·61	·84	·26	10·73	2·99	

BRITISH STANDARD SECTIONS

UNEQUAL ANGLES

(continued)

1	2	3	4	5	6	7	8
Reference No.	*Size* $A \times B$	*Thickness* t	*Minimum Thickness Rolled*	*Radii* Root r_1	*Radii* Toe r_2	*Calculated Weight per foot* w	*Sectional Area* a
	in.	in.	in.	in.		lb.	in.^{2}
BSUA 122	8 × 6	**·5**	·45	·54	·38	**22·95**	**6·75**
		·5625				*25·70*	*7·56*
		·625				*28·42*	*8·36*
		·6875				*31·12*	*9·15*
		·75				**33·79**	**9·94**
		·8125				*36·43*	*10·72*
		·875				*39·05*	*11·48*
BSUA 123	9 × 4	**·5**	·45	·51	·36	**21·25**	**6·25**
		·5625				*23·79*	*7·00*
		·625				*26·30*	*7·73*
		·6875				*28·78*	*8·47*
		·75				**31·24**	**9·19**
		·8125				*33·67*	*9·90*
		·875				*36·07*	*10·61*

Remarks

1. The thicknesses printed in heavy type are the Standards. For the information of designers of structural steelwork the properties are given of sections differing from the standard thicknesses in steps of 0·0625 inch, but other intermediate thicknesses can be obtained.

2. The dimensions, thickness and profile of Standard Unequal Angles shall be in accordance with the accompanying list and sketch, but finished sections in which the angle between the flanges is not less than 89° nor more than 91° shall be accepted as conforming to the Standard.

BRITISH STANDARD SECTIONS

UNEQUAL ANGLES

(continued)

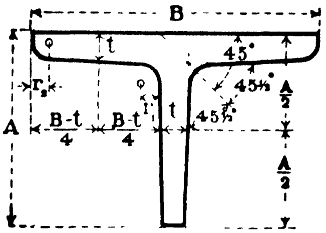
9	10	11	12	13	14	15	16	17	18	19	20	21	22
Centre of Gravity		*Moments of Inertia*				*Radii of Gyration*				*Tan. of Angle*	*Moduli of Section*		*Reference No.*
				Max.	*Min.*			*Max.*	*Min.*				
c_x	c_y	J_x	J_y	J_u	J_v	i_x	i_y	i_u	i_v	α	Z_x	Z_y	
in.		in.[4]		in.[4]		in.		in.			in.[3]		
2·44	**1·45**	**43·47**	**21·08**	**53·28**	**11·26**	**2·54**	**1·77**	**2·81**	**1·29**	**·55**	**7·82**	**4·63**	
2·47	1·47	48·43	23·44	59·33	12·54	2·53	1·76	2·80	1·29	·55	8·75	5·18	
2·49	1·50	53·27	25·74	65·21	13·80	2·52	1·75	2·79	1·28	·55	9·67	5·72	
2·52	1·52	57·99	27·97	70·93	15·03	2·52	1·75	2·78	1·28	·55	10·58	6·25	BSUA 122
2·54	**1·55**	**62·60**	**30·14**	**76·49**	**16·24**	**2·51**	**1·74**	**2·77**	**1·28**	**·55**	**11·47**	**6·77**	
2·57	1·57	67·10	32·24	81·90	17·44	2·50	1·73	2·76	1·28	·55	12·35	7·28	
2·59	1·60	71·49	34·29	87·16	18·63	2·49	1·73	2·75	1·27	·54	13·22	7·79	
3·27	**·80**	**52·46**	**6·65**	**54·70**	**4·41**	**2·90**	**1·03**	**2·96**	**·84**	**·22**	**9·16**	**2·08**	
3·30	·82	58·42	7·37	60·88	4·90	2·89	1·03	2·95	·84	·21	10·25	2·32	
3·33	·85	64·23	8·06	66·91	5·38	2·88	1·02	2·94	·83	·21	11·33	2·56	
3·36	·87	69·91	8·73	72·78	5·86	2·87	1·02	2·93	·83	·21	12·39	2·79	BSUA 123
3·38	**·90**	**75·45**	**9·37**	**78·49**	**6·33**	**2·87**	**1·01**	**2·92**	**·83**	**·21**	**13·43**	**3·02**	
3·41	·92	80·86	9·99	84·06	6·79	2·86	1·00	2·91	·83	·21	14·46	3·25	
3·43	·95	86·13	10·60	89·48	7·25	2·85	1·00	2·90	·83	·21	15·47	3·47	

3. Angles ordered to the standard thickness shall be practically accurate in profile; but if the thickness is above the Standards, the flanges will be longer than the Standards. The profile at the back of the toe will be slightly rounded when above the Standards, instead of square, but the radii at the root and toe will remain unchanged. In Unequal Angles the flanges may differ in thickness, but the difference shall not exceed ·05 inch.

4. Angles may be ordered by width of flanges and thickness, or by width of flanges and weight per foot, but not by both thickness and weight per foot.

BRITISH STANDARD SECTIONS

'T'-BARS



a = Sectional Area in sq. in.

w = Weight in lb. per ft. = $3{\cdot}4a$.

$r_1 = 0{\cdot}03\,(A+B)+0{\cdot}12$.

$r_2 = 0{\cdot}7r_1$.

1	2	3	4	5	6	7
Reference No.	*Size*	*Standard Thickness*	*Radii* Root	*Radii* Toe	*Calculated Weight per foot*	*Sectional Area*
	$B \times A$	t	r_1	r_2	w	a
	in.	in.	in.		lb.	in.²
BST 101	1½ × 1½	·25	·21	·15	2·36	·69
BST 102	2 × 2	·25	·24	·17	3·21	·94
BST 103	2½ × 2½	·25	·27	·19	4·07	1·20
BST 104	2½ × 2½	·375	·27	·19	5·92	1·74
BST 105	3 × 3	·375	·30	·21	7·20	2·12
BST 106	4 × 3	·375	·33	·23	8·49	2·50
BST 107	4 × 3	·5	·33	·23	11·09	3·26
BST 108	4 × 4	·375	·36	·25	9·77	2·87
BST 109	4 × 4	·5	·36	·25	12·79	3·76
BST 110	5 × 3	·375	·36	·25	9·79	2·88
BST 111	5 × 3	·5	·36	·25	12·80	3·77
BST 112	5 × 4	·375	·39	·27	11·06	3·25
BST 113	5 × 4	·5	·39	·27	14·50	4·27
BST 114	6 × 3	·375	·39	·27	11·08	3·26
BST 115	6 × 3	·5	·39	·27	14·52	4·27
BST 116	6 × 4	·5	·42	·29	16·22	4·77
BST 117	6 × 4	·625	·42	·29	19·99	5·88
BST 118	6 × 6	·5	·48	·34	19·62	5·77
BST 119	6 × 6	·625	·48	·34	24·23	7·13

Remarks

1. The dimensions, thickness and profile of Standard Tees shall be in accordance with the accompanying list and sketch. The standard thickness of web shall be at a distance half-way between the extreme edge of the web and the farther side of the flange. The standard thickness of flange shall be measured at a distance half-way between the extreme edge of the flange and the nearer side of the web.

BRITISH STANDARD SECTIONS

'T'-BARS

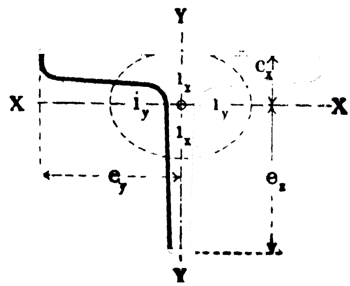
c_x, c_y Distance of centre of gravity from top line and Y axis.

$J = ai^2$ Moment of inertia.

$i = \sqrt{(J/a)}$ Radius of gyration.

e_x, e_y Distance of outer fibres from X and Y axes.

$Z = J/e$ Modulus of section.



8	9	10	11	12	13	14	15	16
Centre of Gravity		Moments of Inertia		Radii of Gyration		Moduli of Section		Reference No.
c_x	c_y	J_x	J_y	i_x	i_y	Z_x	Z_y	
in.		in.^{4}		in.		in.^{3}		
·46	0	·14	·07	·44	·31	·13	·09	BST 101
·58	0	·34	·16	·60	·41	24	·16	BST 102
·70	0	·68	·30	·75	·50	·38	24	BST 103
·75	0	·96	·47	·74	·52	·55	·38	BST 104
·87	0	1·71	·81	·90	·62	·80	·54	BST 105
·77	0	1·86	1·91	·86	·87	·83	·96	BST 106
·82	0	2·37	2·60	·85	·89	1 08	1·30	BST 107
1·10	0	4·19	1·90	1·21	·81	1·45	·95	BST 108
1·16	0	5·40	2·59	1·20	·83	1·90	1·30	BST 109
·69	0	1·97	3·72	·83	1·14	·85	1·49	BST 110
·74	0	2·51	5·04	·82	1·16	1·11	2·01	BST 111
1·00	0	4·47	3·70	1·17	1·07	1·49	1·48	BST 112
1·05	0	5·77	5·02	1·16	1·09	1·96	2·01	BST 113
·63	0	2·06	6·40	·80	1·40	·87	2·13	BST 114
·68	0	2·63	8·67	·78	1·42	1·14	2·89	BST 115
·97	0	6·07	8·64	1·13	1·35	2·00	2·88	BST 116
1·02	0	7·33	10·93	1·12	1·36	2·46	3·64	BST 117
1·63	0	19·04	8·56	1·82	1·22	4·36	2·85	BST 118
1·69	0	23·31	10·87	1·81	1·23	5·40	3·62	BST 119

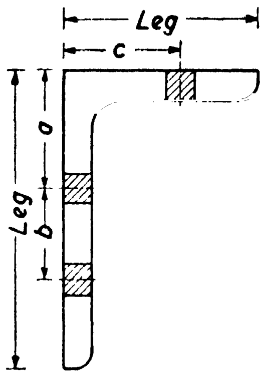
2. The tapers of the flange and web shall be such that the under side of the flange forms an angle of ½° with the horizontal upper side, whilst each side of the web forms an angle of ½° with the vertical centre line as shown in the diagram.

3. Tees ordered to the standard thickness shall be practically accurate in profile.

4. Tees may be ordered by width of flange, depth of section and thickness, or by width of flange, depth of section and weight per foot but not by both thickness and weight per foot.

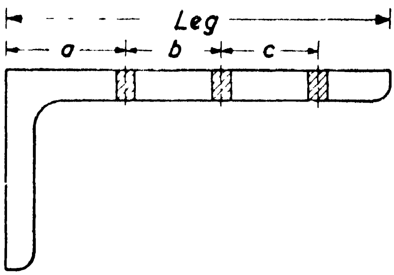
NOTE. The sections listed on pages 880 to 895 inclusive are extracted from B.S. 4A—1934.

SPACING OF HOLES IN ANGLES



Dimensions in inches

Leg	*a*	*b*	*c*
9	3	4	—
8	3	3	4½
7	2½	3	4
6	2¼	2¼	3½
5	2	1¾	3
4½	—	—	2½
4	—	—	2¼
3½	—	—	2
3	—	—	1¾
2½	—	—	1⅜
2	—	—	1⅛
1½	—	—	⅞

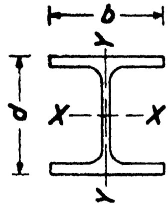


Dimensions in inches

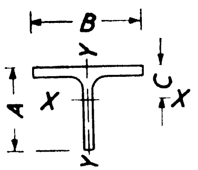
Leg	*a*	*b*	*c*
9	3	2	2
8	2½	2	2

BROAD FLANGE BEAMS WITH PARALLEL FLANGES

DIMENSIONS AND PROPERTIES IN INCH UNITS



Size d×b	*Weight per ft. in lb.*	*Area*	*Thickness*		*Moments of Inertia*		*Section Moduli*		*Radii of Gyration*	
			Web	*Flange*	*About XX*	*About YY*	*About XX*	*About YY*	*About XX*	*About YY*
6 × 6	20	5·88	0·26	0·37	38·84	13·16	12·95	4·38	2·57	1·50
6 × 6	25	7·35	0·31	0·47	47·40	16·82	15·80	5·61	2·54	1·51
7 × 7	34	10·00	0·35	0·55	88·15	31·48	25·19	8·99	2·97	1·77
8 × 8	31	9·12	0·29	0·43	109·60	36·83	27·40	9·20	3·47	2·01
8·12 × 8·03	35	10·29	0·32	0·49	126·33	42·19	31·11	10·51	3·50	2·03
8·25 × 8·08	40	11·76	0·36	0·55	146·57	48·67	35·53	12·05	3·53	2·04
8 × 8	45	13·23	0·39	0·64	152·85	55·13	38·21	13·78	3·40	2·04
10 × 10	49	14·41	0·34	0·56	273·22	93·11	54·64	18·62	4·35	2·54
10·12 × 10·03	54	15·88	0·37	0·62	305·69	103·55	60·41	20·65	4·39	2·56
10 × 10	63	18·53	0·43	0·72	340·90	120·90	68·18	24·18	4·29	2·55
10·06 × 10	66	19·41	0·46	0·75	357·92	125·70	71·15	25·14	4·30	2·54
12·12 × 12	65	19·12	0·39	0·61	533·73	174·93	88·07	29·15	5·28	3·02
12·25 × 12·04	72	21·18	0·43	0·67	597·91	194·95	97·61	32·38	5·31	3·04
12 × 12	75	22·06	0·43	0·71	598·73	206·00	99·79	34·33	5·21	3·06
12·06 × 12	78	22·94	0·47	0·73	621·33	210·56	103·04	35·09	5·20	3·03
12 × 12	83	24·41	0·47	0·80	655·26	229·26	109·21	38·21	5·18	3·06
12·14 × 12	91	26·76	0·55	0·85	720·26	246·34	118·66	41·05	5·19	3·03
14·06 × 12	78	22·94	0·43	0·72	850·95	206·32	121·04	34·39	6·09	3·00
14·18 × 12·02	84	24·71	0·45	0·77	928·35	224·58	130·93	37·35	6·13	3·02
14 × 12	90	26·47	0·50	0·82	956·70	237·97	136·67	39·66	6·01	3·0C
14 × 12	100	29·41	0·55	0·92	1050·85	266·01	150·12	44·33	5·98	3·00
16·13 × 12	91	26·76	0·50	0·79	1269·52	227·52	157·41	37·92	6·89	2·92
16·29 × 12·03	100	29·41	0·54	0·88	1418·28	254·55	174·12	42·32	6·94	2·94
16 × 12	110	32·35	0·55	1·00	1508·25	289·19	188·53	48·20	6·83	2·99
18 × 12	122	35·88	0·59	1·08	2089·71	310·79	232·19	51·80	7·63	2·94
20 × 12	135	39·71	0·63	1·16	2817·72	334·18	281·77	55·70	8·42	2·90
22 × 12	150	44·12	0·65	1·28	3767·90	368·66	342·54	61·44	9·24	2·89
24 × 12	165	48·53	0·70	1·36	4857·49	393·24	404·79	65·54	10·00	2·85



BROAD FLANGE TEES

Cut from Broad Flange Beams with parallel flanges

DIMENSIONS AND PROPERTIES IN INCH UNITS

Size $B \times A$	*Weight per ft in lbs*	*Area*	*Thickness*		*C*	*Moments of Inertia*		*Section Moduli*		*Radii of Gyration*	
			Web	*Flange*		*About XX*	*About YY*	*About XX*	*About YY*	*About XX*	*About YY*
$6 \times 2\frac{15}{16}$	9 94	2 92	0 26	0 37	0 52	1 50	6 58	0 62	2 19	0 71	1 50
$6 \times 2\frac{15}{16}$	12 43	3 66	0 31	0 47	0 55	1 74	8 41	0 73	2 80	0 69	1 51
$7 \times 3\frac{7}{16}$	16 92	4 98	0 35	0 55	0 63	3 16	15 74	1 13	4 50	0 80	1 77
$8 \times 3\frac{15}{16}$	15 44	4 54	0 29	0 43	0 66	4 11	18 41	1 25	4 60	0 95	2 01
$8\ 03 \times 4$	17 43	5 13	0 32	0 49	0 68	4 66	21 10	1 40	5 25	0 95	2 03
$8\ 08 \times 4\frac{1}{16}$	19 72	5 86	0 36	0 55	0 72	5 47	24 33	1 64	6 02	0 97	2 04
$8 \times 3\frac{15}{16}$	22 41	6 59	0 39	0 64	0 72	5 32	27 56	1 65	6 89	0 90	2 04
$10 \times 4\frac{15}{16}$	24 42	7 18	0 34	0 56	0 80	9 69	46 55	2 34	9 31	1 16	2 54
$10\ 03 \times 5$	26 93	7 92	0 37	0 62	0 82	10 76	51 77	2 58	10 32	1 16	2 56
$10 \times 4\frac{15}{16}$	31 41	9 24	0 43	0 72	0 86	11 83	60 45	2 90	12 09	1 13	2 56
10×5	32 90	9 63	0 46	0 75	0 88	12 74	62 85	3 12	12 57	1 15	2 55
12×6	32 41	9 53	0 39	0 61	0 97	20 00	87 46	3 98	14 57	1 45	3 02
12×6	35 91	10 56	0 43	0 67	1 01	22 48	97 47	4 45	16 19	1 46	3 04
$12 \times 5\frac{15}{16}$	37 40	11 00	0 43	0 71	0 97	21 16	103 00	4 26	17 16	1 39	3 06
12×6	38 90	11 44	0 47	0 73	1 02	23 13	105 28	4 67	17 55	1 42	3 03
$12 \times 5\frac{15}{16}$	41 41	12 18	0 47	0 80	1 00	22 83	114 63	4 62	19 10	1 37	3 07
12×6	45 39	13 35	0 55	0 85	1 07	26 73	123 17	5 41	20 53	1 41	3 04
12×7	38 90	11 44	0 43	0 72	1 18	33 86	103 16	5 85	17 19	1 72	3 00
$12\ 02 \times 7$	41 89	12 32	0 45	0 77	1 20	36 33	112 29	6 23	18 68	1 72	3 02
$12 \times 6\frac{15}{16}$	44 89	13 20	0 50	0 82	1 23	38 19	118 98	6 69	19 83	1 70	3 00
$12 \times 6\frac{15}{16}$	49 88	14 67	0 55	0 92	1 26	41 42	133 00	7 29	22 16	1 68	3 01
12×8	45 39	13 35	0 50	0 79	1 49	58 39	113 76	8 97	18 96	2 09	2 93
$12\ 03 \times 8\frac{1}{16}$	49 88	14 67	0 54	0 88	1 51	63 52	127 27	9 67	21 16	2 08	2 94
$12 \times 7\frac{15}{16}$	54 88	16 14	0 55	1 00	1 45	61 94	144 59	9 55	24 10	1 96	2 99
$12 \times 8\frac{15}{16}$	60 87	17 90	0 59	1 08	1 71	93 36	155 39	12 92	25 90	2 28	2 94
$12 \times 9\frac{15}{16}$	67 37	19 81	0 63	1 16	1 98	134 93	167 09	16 96	27 85	2 61	2 91
$12 \times 10\frac{15}{16}$	74 86	22 02	0 65	1 28	2 21	184 36	184 33	21 13	30 72	2 89	2 89
$12 \times 11\frac{15}{16}$	82 35	24 22	0 70	1 36	2 52	253 47	196 62	26 92	32 77	3 23	2 85

The Broad Flange Sections are rolled by the South Durham Steel and Iron Co Ltd, Middlesbrough, who have supplied the Tables of Properties

TUBES

Approx. Outside Diam.	Thickness		Weight Plain Ends	Area of Cross-Section	Internal Volume	Surface		Moment of Inertia I	Modulus of Section Z	Powers of Radius of Gyration	
						External	Internal			r	r²
in.	S.W.G. or Frcs.	in.	lb./ft.	in.²	ft.³/ft.	ft.²/ft.	ft.²/ft.	in.⁴	in.³	in.	in.²
$\frac{27}{32}$	**14**	**0·080**	**0·643**	**0·192**	**0·0025**	**0·221**	**0·179**	**0·014**	**0·034**	**0·272**	**0·074**
	13	0·092	0·739	0·217	0·0024	0·221	0·173	0·016	0·037	0·268	0·072
	12	**0·104**	**0·825**	**0·242**	**0·0022**	**0·221**	**0·166**	**0·017**	**0·040**	**0·264**	**0·070**
	11	0·116	0·902	0·265	0·0020	0·221	0·160	0·018	0·043	0·261	0·068
	10	**0·128**	**0·982**	**0·288**	**0·0019**	**0·221**	**0·154**	**0·019**	**0·045**	**0·257**	**0·066**
	9	0·144	1·076	0·317	0·0017	0·221	0·145	0·020	0·048	0·253	0·064
	8	0·160	1·169	0·344	0·0015	0·221	0·137	0·021	0·050	0·248	0·062
$1\frac{1}{16}$	**13**	**0·092**	**0·941**	**0·281**	**0·0042**	**0·278**	**0·230**	**0·033**	**0·063**	**0·345**	**0·119**
	12	0·104	1·065	0·313	0·0040	0·278	0·224	0·036	0·068	0·341	0·116
	11	**0·116**	**1·173**	**0·345**	**0·0038**	**0·278**	**0·217**	**0·039**	**0·074**	**0·337**	**0·114**
	10	0·128	1·278	0·376	0·0035	0·278	0·211	0·042	0·079	0·333	0·111
	9	**0·144**	**1·413**	**0·416**	**0·0033**	**0·278**	**0·203**	**0·045**	**0·085**	**0·329**	**0·108**
	8	0·160	1·542	0·454	0·0030	0·278	0·194	0·048	0·090	0·324	0·105
	7	0·176	1·667	0·490	0·0028	0·278	0·186	0·050	0·094	0·320	0·102
$1\frac{11}{32}$	**12**	**0·104**	**1·349**	**0·405**	**0·0070**	**0·352**	**0·297**	**0·078**	**0·117**	**0·440**	**0·193**
	11	0·116	1·521	0·447	0·0067	0·352	0·291	0·085	0·127	0·436	0·190
	10	**0·128**	**1·651**	**0·489**	**0·0065**	**0·352**	**0·285**	**0·091**	**0·136**	**0·432**	**0·187**
	9	0·144	1·845	0·543	0·0061	0·352	0·276	0·099	0·147	0·427	0·183
	8	**0·160**	**2·009**	**0·595**	**0·0057**	**0·352**	**0·268**	**0·106**	**0·158**	**0·422**	**0·178**
	7	0·176	2·195	0·646	0·0054	0·352	0·260	0·113	0·168	0·418	0·174
	6	**0·192**	**2·362**	**0·695**	**0·0050**	**0·352**	**0·251**	**0·118**	**0·176**	**0·413**	**0·170**
$1\frac{11}{16}$	**12**	**0·104**	**1·729**	**0·517**	**0·012**	**0·442**	**0·387**	**0·163**	**0·193**	**0·561**	**0·315**
	11	0·116	1·947	0·573	0·012	0·442	0·381	0·178	0·211	0·557	0·310
	10	0·128	2·132	0·627	0·011	0·442	0·375	0·192	0·227	0·553	0·306
	9	**0·144**	**2·357**	**0·698**	**0·011**	**0·442**	**0·366**	**0·210**	**0·249**	**0·548**	**0·300**
	8	0·160	2·611	0·768	0·010	0·442	0·358	0·226	0·268	0·543	0·295
	7	**0·176**	**2·821**	**0·836**	**0·010**	**0·442**	**0·350**	**0·242**	**0·287**	**0·538**	**0·289**
	6	0·192	3·067	0·902	0·009	0·442	0·341	0·256	0·304	0·533	0·284
	5	**0·212**	**3·341**	**0·983**	**0·009**	**0·442**	**0·331**	**0·273**	**0·323**	**0·527**	**0·278**
$1\frac{29}{32}$	**11**	**0·116**	**2·201**	**0·652**	**0·015**	**0·499**	**0·438**	**0·262**	**0·275**	**0·634**	**0·402**
	10	0·128	2·431	0·715	0·015	0·499	0·432	0·284	0·298	0·630	0·397
	9	0·144	2·711	0·797	0·014	0·499	0·424	0·312	0·327	0·625	0·391
	8	**0·160**	**2·988**	**0·878**	**0·014**	**0·499**	**0·415**	**0·377**	**0·354**	**0·620**	**0·384**
	7	0·176	3·253	0·957	0·013	0·499	0·407	0·362	0·380	0·615	0·378
	6	**0·192**	**3·520**	**1·03**	**0·013**	**0·499**	**0·399**	**0·385**	**0·404**	**0·610**	**0·372**
	5	0·212	3·837	1·13	0·012	0·499	0·388	0·411	0·431	0·604	0·364
	4	**0·232**	**4·149**	**1·22**	**0·011**	**0·499**	**0·378**	**0·436**	**0·457**	**0·598**	**0·357**

The items in bold type refer to standard sizes of tubes, designs should be based only on these sizes.

TUBES

Approx. Outside Diam.	*Thickness*		*Weight Plain Ends*	*Area of Cross-Section*	*Internal Volume*	*Surface*		*Moment of Inertia I*	*Modulus of Section Z*	*Powers of Radius of Gyration*	
						External	*Internal*			*r*	*r*²
in.	S.W.G. or Frcs.	in.	lb./ft.	in.²	ft.³/ft.	ft.²/ft.	ft.²/ft.	in.⁴	in.³	in.	in.²
2⅜	**11**	**1·116**	**2·779**	**0·823**	**0·025**	**0·622**	**0·561**	**0·527**	**0·443**	**0·800**	**0·640**
	10	0·128	3·072	0·904	0·024	0·622	0·555	0·572	0·482	0·796	0·633
	9	0·144	3·432	1·01	0·024	0·622	0·546	0·631	0·531	0·790	0·625
	8	**0·160**	**3·795**	**1·11**	**0·023**	**0·622**	**0·538**	**0·686**	**0·578**	**0·785**	**0·616**
	7	0·176	4·134	1·22	0·022	0·622	0·530	0·740	0·623	0·780	0·608
	6	**0·192**	**4·488**	**1·32**	**0·022**	**0·622**	**0·521**	**0·790**	**0·666**	**0·775**	**0·600**
	5	0·212	4·898	1·44	0·021	0·622	0·511	0·851	0·716	0·768	0·590
	4	0·232	5·311	1·56	0·020	0·622	0·500	0·907	0·764	0·762	0·581
	¼ in.	**0·25**	**5·675**	**1·67**	**0·019**	**0·622**	**0·491**	**0·955**	**0·804**	**0·756**	**0·572**
3	**10**	**0·128**	**3·893**	**1·15**	**0·041**	**0·785**	**0·718**	**1·19**	**0·795**	**1·02**	**1·03**
	9	0·144	4·393	1·29	0·040	0·785	0·710	1·32	0·880	1·01	1·02
	8	0·160	4·854	1·43	0·039	0·785	0·702	1·44	0·963	1·01	1·01
	7	**0·176**	**5·301**	**1·56**	**0·038**	**0·785**	**0·693**	**1·56**	**1·04**	**1·00**	**1·00**
	6	0·192	5·759	1·69	0·037	0·785	0·685	1·68	1·12	1·00	0·990
	5	**0·212**	**6·304**	**1·86**	**0·036**	**0·785**	**0·674**	**1·81**	**1·21**	**0·988**	**0·977**
	4	0·232	6·859	2·02	0·035	0·785	0·664	1·95	1·30	0·982	0·964
	¼ in.	0·25	7·343	**2·16**	**0·034**	**0·785**	**0·654**	**2·06**	1·37	**0·976**	**0·953**
3½	**10**	**0·128**	**4·577**	**1·36**	**0·057**	**0·916**	**0·849**	**1·93**	**1·10**	**1·19**	**1·42**
	9	0·144	5·162	1·52	0·056	0·916	0·841	2·14	1·22	1·19	1·41
	8	0·160	5·708	1·68	0·055	0·916	0·833	2·35	1·34	1·18	1·40
	7	**0·176**	**6·246**	**1·84**	**0·054**	**0·916**	**0·824**	**2·55**	**1·45**	**1·18**	**1·38**
	6	0·192	6·784	2·00	0·053	0·916	0·816	2·74	1·56	1·17	1·37
	5	**0·212**	**7·442**	**2·19**	**0·052**	**0·916**	**0·805**	**2·97**	**1·70**	**1·16**	**1·36**
	4	0·232	8·098	2·38	0·050	0·916	0·795	3·20	1·83	1·16	1·34
	¼ in.	**0·25**	**8·679**	**2·55**	**0·049**	**0·916**	**0·785**	**3·39**	**1·94**	**1·15**	**1·33**
4	**9**	**0·144**	**5·878**	**1·74**	**0·075**	**1·05**	**0·972**	**3·25**	**1·62**	**1·36**	**1·86**
	8	0·160	6·563	1·93	0·074	1·05	0·963	3·56	1·78	1·36	1·85
	7	**0·176**	**7·172**	**2·11**	**0·073**	**1·05**	**0·955**	**3·87**	**1·94**	**1·35**	**1·83**
	6	0·192	7·810	2·30	0·071	1·05	0·947	4·17	2·09	1·35	1·82
	5	**0·212**	**8·557**	**2·52**	**0·070**	**1·05**	**0·936**	**4·54**	**2·27**	**1·34**	**1·80**
	4	0·232	9·337	2·75	0·068	1·05	0·926	4·89	2·47	1·33	1·78
	¼ in.	**0·25**	**10·014**	**2·95**	**0·067**	**1·05**	**0·916**	**5·20**	**2·60**	**1·33**	**1·77**
4½	**9**	**0·144**	**6·647**	**1·97**	**0·097**	**1·18**	**1·10**	**4·68**	**2·08**	**1·54**	**2·37**
	8	0·160	7·417	2·18	0·095	1·18	1·09	5·14	2·29	1·54	2·36
	7	**0·176**	**8·117**	**2·39**	**0·094**	**1·18**	**1·09**	**5·60**	**2·49**	**1·53**	**2·34**
	8	0·192	8·835	2·60	0·092	1·18	1·08	6·04	2·68	1·52	2·32
	5	**0·212**	**9·695**	**2·86**	**0·091**	**1·18**	**1·07**	**6·58**	**2·92**	**1·52**	**2·30**
	4	0·232	10·576	3·11	0·089	1·18	1·06	7·10	3·16	1·51	2·28
	¼ in.	**0·25**	**11·349**	**3·34**	**0·087**	**1·18**	**1·05**	**7·56**	**3·36**	**1·51**	**2·27**
5½	**7**	**0·176**	**10·006**	**2·94**	**0·145**	**1·44**	**1·35**	**10·4**	**3·80**	**1·88**	**3·55**
	6	0·192	10·886	3·20	0·143	1·44	1·34	11·3	4·11	1·88	3·53
	5	**0·212**	**11·971**	**3·52**	**0·141**	**1·44**	**1·33**	**12·3**	**4·48**	**1·87**	**3·50**
	4	0·232	13·055	3·84	0·138	1·44	1·32	13·3	4·85	1·86	3·48
	¼ in.	**0·25**	**14·019**	**4·12**	**0·136**	**1·44**	**1·31**	**14·2**	**5·18**	**1·86**	**3·45**
	9/32 in.	0 28125	15·678	4·61	0·133	1·44	1·29	15·7	5·73	1·85	**3·41**

The items in bold type refer to standard sizes of tubes, designs should be based only on these sizes.

TUBES

Approx. Outside Diam.	*Thickness*		*Weight Plain Ends*	*Area of Cross-Section*	*Internal Volume*	*Surface*		*Moment of Inertia I*	*Modulus of Section Z*	*Powers of Radius of Gyration*	
						External	*Internal*			*r*	*r²*
in.	S.W.G. or Frcs.	in.	lb./ft.	in.²	ft.³/ft.	ft.²/ft.	ft.²/ft.	in.⁴	in.³	in.	in.²
6½	**7**	**0·176**	**11·891**	**3·50**	**0·206**	**1·70**	**1·61**	**17·5**	**5·38**	**2·24**	**5·00**
	6	0·192	12·937	3·80	0·204	1·70	1·60	18·9	5·83	2·23	4·98
	5	**0·212**	**14·241**	**4·19**	**0·201**	**1·70**	**1·59**	**20·7**	**6·38**	**2·22**	**4·95**
	4	0·232	15·532	4·57	0·199	1·70	1·58	22·5	6·91	2·22	4·92
	1/4 in.	**0·25**	**16·690**	**4·91**	**0·196**	**1·70**	**1·57**	**24·0**	**7·39**	**2·21**	**4·89**
	9/32 in.	**0·28125**	**18·682**	**5·49**	**0·192**	**1·70**	**1·55**	**26·6**	**8·19**	**2·20**	**4·84**
6⅝	**7**	**0·176**	**12·124**	**3·57**	**0·215**	**1·73**	**1·64**	**18·6**	**5·60**	**2·28**	**5·20**
	6	0·192	13·193	3·88	0·212	1·73	1·63	20·1	6·07	2·28	5·18
	5	**0·212**	**14·522**	**4·27**	**0·210**	**1·73**	**1·62**	**22·0**	**6·64**	**2·27**	**5·15**
	1/4 in.	**0·25**	**17·024**	**5·01**	**0·205**	**1·73**	**1·60**	**25·5**	**7·69**	**2·26**	**5·09**
	9/32 in.	**0·28125**	**19·058**	**5·61**	**0·200**	**1·73**	**1·59**	**28·3**	**8·53**	**2·25**	**5·04**
7⅝	**6**	**0·192**	**15·244**	**4·48**	**0·286**	**2·00**	**1·90**	**31·0**	**8·13**	**2·63**	**6·91**
	5	**0·212**	**16·786**	**4·94**	**0·283**	**2·00**	**1·89**	**33·9**	**8·90**	**2·62**	**6·87**
	1/4 in.	**0·25**	**19·694**	**5·79**	**0·277**	**2·00**	**1·87**	**39·4**	**10·3**	**2·61**	**6·81**
	9/32 in.	**0·28125**	**22·062**	**6·49**	**0·272**	**2·00**	**1·85**	**43·8**	**11·5**	**2·60**	**6·75**
8⅝	**6**	**0·192**	**17·295**	**5·09**	**0·370**	**2·26**	**2·16**	**45·2**	**10·5**	**2·98**	**8·89**
	5	**0·212**	**19·051**	**5·60**	**0·367**	**2·26**	**2·15**	**49·6**	**11·5**	**2·98**	**8·85**
	1/4 in.	**0·25**	**22·364**	**6·58**	**0·360**	**2·26**	**2·13**	**57·7**	**13·4**	**2·96**	**8·78**
	9/32 in.	**0·28125**	**25·066**	**7·37**	**0·355**	**2·26**	**2·11**	**64·2**	**14·9**	**2·95**	**8·71**
9⅝	**6**	**0·192**	**19·346**	**5·69**	**0·465**	**2·52**	**2·42**	**63·3**	**13·2**	**3·34**	**11·1**
	5	0·212	21·315	6·27	0·462	2·52	2·41	69·5	14·4	3·33	11·1
	4	**0·232**	**23·277**	**6·85**	**0·458**	**2·52**	**2·40**	**75·6**	**15·7**	**3·32**	**11·0**
	¼ in.	0·25	25·035	7·36	0·454	2·52	2·39	81·0	16·8	3·32	11·0
	9/32 in.	**0·28125**	**28·070**	**8·26**	**0·448**	**2·52**	**2·37**	**90·2**	**18·7**	**3·31**	**10·9**
	5/16 in.	0·3125	31·085	9·14	0·442	2·52	2·36	99·2	20·6	3·29	10·9
	3/8 in.	**0·375**	**37·051**	**10·9**	**0·430**	**2·52**	**2·32**	**116·7**	**24·3**	**3·27**	**10·7**
10¾	**6**	**0·192**	**21·653**	**6·37**	**0·586**	**2·81**	**2·71**	**88·8**	**16·5**	**3·73**	**13·9**
	5	0·212	23·863	7·02	0·582	2·81	2·70	97·5	18·1	3·73	13·9
	1/4 in.	**0·25**	**28·039**	**8·25**	**0·573**	**2·81**	**2·68**	**114**	**21·2**	**3·71**	**13·8**
	9/32 in.	**0·28125**	**31·450**	**9·25**	**0·566**	**2·81**	**2·67**	**127**	**23·6**	**3·70**	**13·7**
	5/16 in.	0·3125	34·840	10·2	0·559	2·81	2·65	140	26·0	3·69	13·6
	3/8 in.	**0·375**	**41·557**	**12·2**	**0·545**	**2·81**	**2·62**	**165**	**30·6**	**3·67**	**13·5**
11¾	**6**	**0·192**	**23·704**	**6·97**	**0·705**	**3·08**	**2·98**	**116**	**19·8**	**4·09**	**16·7**
	1/4 in.	**0·25**	**30·709**	**9·03**	**0·690**	**3·08**	**2·95**	**149**	**25·4**	**4·07**	**16·5**
	9/32 in.	**0·28125**	**34·454**	**10·1**	**0·683**	**3·08**	**2·93**	**167**	**28·4**	**4·06**	**16·5**
	5/16 in.	0·3125	38·178	11·2	0·675	3·08	2·91	184	31·3	4·05	16·4
	3/8 in.	**0·375**	**45·563**	**13·4**	**0·660**	**3·08**	**2·88**	**217**	**36·9**	**4·02**	**16·2**
12¾	**5**	**0·212**	**28·392**	**8·35**	**0·829**	**3·34**	**3·23**	**164**	**25·7**	**4·43**	**19·7**
	¼ in.	0·25	33·379	9·82	0·818	3·34	3·21	192	30·1	4·42	19·5
	9/32 in.	**0·28125**	**37·458**	**11·0**	**0·810**	**3·34**	**3·19**	**214**	**33·6**	**4·41**	**19·4**
	5/16 in.	**0·3125**	**41·516**	**12·2**	**0·802**	**3·34**	**3·17**	**236**	**37·1**	**4·40**	**19·3**
	3/8 in.	**0·375**	**49·568**	**14·6**	**0·785**	**3·34**	**3·14**	**279**	**43·8**	**4·38**	**19·2**

The items in bold type refer to standard sizes of tubes, designs should be based only on these sizes.

The above Tables of Properties of Tubes for Structural Purposes have been supplied by Tubewrights Ltd., London.

BRITISH STANDARDS

(*obtainable from British Standards Institution*, 2, *Park St., London, W.*1)

4 *Dimensions and Properties of Channels and Beams for Structural Purposes*

4A *Dimensions and Properties of Equal Angles, Unequal Angles and T Bars for Structural Purposes.*

6 *Dimensions and Properties of Bulb Angles and Bulb Plates for Structural Purposes.*

15 *Structural Steel.*

24 *Part* 6, *Steel Plates, Sections, Bars and Rivets for Locomotive Boilers, Locomotives, Carriages and Wagons.*

153 *Girder Bridges.*

275 *Dimensions of Rivets.*

350 *Conversion Factors and Tables.*

449 *The Use of Structural Steel in Building.*

466 *Electric Overhead Travelling Cranes for General Use in Factories, Workshops and Warehouses.*

499 (*Section* 7) *Scheme of Symbols for Welding.*

548 *High Tensile Structural Steel for Bridges, etc., and General Building Construction.*

639 *Covered Electrodes for the Metal-arc Welding of Mild Steel.*

648 *Schedule of Weights of Building Materials.*

785 *Rolled Steel Bars and Hard Drawn Steel Wire for Concrete Reinforcement.*

798 *Galvanised Corrugated Steel Sheets.*

877 *Foamed Blast-furnace Slag for Concrete Aggregate.*

916 *Black Bolts, Screws and Nuts.*

938 *Metal-arc Welding as applied to Tubular Steel Structural Members*

968 *High Tensile* (*Fusion Welding Quality*) *Structural Steel for Bridges, etc., and General Building Construction.*

990 *Metal Casement Windows and Casement Doors for Domestic Buildings.*

1083 *Precision Hexagon Bolts, Screws, Nuts and Plain Washers.*

1144 *Cold Twisted Steel Bars for Concrete Reinforcement.*

1190 *Hollow Clay Building Blocks.*

1221 *Steel Fabric for Concrete Reinforcement.*

1449 *Steel Plate, Sheet and Strip.*

1775 *Steel Tubes for Mechanical, Structural and General Engineering Purposes.*

1787 *Steel Windows for Industrial Buildings.*

1856 *General Requirements for the Metal-arc Welding of Mild Steel.*

2503 *Steel Windows for Agricultural Use.*

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